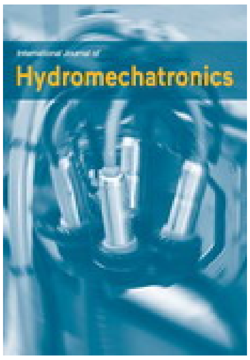

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Abstract: The present work proposes a novel remaining useful life (RUL) prediction method for milling tools by improving inverse Gaussian (IG) stochastic degradation process, to address the insufficient accuracy of existing RUL predicting methods under limited training sample data. In contrast to the standard IG process, which establishes the degradation increment relative to the initial state of degradation, the proposed improved IG process adopts an assumption that the degradation increment is more greatly affected by the current state of degradation rather than the initial state. This yields a three-parameter increment of wear, and the parameters are estimated by maximum likelihood estimation. Finally, the RUL of tool is predicted based on slice sampling technology. Applications of the proposed RUL prediction method to a benchmark milling tool wear dataset and a milling tool wear experiments demonstrate that the method obtains superior RUL prediction performance relative to two other state-of-the-art methods.

Keywords: milling tool; remaining useful life; inverse Gaussian process.

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1 Introduction

Milling tools suffer gradual wear during the milling process owing to the combined effects of high pressure at the tool/workpiece interface, high temperature, and vibrations and mechanical impacts (Mohanraj et al., 2024). The accumulated tool wear eventually passes a threshold value, after which the machining quality declines seriously (Sun et al., 2022; Samin et al., 2023). Accordingly, this threshold wear value represents the point at which the milling tool has reached the end of its useful life, and must be replaced. The substantial impact of tool wear on machining quality necessitates the implementation of a reliable means of estimating the remaining useful life (RUL) of milling tools, which serves as an important component of prognostics and health management (PHM) applications in the cutting process (Kumar, 2025; Zhou et al., 2022a). Conventional RUL methods are based on a processing or experience index, such as cutting time or the number of pieces processed. However, these methods rather crudely predict the RUL of milling tools, and increase production costs by implementing milling tool replacement either too late or too early. Here, replacing the tool too late results in increased production costs owing to the need to scrap finished workpieces due to an unacceptably low machining quality, while replacing the tool too early results in increased production costs owing to tool wastage and increased downtime, which affects the processing efficiency (Wang et al., 2023)? As such, tools are generally replaced well before the end of their RUL because the latter costs are deemed more acceptable. In fact, past studies have demonstrated that only 50%–80% of available tool lifetime is used in actual industrial machining processes (Zhu et al., 2021; Samin et al., 2019). Therefore, increasingly accurate RUL predictions for cutting tools can significantly improve tool utilization and reduce processing costs (Xiao et al., 2025; Hei et al., 2025).

Numerous alternative approaches to conventional RUL methods based on experience indexes have been developed using time-series tool condition monitoring (TCM) data obtained during the machining process from various sensors such as cutting force, vibration, spindle motor current, and acoustic emission (AE) sensors, or data from multiple types of sensors. These data are then employed within a suitable tool wear model for predicting the RUL of milling tools. The increasing development of deep learning in recent years has led to increasing interest in RUL methods based on deep learning models (Yin et al., 2024; Fu et al., 2024; Zhang et al., 2024), such as recurrent neural networks (RNNs) (Guo et al., 2017), long short-term memory (LSTM) networks (Liu et al., 2021; Zheng et al., 2022), and gated recurrent networks (GRNs) (Dong et al., 2019; Kumar et al., 2023). These methods have been demonstrated to be quite effective at capturing the long-term dependencies of tool wear within time series data, and can provide highly accurate RUL prediction for milling tools. However, these methods require extensive training sample data in conjunction with the tedious fine tuning of model parameters (Yan et al., 2023; Zhou et al., 2022b; Hei et al., 2024; Wang et al., 2025). Here, the need for large training sample sets is particularly problematic because machining processes can employ a large array of milling tools composed of different materials with different designs in addition to a vast array of possible workpiece materials. As a result, the extensive training sample required by these highly accurate deep-learning methods are not always available. Accordingly, other RUL prediction methods must be employed when restricted to conditions with limited training sample data.

This issue has been addressed by the application of tool degradation data analysis methods based on stochastic processes, which aim to capture potential tool failure processes through degradation data, and are widely applied RUL prediction methods in many fields (Guo et al., 2018), due to their use of probability as a means of describing the uncertainties inherent in stochastic processes and their limited training data requirements

(Bian and Gebraeel, 2014). Here, stochastic dynamics are the most common feature of practical degradation processes due to the uncertainty in the working environment, random errors in measurements, and the variability of the system. The most commonly employed continuous stochastic degradation processes include the Wiener process (Zhang et al., 2018), the Gamma process (Ling et al., 2019), which has been widely used recently in material degradation or damage modelling such as batch production fault prognosis, damage detection, and the inverse Gaussian (IG) process (Wu et al., 2020; Lu et al., 2019). The Wiener process employs a drift model, where the increment of degradation is assumed to be an independent normal distribution, and the mean and variance of the distribution are functions of time. The advantages of the Wiener process are that multiple observable environmental factors, such as temperature and humidity, can be considered (Wang et al., 2014). However, the Wiener process is only applicable to degradation processes that are not monotonic, which limits its application in monotonically increasing degradation conditions, such as are encountered in tool wear. In contrast, gamma and IG processes are more suitable for monotonic degradation processes because they inherently impose monotonic constraints on the sample paths (Chen et al., 2015). Here, the gamma process employs an independent non-negative random variable as the increment of degradation, which follows a gamma distribution and has a time-dependent scale function. The design of appropriate scale functions enables explanatory variables and random errors to be incorporated into the model. However, the gamma process is not always effective for monotonically increasing degenerate processes like tool wear, particularly when the increment of degradation does not follow a gamma distribution. This issue associated with the gamma process is addressed by the IG process by presenting a clear physical interpretation for degradation as the result of small accumulated damages, and the process applies explicit analytical formulas for the probability density function (PDF) of RUL distributions (Wang and Xu, 2010). This has led to a number of applications of the IG process to degenerate processes. For example, Ye et al. (2014) found that accelerated degradation under constant stress follows an IG process, while Boskoski and Juricic (2016) employed an IG distribution to describe the repetitive vibrational patterns for both single and multiple localized surface bearing faults. However, despite the many advantages of the IG process, very few studies have applied this process to the degradation process of milling tools for the purpose of conducting RUL prediction.

