A comparative analysis of game theory techniques for study of energy interactions in interconnected microgrids

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Abstract: The growing energy needs of society can be met with one or more microgrids working in conjunction. A need of a trading and scheduling mechanism for energy exchange among microgrids for sustainable development of the consumers is required. Game theory techniques have been widely utilised to study this interaction among microgrids. This paper seeks to analyse the different game theory-based energy trading techniques to elaborate on such models' efficiencies. The study is based on renewable energy generation and consumption in Denmark. Lexicographic egalitarian solution is proposed as a bargaining solution for two participating microgrids in an energy trading game. A comparison is performed between the proposed model and conventional techniques for inter-microgrid trading. The proposed bargaining solution depicts a fairness index of 0.974 compared to 0.946 for the Nash solution and 0.954 for the Kalai-Smorodinsky solution. The results of the proposed study provide a better insight into the various aspects of energy sharing algorithms and will help improve the utilisation of green energy.

Keywords: microgrid; game theory; distributed generation; energy trading; bargaining game.

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1 Introduction

The current energy infrastructure consists of centralised generation plants transferring energy to remote locations through a good distribution network. The rising energy expenditure needs the distribution network to be capable of supplying energy to remote areas (Kaundinya et al., 2009). It leads to increased capital cost, low flexibility, and power losses in the distribution network (Bayod-Rújula, 2009). Advanced power infrastructure leads its way in providing solutions to manage such complexities. Among such solutions, microgrids are emerging as a promising solution ensuring long-term sustainability for the power network.

A microgrid is an entity consisting of multiple distributed energy resources and loads acting independently or in conjunction with the primary grid (Venkatraman and Khaitan, 2015; Stadler et al., 2016). They also encompass an advanced control module. Microgrids propose many benefits over the conventional approach, including enhanced reliability, efficient resource utilisation and improved energy management. Microgrids may function interactively with other microgrids. It decreases the load on the particular microgrid. The excess energy generated by the microgrids can be exchanged with other microgrids facing a shortage of energy (Gregoratti and Matamoros, 2014). However, this energy exchange practice will lead to dissatisfaction among the participating parties without a proper strategy. Proper energy trading procedures need to be implemented to enable a fair exchange of pricing among microgrids (Kasbekar and Sarkar, 2012).

Energy trading is essential in laying down proper procedures for energy transfer from one microgrid to another. Microgrids can engage in energy trading when the local production is unable to satisfy the demand. Microgrids can either borrow energy from the utility or engage in energy exchange with other microgrids in such a case (Wang and Huang, 2016a). Sharing energy from the grid may prove costly to the consumer. Additionally, it will result in a loss of power quality in the form of voltage fluctuations (Mohd et al., 2008). The supply may face power outages from the primary grid.

Game theory techniques are efficient in dealing with situations comprising multiple participating players (Mei, 2018) and various application areas such as electric vehicle station trading (Wang et al., 2020), wireless virtual sensor networks (Pandremmenou et al., 2013b) and vehicle energy exchanges (Zhang et al., 2016b). The microgrids can act as a supplier or as a consumer of energy. The microgrids behave as players in a game and deploy such strategies to obtain maximum benefit. They undergo a bargaining game, just like typical participants of a game. Microgrids having excess power will circulate the price at which it is willing to sell power. Microgrids having a deficit of power will now acquaint itself with prospective energy sellers. The microgrid with the lowest energy cost will be chosen among all microgrids. Energy is imported from the grid if enough energy is not available with the microgrid cluster. It results in a trade war among microgrids where several microgrids with an excess of power compete against each other. The microgrid can either be selected as an energy supplier or earn profits from it.

There is a tradeoff between the microgrids.

The payoff to the respective microgrids involved in energy trading depends on the game. The type of game involved and the respective parties' utilities affect the net profit curve of the participating microgrids. An equilibrium point is reached when the participating players agree to the offered rate and reach an agreement. This agreement point shifts in space when the utility and the game category are changed. This represents the bargaining solution of the selected game. The bargaining solutions were proposed by Nash (1950), Kalai and Smorodinsky (1975) and Kalai (1977).

A peer-to-peer (P2P) energy-trading market can be modelled as a non-cooperative game at the distribution level. Many attempts have been made until now to establish such a game for distributed networks such as microgrids (MG). The P2P trading network is rapidly evolving its services into the distributed smart grid network (Zhang et al., 2018). The peers perform multi directional energy trading, i.e., buying and selling directly among each other independent of the utility companies (Long et al., 2017; Zhang et al., 2016a). The game and auction theories have various applications in development of energy management for smart grids (Tushar et al., 2018b; Alsalloum et al., 2020). Non-cooperative games are used considerably for scheduling such energy management methods and for trading this surplus energy with buyers in order to earn income (Paudel et al., 2018). The integrated multimicrogrid (IMMG) framework takes advantage of several energy sources and manages energy sharing/trading between MG's and the primary grid efficiently to boost the security, reliability, and energy productivity of the system using game theory techniques (Fan et al., 2018b; Kong et al., 2020). A case study for such a system in Beja, Tunisia was able to prove that implementing such a game theory technique for real time pricing measures and energy management led to a saving in real power of up to 20% and reduced carbon emissions (Maddouri et al., 2020). Likewise, Ali et al. (2020) performed a case study for a clustered microgrid in a town of Mount Magnet in Western Australia. Another such analysis was performed by Oladejo and Folly (2019) for a fair profit situation in a grid connected microgrid. It is observed that the cooperative game theory increased profits independent of transfer prices. The application of a cooperative game for obtaining the Nash equilibrium and selection of the best pair of players for obtaining the maximum profit shows the effectiveness of game theory techniques in multi-player scenarios.

Game theory techniques find vast applications in energy trading in the distribution network. A Bayesian-Stackelberg game model was utilised by He and Wei (2016) allowing sellers to lead the game while buyers provide the bidding price. This was also seen in several past works as there is no assurance of fairness in this technique (Zhang et al., 2015b, 2015a; Marzband et al., 2018). Cooperative game theory techniques have been widely utilised by several researchers where several interconnected microgrids may act independently or act as a grand coalition for the objective of reducing operation cost and increasing reliability. One such grand coalition was proposed by Du et al. (2018) where a cooperation game was setup between interconnected microgrids for better utilisation efficiency which was explored to further increase individual microgrid utility (Mei et al., 2019). However, this approach failed to take into account the transmission power losses and focused more on linear models for power loss. This was improved by Querini et al. (2020) where a canonical coalition model for multiple microgrids rather than a dual microgrid approach was proposed. Several of the works focused on the cooperative trading model between interconnected microgrids along with utilising the Nash equilibrium model to achieve the common goal of attaining optimum payoff (Wu et al., 2016; Wang and Huang, 2016a; Oladejo and Folly, 2019; Tushar et al., 2018a). One of the limitations with the application of cooperative methods is that they suffer from a disadvantage where withdrawal of a player from the game leads to loss in profit. Some other techniques like reinforcement learning (Xiao et al., 2018), chance constrained programming (Daneshvar et al., 2020), secondary market utilisation or transactive multi-resource trading methodologies (Wu et al., 2016) were utilised to maximise revenue for inter-microgrid trading.

