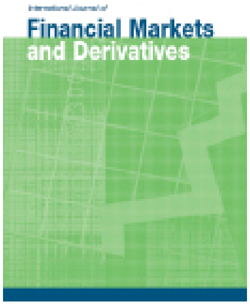

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Jimmy E. Hilliard, Jitka Hilliard, Yufei Wu

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The sensitivity of variance risk premium estimates to grid and strike fineness: simulating the Heston model with jumps

Jimmy E. Hilliard* and Jitka Hilliard

Department of Finance,
Raymond J. Harbert College of Business,
Auburn University,
Auburn, Alabama, USA
Email: jim.hilliard@auburn.edu
Email: jitka.hilliard@auburn.edu
*Corresponding author

Yufei Wu

School of Creative Enterprise and Market Systems,
Sorrell College of Business,
Troy University,
Troy, Alabama, USA
Email: wuy@troy.edu

Abstract: We investigate variance risk premiums in the Heston stochastic volatility model with Merton jumps. Risk neutral variance estimates are derived from out-of-the-money option-based integrands. In a controlled simulation setting with model-generated option prices and evenly spaced strikes, our results suggest that 12 or fewer option strikes are sufficient to obtain accurate model-free variance estimates – a finding with practical relevance for researchers working with illiquid underlyings where the available strike grid may be sparse. We find that sample variance is a better estimate of integrated variance than finite quadratic variation under a coarse grid. Model free methods give better estimates than averaged Black-Scholes implied volatilities.

Keywords: variance risk premium; Heston model; Merton jumps; robustness.

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Biographical notes: Jimmy E. Hilliard is the Harbert Eminent Scholar and Professor of Finance at the Harbert College of Business at Auburn University. His research focuses on investments, options, and futures.

Jitka Hilliard is a Professor of Finance and the J.K. Lowder Family Professor at the Harbert College of Business at Auburn University. Her research focuses on investments, options and futures, and financial literacy.

Yufei Wu is an Assistant Professor at the Sorrell College of Business at Troy University. His research focuses on investments, options, futures, and corporate finance.

1 Background

Motivation and contribution

Risk-neutral parameter estimates are fundamentally important for pricing derivative securities and for measuring the variance risk premium (VRP), defined as the difference between the physical and risk-neutral expectations of realised variance. The VRP on S&P 500 index options is consistently negative: investors willingly pay above the physical expected value for variance exposure because such payoffs diversify returns on equity portfolios in unfavorable states of nature (Bakshi and Kapadia, 2003; Carr and Wu, 2009). Accurate measurement of the VRP requires both a reliable estimate of realised (physical) variance from return data and a reliable estimate of risk-neutral variance from option prices.

The model-free approach (Britten-Jones and Neuberger, 2000; Carr and Wu, 2009; Jiang and Tian, 2005; Rompolis and Tzavalis, 2017) integrates out-of-the-money option prices over the available strike grid. In theory this integral extends over a continuum of strikes. In practice, the strike grid is finite and may be sparse for illiquid underlyings. Prior work has analysed truncation error (bounded strike range) and discretisation error (finite grid), but a direct simulation-based assessment of the minimum number of strikes required for accurate VRP estimation under SVJ dynamics has not been provided.

Our contribution is to fill this gap using efficient exact and almost exact simulation methods (Broadie and Kaya, 2006; Oosterlee and Grzelak, 2020) and the Fourier-cosine (COS) option pricing method (Fang and Oosterlee, 2009). We systematically vary the number of option strikes from 6 to 24 and find that 12 strikes are sufficient to reduce model-free bias below 0.002 across a wide range of VolVol and jump-intensity scenarios. These findings directly inform empirical researchers who must construct VRP estimates from sparse option panels.

Terminology and notation

To avoid ambiguity we formally define all variance measures used in the paper. Quadratic variation (QV): the limit of the sum of squared log-returns as the sampling mesh tends to zero; converges to integrated variance for semi-martingales. Realised variance (RV): the finite-grid approximation to QV, the sum of squared log-returns on a positive-mesh grid. Sample variance (SV): RV minus n times the squared mean return, divided by $n-1$; the standard unbiased variance estimator. Integrated variance ($\overline{QV}$): the time-averaged instantaneous variance $\frac{1}{\tau}\int_0^\tau V(s)ds$, the theoretical target. Model-free implied variance ($\widehat{IV}_{MF}$): the option-based estimate of risk-neutral integrated variance from the OTM put-call integrand. Black-Scholes implied variance ($\widehat{IV}_{BS}$): the cross-strike average of Black-Scholes implied variances, used as a misspecification benchmark.

Risk neutral parameter estimates are fundamentally important for pricing derivative securities. Equilibrium pricing models for well posed payoffs $H(S_T)$ on an underlying security S_t satisfy

$$C = e^{-rT} E^{Q}_{S_T} \left[H(S_T) \mid F_t \right], \tag{1}$$

where T is the time to contract expiration, S_t is a semi-martingale, $E^{Q}_{S_T}$ is expectation under the risk-neutral measure Q, F_t is the available information at $t \leq T$, and r is the default-free interest rate. When markets are complete and S_t can be hedged, C is the no-arbitrage solution.

Broadie and Jain (2008) and Carr and Wu (2009) and others consider a variance swap on $[t, T]$ with payoff $H_T = (RV_{t,T} - K)L$, where RV is realised variance, K is the strike price, and L is the swap principal. The zero-cost swap requires $E^{Q}(RV_{t,T}) = K$.

We use the standard definition that quadratic variation is the sum of squared returns on a fine grid (mesh $\to 0$) and that realised variation is a coarse-grid version of quadratic variation. The sample variance is realised variance minus the squared mean return and is denoted $SV_{t,T}$. With n equally spaced return observations on $[t, T]$:

$$RV_{t,T} = \sum_{t=1}^{n} r_t^2, \;\; SV_{t,T} = \frac{1}{n-1} \sum_{t=1}^{n} (r_t - \bar{r})^2, \tag{2}$$

where $\bar{r}$ is the mean of r_t. Equally spaced daily data can be annualised by trading (252) or calendar (360) days. The relationship between realised variance and sample variance is

$$RV_{t,T} = (n-1) SV_{t,T} + n\bar{r}^2, \tag{3}$$

or $SV_{t,T} \approx RV_{t,T}/(n-1)$ for small sampling intervals. For stochastic volatility models, quadratic variation is a consistent estimator of integrated volatility.