The present work addresses the above-discussed issues by proposing a novel RUL prediction method that estimates milling tool wear according to an improved IG stochastic degradation process that does not require extensive training data. Here, the improved IG process adopts an assumption that the degradation increment is more greatly affected by the current state of degradation rather than the initial state, as is assumed by the standard IG process. This yields a three-parameter increment of wear, and the parameters are estimated by maximum likelihood estimation (MLE). Finally, the RUL is predicted based on slice sampling technology. Applications of the proposed RUL prediction method to a benchmark milling TCM dataset and our milling tool wear experiments conducted specifically for this study demonstrate that the method obtains superior RUL prediction performance relative to two other state-of-the-art methods.

The remainder of this paper is organized as follows. Section 2 introduces the basic working principles of the improved IG processes. Section 3 describes the theoretical framework of the proposed RUL prediction method. The RUL prediction performance of the proposed method is compared with those of two current state-of-the-art methods based on two datasets in Section 4. Finally, conclusions are given in Section 5.

2 Improved IG process modelling

2.1 Principles of improved IG process

A degradation process $\{y(t), t \geq 0\}$ that follows an IG process can be defined in terms a wear variable $y(t)$ varying over time t from an initial state at $t = 0$ according to the increment of wear $\Delta y(t) = y(t) - y(0) \approx IG(\Lambda(t) - \Lambda(0), \lambda(\Lambda(t) - \Lambda(0))^2)$, where λ is a scale parameter and $\Lambda(t)$ is a shape function (Ye et al., 2014). In addition, the following properties must be met:

1 $y(0) = 0$ when $t = 0$

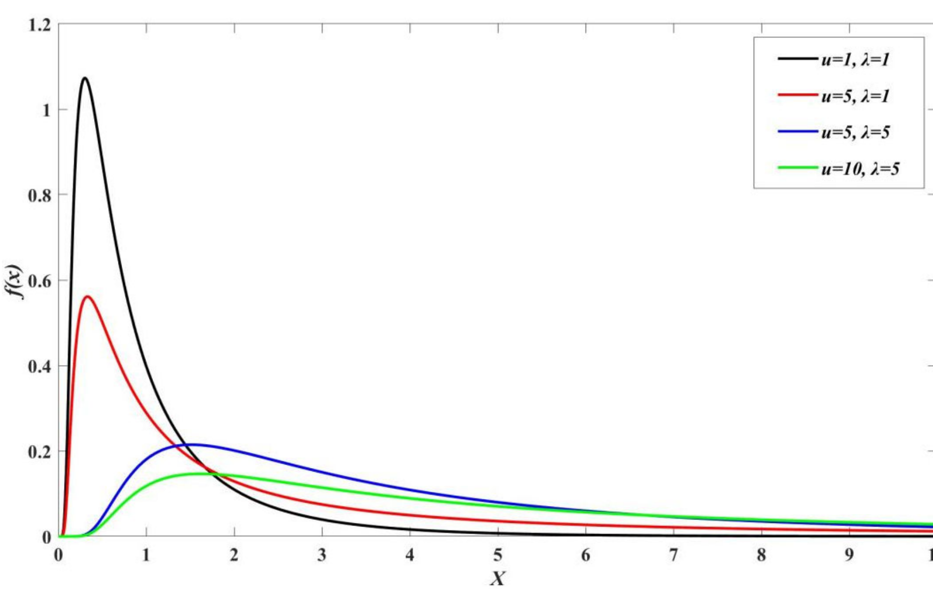
2 $y(t)$ is a monotonically increasing function of t

3 for two disjoint time intervals (t_1, t_2) and (t_3, t_4) (i.e., $[t_1, t_2] \cap [t_3, t_4] = \Phi$), $y(t_2) - y(t_1)$ and $y(t_4) - y(t_3)$ are mutually independent

4 the increment $y(t) - y(0)$ follows an IG distribution and its PDF is

$$f\left(y(t) \middle| \Lambda(t), \lambda\Lambda^2(t)\right) = \sqrt{\frac{\lambda\Lambda^2(t)}{2\pi y(t)^3}} \exp\left\{-\frac{\lambda\left(y(t) - \Lambda(t)\right)^2}{2y(t)\Lambda^2(t)}\right\} \quad (1)$$

Figure 1 PDFs of IG processes with different values for parameters μ and λ (see online version for colours)



The shape function of a tool degradation process can be defined as $\Lambda(t) = \mu t$, as has been proposed for a fatigue degradation process caused by crack growth (Peng et al., 2014).

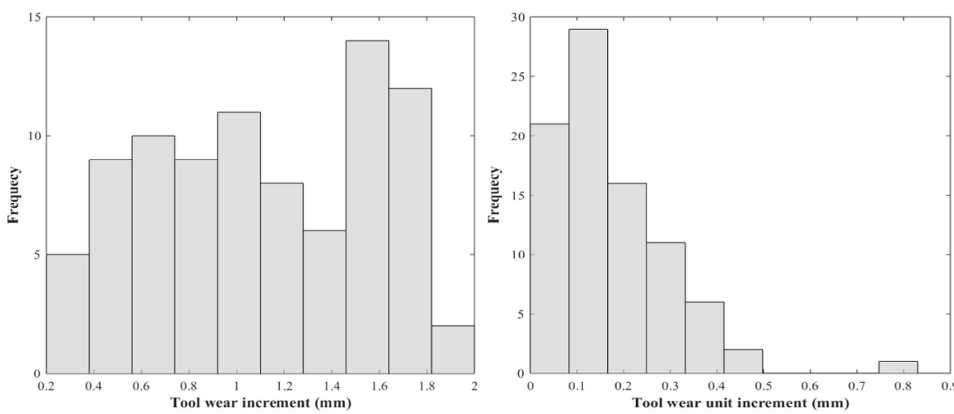
Accordingly, the model parameters of an IG process with $\Lambda(t) = \mu t$ are $\{\mu, \lambda\}$, and the PDFs of IG processes with different parameter values are presented in Figure 1.

