Inventory model for non-instantaneous deteriorating item with random pre-deterioration period

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Abstract: The paper studies an inventory model for non-instantaneous deteriorating item where the deterioration of the item is initiated at a random time point. It is assumed that no shortages are allowed and demand occurs uniformly but at different rates during pre- and post-deterioration periods. The optimum order quantity and reorder intervals are determined so as to minimise the total expected cost per unit length of an inventory cycle. Numerical examples are cited and a sensitivity analysis is carried out to study the effect of model parameters on the optimum policy.

Keywords: inventory control; periodic review model; non-instantaneous deteriorating item; random pre-deterioration period; uniform demand rate.

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1 Introduction

Depletion in stock takes place owing to demand. However, many products, like volatile liquids, agricultural items, films, blood, drugs, fashion goods, electrical components etc., undergo deterioration through evaporation, spoilage, dryness etc. during their normal storage period and thereby causes depletion in stock. Hence, while developing inventory policies for such products, the loss due to deterioration should not be ignored. The earliest work along this line is due to Ghare and Schrader (1963) who developed the EOQ model for an exponentially decaying inventory. Thereafter, many authors discussed inventory models for deteriorating items under different setups, like Covert and Philip (1973), Philip (1974), Sarkar and Sarkar (2013), Shah et al. (2013), Sicilia et al. (2014), Qin et al. (2014), Maity and Pal (2015a, 2015b), and Pervin et al. (2015), to name a few.

Generally, it is assumed that deterioration starts as soon as the items arrive in inventory. However, in real life, most items retain their quality or original condition for a certain span of time before deteriorating. This phenomenon has been termed as 'non-instantaneous deterioration' by Wu et al. (2006), and can be commonly observed in products like fruits, vegetables and fashion items. For such items the assumption that the deterioration starts from the instant of arrival in stock may cause retailers to make inappropriate replenishment policies. Liu and Shi (1999) classified inventory models into two categories, viz. decay models and finite lifetime models. Castro and Alfa (2004) proposed a lifetime replacement policy in discrete time for a single unit system. Ouyang et al. (2006) developed an inventory model for non-instantaneous deteriorating items with permissible delay in payments. Chang et al. (2010) developed optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand. Samanta and Pal (2015) studied a periodic review inventory policy for non-instantaneous deteriorating items with time dependent deterioration rate. Pal and Chandra (2014) investigated inventory policy for non-instantaneous deteriorating items with stock and time dependent demand, price discount and partial backlogging. In all these studies it has been assumed that the pre-deterioration period is fixed. But, in real life such an assumption is not true for most items in inventory. However, no study has come to our notice where the pre-deterioration period is treated as a random variable.

In this paper we consider a periodic review inventory model for non-instantaneous deteriorating items where the pre-deterioration period is random, and the demand rate during the pre-deterioration period is greater than that in the post-deterioration period. The paper is organised as follows. Section 2 gives the assumptions and notations used in the model. Section 3 analyses the model and discusses the optimal solution procedure. Numerical examples are cited and a sensitivity analysis is carried out in Section 4. Finally, in Section 5 some concluding remarks are made.

2 Assumptions and notations

The following assumptions are made in the model:

1 Demand occurs at a uniform rate. The demand rate in pre-deterioration period is greater than that in the post-deterioration period.

2 Shortages are not allowed.

3 During post-deterioration period, the failure time of an item has an exponential distribution.

4 The time point μ from which an item starts to deteriorate is a random variable following Rect. $(0, a)$ distribution, $a > 0$.

5 Replenishment is instantaneous on ordering.

6 No shortage is allowed.

The notations used in the study are as follows:

D_1 pre-deterioration demand rate

D_2 post-deterioration demand rate, $D_1 \geq D_2$

θ constant deterioration rate in post-deterioration period, $0 < \theta < 1$

T length of reorder interval

M length of pre-deterioration period

Q optimal order quantity

C_s ordering cost per unit ordered

C_1 purchase cost per unit

C_2 deterioration cost per unit deterioration

P a fraction such that the carrying cost per item per unit time is pC_1

$I(t)$ inventory level at time t.

3 The model and its analysis

The inventory policy is to place an order for Q units at the beginning of each reorder interval of length T, such that the stock level is zero at the end of the interval. Q and T are the decision variables and are so determined that the total expected cost per unit length of a reorder interval is minimised.