Different bargaining games are utilised for solving the problem of energy sharing between multiple energy producers and consumers. Every bargaining solution tries to maximise the payoff to the microgrid and increase profits. Hence, a comparison of such techniques for obtaining the optimum solution is of the utmost importance to provide better cost savings to customers. The Nash bargaining solution is extensively used in several research works to obtain the bargaining solution between interconnected microgrids (Wang and Huang, 2015, 2016b; Fan et al., 2018a; Vakili et al., 2018). A comparison of Nash, Kalai-Smorodinsky, Egalitarian, and other bargaining games in terms of fairness and efficiency was performed by Fourati et al. (2016), but the research is focused on optimal power allocation for cognitive radio networks. The study by Bhatti and Broadwater (2019) for residential microgrids utilising Nash equilibrium can accommodate any number of players and utilise a market reputation index as an incentive mechanism to improve their efficiency and reliability. An effort was performed by Garcia et al. (2020) to improve the bargaining game by proposing a novel bargaining game based on the Kalai-Smorodinsky solution to lower the concessions of the energy trade.

Following the examination of existing gaps in knowledge and an extensive review, the contributions of this paper can now be summarised. The objective of the paper is to propose three different bargaining models leading to three different solutions. Each solution will lead to a different payoff to the participating microgrids. The solutions are compared to each other based on their efficiency in offering the best energy rate to the consumer microgrid. The study will reveal the different strategies to obtain the equilibrium point of the game. The usefulness of the game theory in simplifying this energy trade is demonstrated.

The main contributions of the paper are as follows:

- A cooperative game model is proposed for an interconnected microgrid setup based on real-time data for renewable generations at different locations in Denmark.
- A bargaining method is proposed, i.e., lexicographic Egalitarian solution, which maximises fairness and efficiency for the energy trading setup.
- An analysis of Nash and the proposed bargaining solution is performed in the context of the energy trade among interconnected microgrids. The fairness and efficiency of such games are evaluated.

The remainder of the paper is organised as follows: Section 2 denotes the interconnected microgrid system model, including the generation and load data for the study. Section 3 formulates the trading problem and discusses the application of different bargaining models for inter-microgrid trade. Section 4 includes the result and discussion. Section 5 includes the verification and validation of the proposed models, followed by Section 6, which includes the conclusions of the study.

2 System model

The system model consists of n microgrids interconnected to each other ($j = 1, 2, \dots, n$). These microgrids are also connected to the grid supply. The microgrids can purchase or sell energy to the main grid. The main grid sells energy to the microgrids at a rate of x per unit. The grid also buys energy from the microgrids at a rate of y per unit.

We assume that there are e microgrids having an excess of energy and d microgrids having a deficit of energy. The microgrids having an excess of energy sells energy at a rate of a per unit. The microgrids purchasing energy do so at a rate of b per unit. The energy surplus s is calculated as the difference of generation g and consumption c.

The energy surplus s can be calculated by the following formula:

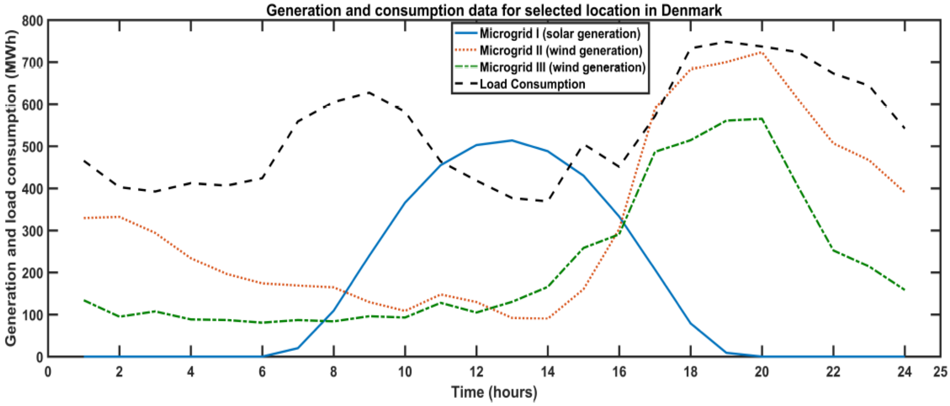
$$E_{surplus(Mg_j)} = G_{Mg_j} - C_{Mg_j} \tag{1}$$

If $E_{surplus(Mg_j)}$ is positive, then a microgrid is said to have an excess of energy else for negative values, energy is in deficit.

It is assumed that two microgrids with deficit of energy can utilise the excess energy from a third microgrid with energy excess. For our study, we will assume four microgrids participating in a game over an eight-hour period. We will analyse the bargaining solutions for different cases.

Analysis of the game among microgrids is done by assuming three microgrids comprising three different sources of generation. Microgrid I generates energy majorly using photo voltaic cells. Microgrids II and III generate energy primarily using wind turbines. The different modes of generation are assumed in order to create variations in generation output at different points of time. It would lead to the study of various cases of energy trade. The generation of three microgrids and the load is plotted in Figure 1.

Figure 1 Generation data for microgrids and load demand (see online version for colours)



The generation data of the microgrids over an eight-hour period has been listed out in Table 1. It is assumed that each microgrids I, II and III has uniform load requirements for every hour of the day.