However, as shown in Figure 2, our preliminary work involving a milling tool wear experiment dataset demonstrates that the degradation increment $\Delta y(t) = y(t) - y(0)$ at a given t does not follow an IG distribution [shown in Figure 2(a)], while the degradation increment $\Delta y(t) = y(t + 1) - y(t)$ does [shown in Figure 2(b)]. Accordingly, we can conclude that the degradation increment is more greatly affected by the current degradation state rather than the initial degradation state. Therefore, we propose an improved IG process that describes the tool wear degradation process better by assuming that $\Delta y(t)$ follows the IG process in accordance with the wear state $y(t)$. Based on this assumption, we define the shape function as a linear function of the current wear state: $\Lambda(y(t)) = \mu_0 + \mu_1 y(t)$, and the PDF of $\Delta y(t)$ is

$$f\left(\Delta y(t) \middle| \Lambda(y(t)), \lambda\Lambda^2\left(y(t)\right)\right) = \sqrt{\frac{\lambda\Lambda^2\left(y(t)\right)}{2\pi\Delta y(t)^3}} \exp\left\{-\frac{\lambda\left(\Delta y(t) - \Lambda y(t)\right)^2}{2\Delta y(t)\Lambda^2 y(t)}\right\} \tag{2}$$

We also note that the unit of time can be defined according to the cutting time for machining a basic task (e.g., the cutting time for milling a surface or milling a certain shape) or the cutting distance (e.g., the time required for milling a length of 10 cm) to avoid the influence of different feed rates on the RUL prediction results.

Figure 2 Tool wear histograms obtained under two different time intervals for a milling tool wear experiment dataset, (a) $\Delta y(t) = y(t) - y(0)$ (b) $\Delta y(t) = y(t + 1) - y(t)$



(a) (b)

2.2 Parameter estimation of improved IG model

The proposed improved IG process has three parameters $\{\mu_0, \mu_1, \lambda\}$, which can be estimated through MLE according to the following process.

Suppose the wear state of a milling tool is measured T unit times, and the observed wear data are $Y = \{y_1, y_2, \ldots, y_T\}$, and the initial wear state $y_0 = 0$. For convenience of description, the shape function $\Lambda(y(t))$ defined in the preceding subsection is abbreviated here as $\Lambda(y_i)$. Then, the log-likelihood function of the improved IG process is

$$\begin{aligned}\ln L(\mu_0, \mu_1, \lambda) &= \sum_{i=0}^{t-1} \ln f(y_{i+1} - y_i) \\ &= \sum_{i=0}^{t-1} \left[\frac{1}{2} \ln \frac{\lambda(\mu_0 + \mu_1 y_i)^2}{2\pi(\Delta y_i)^3} - \frac{\lambda(\Delta y_i - \mu_0 - \mu_1 y_i)^2}{2\Delta y_i \times (\mu_0 + \mu_1 y_i)^2} \right]\end{aligned} \tag{3}$$

We then calculate the partial derivatives of the three parameters, and set them equal to 0, as follows.

$$\frac{\partial \ln L}{\partial \mu_0} = \sum_{i=0}^{t-1} \left[\frac{(\mu_0 + \mu_1 y_i + \mu_2 c_i)^2 + \lambda(\Delta y_i - \mu_0 - \mu_1 y_i - \mu_2 c_i)}{(\mu_0 + \mu_1 y_i + \mu_2 c_i)^3} \right] = 0 \tag{4}$$

$$\frac{\partial \ln L}{\partial \mu_1} = \sum_{i=0}^{t-1} \left[\frac{y_i(\mu_0 + \mu_1 y_i + \mu_2 c_i)^2 + \lambda y_i(\Delta y_i - \mu_0 - \mu_1 y_i - \mu_2 c_i)}{(\mu_0 + \mu_1 y_i + \mu_2 c_i)^3} \right] = 0 \tag{5}$$

$$\frac{\partial \ln L}{\partial \lambda} = \sum_{i=0}^{t-1} \left[\frac{1}{2\lambda} - \frac{(\Delta y_i - \mu_0 - \mu_1 y_i - \mu_2 c_i)^2}{2\Delta y_i(\mu_0 + \mu_1 y_i + \mu_2 c_i)^2} \right] = 0 \tag{6}$$

Finally, the MLE estimators of the three model parameters $\{\mu_0^*, \mu_1^*, \lambda^*\}$ can be obtained by solving the above equations. Here, we employ the differential evolution (DE) algorithm, in which the objective function is the sum of the absolute values of the expressions on the left sides of the three equations, the scaling factor and cross factor are set to 0.5 and 0.3, respectively, and the upper limit of estimation error is set to 10^{-5}.

3 RUL prediction method for milling tools

The framework of the proposed RUL prediction method based on the improved IG process is illustrated in Figure 3, and includes the model training phase and RUL prediction phase. Here, the parameters of the improved IG model are determined in the model training phase in conjunction with a tool wear degradation process training dataset, while the RUL value and 95% confidence interval for the milling tool are predicted in the RUL prediction phase.

The procedure of the proposed method is given in detail as follows.

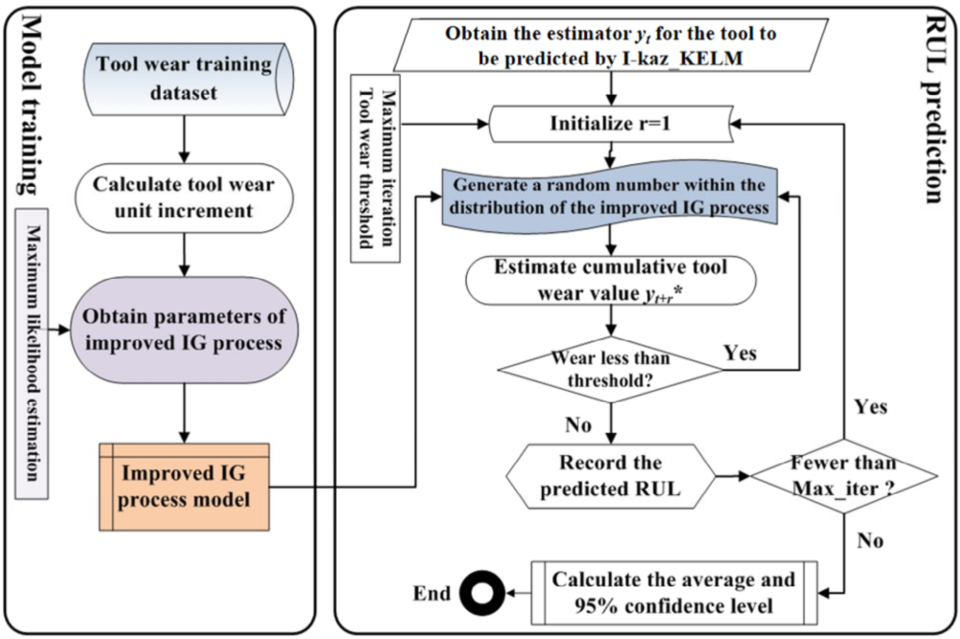
3.1 Model training

Step 1 Collect training samples $\{y_{ij}, i = 1, 2, \ldots, N; j = 1, 2, \ldots, T_i\}$, where y_{ij} denotes the wear value of the i^{th} tool after the j^{th} milling interval, N denotes the number of tools for which data are included within the training samples, and T_i denotes the number of milling intervals for which data are included for the i^{th} tool.