1.1 The VRP

The VRP, inherent in a variance swap, is defined as $E^{P}[RV_{t,T}] - E^{Q}[RV_{t,T}]$, where P is the physical measure. The estimated VRP on S&P 500 index returns (SPX) is generally observed to be negative, meaning participants are willing to pay in excess of physical expected value for positions with increasing payoffs when variance increases. These payoffs diversify returns on widely held traded forms of the S&P 500 index, consistent with high prices for assets that pay off in otherwise unfavorable states of nature (Bakshi and Kapadia, 2003; Carr and Wu, 2009).

1.2 Preview

We focus on model-free estimates of risk-neutral variances, sample variances, and the VRP. Model-free estimates require a database of asset returns and option prices with common maturity over a fine grid of strikes. To address the liquidity issue, we use simulations to gauge estimation bias and the sensitivity of VRPs to the number of option strikes. We also compare Black-Scholes implied volatilities with model-free estimates.

Our simulations suggest that accurate model-free variance estimates require only a few option strikes. Realised variance estimates had more bias than sample variance estimates when observations were taken at daily frequencies. There was no notable difference with sampling at minute-by-minute intervals. Bias and VRP estimates were especially accurate when minute-by-minute realisations were generated. As expected by construction (SVJ model), Black-Scholes implied variances averaged over strikes were more biased and had greater volatility than model-free estimates.

1.3 Simulation

Broadie and Jain (2008) evaluate biases when quadratic variation results are approximated using coarse sampling grids. They show that realised variance is a consistent estimator of integrated volatility for the Merton-Heston model (hereafter SVJ). The coefficient of the variance innovation (VolVol) and the correlation between return and variance innovations are noticeably absent in the quadratic variation of the SVJ model. However, consistent with their derivations, we expect that biases will be present when we generate a finite number of returns with non-zero diffusion correlations and different levels of VolVol.

The simulation experiment proceeds as follows:

1 choose a stochastic variance model

2 generate underlying simulations for a number of parameter choices

3 use the characteristic function approach to generate option prices at different strikes

4 evaluate the bias in model-free variance estimates and errors in the model-free estimates of VRPs.

1.4 Return and variance processes

We generate physical and risk-neutral realisations from the Merton (1976) jump diffusion model extended to include Heston (1993) stochastic volatility. The SVJ process with physical measure P can be written

$$dS = \left(\mu - \lambda_J \bar{J}\right) Sdt + \sqrt{V} SdW_s^P + JSdq, \tag{4}$$

$$dV = [\kappa(\theta - V)]\, dt + b\sqrt{V} dW_v^P, \tag{5}$$

where μ is instantaneous drift, V is stochastic variance, $\ln(1+J) \sim N(\mu_J - \sigma_J^2/2, \sigma_J^2)$, λ is the Poisson arrival intensity, and $\bar{J} = e^{\mu_J} - 1$ is the expected jump magnitude. The parameters $\kappa \geq 0$, θ, and b are the speed of adjustment, long-run mean, and

VolVol coefficient of the variance process. The Brownian motions dW_s^P and dW_v^P have instantaneous correlation ρ. Jumps are assumed to carry no systematic risk and are independent of diffusion innovations. The Feller condition $2\kappa\theta > b^2$ guarantees positive variance. Correlations are incorporated via $dW_s^P = \rho dW_v^P + \sqrt{1-\rho^2} d\tilde{W}_s^P$.

For the risk-neutral measure Q we assume there are $m \geq 2$ derivatives dependent on V and write

$$dV = \left[\kappa^P(\theta^P - V) - \lambda(V,t) b\sqrt{V}\right] dt + b\sqrt{V} dW_v^Q,$$

where $\lambda(V,t)b\sqrt{V}$ is the market price of variance risk. We adopt the proportional specification $\lambda(V,t) = \alpha^* \sqrt{V}/b$, where α^* is a constant. Substituting gives

$$dV = \left[\kappa^P(\theta^P - V) - \alpha^* V\right] dt + b\sqrt{V} dW_v^Q \tag{6}$$

$$= (\kappa^P + \alpha^*) \left[\frac{\kappa^P \theta^P}{\kappa^P + \alpha^*} - V\right] dt + b\sqrt{V} dW_v^Q, \tag{7}$$

so that the risk-neutral speed of adjustment is $\kappa^Q = \kappa^P + \alpha^*$ and the risk-neutral long-run mean is $\theta^Q = \kappa^P\theta^P/(\kappa^P + \alpha^*)$. Correlations ρ can be explicitly incorporated by letting $\alpha^* = \rho\kappa^P$, where ρ is a constant. Wong and Heyde (2006) develop conditions under which the risk-neutral process remains an equivalent local martingale; for the Heston model an ELMM requires $|\rho| \leq \kappa/b$. Our simulations satisfy this condition.[1]

1.5 Realised variances

Realised variances are frequently analysed using results from the theory of quadratic variation. The continuous quadratic variation of returns converges to expected integrated variance when returns and stochastic variances are semi-martingales, i.e., $QV \to \frac{1}{\tau}\int_0^\tau V(t)ds$ (Barndorff-Nielsen and Shephard, 2002). We typically norm the result for interval length and write $QV \to \frac{1}{\tau}\int_0^\tau V(t)ds$. The result for the non-jump component of the SVJ processes is (Broadie and Jain, 2008)

$$\overline{QV} := \frac{1}{\tau}\int_0^\tau V(s)ds = \theta + \frac{(V_0 - \theta)(1 - e^{-\kappa\tau})}{\kappa\tau}. \tag{8}$$

Notably, the result does not depend on ρ or on b. The expected integrated variance of the uncorrelated jump term is

$$QV_J = \lambda(\sigma_J^2 + \bar{J}^2), \tag{9}$$

giving $QV = \overline{QV} + QV_J$:

$$QV = \theta + \frac{(V_0 - \theta)(1 - e^{-\kappa\tau})}{\kappa\tau} + \lambda(\sigma_J^2 + \bar{J}^2). \tag{10}$$

The risk-neutral integrated variance follows with κ^Q and θ^Q replacing the physical parameters.

Under discrete sampling, Broadie and Jain (2008) show that estimates of quadratic variation include a term depending on ρ, b, and other factors not shown in equation (10), but the additional contribution is $O(1/n)$ where n is the number of grid points. Subtracting means from quadratic variation estimates has negligible effect.

