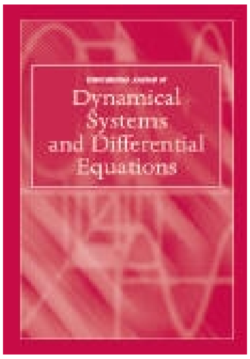

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Nonlinear programming differential equation method for architectural landscape spatial structure engineering in dynamics systems

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Abstract: In response to the problems of excessive model simplification and difficulty in parameter adjustment in current architectural landscape spatial structure planning, nonlinear programming differential equations (NPDEs) were applied to improve computational efficiency. Firstly, key features of architectural landscape spatial structure were defined, and nonlinear partial differential equations were used to simulate the evolution of spatial form over time. Secondly, architectural landscape observation data can be collected, and regression analysis can be used to identify key parameters of differential equations. Afterwards, the simulated annealing (SA) algorithm can be used to find the optimal parameter values. Then, the finite element method (FEM) can be applied to solve nonlinear differential equations. Finally, the paper presented the architectural landscape spatial structure under multiple feasible solutions (Pareto optimal solution set) and compared the key indicators of the paper's method with those of traditional models.

Keywords: architectural landscape; spatial structure engineering; nonlinear programming differential equations; regression analysis; simulated annealing.

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1 Introduction

In modern urban planning and architectural design, with the acceleration of urbanisation and the increase in population density, the planning and design of architectural landscape spatial structures are increasingly valued. Architectural landscape not only needs to meet the basic needs of visual aesthetics and functional practicality but also plays a key role in cultural inheritance and promoting social interaction (Moneta, 2020; Huang, 2024). When planning and designing, comprehensive consideration should be given to its impact on environmental sustainability (Poon, 2021; Philokyprou and Michael, 2021) and economic benefits (King et al., 2022; Cabanek et al., 2020). Facing increasingly complex urban environments and diverse needs, current planning methods often fail to efficiently solve practical problems due to overly simplified models or inflexible parameter adjustments, which limit the innovation and practicality of architectural landscape design. Developing a new type of method has become an urgent task at present.

This article uses partial differential equations to describe the spatial form, functional zoning, and dynamic changes in pedestrian flow in architectural landscape spatial structure engineering. By collecting relevant data on the architectural landscape and conducting regression analysis to identify key parameters, a nonlinear regression model is constructed. To adapt to different building scenarios, the SA algorithm can be used for parameter optimisation. For nonlinear partial differential equations, FEM can be used for numerical solutions, and the stability of numerical solutions can be obtained by iteratively solving algebraic equations. In evaluating results, the Pareto optimal solution set of architectural landscape spatial structure under multiple feasible solutions was presented, and model performance was compared. The results showed that the NPDE method has advantages under various indicators. The experimental results show that in different types of building planning problems, the average adjustment time of the NPDEs method is significantly shorter than that of traditional methods. It maintains high stability and prediction accuracy under different data volumes and spatial node numbers, demonstrating its efficiency and accuracy in architectural landscape spatial planning. The current methods of architectural landscape planning suffer from problems of excessive model simplification and difficulty in parameter adjustment, which restrict their innovation and practicality. In addressing this challenge, this paper innovates in two main aspects: firstly, by simplifying the model to reduce computational complexity, accelerate computation speed, and enhance practicality. Secondly, by employing the SA algorithm to search for the optimal solution in a large parameter space, thus avoiding getting trapped in local optima and improving computational efficiency. Simultaneously, integrating these methods with nonlinear programming partial differential equations enables a more accurate description of the spatial form and dynamic changes of architectural landscapes, further enhancing computational efficiency and model reliability. The comprehensive application of these methods can effectively address the current issues in architectural landscape planning. The main contributions of this paper are as follows: introducing the NPDE method to address the challenges of oversimplified models and difficult parameter adjustments in current architectural landscape spatial structure planning, significantly improving computational efficiency and model applicability.