Step 2 Calculate the unit degradation increment $\Delta y_{ij} = y_i(j + 1) - y_{ij}$, $i = 1, 2, \ldots, N$; $j = 1, 2, \ldots, T_i - 1$.

Step 3 Solve equations (4)–(6), and obtain the estimators of the three model parameters $\{\mu_0^*, \mu_1^*, \lambda^*\}$.

Figure 3 Framework of the proposed RUL prediction method based on the improved IG process (see online version for colours)



3.2 RUL prediction

Step 4 Obtain the estimator y_t of the tool being subjected to be predicted RUL prediction online through an TCM regression model [such as the I-kaz_KELM (Gao et al., 2024), in which ten dimensionless statistical features from the time domain and frequency domain are extracted as input parameters for the regression algorithm, and obtain y_t with small error]. Set the value of the maximum number of iterations (*Max_iter*), tool wear threshold (*Max_wear*), and the unit time interval (*unitTime*). Let the estimated shape function be given as $\Lambda^*(y_t) = \mu_0^* + \mu_1^* y_t$, and set the iteration number $iter = 1$.

Step 5 Set the number of unit time increments $r = 1$.

Step 6 Generate a random number Δy_{t+r-1}^* within the distribution of the improved IG process $IG(\Lambda^*(y_{t+r-1}), \lambda^*(\Lambda^*(y_{t+r-1})^2))$ using a random sampling technique. Here, the slice sampling technique (Neal, 2003) is employed in this paper using the slicesample function in MATLAB.

Step 7 Let $y_{t+r}^* = y_{t+r-1} + \Delta y_{t+r-1}^*$.

Step 8 If $y_{t+r}^* < Max\_wear$, set $r = r + 1$ and return to step 6.

Step 9 If $y^*_{t+r} \geq Max\_wear$, set the number of unit time increments at the $iter^{\text{th}}$ prediction of the tool wear state y_t as $RUL^t_{iter} = r$, and then set $iter = iter + 1$.

Step 10 If $iter \leq Max\_iter$, return to step 5.

Step 11 If $iter > Max\_iter$, discontinue iterations. Thus, the final estimator for the RUL of the tool wear state y_t is $RUL^*(t) = RUL^t_{avg} \times unitTime$, RUL^t_{avg} is the average of RUL^t_{iter}, $iter = \{1, 2, \ldots, Max\_iter\}$, and its 95% confidence interval is $(RUL_{avg} - Z_{0.025}, RUL_{avg} + Z_{0.025}) \times unitTime$, where $Z_{0.025}$ denotes the upper 0.025 quantile of the normal distribution followed by the set $\{RUL^t_{iter}\}$.

We note here that the proposed method requires the generation of random numbers according to the IG process in step 6, and that slice sampling technology is employed because it is very simple and effective in the case of single variable distributions, and its sampling efficiency is greater than that of the Gibbs sampling or simple Metropolis-Hasting sampling algorithm (Neal, 2003; Kirschenmann et al., 2018).

4 Experimental investigation

4.1 Description of PHM 2010 TCM benchmark dataset

The benchmark dataset employed for initially testing the proposed RUL prediction method derives from the tool wear prediction problem reported in the 2010 PHM Data Challenge (The Prognostics and Health Management Society, 2010). The dataset was collected from a high-speed computer numerical control (CNC) milling machine under dry milling operations with the operational parameters listed in Table 1. The sensors included a Kistler 3-component dynamometer, three Kistler piezoelectric accelerometers, and a Kistler AE sensor. The sensor output signals were collected by means of a data-acquisition system (National Instruments PCI-1200). The corresponding flank wear of each flute of the milling tool was measured offline using a LEICA MZ12 optical microscope after finishing each surface. The dataset is composed of three cutter records, denoted as C1, C4, and C6, where each record contained 315 data samples.

Table 1 Operational parameters employed in the PHM 2010 milling experiments

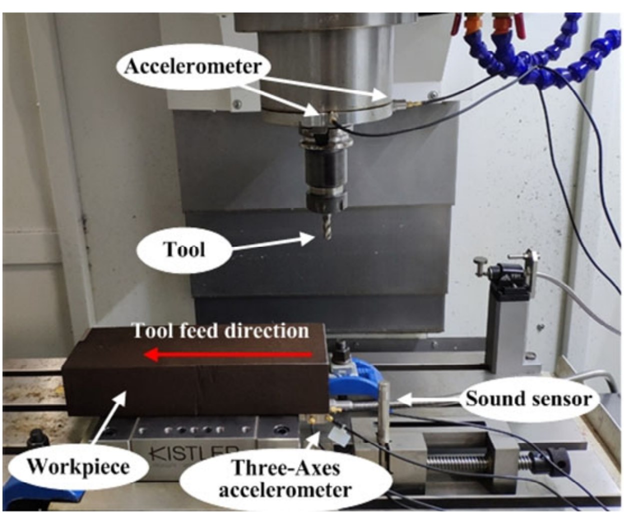
Operational parameter	*Value*	*Operational parameter*	*Value*
CNC machine	Roders Tech RFM 760	Spindle speed	10,400 RPM
Workpiece material	Inconel 718 (jet engines)	Feed rate	1555 mm/min
Cutter	Three-flute ball nose milling cutter	Y depth of cut (radial)	0.125 mm
Number of sensors	5	Z depth of cut (axial)	0.2 mm
Number of sensor channels	7	Data sampling rate	50 kHz/channel

4.2 Milling TCM experimental setup

The experimental setup employed for conducting the TCM of milling tools and RUL prediction under various operating conditions is illustrated in Figure 4. The setup employed a DMTG VDL850A vertical machining center and an uncoated three-flute tungsten steel end milling cutter (Φ 10 mm). The material of the workpiece was #45 steel with dimensions of 300 mm × 100 mm × 80 mm. Sensors were applied to measure vibration, current, and sound emitted during the machining process, where a three-axis piezoelectric accelerometer and two piezoelectric accelerometers were mounted to measure the vibrations of the workpiece and the spindle, respectively; Three current sensors were clamped on the spindle motor wires to measure the three-phase current of the motor. And a microphone was fixed near the workpiece to measure the generated sound. These signals were collected by means of a data-acquisition system with a sampling rate of 12 kHz. The length of the flank wear of the secondary cutting edge on the three flutes of the milling cutter was employed as the tool wear criterion in the experiments, and the wear length value at a given time was defined as the maximum wear length of the three flutes, which were measured offline using an optical microscope each time after milling a complete surface.