Simulations via Euler discretisation of equations (4)–(5) are problematic because satisfying the Feller condition for continuous returns does not guarantee positive variance with discrete samples. Broadie and Kaya (2006) develop a sampling scheme based on the exact distribution; Oosterlee and Grzelak (2020) find this computationally expensive and develop the almost exact method, using Euler discretisation of the price process and exact simulation of the variance process. An almost exact sample from the variance process is

$$V_i = \frac{b^2}{4}(1 - e^{-\kappa\Delta})\chi_d^2\left(\frac{4e^{-\kappa\Delta}}{b^2(1 - e^{-\kappa\Delta})}V_{i-1}\right), \tag{11}$$

where Δ is the sampling interval and $\chi_d^2(\nu)$ is the non-central chi-square with $d = 4\kappa\theta/b^2$ degrees of freedom and non-centrality parameter $\nu = 4e^{-\kappa\Delta}V_{i-1}/[b^2(1 - e^{-\kappa\Delta})]$.

We simulate the non-central χ^2 by first generating $\tilde{P}$ from a Poisson distribution with intensity $\nu/2$ and then computing

$$\chi_d^2(\nu) = \chi_{d+2\tilde{P}}^2, \tag{12}$$

where $\chi_{d+2\tilde{P}}^2$ is central chi-square with $d + 2\tilde{P}$ degrees of freedom (Broadie and Kaya, 2006; Glasserman, 2003).

We simulate the log stock price process $y = \ln(S)$ with Euler discretisation as follows:

$$y_i = y_{i-1} + \left(\mu - d - \frac{V_i}{2}\right)\Delta + \sqrt{V_i}\left(\rho\eta_v + \sqrt{1-\rho^2}\eta_s\right) + \ln(1 + J)q_i, \tag{13}$$

$$\eta_v\sqrt{V_i} = \frac{V_i - V_{i-1} - \kappa(\theta - V_{i-1})\Delta}{b}, \tag{14}$$

where η_s and η_v correspond to innovations in the stock process and variance process. Note that η_v is the random increment in V between t_i and t_{i-1}.[2]

1.6 Simulated option prices

Option prices are derived from simulated underlying prices using the characteristic function method. Closed form characteristic functions for the SVJ model can be inverted to give the probability density function and option prices obtained by the standard risk neutral expectation of the payoff function. We use the SVJ characteristic function given in Kangro et al. (2004) and obtain option prices by the COS inversion method of Fang and Oosterlee (2009). In the COS method, the product of the probability density function and the payoff is transformed into the products of cosine basis functions and the option (payoff) function. The result is an accurate approximation and rapid convergence. We validate the algorithm by pricing a SVJ model with parameters published by Duffie et al. (2000) and Broadie and Kaya (2006). The COS algorithm exactly reproduces the option value in their example.

Table 1 Model free estimates over number of strikes, volatility, and arrivals

Intensity strikes cols	*Risk neutral variances* $\lambda = 0.0$					*Risk neutral variances* $\lambda = 0.5$				
	IV_N *(1)*	$\widehat{IV}_{MF}$ *(2)*	$MFBias$ *(2)–(1)*	$\widehat{IV}_{BS}$ *(4)*	$BSBias$ *(4)–(1)*	IV_N *(5)*	$\widehat{IV}_{MF}$ *(6)*	$MFBias$ *(6)–(5)*	$\widehat{IV}_{BS}$ *(8)*	$BSBias$ *(8)–(5)*
Panel A: VolVol = 0.0										
6	0.0143	0.0069	–0.0074	0.0142	0.0000	0.0327	0.0218	–0.0110	0.0618	0.0291
8	0.0143	0.0095	–0.0047	0.0134	–0.0009	0.0327	0.0268	–0.0059	0.0603	0.0276
10	0.0143	0.0123	–0.0019	0.0141	–0.0001	0.0327	0.0306	–0.0021	0.0581	0.0254
12	0.0143	0.0145	0.0003	0.0137	–0.0005	0.0327	0.0334	0.0007	0.0569	0.0242
18	0.0143	0.0141	–0.0002	0.0141	–0.0002	0.0327	0.0336	0.0009	0.0571	0.0244
24	0.0143	0.0137	–0.0006	0.0138	–0.0004	0.0327	0.0336	0.0009	0.0575	0.0248
Panel B: VolVol = 0.25										
6	0.0143	0.0066	–0.0077	0.0181	0.0038	0.0327	0.0214	–0.0113	0.0626	0.0299
8	0.0143	0.0097	–0.0046	0.0164	0.0022	0.0327	0.0270	–0.0057	0.0604	0.0277
10	0.0143	0.0125	–0.0018	0.0168	0.0026	0.0327	0.0309	–0.0018	0.0583	0.0256
12	0.0143	0.0147	0.0004	0.0169	0.0026	0.0327	0.0337	0.0009	0.0573	0.0246
18	0.0143	0.0142	–0.0001	0.0168	0.0025	0.0327	0.0339	0.0012	0.0574	0.0246
24	0.0143	0.0138	–0.0004	0.0168	0.0025	0.0327	0.0338	0.0011	0.0577	0.0250
Panel C: VolVol = 0.50										
6	0.0143	0.0064	–0.0079	0.0229	0.0087	0.0327	0.0209	–0.0118	0.0623	0.0296
8	0.0143	0.0099	–0.0044	0.0212	0.0069	0.0327	0.0270	–0.0057	0.0605	0.0278
10	0.0143	0.0126	–0.0016	0.0207	0.0065	0.0327	0.0310	–0.0017	0.0585	0.0258
12	0.0143	0.0148	0.0006	0.0212	0.0069	0.0327	0.0338	0.0011	0.0574	0.0247
18	0.0143	0.0143	0.0001	0.0219	0.0077	0.0327	0.0341	0.0014	0.0575	0.0248
24	0.0143	0.0139	–0.0003	0.0212	0.0069	0.0327	0.0340	0.0013	0.0579	0.0252
Panel D: VolVol = 0.75										
6	0.0143	0.0070	–0.0073	0.0278	0.0136	0.0327	0.0211	–0.0116	0.0623	0.0296
8	0.0143	0.0105	–0.0038	0.0256	0.0113	0.0327	0.0275	–0.0052	0.0609	0.0281
10	0.0143	0.0131	–0.0011	0.0262	0.0120	0.0327	0.0315	–0.0013	0.0589	0.0262
12	0.0143	0.0151	0.0009	0.0254	0.0111	0.0327	0.0342	0.0015	0.0577	0.0250
18	0.0143	0.0144	0.0002	0.0250	0.0108	0.0327	0.0343	0.0016	0.0579	0.0252
24	0.0143	0.0140	–0.0002	0.0259	0.0117	0.0327	0.0342	0.0015	0.0583	0.0256