Using partial differential equations to simulate the spatial evolution of architectural landscapes over time, combined with landscape observation data for regression analysis, effectively identifying key parameters to better meet practical needs.

Optimising parameters using simulated annealing algorithm and solving nonlinear differential equations with finite element methods, enabling efficient and precise description of complex architectural scenarios.

The introduced methods have significantly reduced average adjustment times across various types of architectural planning problems, while maintaining high stability and prediction accuracy, greatly enhancing computational efficiency and user satisfaction in architectural landscape spatial structure engineering.

2 Related work

Currently, many scholars have conducted research and attempts on the spatial structure planning of architectural landscapes. Some scholars have attempted to apply optimisation models to improve planning efficiency (Li and Fan, 2022; Liu et al., 2021). Li et al. (2021) validated the impact of topological depth (path coefficient: 0.98) on landscape efficiency using structural equation modelling (SEM). Guan et al. (2022) constructed an ecological security pattern optimisation model for the main urban area of Chongqing using the granularity inversion method, minimum cumulative resistance model, and spatial network analysis method. Wang (2024) pointed out the role of computer-aided design (CAD) models in landscape architecture spatial structure planning, reducing the design cycle by about 20%. Other scholars choose to use algorithms for design parameter optimisation (Deng et al., 2022; Wang et al., 2020). In the context of sustained urban densification, van Ameijde et al. (2022) proposed using evolutionary algorithms to achieve multi-objective optimisation of architectural landscape spatial structure. Taking parking space planning as an example, Zaki et al. (2023) proposed using models such as AlexNet and ResNet-50 for spatial structure planning optimisation. To achieve a precise classification of architectural landscape spatial structure, Alymani et al. (2023) designed the deep graph convolutional neural network (DGCNN) and optimised its parameters.

Although existing methods have made some progress, the common problem is the lack of practicality and reliability of the model, which fails to comprehensively solve the complex dynamic problems in architectural landscape planning.

In recent years, research in the field of architectural landscape spatial structure engineering has gradually introduced interdisciplinary innovative approaches. Peng (2021) explored landscape element optimisation strategies based on artificial intelligence and complex network theory, analysing through simulations the environmental impacts of different element combinations. On the other hand, Goel et al. (2023) proposed a spatial structure design model based on convolutional neural networks using deep learning algorithms, capable of rapidly responding to market demand changes and performing real-time optimisations. Lv et al. (2022) developed a comprehensive assessment model driven by big data, integrating data mining techniques to forecast long-term development trends in urban landscape design. Additionally, Vargas-Hernandez and Zdunek-Wielgołaska (2021) combined complex system theory with urban ecology to study the potential of dynamic adjustment strategies for urban green space systems to improve air quality. In the context of digital innovation, Fukuda et al. (2021) introduced an architectural landscape simulation platform based on virtual reality technology, effectively shortening design validation cycles and enhancing decision-making efficiency. Simultaneously, Xiang et al. (2022) used multi-agent system modelling methods to investigate the influence mechanisms of different social and cultural factors on landscape planning decisions. Current research demonstrates the potential of interdisciplinary approaches in optimising architectural landscape spatial structure engineering (Zhang et al., 2021b; Li and Fan, 2022), providing new perspectives and solutions for future planning and design.