The fourteen operational conditions listed in Table 2 were adopted with random combinations of three operational parameters: spindle speed ranging from 2,300 to 2,500 RPM, depth of cut ranging from 0.4 to 0.6 mm, and feed rate ranging from 400 to 500 mm/min. Each case began with a new tool, and complete surfaces were milled until a given tool attained a limiting wear length of at least 1.7 mm. As a result, most of the tools completed 10 milling passes, while the tool in case 7 completed 11 passes and the tool in case 14 completed seven passes. An example of the tool wear observed at different numbers of milling passes is presented in Figure 5 for the tool subjected to milling condition 5.

Figure 4 Experimental setup employed for milling TCM experiments (see online version for colours)



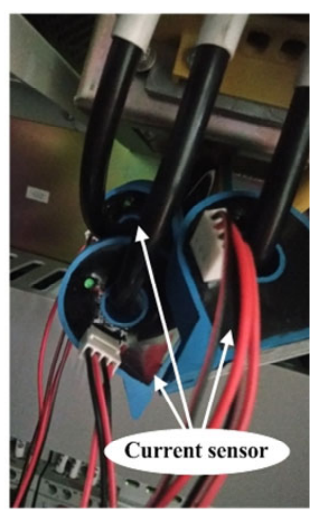
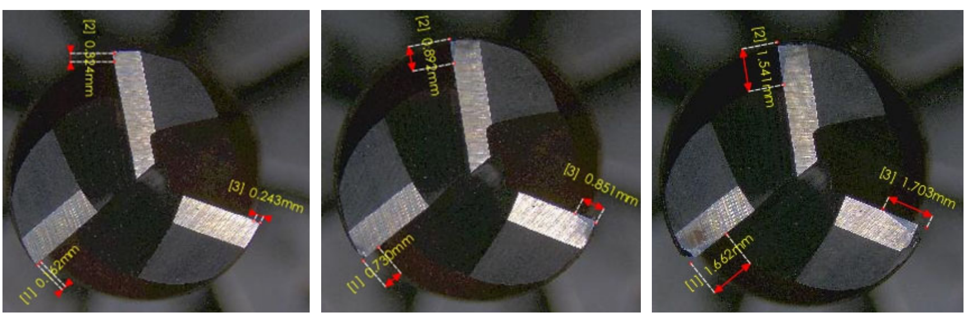


Table 2 Cutting parameters in the milling TCM experiments

Case	*Spindle speed (RPM)*	*Depth of cut (mm)*	*Feed rate (mm/min)*
1	2,300	0.4	400
2	2,300	0.5	450
3	2,300	0.6	500
4	2,400	0.4	450
5	2,400	0.5	500
6	2,400	0.6	400
7	2,500	0.4	500
8	2,500	0.5	400
9	2,500	0.6	450
10	2,300	0.4	500
11	2,300	0.6	400
12	2,500	0.6	500
13	2,500	0.4	400
14	2,500	0.6	400

Figure 5 Samples of the flank wear of the secondary cutting edges (milling condition 5), (a) first pass (b) fifth pass (c) tenth pass (see online version for colours)



(a) (b) (c)

4.3 Description of compared methods

Two current state-of-the-art methods are employed to compare the RUL prediction performance of the proposed method, including the standard IG process discussed in Section 2 and the power regression proposed by Benkedjouh et al. (2013). As discussed, the standard IG process employs the shape function $\Lambda(t) = \mu_t$, and the two model parameters $\{\mu, \lambda\}$ are estimated using MLE with the DE algorithm. The power regression method establishes the following power regression function of tool wear with respect to cutting time: $y(t) = a_0 \times t^{a_1} + a_2$, where a_0, a_1, and a_2 are parameters estimated using MLE. Correspondingly, this yields the cutting time as a function of the tool wear: $t = (a_0^{-1}[y(t) - a_2])^{(1/a_1)}$, and the RUL can be calculated as $T_f - t$, where T_f is the cutting time threshold corresponding to the tool wear threshold.

The prediction performance of these methods were evaluated based on the mean absolute error (MAE), mean absolute percentage error (MAPE), root mean square error (RMSE), the average length of the 95% confidence interval (ALCI), and RUL prediction accuracy (RUL_PA). The performance index RUL_PA is defined as follows (The Prognostics and Health Management Society, 2010):

$$RUL\_PA = \frac{1}{N}\sum_{i=1}^{N}\exp\left\{-\frac{|RUL(i) - RUL^*(i)|}{RUL(i)}\right\} \tag{7}$$

where $RUL(i)$ and $RUL^*(i)$ respectively denote the actual and estimated RUL values of the milling tool and N denotes the number of testing samples. Here, the prediction accuracy increases as the MAE, MAPE, RMSE, and ALCI approach 0, while RUL_PA has a value in the range (0, 1], and the prediction accuracy increases as its value approaches 1.

5 Results and analysis

5.1 Analysis with the PHM 2010 TCM benchmark dataset

Figure 6 presents the histogram of tool wear unit increments obtained from the three cutter records in the database and the corresponding PDF of the IG process with parameters μ = 0.5045 and λ = 1.8582 established according to the mean value and standard deviation of the histogram data. It can be seen that the PDF of the IG process is quite consistent with the histogram of tool wear unit increments. These results demonstrate that the tool wear unit increment $\Delta y(t)$ follows the IG process with respect to the wear state $y(t)$ reasonably well.

For simplicity, records C1 and C4 were selected as the training sample for use in the model training stage of the proposed method, and C6 was selected as the testing sample in the RUL prediction stage. The obtained estimators were $\mu_0^* = -104.65$, $\mu_1^* = 377.28$, and $\lambda^* = 0.1431$. The value of Max_iter was set to 100, Max_wear was set to 170×10^{-3} mm because this was approximately the maximum tool wear observed in record C1, and the value of unitTime was set to the cutting time required for milling a surface. The predicted RUL obtained by the proposed method for the C6 testing set along with the lower and upper 95% confidence interval bounds and the actual RUL are presented in Figure 7 with respect to tool wear. It can be found that the RUL prediction results based on the proposed method are in good agreement with the actual RUL values, where the actual values reside within the 95% confidence interval for 65.4% of the available RUL data, while the remaining 34.6% of the available RUL data resides within close proximity to the established bounds.