Notes: Theoretical European option prices from the SVJ model are computed using Fourier transforms and the COS pricing formulas with the integral approximated numerically with the number of strikes as noted. The Poisson arrival rate is λ and the risk-free rate is r_f = 0.0319. Jump parameters are μ_J = –0.15 and σ_J = 0.1278. Stochastic volatility parameters are κ = 3.99, θ = 0.014; VolVol is the variance in the Heston model and the price of risk is $\lambda(V,t) = \alpha\sqrt{V}/b = -0.50\sqrt{V}/\hat{b}$. Parameters are annualised. Black-Scholes implied variances are from SVJ prices. Biases are computed as the risk neutral variance estimate minus the known parameter ($\widehat{IV} - IV_N$).

Table 2 VRPs and biases: decomposition

λ	*Theory*			*Sample*		*Model free*			*Black-Scholes*		
col.	QV_P *(1)*	QV_R *(2)*	VRP *(1)–(2)*	RV *(4)*	$Bias$ *(4)–(1)*	$\widehat{IV}_{MF}$ *(6)*	$\widehat{VRP}_{MF}$ *(4)–(6)*	$MFBias$ *(6)–(2)*	$\widehat{IV}_{BS}$ *(9)*	$\widehat{VRP}_{BS}$ *(4)–(9)*	$BSBias$ *(9)–(2)*
Panel A: VolVol = 0.0											
0	0.0140	0.0160	–0.0020	0.0142	0.0002	0.0142	0.0000	–0.0018	0.0329	–0.0187	0.0169
0.11	0.0181	0.0201	–0.0020	0.0135	–0.0045	0.0185	–0.0050	–0.0015	0.0414	–0.0279	0.0213
0.25	0.0232	0.0252	–0.0020	0.0138	–0.0094	0.0241	–0.0103	–0.0012	0.0478	–0.0340	0.0226
0.5	0.0324	0.0345	–0.0020	0.0418	0.0094	0.0339	0.0079	–0.0006	0.0581	–0.0163	0.0237
1	0.0509	0.0529	–0.0020	0.0780	0.0271	0.0532	0.0248	0.0003	0.0743	0.0038	0.0214
3	0.1247	0.1267	–0.0020	0.1768	0.0521	0.1274	0.0494	0.0007	0.1337	0.0430	0.0070
Panel B: VolVol = 0.25											
0	0.0140	0.0160	–0.0020	0.0152	0.0012	0.0142	0.0009	–0.0018	0.0175	–0.0024	0.0015
0.11	0.0181	0.0201	–0.0020	0.0136	–0.0044	0.0187	–0.0050	–0.0014	0.0385	–0.0249	0.0184
0.25	0.0232	0.0252	–0.0020	0.0139	–0.0093	0.0242	–0.0103	–0.0010	0.0468	–0.0329	0.0215
0.5	0.0324	0.0345	–0.0020	0.0175	–0.0150	0.0340	–0.0165	–0.0005	0.0573	–0.0398	0.0228
1	0.0509	0.0529	–0.0020	0.0180	–0.0329	0.0533	–0.0354	0.0004	0.0734	–0.0554	0.0204
3	0.1247	0.1267	–0.0020	0.1339	0.0092	0.1274	0.0065	0.0007	0.1333	0.0007	0.0065
Panel C: VolVol = 0.50											
0	0.0140	0.0160	–0.0020	0.0141	0.0001	0.0143	–0.0003	–0.0017	0.0220	–0.0079	0.0060
0.11	0.0181	0.0201	–0.0020	0.0172	–0.0008	0.0189	–0.0016	–0.0012	0.0390	–0.0217	0.0189
0.25	0.0232	0.0252	–0.0020	0.0318	0.0085	0.0244	0.0074	–0.0008	0.0471	–0.0153	0.0218
0.5	0.0324	0.0345	–0.0020	0.0356	0.0031	0.0342	0.0014	–0.0003	0.0575	–0.0219	0.0230
1	0.0509	0.0529	–0.0020	0.0424	–0.0085	0.0535	–0.0111	0.0006	0.0733	–0.0309	0.0204
3	0.1247	0.1267	–0.0020	0.1817	0.0570	0.1276	0.0541	0.0009	0.1332	0.0485	0.0065
Panel D: VolVol = 0.75											
0	0.0140	0.0160	–0.0020	0.0119	–0.0021	0.0145	–0.0025	–0.0015	0.0261	–0.0142	0.0101
0.11	0.0181	0.0201	–0.0020	0.0297	0.0116	0.0191	0.0106	–0.0010	0.0398	–0.0102	0.0198
0.25	0.0232	0.0252	–0.0020	0.0152	–0.0081	0.0246	–0.0095	–0.0006	0.0477	–0.0325	0.0224
0.5	0.0324	0.0345	–0.0020	0.0176	–0.0149	0.0344	–0.0169	0.0000	0.0579	–0.0403	0.0234
1	0.0509	0.0529	–0.0020	0.0627	0.0118	0.0537	0.0090	0.0008	0.0733	–0.0106	0.0204
3	0.1247	0.1267	–0.0020	0.0350	–0.0897	0.1277	–0.0927	0.0010	0.1332	–0.0982	0.0065
Avg.	0.0439	0.0459	–0.0020	0.0435	–0.0003	0.0454	–0.0019	–0.0005	0.0627	–0.0192	0.0168
Std.	0.0389	0.0389	0.0000	0.0501	0.0273	0.0397	0.0273	0.0009	0.0358	0.0289	0.0072

Notes: Theoretical option prices and sample estimates are from the Merton jump diffusion model with Heston volatility (SVJ model). The at-the-money strike is 100 and r_f = 0.0319. Jump parameters are μ_J = –0.15 and σ_J = 0.1278. Stochastic volatility parameters are κ = 3.99, θ = 0.014 and the market price of risk is $\lambda(V,t) = \alpha\sqrt{V}/b = -0.50\sqrt{V}/\hat{b}$. Options are European and expire in 30 days. Theoretical underlying prices from the SVJ model computed using Fourier transforms and the COS pricing formulas with the integral approximated numerically using 256 coefficients. Risk neutral estimates were calculated using out-of-the-money put and call options with 18 evenly spaced strikes. Samples were generated over daily intervals. There were 60 iterations of each sample path. Variance risk premiums are calculated as the known average physical integrated variance minus the known average risk neutral variance ($VRP = QV_P - QV_R$). Sample variances are averaged over iterations and denoted by RV. The variance risk premium estimate is the average sample variance minus the risk neutral variance estimate ($\widehat{VRP} = RV - \widehat{IV}$). Biases are computed as the risk neutral variance estimate minus the known parameter ($\widehat{IV} - QV_R$).