Table 1 Generation and load data for microgrids

Hour	*Microgrid I (PV generation in MW)*	*Microgrid II (wind generation in MW)*	*Microgrid III (wind generation in MW)*	*Load consumption for M1 and M2 (MW)*
1	0	329.4	133.8	247.0
2	0	332.2	94.8	238.7
3	0	294.1	107.4	234.6
4	0	233.9	88.3	225.1
5	0	196.5	86.9	222.1
6	0	174.0	80.7	211.4
7	20	168.9	86.8	211.5
8	109	164.5	83.7	204.9
9	240	129.5	95.9	209.7
10	366	108.5	92.8	227.1
11	455	147.6	127.7	255.3
12	503	129.6	104.7	255.7
13	514	91.8	130.1	270.3
14	488	89.9	165.7	263.9
15	430	160.2	258.4	271.1
16	333	303.3	290.7	269.6
17	207	589.6	486.5	284.4
18	79	682.8	514.6	328.0
19	9	700.1	561.1	331.0
20	0	723.7	565.4	297.4
21	0	612.5	405.2	275.5
22	0	507.0	252.4	260.9
23	0	466.6	214.1	243.1
24	0	390.6	158.3	233.9

3 Problem formulation

3.1 Energy distribution

The energy deficient microgrids share the excess energy generated by other microgrids running on alternate sources of energy. The load distribution was assumed as consistent for all microgrids. The remaining energy not fulfilled from the excess is imported from the primary grid. The generation data for the solar PV and wind generating stations have been taken from Energinet, Denmark (Muehlenpfordt, 2019). The load consumption residential data is taken for the location Energi Fyn Nyborg in Denmark (Muehlenpfordt, 2019).

The hourly net energy surplus and deficit in the interconnected microgrids and the net grid imported energy for a day is shown in Table 2. The excess energy available at a particular hour with a microgrid is also discussed. The interconnected microgrid setup can be seen in Figure 2. The flowchart depicting the status of the interconnected microgrids in the energy trading problem is shown in Figure 3.

Table 2 Microgrid trading

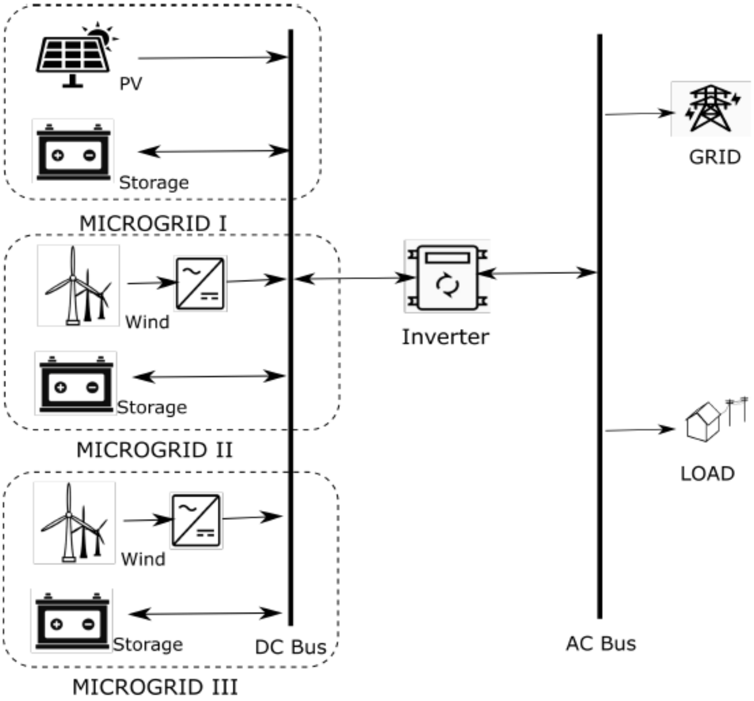
Hour	*Microgrid I surplus (S) / deficit (D)*	*Microgrid II surplus (S) / deficit (D)*	*Microgrid III surplus (S) / deficit (D)*	*Energy imported from grid (MW)*	*Unutilised energy*
1	247.0 D	82.4 S	113.2 D	164.6	0
2	238.7 D	93.5 S	143.9 D	145.2	0
3	234.6 D	59.5 S	127.2 D	175.1	0
4	225.1 D	8.80 S	136.8 D	216.3	0
5	222.1 D	25.6 D	135.2 D	247.7	0
6	211.4 D	37.4 D	130.7 D	248.8	0
7	191.5 D	42.6 D	124.7 D	234.1	0
8	95.90 D	40.4 D	121.2 D	136.3	0
9	30.30 S	80.2 D	113.8 D	49.9	0
10	138.9 S	118.6 D	134.3 D	0	20.3
11	199.7 S	107.7 D	127.6 D	0	92
12	247.3 S	126.1 D	151.0 D	0	121.2
13	243.7 S	178.5 D	140.2 D	0	65.2
14	224.1 S	174.0 D	98.2 D	0	50.1
15	158.9 S	110.9 D	12.7 D	0	48
16	63.40 S	33.70 S	21.1 S	0	97.1
17	77.40 D	305.2 S	202.1 S	0	227.8
18	249.0 D	354.8 S	186.6 S	0	105.8
19	322.0 D	369.1 S	230.1 S	0	47.1
20	297.4 D	426.3 S	268.0 S	0	128.9
21	275.5 D	337.0 S	129.7 S	0	61.5
22	260.9 D	246.1 S	8.5 D	14.8	0
23	243.1 D	223.5 S	29.0 D	19.6	0
24	233.9 D	156.7	75.6 D	77.2	0

3.2 Games among interconnected microgrids

With multiple partners bargaining over a claim, game theory techniques can help reach a final judgement. In our case, the players participating in the game are interconnected microgrids. We have assumed that three microgrids, M_1,M_2 and M_3, are interconnected. They all are actively exchanging energies among each other. Each microgrid has a utility which is taken as the demand of the microgrid. The microgrids generate energy according to the type of generation unit. Microgrids generate excesses and deficit of energies accordingly. At a particular instant of time, a particular microgrid may have an excess of energy. This energy can be shared by other microgrids that are in a deficit of energy. The microgrids deficit in energy each wants to maximise the energy it can borrow from the local microgrid. The grid poses a costly energy alternative to the energy problem.

Two microgrids bargaining over some excess energy value will try to obtain the maximum portion of this excess energy. Hence, proper rules need to be established for this energy transfer in order to avoid disputes. Each microgrid has a utility value based on energy demand. More the energy demand, more is the utility of the particular microgrid.

Figure 2 Interconnected microgrids



Some axioms are specified, which the bargaining games must satisfy to obtain the solution:

1 Individual rationality

The bargaining game should lead to a solution that must be good enough for the respective players than a refusal to participate in the game:

$$\left(u_1^*, u_2^*\right) \geq (0, 0).$$

2 Feasibility

We cannot bargain on a non-existent quantity. If a microgrid does not have excess energy, the game will not exist. (u_1^*, u_2^*) should belong to set X.

3 Pareto optimality

It is a condition that ensures the supremacy of the bargaining solution (u_1^*, u_2^*). It is a condition where it is impossible to make any criterion better without making other criteria worse than the former one.