Since the pre-deterioration period μ is random, following the Rect. $(0, a)$ distribution, the inventory situation in any reorder interval will depend on whether

1 $T \geq a$

2 $T < a$.

Case 1: $T \geq a$

In this case, the demand rate is D_1 in the interval $(0, \mu)$, and changes to D_2 in the interval (μ, T).

The inventory level $I(t)$ at time t in the interval $(0, T)$, therefore, satisfies the following differential equations:

$$\frac{dI(t)}{dt} = -D_1, \qquad 0 \leq t \leq \mu$$

$$\frac{dI(t)}{dt} + \theta I(t) = -D_2 \qquad \mu \leq t \leq T.$$

The boundary conditions are $I(0) = Q$ and $I(T) = 0$, which give

$$I(t) = Q - D_1 t, \qquad 0 \le t \le \mu \tag{3.1}$$

$$I(t) = \frac{D_2}{\theta}(e^{\theta(T-t)} - 1) \qquad \mu \le t \le T \tag{3.2}$$

From (2.1) and (2.2), we have

$$Q = D_1\mu + \frac{D_2}{\theta}(e^{\theta(T-t)} - 1) \tag{3.3}$$

(2.3) gives a relationship between the order quantity Q and the reorder interval T for given μ. Hence for given μ, we can express the total cost over a reorder interval as a function of T.

3.1 Cost function

The different components of the cost function in a reorder interval (0, T) for given μ are as follows:

1 Ordering cost = C_s

2 Purchase cost $(P_\mu) = C_1 Q = C_1 D_1 \mu + C_1 \frac{D_2}{\theta}(e^{\theta(T-\mu)} - 1)$ (3.4)

3 Deterioration cost $(D_\mu^*) = C_2 \int_\mu^T I(t)dt = \frac{C_2 D_2}{\theta}\left[e^{\theta(T-\mu)} - 1 - \theta(T\mu)\right]$ (3.5)

4 Holding cost $= (H\mu) = C_1 p \int_0^T I(t)dt$

$$= C_1 p\left[\frac{D_1\mu_2}{2} + \frac{D_2}{\theta_2}(\mu\theta + 1)(e^{\theta(T-t)} - 1) - \frac{D_2}{\theta}(T-\mu)\right] \tag{3.6}$$

Hence, the cost per unit length of a reorder cycle, for given μ, is obtained as

$$C_1(T, \mu) = \frac{1}{T}\left[C_s + P + D^* + H\right]$$

$$= \frac{1}{T}\begin{bmatrix} Cs + C_1 D_1 \mu + C_1 \frac{D_2}{\theta}(e^{\theta(T-\mu)} - 1) \\ + \frac{C_2 D_2}{\theta} e^{\theta(T-\mu)} - 1 - C_2 D_2 (T-\mu) + C_1 p \frac{D_1 \mu_2}{2} \\ + C_1 \frac{D_2}{\theta_2}(\mu\theta + 1)(e^{\theta(T-t)} - 1) - C_1 p \frac{D_2}{\theta}(T-\mu) \end{bmatrix}. \tag{3.7}$$

And the expected cost per unit length of a reorder cycle is

$$C_1(T) = \int_0^a C_1(T, \mu) f(\mu) d\mu$$
$$= \frac{1}{Ta}\left[C^* - A_1 T + A_2\left(e^{\theta T} - e^{\theta(T-a)}\right) - A_3 e^{\theta(T-a)}\right] \tag{3.8}$$
$$= \frac{N_1(T)}{T}, \text{ say}$$

where

$$C^* = C_s a + C_1 D_1 a^2/2 + C_1 p D_1 a^3/6 + C_2 D_2 a^2 \theta/2 - D_2 a\left(C_1 + C_2 + C_1 p/\theta\right)/\theta$$

$$A_1 = D_2 a\left(C_2 \theta + C_1 p/\theta\right)$$

$$A_2 = D_2\left(C_1 + C_2 + 2C_1 p/\theta\right)/\theta^2$$

$$A_3 = C_1 p D_2 a/2$$

$$N_1(T) = \frac{1}{a}\left[C^* - A_1 T + A_2\left(e^{\theta T} - e^{\theta(T-a)}\right) - A_3 e^{\theta(T-a)}\right].$$

3.2 Solution procedure

The optimal value of T that minimises $C(T)$ is a solution to

$$\frac{\partial C(T)}{\partial T} = 0,$$

which gives

$$(\theta T - 1)e^{\theta T} = C^* / \left\{A_2 - \left(A_2 + A_3\right)e^{-\theta a}\right\} \tag{3.9}$$

$(\theta T - 1)$ is an increasing function of T varying in the range $[-1, \infty)$. Hence, there exists a unique T satisfying (3.9) provided $C^*/\{A_2 - (A_2 + A_3)e^{-\theta a}\} \geq -1$.