Table 3 VRPs and biases: sensitivity to sampling interval

λ	*Theory*			*Sample*			*Model free*			*Black-Scholes*		
col.	QV_P *(1)*	QV_R *(2)*	VRP *(1)–(2)*	RV *(4)*	$RVBias$ *(4)–(1)*	$\hat{\sigma}^2_{RV}$ *(6)*	$\widehat{IV}_{MF}$ *(7)*	$\widehat{VRP}_{MF}$ *(4)–(7)*	$MFBias$ *(7)–(2)*	$\widehat{IV}_{BS}$ *(10)*	$\widehat{VRP}_{BS}$ *(4)–(10)*	$BSBias$ *(10)–(2)*
						Panel A: Δ = *10/360*						
0	0.0140	0.0160	–0.0020	0.0130	–0.0010	0.0006	0.0143	–0.0013	–0.0017	0.0220	–0.0089	0.0060
0.11	0.0181	0.0201	–0.0020	0.0187	0.0006	0.0136	0.0189	–0.0002	–0.0012	0.0390	–0.0203	0.0189
0.25	0.0232	0.0252	–0.0020	0.0234	0.0002	0.0215	0.0244	–0.0010	–0.0008	0.0471	–0.0237	0.0218
0.5	0.0324	0.0345	–0.0020	0.0338	0.0013	0.0420	0.0342	–0.0004	–0.0003	0.0575	–0.0237	0.0230
1	0.0509	0.0529	–0.0020	0.0521	0.0012	0.0710	0.0535	–0.0014	0.0006	0.0733	–0.0212	0.0204
3	0.1247	0.1267	–0.0020	0.1336	0.0089	0.2405	0.1276	0.0061	0.0009	0.1332	0.0005	0.0065
Average	0.0110	0.0115	–0.0005	0.0114	0.0005	0.0521	0.0114	0.0001	–0.0001	0.0155	–0.0041	0.0040
						Panel B: Δ = *5/360*						
0	0.0140	0.0160	–0.0020	0.0134	–0.0006	0.0002	0.0143	–0.0010	–0.0017	0.0220	–0.0086	0.0060
0.11	0.0181	0.0201	–0.0020	0.0191	0.0011	0.0090	0.0189	0.0002	–0.0012	0.0390	–0.0198	0.0189
0.25	0.0232	0.0252	–0.0020	0.0233	0.0000	0.0146	0.0244	–0.0011	–0.0008	0.0471	–0.0238	0.0218
0.5	0.0324	0.0345	–0.0020	0.0340	0.0015	0.0304	0.0342	–0.0002	–0.0003	0.0575	–0.0235	0.0230
1	0.0509	0.0529	–0.0020	0.0543	0.0034	0.0617	0.0535	0.0008	0.0006	0.0733	–0.0190	0.0204
3	0.1247	0.1267	–0.0020	0.1354	0.0107	0.1795	0.1276	0.0079	0.0009	0.1332	0.0023	0.0065
Average	0.0110	0.0115	–0.0005	0.0116	0.0007	0.0449	0.0114	0.0003	–0.0001	0.0155	–0.0039	0.0040

Notes: Theoretical option prices and sample estimates are from the Merton jump diffusion model with Heston volatility (SVJ model). The at-the-money strike is 100 and r_f = 0.0319. Jump parameters are μ_J = –0.15 and σ_J = 0.1278. Stochastic volatility parameters are κ = 3.99, θ = 0.014 and the price of volatility risk is $\lambda(V,t) = \alpha\sqrt{V}/b = -0.50\sqrt{V}/\hat{b}$ where $b = VolVol$ = 0.50. Options are European and expire in 30 days. Theoretical underlying prices from the SVJ model are computed using Fourier transforms and the COS pricing formulas with the integral approximated numerically using 256 coefficients. Risk neutral estimates were calculated using out-of-the-money put and call options with 18 evenly spaced strikes. There were 50,000 iterations of each sample path. Observations were generated at Δ-length intervals, and results are reported in panels A through D. Sample variances are averaged over iterations and denoted by RV. Biases of sample variances ($RVBias$) are computed as $RV - QV_P$. Variance risk premiums are calculated as the known average physical integrated variance minus the known average risk neutral variance ($VRP = QV_P - QV_R$).

Table 3 VRPs and biases: sensitivity to sampling interval (continued)

λ	*Theory*			*Sample*			*Model free*			*Black-Scholes*		
col.	QV_P *(1)*	QV_R *(2)*	VRP *(1)–(2)*	RV *(4)*	$RVBias$ *(4)–(1)*	$\hat{\sigma}^2_{RV}$ *(6)*	$\widehat{IV}_{MF}$ *(7)*	$\widehat{VRP}_{MF}$ *(4)–(7)*	$MFBias$ *(7)–(2)*	$\widehat{IV}_{BS}$ *(10)*	$\widehat{VRP}_{BS}$ *(4)–(10)*	$BSBias$ *(10)–(2)*
							Panel C: Δ = *1/360*					
0	0.0140	0.0160	–0.0020	0.0139	–0.0001	0.0001	0.0143	–0.0004	–0.0017	0.0220	–0.0080	0.0060
0.11	0.0181	0.0201	–0.0020	0.0188	0.0007	0.0079	0.0189	–0.0001	–0.0012	0.0390	–0.0202	0.0189
0.25	0.0232	0.0252	–0.0020	0.0251	0.0019	0.0154	0.0244	0.0007	–0.0008	0.0471	–0.0220	0.0218
0.5	0.0324	0.0345	–0.0020	0.0344	0.0019	0.0282	0.0342	0.0002	–0.0003	0.0575	–0.0231	0.0230
1	0.0509	0.0529	–0.0020	0.0573	0.0064	0.0593	0.0535	0.0038	0.0006	0.0733	–0.0160	0.0204
3	0.1247	0.1267	–0.0020	0.1403	0.0156	0.1747	0.1276	0.0127	0.0009	0.1332	0.0071	0.0065
Average	0.0110	0.0115	–0.0005	0.0121	0.0011	0.0439	0.0114	0.0007	–0.0001	0.0155	–0.0034	0.0040
							Panel D: Δ = *0.5/360*					
0	0.0140	0.0160	–0.0020	0.0139	–0.0001	0.0001	0.0143	–0.0005	–0.0017	0.0220	–0.0081	0.0060
0.11	0.0181	0.0201	–0.0020	0.0188	0.0008	0.0072	0.0189	–0.0001	–0.0012	0.0390	–0.0201	0.0189
0.25	0.0232	0.0252	–0.0020	0.0241	0.0009	0.0147	0.0244	–0.0002	–0.0008	0.0471	–0.0229	0.0218
0.5	0.0324	0.0345	–0.0020	0.0349	0.0024	0.0269	0.0342	0.0007	–0.0003	0.0575	–0.0226	0.0230
1	0.0509	0.0529	–0.0020	0.0562	0.0053	0.0594	0.0535	0.0027	0.0006	0.0733	–0.0171	0.0204
3	0.1247	0.1267	–0.0020	0.1398	0.0151	0.1616	0.1276	0.0122	0.0009	0.1332	0.0066	0.0065
Average	0.0110	0.0115	–0.0005	0.0120	0.0010	0.0427	0.0114	0.0006	–0.0001	0.0155	–0.0035	0.0040