4 Independence of irrelevant alternatives

If *Y* is a subset of *X*, if *X*'s solution lies within *Y*, then this is the solution to the complete bargaining game. When removed from the main set, these irrelevant sets will not change the solution of the bargaining game. The expansion of this set will not affect the solution.

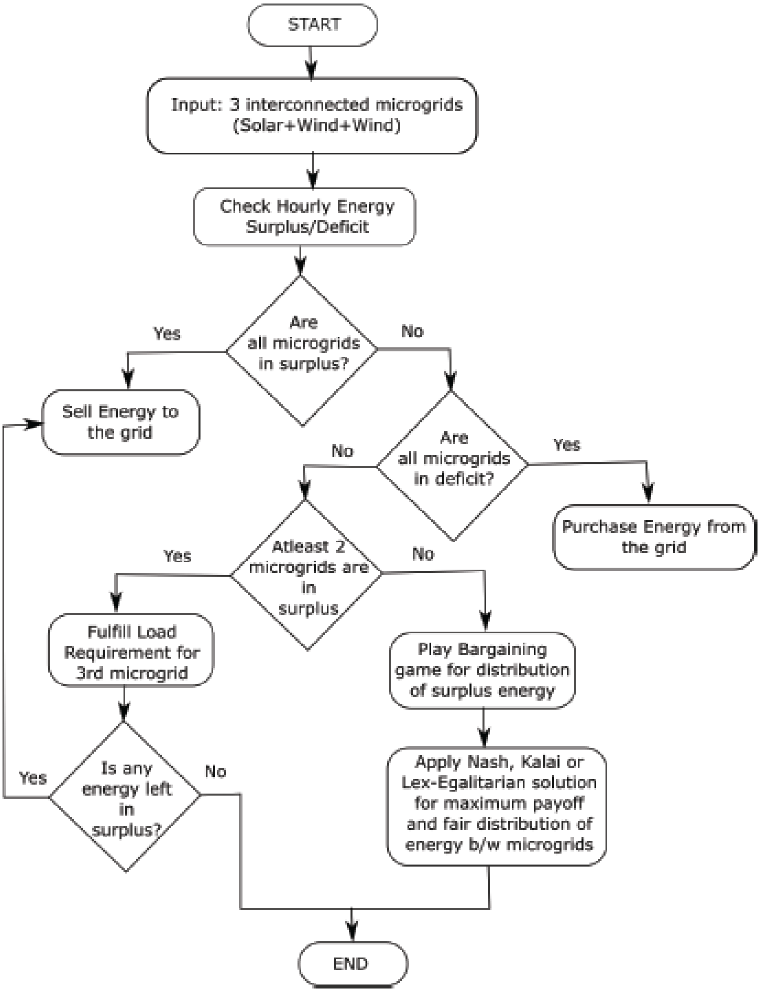
5 Independence of linear transformations

A change in units of transformation will not change the solution of the game.

6 Symmetry

If two players bargain over a similar amount and make similar demands, they have identical payoffs.

Figure 3 Flowchart for energy exchange between microgrids



3.2.1 Nash bargaining solution

The Nash bargaining solution is used to solve the problem of two-player bargaining. When there are multiple partners bargaining over a claim, game theory techniques can effectively reach a final judgement. Nash considered a set of outcomes as a coordinate point which represents the solution of the game. The microgrids need to accept a proposal regarding energy share else; they would both have to agree to an inferior proposal. This inferior proposal can also be termed as a disagreement point initially assumed as (0, 0). Both microgrids get nothing if they disagree with a proposal.

The components of the bargaining game are as follows:

- two players
- set X is the set of realisable outcomes
- outcomes are denoted by $U = (u_1, u_2)$ where u_1 and u_2 are the utilities of microgrids
- the disagreement point D is at $(d_1, d_2) = (0, 0)$
- the Nash bargaining solution is denoted by (u_1^*, u_2^*).

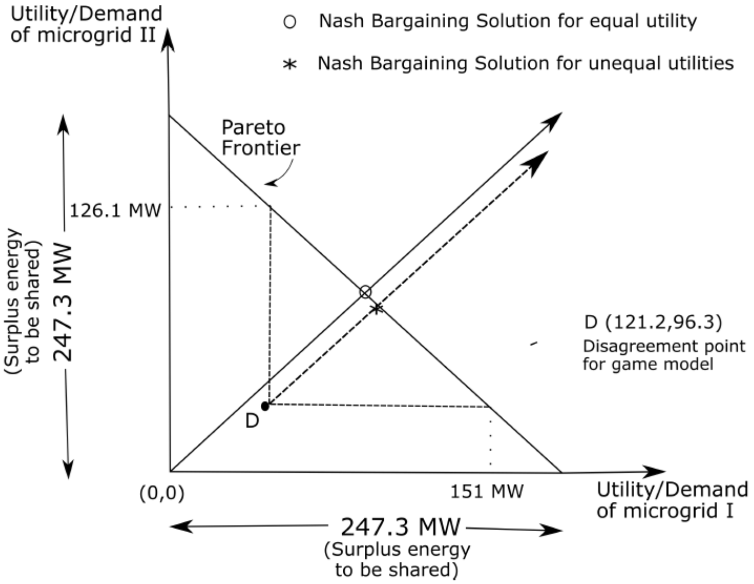
Nash Bargaining solution is a unique outcome $(u_1^*, u_2^*) \subset U$.

A Nash bargaining solution is an outcome $(u_1^*, u_2^*) \in U \cup \{D\}$ that satisfies four axioms discussed earlier. The Nash solution should satisfy Pareto efficiency, symmetry, irrelevant alternatives and linear transformation independence.

The bargaining solution should satisfy:

$$\max_{u_1^*, u_2^*} (u_1^* - d_1)(u_2^* - d_2) \text{ subject to } u_1^*, u_2^* \geq d_1, d_2 \tag{2}$$

Figure 4 Nash bargaining solution for bargaining parties



The Nash bargaining solution to share the excess energy of the microgrid is depicted in Figure 4. It is observed that an excess of energy with microgrid M_3 has to be shared between two parties, namely M_1 and M_2. The utility of microgrid M_1 and M_2 depends on the energy deficit. Microgrid with the highest utility will receive the top portion of the excess energy.

The Nash bargaining solution can be used to calculate the disagreement point (d_1, d_2). The line extending from the disagreement point to the set boundary will give the Nash bargaining solution. In case the utility of both microgrids demanding energy is the same, the Nash solution will lead to equal payoffs to both microgrids.