NPDEs are a class of equations involving unknown functions and their derivatives. Its equations are in a nonlinear state in form and are used to describe the dynamic changes of various complex systems (Quaranta et al., 2020; Zhang et al., 2021a). Existing research has attempted to apply the solution for NPDEs in multiple fields (Liu et al., 2020; Lu et al., 2020). Zhang et al. (2021c) established a mixed integer nonlinear program (MINLP) model to address the issue of long logistics delivery times in the supply chain. To improve the economic feasibility of chemical organic solvent disposal, Chea et al. (2020) formulated it as a mixed integer nonlinear programming optimisation problem. For biology, Zheng et al. (2022) proposed a mosquito population replacement model consisting of two differential equations. Luo et al. (2022) studied the non-consumptive and consumptive effects of heterochromatic ladybirds on host aphids based on NPDEs. In the field of engineering, Sabir et al. (2021) designed a novel stochastic computing framework called fractional Meyer wavelet artificial neural network (FMW-ANN) for the nonlinear singular fractional Lane-Emden (NS-FLE) differential equation. Bukhari et al. (2020) proposed an integrated bimodal computational paradigm based on the nonlinear autoregressive radial basis functions (NAR-RBF) neural network model. In the planning of architectural landscape spatial structure, considering various factors and constraints of architectural design, methods can simulate and optimise the changes and development of spatial structure (Hu et al., 2021; Zhou, 2023). By applying NPDEs, designers can accurately predict and adjust the layout of building elements to meet environmental, social, and economic needs (Wu et al., 2023; Qin et al., 2021). It ensures the sustainability and functionality of building projects while improving planning efficiency and quality.

3 Establishment of differential equation models

To construct the simulation of the spatial form evolution over time for nonlinear partial differential equations, the first step is to identify the fundamental elements of the model. Based on the actual conditions within the space, including the geometric configurations of architectural landscapes, the functional zoning of different spatial uses, and the movement paths of people within the space, a mathematical model for simulating the spatial form changes is established. In the process of model construction, partial differential equations are utilised to represent the patterns of these changes and evolutions. This paper constructs corresponding partial differential equation models based on the rate of change in spatial form, the influence of external factors on space, the dynamical functions of functional zoning, and the movement patterns of people. During the model construction process, the interactions and influences between different factors are considered to ensure the integrity and accuracy of the model. Through formalisation and parameter setting, the process of spatial form evolution over time is simulated more accurately.

The function $f(x, y, z, t)$ can be used to represent the morphological changes of any point in space at time t, where x, y, and z represent spatial coordinates. The spatial morphological changes can be expressed using partial differential equations:

$$\frac{\partial f}{\partial t} = \nabla \cdot (Df) + S(x, y, z, t) \tag{1}$$

Among them, ∇ represents the gradient operator; D is the diffusion coefficient, representing the rate of spatial morphology change; $S(x, y, z, t)$ is the source term, representing the influence of external factors on spatial morphology. Let $g(x, y, z, t)$ represent the functional zoning type of (x, y, z) at time t, and describe the evolution of functional zoning using partial differential equations in the following form:

$$\frac{\partial g}{\partial t} = \Phi(g, \nabla g, x, y, z, t) \tag{2}$$

Here, Φ represents the dynamic function of the functional zoning change, which depends on the current state g, state space change ∇g, and location (x, y, z). Let $h(x, y, z, t)$ represent the pedestrian flow density at point (x, y, z) at time t, and the dynamic changes in pedestrian flow are described by the following continuity equation:

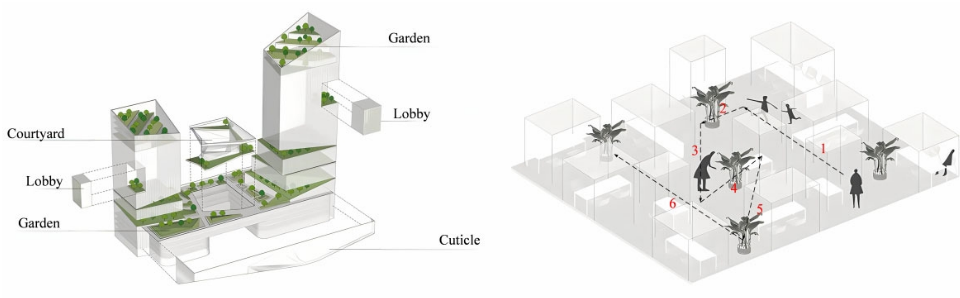
$$\frac{\partial h}{\partial t} + \nabla \cdot (vh) = 0 \tag{3}$$

Here, v represents the velocity field of human flow, which depends on spatial form and functional zoning.