Notes: Theoretical option prices and sample estimates are from the Merton jump diffusion model with Heston volatility (SVJ model). The at-the-money strike is 100 and r_f = 0.0319. Jump parameters are μ_J = –0.15 and σ_J = 0.1278. Stochastic volatility parameters are κ = 3.99, θ = 0.014 and the price of volatility risk is $\lambda(V,t) = \alpha\sqrt{V}/b = -0.50\sqrt{V}/\hat{b}$ where $b = VolVol$ = 0.50. Options are European and expire in 30 days. Theoretical underlying prices from the SVJ model are computed using Fourier transforms and the COS pricing formulas with the integral approximated numerically using 256 coefficients. Risk neutral estimates were calculated using out-of-the-money put and call options with 18 evenly spaced strikes. There were 50,000 iterations of each sample path. Observations were generated at Δ-length intervals, and results are reported in panels A through D. Sample variances are averaged over iterations and denoted by RV. Biases of sample variances ($RVBias$) are computed as $RV - QV_P$. Variance risk premiums are calculated as the known average physical integrated variance minus the known average risk neutral variance ($VRP = QV_P - QV_R$).

1.6.1 Implementation details

We use 256 Fourier-cosine coefficients. The COS integration interval is set to ± 12 standard deviations of the log-return distribution, yielding negligible truncation error across all parameter configurations tested. Strikes are evenly spaced over ± 8 standard deviations from the at-the-money strike; no interpolation or extrapolation of option prices is performed. We use out-of-the-money puts and calls throughout (Carr and Wu, 2009), averaging the put and call prices at the at-the-money strike. Numerical integration of the model-free integrand uses the midpoint (rectangular) rule; the smooth, unimodal shape of the integrand (Figure 2) implies rapid convergence of this simple rule.

Using a wide set of option strikes and maturities, we estimate model free risk neutral moments.

1.7 Model free risk neutral variance and moments

Model free risk-neutral variances have been extensively analysed by Britten-Jones and Neuberger (2000), Carr and Wu (2009), Jiang and Tian (2005), Bollerslev et al. (2011), Bakshi and Madan (2006), Rompolis and Tzavalis (2017) (RT) and others.[3] We specifically follow the results of RT, who generalise earlier results and compute non-central risk-neutral moment estimates of order i, μ_i, as follows:

$$\begin{aligned}
\mu_1 &= e^{(r-q)\tau} - 1 - e^{r\tau}\left[\int_{S_t}^{\infty}\frac{C(\tau,K)}{K^2}dK + \int_0^{S_t}\frac{P(\tau,K)}{K^2}dK\right],\\
\mu_m &= me^{r\tau}\int T(m,t)\left[\frac{C(\tau,K)}{K^2}+\frac{P(\tau,K)}{K^2}\right]dK,\ m\geq 2,\\
T(m,t) &= \left[\ln\frac{K}{S_t}\right]^{m-2}\left[m-1-\ln\frac{K}{S_t}\right].
\end{aligned} \tag{15}$$

where C is call price, K is strike price, S_t is the underlying price, r is the risk-free interest rate, and q is the dividend rate. Using risk-neutral moments and approximations given in RT, the implied variance (IV) is estimated as

$$\widehat{IV} = \frac{2}{\tau}\mu_1 + JT + DT, \tag{16}$$

$$DT = \frac{2}{\tau}\left[1 + (r-q)\tau - e^{(r-q)\tau}\right], \tag{17}$$

$$JT = -\frac{2}{\tau}\left[\frac{\mu_3}{3!}+\frac{\mu_4}{4!}\right], \tag{18}$$

where JT is the correction term for jumps when the underlying is equity and $DT = 0$ for a futures contract.

1.8 Simulation results

We decompose the theoretical VRP into risk premium estimates and biases as follows:

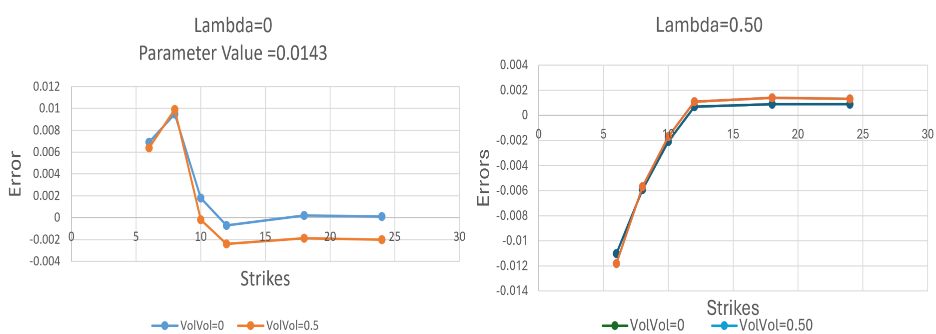
$$VRP = QV_P - QV_N$$

$$
\begin{aligned}
&= (SV - \widehat{IV}) + (\widehat{IV} - QV_N) - (SV - QV_P) \\
&= \widehat{VRP} + MFBias - SVBias, \qquad (19)
\end{aligned}
$$

where QV_P (QV_N) is the time-averaged integrated variance for the physical (risk-neutral) process. Estimates of the sample variance of physical returns and option estimate of implied risk-neutral variances are denoted by SV and $\widehat{IV}$, respectively. Model free bias is $MFBias = \widehat{IV} - QV_N$ and estimates of physical bias is $SVBias = SV - QV_P$. In the simulations, VRP parameters are known and all three components can be computed. Only estimates $\widehat{VRP} = (SV - \widehat{IV})$ are available when real data is used. We use sample variances (SV) instead of realised variances (RV) and document superior performance for fine grids below.