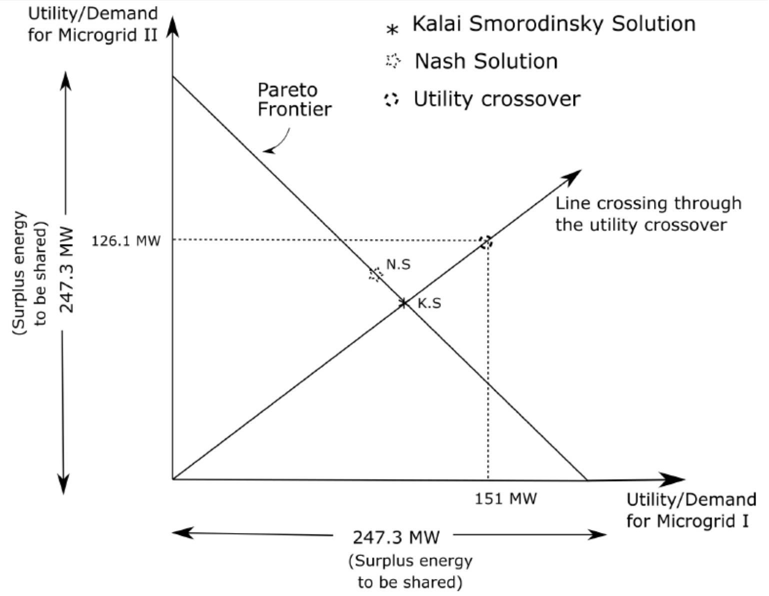
The microgrid with the higher utility will get a greater share of the excess energy. For example, the Nash bargaining solution for the two microgrids sharing 15 kW excess energy calculates (6.5, 8.5).

It means the microgrid demanding 10 kW receives the share of 6.5 kW while the second microgrid gets 8.5 kW out of the total excess of 15 kW. The rest of the deficit energy is imported from the grid.

3.2.2 Kalai-Smorodinsky solution

The lack of flexibility in the Nash bargaining solution leads to a new solution proposed by Kalai. It was proposed for the case when the outcome set X is expanded. On expansion, the solution outcomes decrease fast, which causes the solution to change. A new axiom 'monotonicity' was proposed to counter this situation when $X \subset Z$. It ensures that the new bargaining solution of the expanded set Z will be adjusted accordingly.

Figure 5 Kalai-Smorodinsky solution



The K-S solution is obtained using following steps:

- The feasible set X is established
- The maximum utility is calculated for microgrid I restricting the payoff to be more than the demand. The maximum utility is calculated for microgrid II restricting the payoff to be more than the demand.

  The K-S bargaining solution is the point where the line through origin crosses the boundary of X. The Kalai-Smorodisnky (KS) solution for two participating microgrids bargaining over a third microgrid's excess energy is shown in Figure 5.

3.2.3 Egalitarian solution

The Egalitarian solution tries to attain equal payoffs to both microgrids. It represents a point in the set where all microgrids attain an equal and maximal increase in utility concerning the disagreement point. The Egalitarian solution is found as the intersection of the Pareto optimal curve and line at 45°.

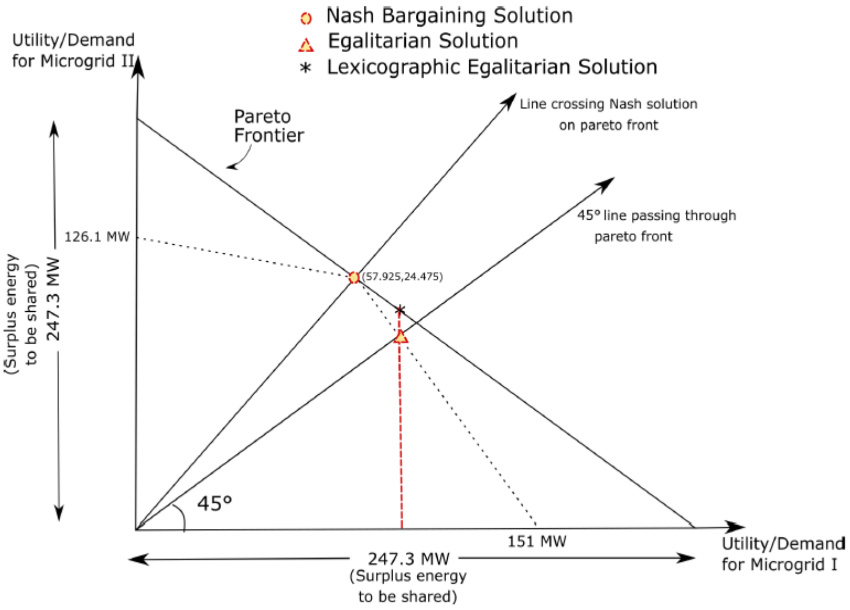
$$\left(u_1^*, u_2^*\right) = \arg\max\left(u > d \cap u_1 = d_1 = u_2 - d_2\right) \tag{3}$$

This solution did not satisfy scale invariance property, and hence, an extension of Chun (1989). It is a point that is lexicographically maximum and violates the Egalitarian rule.

In an Egalitarian lexicographic solution, some microgrids may receive lesser utilities than other microgrids but still receive at least the utilities they would attain at the Egalitarian outcome.

The solution is depicted in Figure 6.

Figure 6 Egalitarian solution (see online version for colours)



4 Results and discussion

The microgrids undergo energy trading for optimum utilisation of energy using any of the bargaining solutions. A method for sharing of surplus energy among two microgrids deficient in energy is solved by the Nash bargaining solution depicted in Figure 4. It discusses the Nash solution for surplus energy sharing been two players short in energy.

The algorithm denotes energy trade at hour 8 in 24 hours. A total surplus of 247.3 MW is to be shared between the two microgrids according to their respective utility. The demand of the respective microgrid serves as the utility in such a case of the bargain. Microgrid I possesses a utility/demand of 151 MW from the total surplus, while microgrid II has a utility/demand of 126.1 MW. The requirement of utility 1 exceed that of utility 2, leading to more significant energy share to microgrid I. The point of disagreement between the energy trade for the two microgrids was found to be (121.2, 96.3), which represents the worst solution of the game. The line extending from the disagreement point to the Nash frontier represents the Nash solution of the game. It is observed that the Nash solution fulfils the requirement of proportional fairness in the game. The NBS for the discussed game is found to (97.36, 102.33), which means 97.33 MW is allocated to microgrid II, and microgrid I receive 102.33 MW of power out of the total surplus of 247.3 MW.