As shown in Figure 1, the key features of the architectural landscape spatial structure can be defined, where the spatial form involves the geometric configuration of the building, and the functional zoning focuses on the distribution of different spatial uses. The movement path of human flow in space can be described through the flow line, and the features can be quantified as mathematical variables.

In the establishment of differential equation models, it is assumed that the variation in spatial form is influenced by the diffusion coefficient, which represents the rate of spatial form change and the influence of external factors. In the description of functional zoning evolution, it is assumed that the dynamic changes in functional zoning are determined by dynamical functions dependent on the current state, state space changes, and location. The change in pedestrian flow density is assumed to be influenced by the pedestrian flow velocity field, which depends on spatial form and functional zoning. Under these assumptions, corresponding partial differential equation models are constructed to describe the morphological, functional, and dynamic changes in architectural landscape spaces.

Figure 1 Architectural landscape space form, functional planning, and human flow (see online version for colours)



Compared with traditional models, the nonlinear programming differential equation method has some obvious advantages and disadvantages. The nonlinear programming differential equation method can better describe nonlinear behaviours in complex systems, such as material nonlinearity and deformation nonlinearity in building structures. By employing finite element methods, the continuous domain can be discretised into a finite number of elements, thus more accurately capturing the behaviour of the system. Efficient algorithms like the Newton-Raphson method can be used to solve nonlinear algebraic equation systems, which can improve the speed and stability of solving, making the model more feasible in practical applications. However, the nonlinear programming differential equation method also faces some challenges and limitations. A large amount of computational resources and time are required during the solving process, especially when dealing with large-scale systems, resulting in high computational complexity. The selection of model parameters and the setting of initial conditions have a significant impact on the results. For certain specific nonlinear problems, there are difficulties in convergence and issues related to multiple solutions, which require further research and improvement.

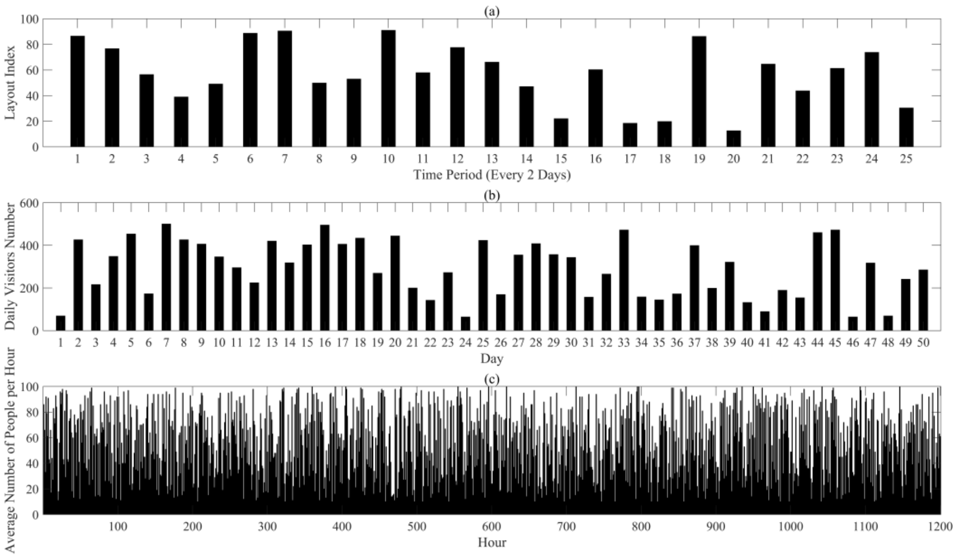
4 Parameter identification and adjustment

The regression analysis method is employed to identify and adjust key parameters in architectural landscape planning. In this study, critical parameters are identified from collected architectural landscape observational data through regression analysis, which directly impacts the spatial structural changes of architectural landscapes, influencing overall landscape perception and functionality. To adapt to different architectural scenarios and personalised needs, model parameters are adjusted. In this research, the simulated annealing algorithm is utilised for parameter optimisation, searching for the global optimal solution in the parameter space.