Theorem 3.1: If $C^*/\{A_2 - (A_2 + A_3)e^{-\theta a}\} \geq -1$, $C(T)$ is convex in T.

Proof: For $C_1(T)$ to be convex in T we must have $\frac{\partial^2 C_1(T)}{\partial T^2} \geq 0$, which gives

$$\frac{\partial^2 N(T)}{\partial T^2} - \frac{2}{aT^2}(\theta T - 1)\left\{A_2 e^{\theta T} - \left(A_2 + A_3\right)e^{\theta(T-a)}\right\} + \frac{2C^*}{aT^2} \geq 0,$$
$$\text{i.e., } \left\{(\theta T - 1)^2 + 1\right\}\left\{A_2 e^{\theta T} - \left(A_2 + A_3\right)e^{\theta(T-a)}\right\} + 2C^* \geq 0, \tag{3.10}$$
$$\text{i.e., } D(T)e^{\theta T} + 2C^* \geq 0,$$

where $D(T) = \{(\theta T - 1)^2 + 1\}\{A_2 - (A_2 + A_3)e{-}\theta a\}$.

If $C^*/\{A_2 - (A_2 + A_3)e^{-\theta a}\} \geq -1$, $\frac{\partial C(T)}{\partial T} = 0$ has a unique solution, and at this unique solution L.H.S. of (3.10) $= (\theta T - 1)^2 - 2(\theta T - 1) + 1 = (\theta T - 2)^2$, which is always ≥ 0.

Hence, the function $C_1(T)$ is convex in T. Using Theorem 3.1, we, therefore propose the following algorithm to find optimal T which minimises the total expected cost $C(T)$:

- if $C^*/\{A_2 - (A_2 + A_3)e^{-\theta a}\} \geq -1$, optimal T is the unique solution to (3.9) provided the solution is $\geq a$, else it is $T = a$
- if $C^*/\{A_2 - (A_2 + A_3)\, e^{-\theta a}\} < -1$, optimal $T = a$.

Case 2: $T < a$

In this situation, the items in inventory deteriorate during a reorder interval if $\mu \in [0, T)$, and do not deteriorate if $\mu \in [T, a]$.

For, $\mu \in [0, T)$, proceeding as in Case 1, we obtain the inventory level $I(t)$ at time t in the interval $[0, T]$ as

$$\begin{aligned} I(t) &= Q - D_1 t, && 0 \leq t \leq \mu \\ &= \frac{D^2}{\theta}\left(e^{\theta(T-t)} - 1\right), && \mu \leq t < T \end{aligned} \tag{3.11}$$

and the cost $C_{21}(T, \mu)$ per unit length of a reorder cycle is given by (3.7).

For $\mu \in [T, a]$, the differential equation for $I(t)$, $t \in [0, T]$, is given by

$$\frac{dI(t)}{dt} = -D_1, \quad 0 \leq t \leq T.$$

with boundary conditions $I(0) = Q$ and $I(T) = 0$, which gives

$$I(t) = D_1(T - t), \quad 0 \leq t \leq T. \tag{3.12}$$

Hence, the order quantity is $Q = I(0) = D_1 T$.