KS bargaining game is applied to the same problem discussed above to determine the KS solution of the game. The KS solution lies on the line passing through the utility crossover and depicted in Figure 5. The KS for the discussed game if found to (112.56, 134.76), which means 112.56 MW is allocated to microgrid II, and microgrid I receive 134.76 MW of power out of the total surplus of 247.3 MW. Similarly, lexicographic Egalitarian (ES) method is utilised to obtain the bargaining solution as depicted in Figure 6. The ES for the discussed game is found to (120.53, 126.76), which means 120.53 MW is allocated to microgrid II, and microgrid I receive 126.76 MW of power out of the total surplus of 247.3 MW.

Table 3 depicts the surplus energy in any one of the microgrids along with the respective utilities or energy demands from the deficient microgrids. The share of energy for each microgrid participating in the game according to the selected technique is also shown.

The energy trading schedule for the three participating microgrids is shown in Figure 7. Positive values of the trade mean that the energy is purchased from the grid, while negative values denote the selling of energy to the grid. It should be noted that without energy trading, all the individual microgrids sell their excess surplus energy to the grid. Energy purchase from the grid is made if the individual microgrid is unable to fulfil the local energy demand. It can be seen that the microgrid I sell much energy from the primary grid during the period of 11 am to 5 pm and purchases energy in the period from 12:00 to 9 am and 7 to 12 pm. Microgrids II and III make most of their purchases in evening hours from 10 am to 4 pm. Microgrid I is the leading buyer of energy to the grid in the period of 6 to 12 pm and microgrid II is the leading seller of energy during 6 to 12 pm This can also be seen from the fact that the solar microgrid I generate most of its energy during sun hours in a day.

Table 3 Division of excess energy according to Nash bargaining theorem

S. no.	*Excess energy (MWh)*	*Utility I u_1 (MWh)*	*Utility II u_2 (MWh)*	*Nash share of microgrid I/microgrid II (MWh)*	*Kalai-Smorodinsky share of microgrid I/microgrid II (MWh)*	*Lexicographic Kalai Egalitarian share of microgrid I/microgrid II (MWh)*
1	82.4	113.2	247.0	(24.45, 57.95)	(25.9, 56.5)	(29, 53.4)
2	93.5	143.9	238.7	(4.9, 58.6)	(35.17, 58.33)	(35.61, 57.89)
3	59.5	127.2	234.6	(16.325, 43.175)	(20.92, 38.58)	(29.39, 30.11)
4	8.8	136.8	225.1	(3.02, 5.78)	(2.05, 6.75)	(4.87, 3.93)
5	30.3	113.8	80.2	(10.95, 19.35)	(17.77, 12.53)	(16.66, 13.64)
6	138.9	134.3	118.6	(67.49, 71.41)	(73.76, 65.14)	(75.84, 63.06,)
7	199.7	127.6	107.7	(102.33, 97.36)	(108.29, 91.41)	(113.32, 86.38)
8	247.3	151	126.1	(126.76, 120.54)	(134.76, 112.54)	(141.44, 105.86)
9	243.7	140.2	178.5	(117.06, 126.63)	(107.21, 136.49)	(94.66, 149.04)
10	224.1	98.2	174.0	(102.57, 121.52)	(80.85, 143.25)	(42.35, 181.75)
11	158.9	12.7	110.9	(67.17, 91.725)	(16.33, 142.57)	(67.17, 91.72)
12	246.1	8.5	260.9	(91.5, 154.6)	(7.76, 238.34)	(91.5, 154.6)
13	223.5	29	243.1	(84.98, 138.51)	(23.82, 199.68)	(84.98, 138.51)
14	156.7	75.6	233.9	(58.56, 98.13)	(38.28, 118.42)	(58.56, 98.13)

Figure 7 Energy trading schedule between microgrids (see online version for colours)

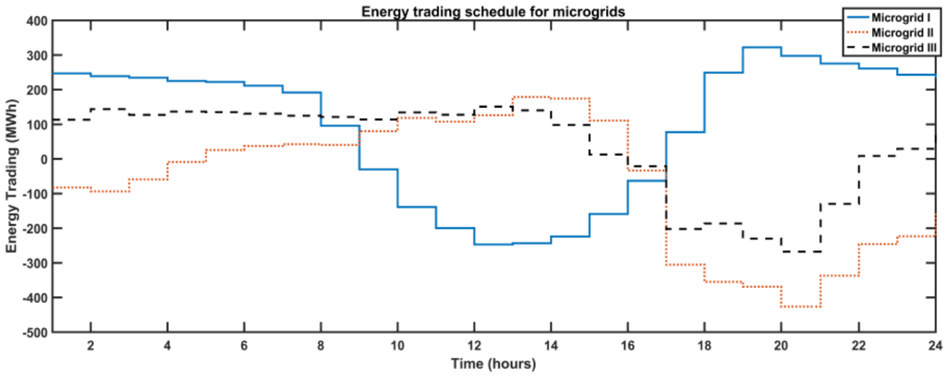
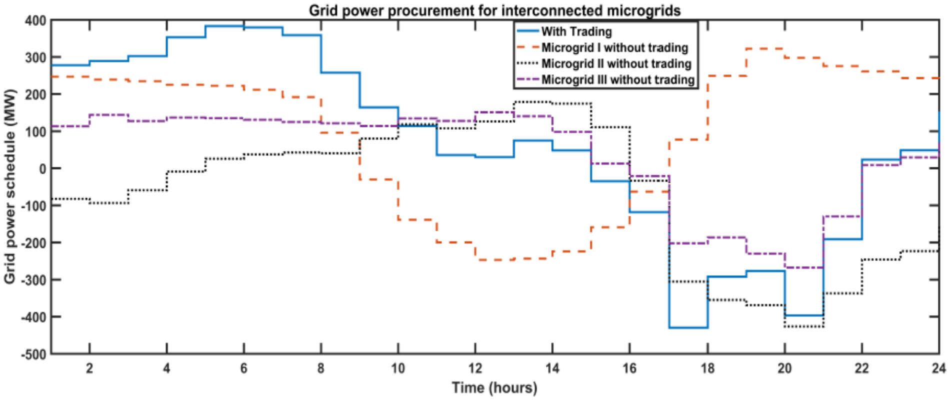


Figure 8 denotes the grid energy procurement schedule for the microgrids with and without trading. Without energy trading, the microgrids function independently to other microgrids. The microgrids fulfil their deficit demand from the main grid directly instead of demanding from other microgrids. It means the microgrids purchase a more considerable amount of energy than when it indulges in energy trading.

