Editorial

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Biographical notes: Herb Kunze is a Professor in the Department of Mathematics and Statistics at the University of Guelph. He received his PhD in Applied Mathematics from the University of Waterloo. His main research interests are applied analysis, dynamical systems, qualitative theory of differential equations and fractal-based methods in analysis.

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This special issue aims to highlight recent research, development and applications of fractals and fractal-based methods.

The idea of modelling the behaviour of phenomena at multiple scales has become a useful tool in both pure and applied mathematics. Fractal-based techniques lie at the heart of multiscale modelling, as fractals are inherently multiscale objects. Fractals have increasingly become a useful tool in real-world applications; they very often describe such phenomena better than traditional mathematical models.

In 1975, Mandelbrot introduced the term ‘fractal’ to characterise spatial or temporal phenomena that are continuous but not differentiable. Fractal properties include self-similarity, complexity, and non-integer dimension. Fractals became very popular once their connection with chaos was discovered in dissipative systems. It was found that attractors in chaotic dissipative systems are typically fractal in nature.

The landmark papers by Hutchinson, Barnsley and Demko showed how systems of contractive maps with associated probabilities (called 'iterated function systems' or IFS by the latter) acting in a parallel manner, either deterministically or probabilistically, could be used to construct fractal sets and measures. These IFS models allow great flexibility in controlling the scale-dependence in the model but at the same time are not overly complex. Therefore the resulting IFS-based methods provide a framework which is at the same time rich enough to describe a wide range of phenomena but also simple enough to allow for concrete analytical results.

Fractal-based methods attempt to discover and exploit inter-scale relationships for modelling, prediction and control of phenomena. Fractal-based image compression was popularised by Barnsley, but research in imaging has moved beyond compression to other types of image processing (denoising, edge detection, deblurring, watermarking, information hiding, etc.) and to the broader notion of multiscale image analysis. Fractal-based methods have also been used to solve a wide variety of inverse problems arising in models described by systems of differential equations in both deterministic and stochastic settings.

This special issue collects together papers from both theoretical and applied perspectives.

A brief summary of the contents:

P. Alonso-Ruiz and U.R. Freiberg consider the famous game Towers of Hanoi, which is related with a family of so-called Hanoi-graphs. Regarding these non-self-similar graphs as geometrical objects, they obtain a sequence of fractals converging to the Sierpinski gasket. They present various convergence results.

M. Demers presents a fractal-based edge detector that takes into account two different criteria: the clustering of near-optimal parent blocks, and the values of certain parameters within the algorithm. This indirect approach to edge detection can give results comparable to well-established edge detection techniques.

K.M. Levere develops a collage-coding technique, based upon the non-linear Lax-Milgram representation theorem, for solving inverse problems for a general class of second-order non-linear elliptic PDEs.

J.L. Véhel and F. Mendivil consider a local version of the theory of fractal strings and associated geometric zeta functions. Such a generalisation allows to describe the asymptotic behaviour of a 'fractal' set in the neighbourhood of any of its points.

S. Marsiglio studies a stochastic, discrete-time, two-sector growth Solow-like model characterised by perpetual growth. Under some conditions, he shows that the capital dynamics can be converted into an iterated function system converging to an invariant distribution supported on a Cantor set.

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