3.3 Cost function

The different components of the cost function over the interval $(0, T)$, for given $\mu \in [T, a]$, are as follows:

1 Purchase cost = $P = C_1 Q = C_1 D_1 T$

2 Holding cost = $H = C_1 p \int_0^T I(t)dt = C_1 p D_1 T^2/2.$

Hence, for given $\mu \in [T, a]$,

$$\begin{aligned} C_{22}(T, \mu) &= \frac{1}{T}[Cs + P + H] \\ &= \frac{1}{T}\left[Cs + C_1\ D_1 T + C_1 p \frac{D_1 T^2}{2}\right]. \end{aligned}$$

The expected cost $C_2(T)$ per unit length of a reorder cycle is, therefore,

$$C_2(T)=\int_0^T C_{21}(T,\mu)f(\mu)d\mu+\int_T^a C_{22}(T,\mu)f(\mu)$$
$$=\frac{1}{Ta}\left[C^{**}+B_1T+B_2T^2+B_3T^3+B_4\left(e^{\theta T}-1\right)\right] \tag{3.13}$$
$$=\frac{N_2(T)}{T},\quad \text{say}$$

where

$$C^{**}=C_s a,\quad B_1=C_1D_1a-D_2\left(C_1+C_2+2C_1p/\theta\right)/\theta,$$

$$B_2=\{C_1p\,D_1a-\left(C_1D_1+C_2D_2\right)-2C_1pD_2/\theta/2,\quad B_3=C_1pD_1/3,$$

$$B_4=D_2\left(C_1+C_2+2C_1p/\theta\right)/\theta^2,$$

$$N_2(T)=\frac{1}{a}\left[C^{**}+B_1T+B_2T^2-B_3T^3+B_4\left(e^{\theta T}-1\right)\right].$$

3.4 Solution procedure

The optimal value of T that minimises $C_2(T)$ satisfies

$$\frac{\partial C_2(T)}{\partial T}=0.$$

which gives

$$C^{**}+2B_3T^3=B_2T^2+B_4\{1+e^{\theta T(\theta T-1)}\}, \tag{3.14}$$

or,

$$g(T)=h(T),$$

where $g(T) = C^{**} + 2B_3T^3$, $h(T) = B_2T^2 + B4\{1 + e\theta^{T(\theta T-1)}\}$.

Lemma 3.1: There exists a unique T (≥ 0) satisfying (3.14).

Proof: We note that C^{**} and B_3 are always positive. Hence, $g(T)$ is a convex increasing function of T with $g(0) = C^{**}(> 0)$. Again, $B_4 > 0$. Then, for $B_2 > 0$, $h(T)$ is an increasing function of T with $h(0) = 0$ and rate of increase $\frac{dh(T)}{dT} = (2B_2 + \theta^2B_4e^{\theta T})T$, which increases more rapidly with increase in T than $\frac{dg(T)}{dT} = 6B_3T^2$. Hence, the two curves $y = g(T)$ and $y = h(T)$ cross each other at a single point on $(0, \infty)$.

If $B_2 < 0$, we can write (3.14) as $g^*(T) = h^*(T)$, where $g^*(T) = C^{**} - B_2T^2 + 2B_3T^3$, $h^*(T) = B_4\{1 + e^{\theta T(\theta T - 1)}\}$. Then, $g^*(T)$ is a convex increasing function of T with $g^*(0) = C^{**}(> 0)$, and $h^*(T)$ is an increasing function of T with $h^*(0) =0$ and rate of increase $\frac{dh^*(T)}{dt} = \theta^2B_4e^{\theta T}T$, which increases more rapidly with increase in T than

$\frac{dg^*(T)}{dT} = -2B_2T + 6B_3T^2$. Hence, the two curves $y = g^*(T)$ and $y = h^*(T)$ cross each other once on $(0, \infty)$.

At $T = T_2$, satisfying (3.14), the expected cost is minimum if

$$\frac{\partial^2}{\partial T^2} C_2(T)\bigg|_{T=T_2} = \frac{1}{T_2^3}\left[2\left(C^{**} - B_4 - B_3T^3\right) + B_4e^{\theta T}\left\{(\theta T - 1)^2 + 1\right\}\right] \geq 0.$$

A comparison of the minimum expected costs obtained in Cases 1 and 2 determines the optimal reorder cycle length to be taken.

4 Numerical examples and sensitivity analysis

Example 4.1: Let us consider an item that can maintain its freshness for at most two months and then starts to deteriorate with a deterioration rate $\theta = 0.2$. Before deterioration starts, the demand rate is 30 items/unit time but it decreases to tem items/unit time when the item starts to deteriorate. The ordering cost is Rs 1,500 per item, purchase cost is Rs. 16 per item, deterioration cost is Rs. 5 per item and the carrying cost is Rs. 4 per item. The problem is to determine the optimal length of an inventory cycle that will minimise the expected cost per unit length of the cycle.