Figure 8 Grid energy procurement for interconnected microgrids with or without energy trading (see online version for colours)



From calculation, it is seen that the microgrid purchases 5,040.4 MW of energy without energy trading and 3,291.9 MW of energy with energy trading leading to lower costs during trading.

Figure 9 and Figure 10 denote the comparative plot of the three assumed game models for energy sharing in microgrids I and II, respectively. It can be observed from Figure 9 and 10 that the three models show close proximity in arriving at the bargaining solution till hour 15. After the 15th hour, the utility/demand ratio between the bargaining models keeps on increasing. For instance, at hour 22, there is a surplus of 246.1 MW of energy and this energy is to be distributed between microgrids I and II with their utility/demand as 260.9 MW and 8.5 MW. This difference between utility values can be well perceived by the Kalai-Smorodinsky solution but gives divergent results for Nash and Lex-Egalitarian solution. Hence, for the Nash and the Lex-Egalitarian equilibrium, the ratio between utilities should be low. This is a limitation of the proposed solution.

Figure 9 Comparative plot of game models for energy share of microgrid I (see online version for colours)

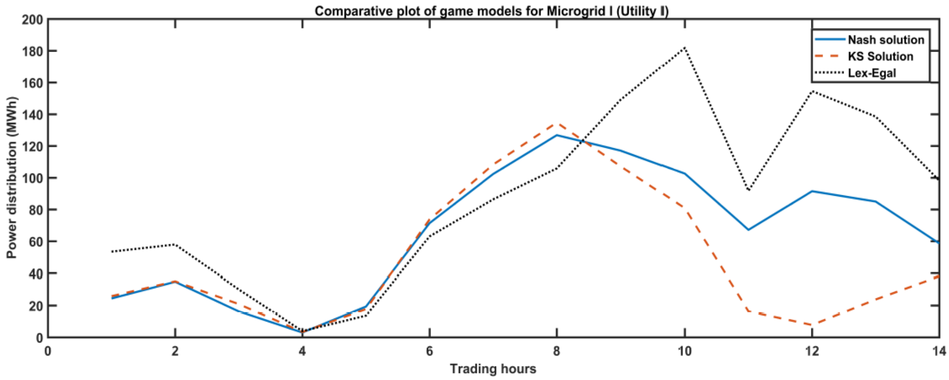
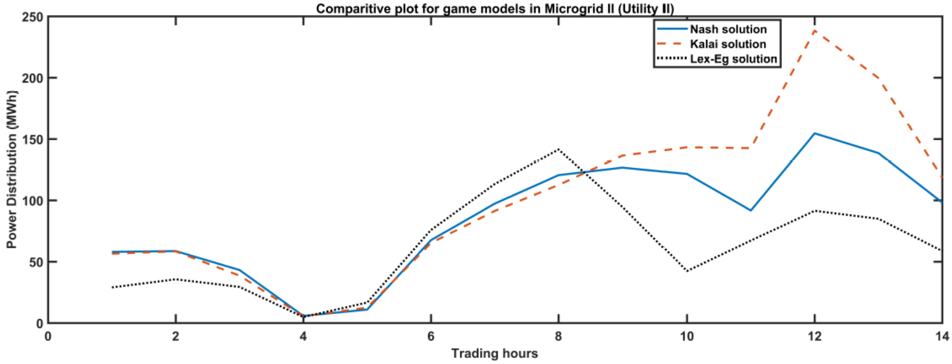


Figure 10 Comparative plot of game models for energy share of microgrid II (see online version for colours)



Dynamic time warping (DTW) (Berndt and Clifford, 1994) technique is utilised to compute the distance metric between two or more time series. The time series are compared with each other for similarity or dissimilarity by converting the data into vectors.

$$f\left(y_j\right) \text{ maps to } f\left(y_k\right) \text{ when } j \leq k \tag{4a}$$

$$f\left(y_j\right) \text{ maps to } f\left(y_k\right) \text{ iff } (j-i) \text{ is within fixed range} \tag{4b}$$

A comparison of the different bargaining models using dynamic warp distance is performed in context of the energy trading model.

It can be seen that the KS bargaining solution closely relates to the Nash solution in terms of the value it assigns to different utilities, while Nash solution is far-off from the Lex-Egaliarian solution. It is depicted in Table 4 where the Nash-Kalai distance calculates to be 24.2 while the Nash-Egalitarian distance is found to be 117.5.

Table 4 Dynamic warp distance between time series for different bargaining models

Utility	*Bargaining game*	*Kalai- Smorodinsky*	*Lex-Egalitarian*
Utility 1	Nash	24.502	117.557
	Kalai	-	118.79
Utility 2	Nash	24.502	117.557
	Kalai	-	120.09

Several other parameters compare the bargaining game solutions such as Pareto-optimal convergence, efficiency and fairness index.

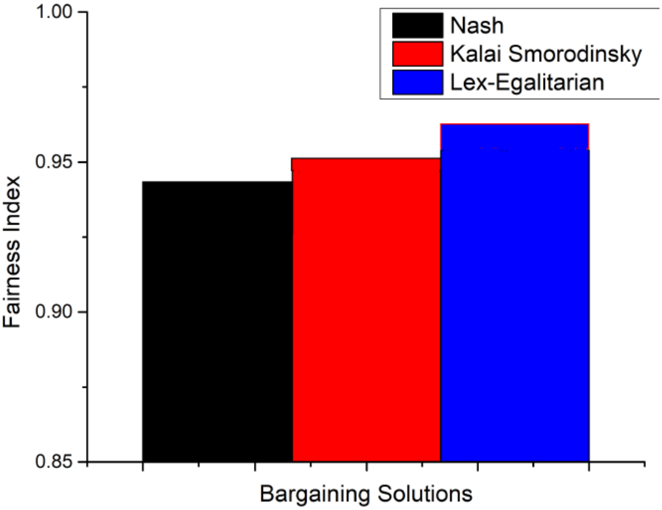
Jain's fairness index (Jain et al., 1984) is depicted to show fairness in the allocation of resources to the two bargaining microgrids on a scale of 0 to 1.

It is given by the following function depicted by equation (5):

$$FI = \frac{\left(\sum_{j=1}^{n} u_j\right)^2}{N * \left(\sum_{j=1}^{n} u_j^2\right)} \tag{5}$$

It is seen from Figure 11 that the distribution of resources is done more fairly when the lexicographic-Egalitarian solution is utilised. The fairness index for the Lex-Egalitarian solution was found to be 0.9747, while it is 0.955 and 0.9457 for K-S and Nash solution, respectively.