For the performance comparison, the estimators of the two parameters employed in the standard IG process obtained for the PHM 2010 dataset were $\mu^* = 143.47$ and $\lambda^* = 100.37$. Meanwhile, the estimators of the three parameters employed in the power regression method for the PHM 2010 dataset were $a_0 = 0.00015$, $a_1 = 2.4467$, and $a_2 = 94.2919$. The predicted tool wear values of the three methods obtained with the C6 testing set along with the lower and upper 95% confidence interval bounds and the actual RUL values are presented in Figure 8, and the prediction performance index values of three methods are listed in Table 3. While the ALCI value obtained by the proposed method employing the improved IG process is twice that obtained by the method based on the standard IG process, the improved IG process greatly increases the RUL prediction performance with respect to the other four indexes. For example, the RMSE obtained by the proposed method is roughly 20% of that obtained by the method based on the standard IG process, while the RUL_PA value is 20% greater. In addition, the proposed method outperformed the power regression method for all indexes, particularly the ALCI, which is only 23% of that of the power regression method. It can be seen that the proposed RUL prediction method based on the improved IG process achieves an overall higher prediction accuracy than that of the other two algorithms for the PHM 2010 TCM dataset.

Figure 6 Histogram of tool wear unit increments along with the corresponding PDF of the IG process (PHM 2010 dataset) (see online version for colours)

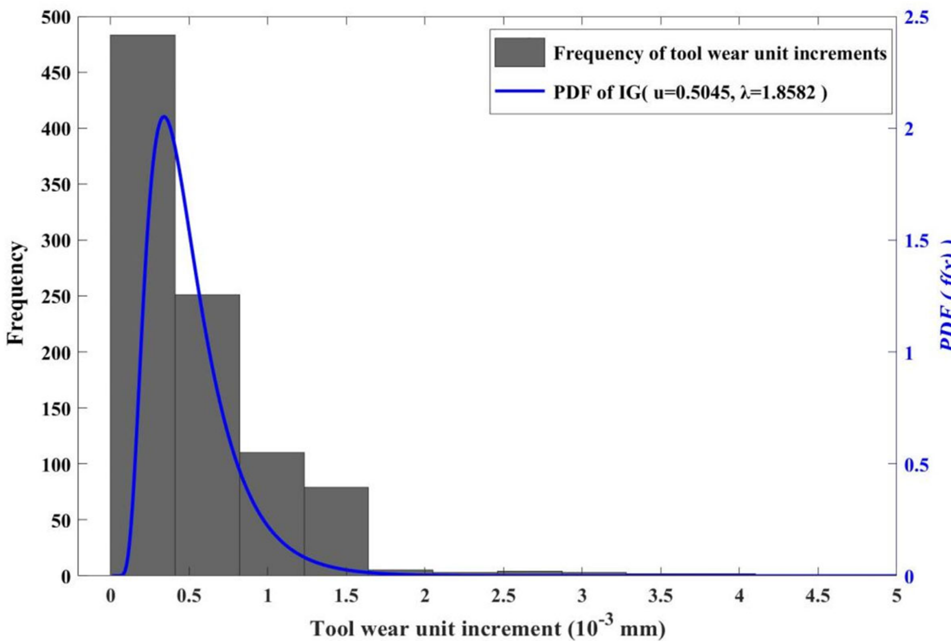


Table 3 Comparison of RUL prediction performances (PHM 2010 TCM dataset)

Method	*MAE*	*MAPE*	*RMSE*	*ALCI*	*RUL_PA*
Standard IG process	51.02	0.5157	64.29	6.75	0.6755
Power regression	12.06	0.0993	13.13	58.11	0.8036
Proposed method	10.53	0.0921	12.78	13.44	0.8730

Figure 7 RUL prediction results of the proposed method with the PHM 2010 dataset (see online version for colours)

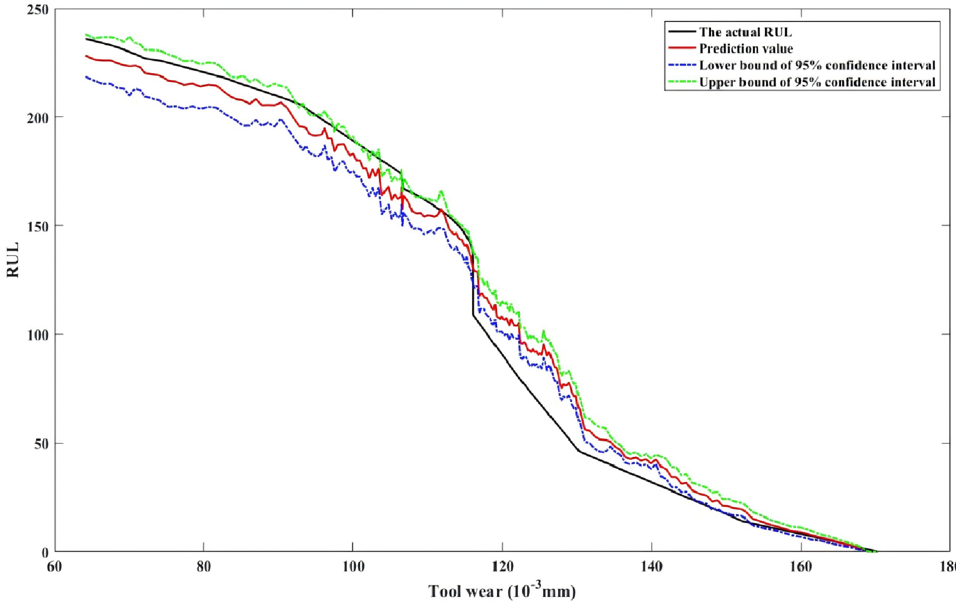
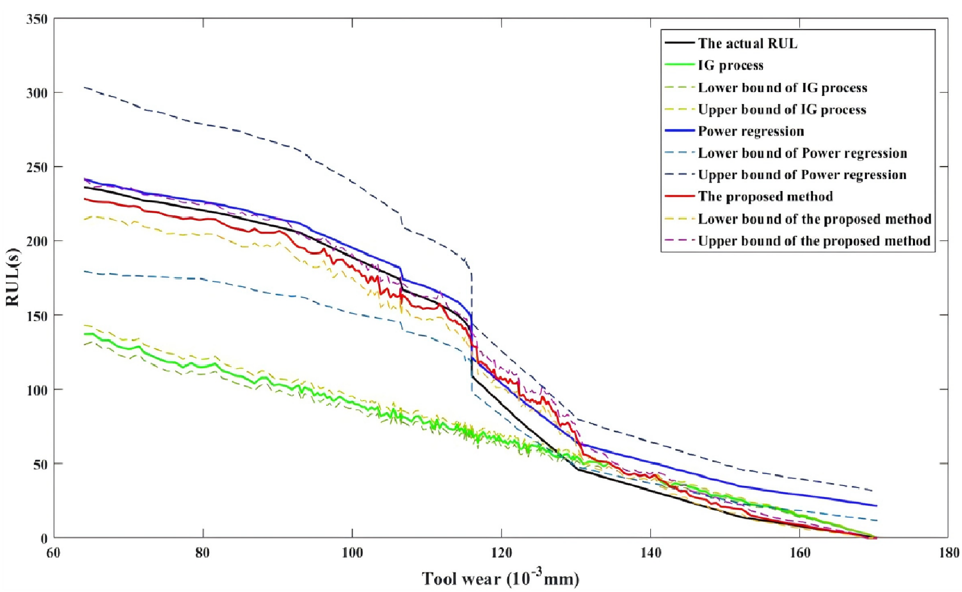