To collect observational data related to architectural landscapes, various approaches have been employed to ensure comprehensiveness and accuracy. Cameras are utilised for 24-hour continuous monitoring of architectural landscape areas, capturing real-time pedestrian flow at every moment. Sensor devices, including infrared sensors and pressure sensors, are installed to detect the specific locations and activity trajectories of individuals, thereby obtaining more detailed information on usage frequency and spatial layout. Feedback from visitors is collected through mobile applications and online survey forms to understand their perceptions and evaluations of the architectural landscape. All collected data undergo rigorous quality control, including data cleaning, outlier handling, and statistical analysis, to ensure the reliability and representativeness of the data.

This article collects observation data related to architectural landscapes, including key information such as spatial layout, usage frequency, and pedestrian flow, as shown in Figure 2.

Figure 2 Architectural landscape observation data, (a) spatial layout (b) usage frequency (c) pedestrian flow



Figures 2(a), 2(b) and 2(c) together show the architectural landscape observation data collected within 50 days. Figure 2(a) shows the number of days (every 2 days) on the horizontal axis and the layout indicators (normalised by multidimensional indicators) on the vertical axis. Figure 2(b) shows the number of days (50 days) on the horizontal axis and the number of daily visitors on the vertical axis. Figure 2(c) shows the hours on the horizontal axis (50 days for a total of 1,200 hours), and the average number of people per hour on the vertical axis. The layout indicators here mainly include multi-dimensional elements such as building height, shape, size, outline, and landscape line direction, materials and colours used, natural and artificial light utilisation, spatial arrangement and organisation, visual/functional focus, texture, and pattern.

In this study, the simulated annealing algorithm was employed to optimise parameters to explore the optimal set of parameters influencing the variation of architectural landscape spatial structures. The objective of this algorithm is to minimise the energy function to find the optimal parameters, thereby avoiding getting trapped in local optima. Through an iterative process, parameters were gradually adjusted to better fit the relationship between architectural landscape features and environmental variables. In the initial stages of the algorithm, a higher temperature setting allowed for a wider search range, and as the temperature gradually decreased, the search space converged, aiding in finding the global optimum solution or an approximate optimal solution. Throughout the iterative process, parameters and energy values underwent fluctuations before gradually stabilising, ultimately successfully obtaining the optimal parameters for the architectural landscape spatial structure in this study. By leveraging the dynamic characteristics of the simulated annealing algorithm, the parameter space was explored and utilised more effectively to meet the complexity and personalised requirements of architectural landscape planning. Identify key parameters from the data through regression analysis, and set the collected architectural landscape observation dataset $\mathcal{D} = (x_i, y_i)_{i=1}^{N}$, where x_i represents independent variables (pedestrian flow, frequency of use, etc.).${}_i y$represents the dependent variable (specific characteristics of spatial layout), to identify the key parameter θ that affects changes in architectural landscape spatial structure through regression analysis. Build the following nonlinear regression model:

$$y = f(\mathbf{x};\boldsymbol{\theta}) + \varepsilon \tag{4}$$

Here, f is a nonlinear function that represents the relationship between architectural landscape features and environmental variables, and ε represents observation noise. Identify the optimal parameter $\boldsymbol{\theta}^*$ by minimising the following loss function:

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{N} \left[y_i - f\left(x_i; \boldsymbol{\theta}\right) \right]^2 \tag{5}$$