Figure 11 Fairness index comparison between different bargaining solutions (see online version for colours)



5 Verification and validation

The results of the study are verified by the previous works performed and recorded in Table 5. Additionally, a sensitivity analysis is performed to analyse the change in energy share among microgrids with constant change in surplus energy.

It can be observed from Figure 12 that for the Nash bargaining solution, the fairness index increases with increase in volume of surplus energy. The fairness index becomes constant after reaching an equilibrium point at 205 MW approx when the fairness index for participating microgrids become constant at the value of 0.9558. This means that the maximum fairness of Nash game is achieved at higher values of approximating a fair game to the players.

Figure 12 Sensitivity of energy share among microgrids with increase in surplus energy for bargaining models (see online version for colours)

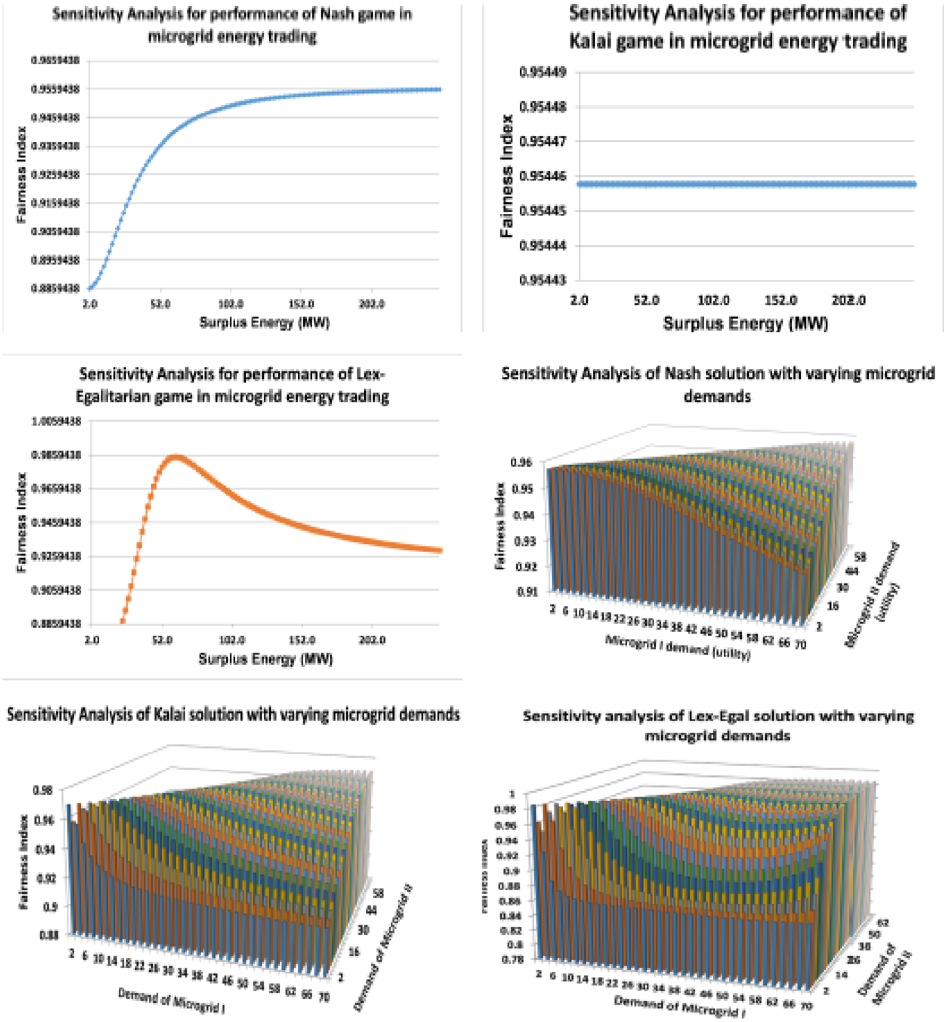


Table 5 Comparison of proposed method with previous works

References	*Methods*	*Fairness index*	*FI of proposed method*
Pandremmenou et al. (2013a)	Nash	0.9992	Nash – 0.9457
Fourati et al. (2016)	Kalai-Smorodisnky	0.9975	Kalai-Smorodinsky – 0.955
Kim (2019)	Nash-Kalai-Smor.-Egal. Soln-Nash with claim, midpoint-constrained Egalitarian and proportional bargaining solutions	0.949 0.600 0.980 0.49	Lex-Egalitarian – 0.9747
Oikonomakou et al. (2017)	Bankruptcy game with Shapley value	0.986	
Ni and Zarakovitis (2011)	Nash	0.9989	

The fairness of the Kalai-Smorodinsky solution however remains independent of the excess energy at a constant value of 0.9544. The fairness of the Lex-Egalitarian reaches a maximum value of 0.9859 at a surplus of 60 MW but the fairness decreases henceforth and becomes constant at a fairness index value of 0.9259.

The sensitivity analysis is repeated for a two input utility value and its effect on the fairness is evaluated. It can be seen that a large diversion between the utility values lead to loss of fairness.

6 Conclusions

In this paper, we studied various game theory techniques for sharing excess energy left with the microgrid. The data is formulated from different generating and load stations in Denmark. The microgrids were studied for 24 hours for excesses and a shortage of energy. The game theory bargaining models were implemented for a maximum payoff to the participating microgrids:

- Nash, Kalai-Smorodinsky and lexicographic Egalitarian game techniques were implemented to find bargaining solutions for the sharing of excess energy.
- The solutions were compared for their efficiency, distance and fairness index.
- The fairness index for the Lex-Egalitarian solution was found to be 0.9747, while it was found to be 0.955 and 0.9457 for K-S and Nash solution, respectively.
- It was however observed that the Lex-Egalitarian and the Nash solution were deviating from the energy balance if the ratio between utilities was too diverse.
- It was found that the Lex-Egalitarian solution is more efficient and fair in its approach to contribute a solution to the division of excess energy.
- A sensitivity analysis proved the effect of surplus energy and utility combination on the fairness of the game. The highest value of fairness was observed in Lex-Egaliatarian solution but it decreased with increasing energy surplus. Nash

bargaining fairness also reached a constant fairness after increasing for a duration of time.

Future research work would focus on the calculation of different cost payoff's for the discussed techniques and optimisation for achieving maximum profit to the microgrids. Furthermore, the research can be expanded for a cluster of interconnected microgrids rather than a three shared network.

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