Solution: For the given model parameters, we have $C^*/\{A_2 - (A_2 + A_3)e^{-\theta a}\} \geq -1$. Hence, the optimal value of T minimising $C_1(T)$, which is obtained from (3.9), is $T = T_1 = 5.02$ months $> a$, and $C_1(T1) =$ Rs. 796.22.

From (3.14) we have $T = T2 = 9.03$ months ($> a$), and $\frac{\partial^2}{\partial T^2} C_2(T)\Big|_{T=T_2} = 94.66 > 0.$

Hence, $C_2(T)$ is a convex function of T. Since $T_2 > a$, optimum T that minimises $C_2(T)$ is $T = a = 2$ months and minimum $C_2(T) = C2(a) =$ Rs. 1,155.08.

Comparing C_1 (T_1) and $C_2(a)$ we have that the optimum reorder interval length is 5.02 months and the minimum expected cost per unit length of a reorder cycle is Rs. 796.22.

Example 4.2: We consider another example with $\theta = 0.5$, $a = 4$ months, ordering cost Rs. 1,000 per item and all other costs same as that in Example 4.1.

Solution: We note that $C^*/\{A_2 - (A_2 + A_3)e^{-\theta a}\} \geq -1$. Solving (3.9) we obtain $T = 3.776 < a$. Hence optimal T minimising $C_1(T)$ is $T_1 = a = 4$ months, and $C_1(T_1) =$ Rs. 880.28.

Equation (3.14) gives $T = T_2 = 3.702$ months ($< a$) with $\frac{\partial^2}{\partial T^2} C_2(T)\Big|_{T=T_2} = 339.538 > 0$

and $C_2(T_2) =$ Rs. 732.16.

Hence, comparing $C_1(T_1)$ and $C_2(T_2)$ we have that the optimum length of the reorder interval is 3.702 months and the minimum expected cost per unit length of a reorder cycle is Rs. 732.16.

Table 1 gives a sensitivity analysis of the model with change in the model parameters.

Table 1 Change in the optimal value of T and the percent change in $C(T)$ with change in the model parameters

C_s	T	$C(T)$	% change in $C(T)$	C_1	T	$C(T)$	% change in $C(T)$
1,300	4.814	755.55	–5.10788	8	5.95	563.69	–29.2042
1,400	4.919	776.09	–2.5282	12	5.39	683.16	–14.1996
1,500	5.02	796.22	0	16	5.02	796.22	0
1,600	5.1169	815.95	2.477958	20	4.75	905.31	13.70099
1,700	5.2105	835.31	4.909447	24	4.54	1011.71	27.06413
C_2				p			
3	5.1185	774.34	–2.74798	0.15	5.4492	733.74	–7.84708
4	5.0685	785.33	–1.36771	0.2	5.218	765.8	–3.82055
5	5.02	796.22	0	0.25	5.02	796.22	0
6	4.9726	807.01	1.355153	0.3	4.839	825.27	3.648489
7	4.9265	817.72	2.700259	0.35	4.6956	853.16	7.15129
a				θ			
1	4.7184	789.09	–0.89548	0.1	6.37	706.67	–11.2469
2	5.02	796.22	0	0.15	5.587	753.71	–5.33898
3	5.3584	808.98	1.602572	0.2	5.02	796.22	0
4	5.7325	826.74	3.833111	0.25	4.588	835.19	4.894376
5	6.11	848.9	6.616262	0.3	4.246	871.32	9.432067

Table 1 we conclude that

1 the model is fairly robust to changes in the model parameters

2 the optimum length of the reorder interval increases with increase in C_S and a, while it is a decreasing function of θ, p, C_1 and C_2.

5 Conclusions

The paper discusses a periodic review inventory model for non-instantaneous deteriorating items when the pre-deterioration period is a random variable. The optimum order quantity and the reorder interval have been obtained so as to minimise the expected cost per unit length of a reorder cycle. The length of the reorder cycle is observed to be an increasing function of the ordering cost and the maximum length of the pre-deterioration period, while it decreases with increase in the deterioration rate, the purchase cost and the carrying cost. The model is also noted to be fairly robust to change in the model parameters. The model can be extended to include shortage and general deterioration rate.

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