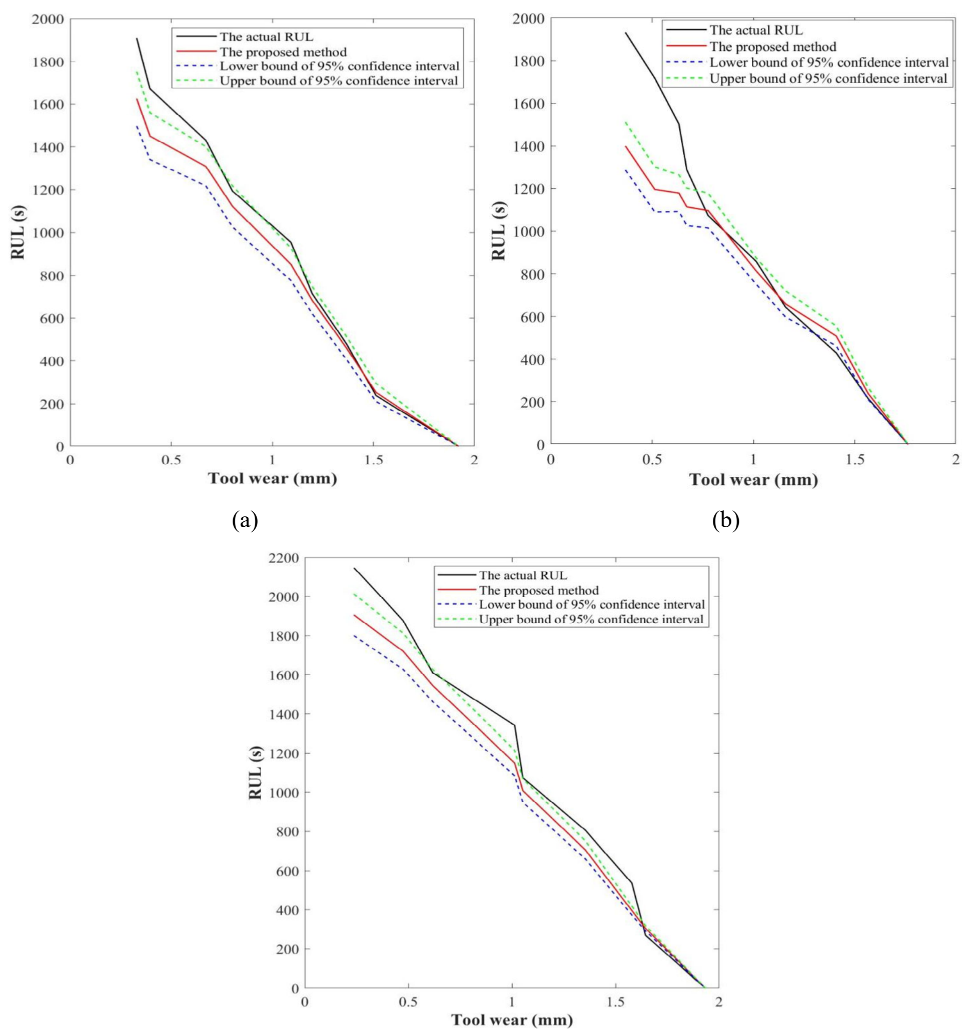
Figure 8 RUL prediction results obtained by three prediction methods (PHM 2010 TCM dataset) (see online version for colours)



5.2 Analysis with the milling TCM experiment

The testing samples used in the RUL prediction stage of the proposed method were collected by randomly selecting one tool from each of the cases employing one of the three feed rates (i.e., 400, 450, and 500 mm/min), and the remaining data were employed as the training samples for use in the model training stage. Accordingly, the test samples included tool 9, tool 12, and tool 13, for which RUL prediction results are presented in the following subsections based a uniform number of 10 milling passes for each milling condition. The obtained estimators were $\mu_0^* = 254.69$, $\mu_1^* = 84.157$, and $\lambda^* = 0.0905$. As before, the parameter values employed were *Max_iter* = 100, *Max_wear* = 1.70 mm, and *unitTime* was set to the cutting time required for a complete milling pass.