Considering the complexity and personalised needs of architectural landscape planning, the model parameter $\boldsymbol{\theta}$ can be adjusted to adapt to specific architectural scenarios. The SA algorithm was used for parameter optimisation in the study, aiming to find the global optimal solution and avoid falling into local optima. The energy function $E(\boldsymbol{\theta})$ can be defined to represent the model error under the current parameters, with the algorithm goal of minimising $E(\boldsymbol{\theta})$. The update process of simulated annealing is expressed as:

1 initialise parameters $\boldsymbol{\theta}_0$ and temperature T_0

2 in step k, randomly select a new parameter state $\boldsymbol{\theta}_{k+1}$ from around $\boldsymbol{\theta}_k$

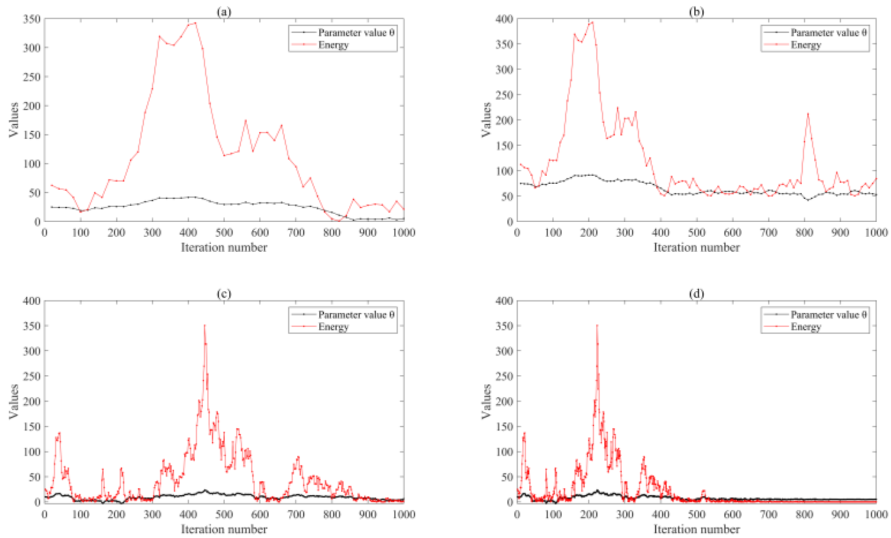
3 calculate the energy difference $\Delta E = E(\theta_{k+1}) - E(\theta_k)$

4 if $\Delta E < 0$, accept the new status; otherwise, the new state can be received with probability $\exp(-\Delta E/T_k)$

5 reduce the temperature and repeat steps 2 to 4 until the convergence condition is met.

Through this process, the optimal parameter set that affects the spatial structure changes of the architectural landscape can be gradually approached, as shown in Figure 3.

Figure 3 Simulates the update process of the degradation algorithm, (a) 50 iterations, (b) 100 iterations, (c) 500 iterations, (d) 1000 iterations (see online version for colours)



Figures 3(a), 3(b), 3(c) and 3(d) show the parameters and energy values of the algorithm at iterations of 50, 100, 500, and 1,000, respectively. It can be seen that SA has dynamic characteristics in the process of searching for the optimal parameters of architectural landscape spatial structure: the initial parameters and energy are relatively fluctuating. As the number of iterations increases, the parameters gradually stabilise. As the energy value decreases, the algorithm gradually shifts from exploration to utilisation, ultimately seeking a stable state with lower energy. In the initial stage, SA allows the algorithm to conduct large-scale searches through high-energy (25) settings. As the energy gradually decreases, the search range gradually converges, which helps to find the global optimal solution/approximate optimal solution. According to Figure 3, SA successfully obtained the optimal parameters of the architectural landscape spatial structure in this paper after the 534th iteration.

5 Numerical solution

In architectural landscape planning, the complexity and scalability of models are important factors to consider. As architectural designs and scales continue to evolve, models need to possess sufficient complexity to capture the intricate relationships between various variables, ensuring the accuracy and reliability of simulation results. Additionally, models need to exhibit a certain level of scalability to accommodate different scales and types of architectural scenarios, effectively handling large-scale data to ensure the efficiency and practicality of the models in real-world applications. In the process of model design and parameter adjustment, a balanced consideration of complexity and scalability is required to achieve accurate modelling and effective optimisation of architectural landscape spatial structures.