Baseline simulation parameters largely correspond to the annualised parameters estimated by Duffie et al. (2000) for S&P 500 returns. They estimated that the Poisson arrival rate was $\hat{\lambda} = 0.11$, the mean jump size was $\hat{\mu}_J = –0.012$ and the jump variance was $\hat{\sigma}_J^2 = 0.015$. Stochastic variance parameters were estimated to be $\hat{\kappa} = 3.99$, $\hat{\theta} = 0.014$, and $\hat{b} = 0.027$. The risk free rate was $r_f = 0.0319$. The extended ranges of λ (up to 3) and VolVol (up to 0.75) are sensitivity checks; $\lambda = 3$ is at the upper end of empirically plausible values and is included only to characterise behaviour in the extreme. To generate risk-neutral realisations, we arbitrarily choose the price of variance risk to be $\lambda(V, t) = -0.50\sqrt{V}/\hat{b}$. The options are European and expire in 30 days. The at-the-money strike and initial stock price is \$100. The initial volatilities for physical and risk neutral realisations, V_0, are the long term means, θ^P and θ^N, respectively.

Figure 1 (a) Risk neutral variances and the number of strikes: no jump scenario ($\lambda = 0$, parameter value = 0.0143) (b) Risk neutral variances and the number of strikes: jump scenario ($\lambda = 0$, parameter value = 0.0327) (see online version for colours)



(a) (b)

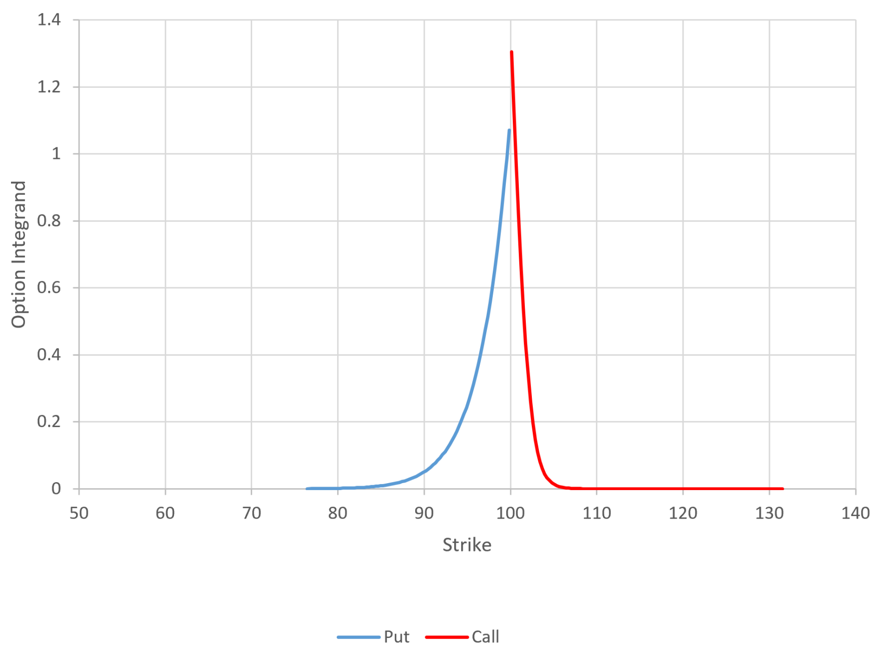
Note: Series shown: VolVol = 0 and VolVol = 0.50.

Results are shown in Tables 1, 2, and 3. Table 1 shows the sensitivity of risk neutral variance estimates to the number of option strikes. Table 2 gives the estimation bias and market risk premium estimates for the baseline simulation while Table 3 shows the sensitivity of physical variance estimates to the fineness of the estimation grid. All tables give model free and Black-Scholes estimates.

1.8.1 Risk neutral variance estimates

The sensitivity of risk neutral variance estimation errors to the number of strikes is shown in Table 1. Strikes are set at fixed spacing and range ± 8 standard deviations from the at-the-money strike. Sensitivity to the number of strikes at a fixed maturity is a limiting factor when option liquidity is small. We evaluate estimates versus theoretical parameters using 6, 8, 10, 12, 18 and 24 total strikes for puts and for calls. Results are given for annual Poisson arrival rates of 0 and 0.50. Model free estimates that have 12 or more strikes are accurate (model free biases are ≤ 0.0009 for $\lambda = 0$ and 0.0016 for $\lambda = 0.50$) with little improvement using 18 or 24 strikes. Graphics are shown in Figures 1(a) and 1(b). Errors approach zero from above when there are no jumps ($\lambda = 0$) while errors approach zero from below when there are jumps ($\lambda = 0.5$). Black-Scholes biases are usually larger by a factor of 10 or more and vary little with the number of strikes since they are averages over strikes.

Figure 2 Baseline model (see online version for colours)



Note: See notes in Table 1.

It may seem curious that accurate model-free estimates can be obtained with only 12 strikes given that the theory is based on a continuous integral over strikes on $[0, \infty)$. The key insight is that the OTM option integrand $C(\tau, K)/K^2$ or $P(\tau, K)/K^2$ is smooth and unimodal, with values approaching zero at strikes of approximately 80 for puts and 110 for calls in our baseline case. This smooth shape implies the midpoint rule converges rapidly, requiring only a small number of strike evaluations. Jiang and Tian (2005) note that truncation error (omitting far-from-the-money strikes) dominates discretisation error, explaining why 12 well-placed strikes achieve high accuracy once the strike range is wide enough. Figure 2 is a graph of out-of-the-money puts and calls for our baseline case with strike at 100. The graph shows the integrand to be smooth with no points of inflection and ordinates approaching zero at strikes of 80 for puts and 110 for calls.