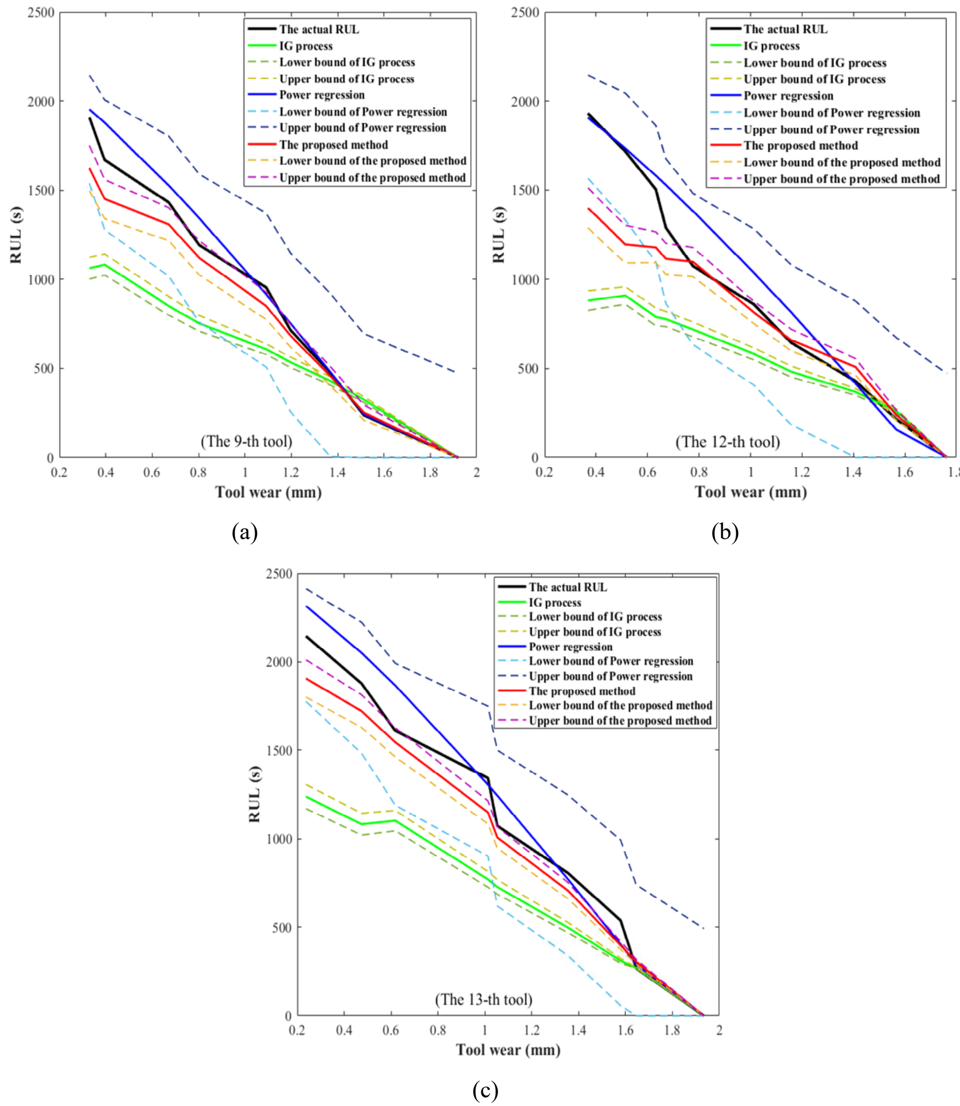
Figure 9 RUL prediction results of the proposed method for the milling TCM experiments, (a) tool 9 (b) tool 12 (c) tool 13 (see online version for colours)



(a) (b) (c)

The RUL prediction results obtained by the proposed method along with the lower and upper 95% confidence interval bounds and actual RUL values for tools 9, 12, and 13 are presented in Figures 9(a), 9(b) and 9(c), respectively. Again, we note that the predicted RUL values are in good agreement with the actual RUL values, where most of the 95% confidence bounds of the predicted RUL values are close to the actual values, except for the 1st and 2nd passes of tools 9 and 12, and the 1st and 4th passes of tool 13. Overall, actual RUL values are found within the 95% confidence prediction intervals for 55.7% of the data.

Figure 10 RUL prediction results of three prediction methods for the milling TCM experiments, (a) tool 9 (b) tool 12 (c) tool 13 (see online version for colours)



For the performance comparison, the estimators of the two parameters employed in the standard IG process obtained for the experimental dataset were $\mu^* = 24.534$ and $\lambda^* = 0.8310$. Meanwhile, the estimators of the three parameters employed in the power regression method for the experimental dataset were $a_0 = 0.0028$, $a_1 = 0.0821$, and $a_2 = 0.1148$. The predicted tool wear values of the three methods along with the lower and upper 95% confidence interval bounds and actual RUL values obtained with the experimental testing set are presented in Figure 10, and the prediction performance index values of the three methods are listed in Table 4. We again note that the ALCI value obtained by the proposed method employing the improved IG process is nearly twice that obtained by the method based on the standard IG process. Nonetheless, the improved IG process greatly increases the RUL prediction performance with respect to the other four indexes. Moreover, the proposed RUL prediction method achieves MAE, MAPE, ALCI, and RUL_PA values that are superior to those of the power regression method, particularly the ALCI, which is only 17% of that of the power regression method. The main reason that the power regression method achieves smaller RMSE values than the proposed method is that the RUL values predicted by the proposed method deviate from the actual values at the first two passes for each tool, resulting in large absolute deviations.

Table 4 Comparison of RUL prediction performances for the milling TCM experiments

Method	*MAE*	*MAPE*	*RMSE*	*ALCI*	*RUL_PA*
Standard IG process	384.22	0.3403	493.56	68.99	0.7171
Power regression	125.43	0.1058	164.84	760.36	0.9007
Proposed method	120.17	0.1035	186.88	130.02	0.9041

6 Conclusions

The present work addressed the insufficient accuracy of existing methods developed for predicting the RUL of milling tools under limited training sample data by proposing an RUL prediction method that estimates tool wear according to an improved IG stochastic degradation process in the absence of extensive training data. Here, preliminary results demonstrated that the degradation increment employed in the IG process is more greatly affected by the current state of degradation rather than the initial state, as is assumed by the standard IG process. The three parameters of the resulting degradation increment were estimated by MLE, and the RUL was predicted based on slice sampling technology. Applications of the proposed RUL prediction method to a benchmark TCM dataset and a dataset derived from milling tool wear experiments conducted specifically for this study demonstrated that the method obtains superior RUL prediction performance relative to existing RUL prediction methods based on the standard IG process and power regression in terms of the MAE, MAPE, and RUL_PA performance indexes. The ALCI obtained by the proposed method is substantially reduced relative to that of the power regression method.

However, the hypothesis of the tool degradation increment from ‘relative initial state’ to ‘relative current state’ in the proposed method is only supported at the level of actual measured data. Providing a clear physical explanation is still important for understanding the evolution process of tool wear. The future work will attempt to provide a physical explanation for the evolution process of tool wear by combining simulation analysis and experimental observation.

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Declarations

All authors declare that they have no conflicts of interest.

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Abbreviations

AE	Acoustic emission
ALCI	Average length of the 95% confidence interval
CNC	Computer numerical control
DE	Differential evolution
GRNs	Gated recurrent networks
IG	Inverse Gaussian
LSTM	Long short-term memory networks
MAE	Mean absolute error
MAPE	Mean absolute percentage error
MLE	Maximum likelihood estimation
PDF	Probability density function
PHM	Prognostics and health management
RMSE	Root mean square error
RNNs	Recurrent neural networks
RUL	Remaining useful life
RUL_PA	RUL prediction accuracy
TCM	Tool condition monitoring