The Black-Scholes average benchmark is included because practitioners sometimes average implied variances from Black-Scholes inversions across available strikes as a simple proxy for risk-neutral variance. Our results quantify the model-misspecification bias this incurs when the true DGP is SVJ. A stronger comparison with VIX-style discretisation or spline-interpolated surfaces is an important avenue for future research.

1.8.2 Bias and VRPs estimates for the baseline case

Table 2 displays results for the baseline case for physical and risk neutral sample variances and includes theoretical integrated variances, model free estimates, Black-Scholes estimates, VRPs, and biases of estimates from theoretical values. To generate realisations, we use daily sampling (Δ = 1/360) over a 30 day horizon. We repeat 60 times, corresponding to 1,800 non-overlapping observations or five years of data. To obtain risk neutral parameter estimates, we generate put and call prices at 18 strikes. The strikes are evenly spaced, with minimum and maximum strikes being approximately eight standard deviations from the at-the-money \$100 strike. In real data applications, the number of strikes can be increased by interpolation and extrapolation (Carr and Wu, 2009).

There are four panels in Table 2, with b, the volatility of volatility (VolVol), ranging from 0 to 0.75. Within each panel, the jump arrival rate ranges from $\lambda = 0$ to $\lambda = 3$. Special cases are $b = 0$ or $\lambda = 0$, corresponding to models with no stochastic volatility or no jumps, respectively. At the other extreme, panel D has $b = 0.75$ and the last row has jump intensity $\lambda = 3$ per year. Averages and sample standard deviations are computed over all panels. Since the price of risk $\lambda(V,t) = -0.50\sqrt{V}/\hat{b}$ is negative, the risk neutral estimates are larger than the physical estimates.

The physical theoretical value, QV_P, is estimated using sample variances SV averaged over 60 iterations. Both physical and risk neutral variances increase with arrival rates. However, the theoretical VRP is negative (–0.002) and constant for all scenarios since physical and risk neutral parameters for the jump process were assumed to be equal.

The biases and VRPs are relatively small when there are no jumps (first row in each panel). Realised variance biases are 0.0002, 0.0012, 0.0001, and –0.0021 in panels A through D, respectively. Similarly, for the no-jump scenario, the risk neutral model free estimates have average bias –0.0018, –0.0018, –0.0017, and –0.0015 in no-jump scenarios. Corresponding Black-Scholes biases are 0.0169, 0.0015, 0.0060, and 0.0101, worse than model free estimates by a factor of 10 in two cases.

Summary statistics over all scenarios show that the average and standard deviation of physical variance biases are –0.0003 and 0.0273. The average and standard deviation of risk neutral biases are –0.0005 and 0.0009. The takeaway is that the reliability (standard deviations) of risk neutral model free estimates are much better than corresponding estimates of physical variance.

1.8.3 Sensitivity of estimates to the grid

We expect that increasing the fineness of the grid will improve bias estimates and therefore the VRP. Broadie and Jain (2008) show that

$$RV \to QV + h(\omega), \qquad h(\omega) = O\left(\frac{1}{n}\right), \tag{20}$$

where ω is a vector of jump parameters. Thus, we expect that estimates will improve as n is increased (sampling interval Δ decreases). In Table 3 we give results when the yearly sampling interval is respectively 10/360, 7/360, 3/360, and 1/360. For a 30-day maturity option, this corresponds to 3, 6 (approx.), 10, and 30 samples per iteration. We average estimates over m = 50,000 iterations. Parameters are the same as in Table 2 except the Heston variance parameter (VolVol) is set to 0.50.

Sample variance biases and standard deviations are reported in Table 3. We confirm the derivations of Broadie and Jain (2008): average biases increase with jump intensity and are positive for models with jumps.[4] But the more notable results are larger bias standard deviations as λ increases (panels A through D) and smaller bias standard deviations as sampling frequency (Δ) increases. The average bias standard deviations are 0.0521, 0.0449, 0.0439, and 0.0427 for sample sizes 3, 6, 30, and 60, respectively. Model free and Black-Scholes risk neutral estimates and biases are not affected by sampling frequency and are thus the same as that reported in panel C of Table 2. VRPs differ, however, because sample variance estimates change.

2 Conclusions

Efficient and accurate Monte Carlo methods were used to simulate the Heston stochastic volatility model with an underlying Merton jump diffusion. We find accurate estimates of model free variance can be obtained with as few as 12 strikes at a common option maturity. Bias estimates are robust to different scenarios and have smaller standard deviations than standard deviations of sample volatility. Sample physical variance estimates can be improved by a finer grid. The reliability of sample variance estimates decline monotonically with increasing jump frequency.

Black-Scholes VRP estimates have larger bias and more volatility than model free methods, as expected, since the simulated data was generated by the Heston model with Merton jumps.

Practical implications and limitations

Our results are obtained in a controlled environment with model-generated option prices, no bid-ask spreads, evenly spaced strikes, and perfectly synchronised data. In real option markets, even when 12 strikes are available, each observed price reflects a bid-ask midpoint that introduces measurement error. Wide bid-ask spreads – common for deep out-of-the-money strikes – can distort the model-free integrand and increase estimation error. Non-synchronous trading introduces an additional noise component. Our findings should therefore be interpreted as a lower bound on the number of strikes required in practice: 12 strikes suffice in an ideal simulation setting, but researchers working with illiquid underlyings may wish to supplement the strike grid through interpolation and extrapolation Carr and Wu (2009) or to assess sensitivity of VRP estimates to alternative grid assumptions.

Declarations

There are no conflicts of interest. No funding was received for this research.

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Notes

1 A unique price of variance risk can be obtained assuming CRRA utility and consumption c that follows $dc/c = \mu_c dt + \sqrt{V} dW_s$. The result using stochastic discount factors and the Girsanov transformation is a unique price of variance risk $\lambda = \gamma\rho\sqrt{V}$, where γ is the coefficient of risk aversion and ρ is the correlation between stock and variance innovations. Results are identical to those in equation (6) if $\alpha^* \sqrt{V}/b \cdot b\sqrt{V} = \gamma\rho V$, i.e., $\alpha^* = \gamma\rho b$. All right-hand-side variables are either preference- or model-specific.

2 More details on this approach are given in Oosterlee and Grzelak (2020).

3 In initial simulations, we obtained identical or near identical results using the methods of either Britten-Jones and Neuberger (2000) or Carr and Wu (2009). BJN use an integral representation and call options while CW use out-of-the-money puts and calls.

4 We estimate integrated variance using sample variance (SV) rather than (RV) since we found that SV produces better estimates for small sample sizes.