Using the finite element method to numerically solve nonlinear partial differential equations involves discretising the continuous domain into a finite number of elements and approximating the field variables within each element to form algebraic equation sets. During the discretisation process, local stiffness matrices and load vectors for the elements are first defined. The definition of the local stiffness matrix involves integration over shape functions, which describe how the field variables change with spatial position within the element. In the solution process of the algebraic equation sets, the Newton-Raphson method is employed to handle nonlinear problems. The method iteratively solves linear systems to update the solution, where the coefficient matrix of the linear system is determined by the stiffness matrix of the current solution and the residual vector. The iteration process continues until convergence criteria are met, where the increment of the solution is less than the specified tolerance error.

In the numerical solution process of nonlinear partial differential equations, there inevitably arises a trade-off and selection among multiple solutions. When using the Newton-Raphson method for iterative solving, each step may involve multiple candidate solutions, and the paper determines the most suitable one to update the current solution. The selection process is influenced by both numerical stability and the accuracy and reliability of the results. The study combines pre-defined tolerance errors and convergence criteria, including iteration step size, residual change rate, etc., to determine proximity to the optimal solution.

For nonlinear partial differential equations, FEM is used for numerical solution in the study, discretising the continuous field into finite elements and approximating the field variable u within each element to form an algebraic equation system. In the discretisation process, define the local stiffness matrix K_e and load vector of the elements, and assemble the global stiffness matrix and global load vector. For each element, the local stiffness matrix is represented as:

$$K_e = \int_{\Omega_e} (\nabla N_i)^T \kappa(u) \nabla N_j d\Omega \tag{6}$$

Here, N_i and N_j are shape functions. In the process of solving algebraic equations, the Newton-Raphson method can be used to handle nonlinear problems, and the following linear systems can be solved in each iteration to update the solution:

$$K(u^n)\delta u^n = -R(u^n) \tag{7}$$

Among them, $R(u^n)$ is the residual vector under the current iteration u^n, and δu^n is the increment of the solution. By iterating until convergence ($\|\delta u^n\| < \varepsilon$, ε is the predetermined tolerance error), the stability of the numerical solution is obtained. Stability analysis evaluates the accuracy of model predictions by comparing numerical results directly with observed data through several consecutive iterations of result changes. The test results are shown in Table 1.

The relative error is calculated by dividing the difference between the observed value and the numerical solution by the observed value, then multiplying by 100 to obtain a percentage representation. This measurement method offers an intuitive assessment of the accuracy of numerical solutions, particularly in architectural design and planning, where high precision is crucial to the outcome. If the relative error of indicators falls within an acceptable range, it validates the effectiveness and reliability of the numerical solution method. Table 1 shows the key indicator results obtained during the numerical solution process. It includes building height, building shape matching, building size ratio, consistency of contour line direction, reflectivity of materials used, colour saturation, natural light utilisation efficiency, uniformity of artificial light brightness, smoothness of spatial organisation, visual focus prominence, functional focus suitability, surface texture consistency, and pattern clarity. These can be compared with observed values. The observed height of the building is 120 meters, and the numerical solution result is 118.8 metres, with a relative error of only 1.0%, demonstrating the high accuracy of the model solution. The observed value of building shape matching is 0.95, and the solution result is 0.94, which once again proves the reliability of the model. The stability and reliability scores of all indicators are above 0.95, indicating the stability and reliability of the model solution. The stability and reliability scores of colour saturation are both 0.99. The numerical solution method adopted has a solid theoretical foundation and has shown good results in practical applications, providing strong mathematical support for the planning and analysis of architectural landscape spatial structures.

Table 1 Model solving test results

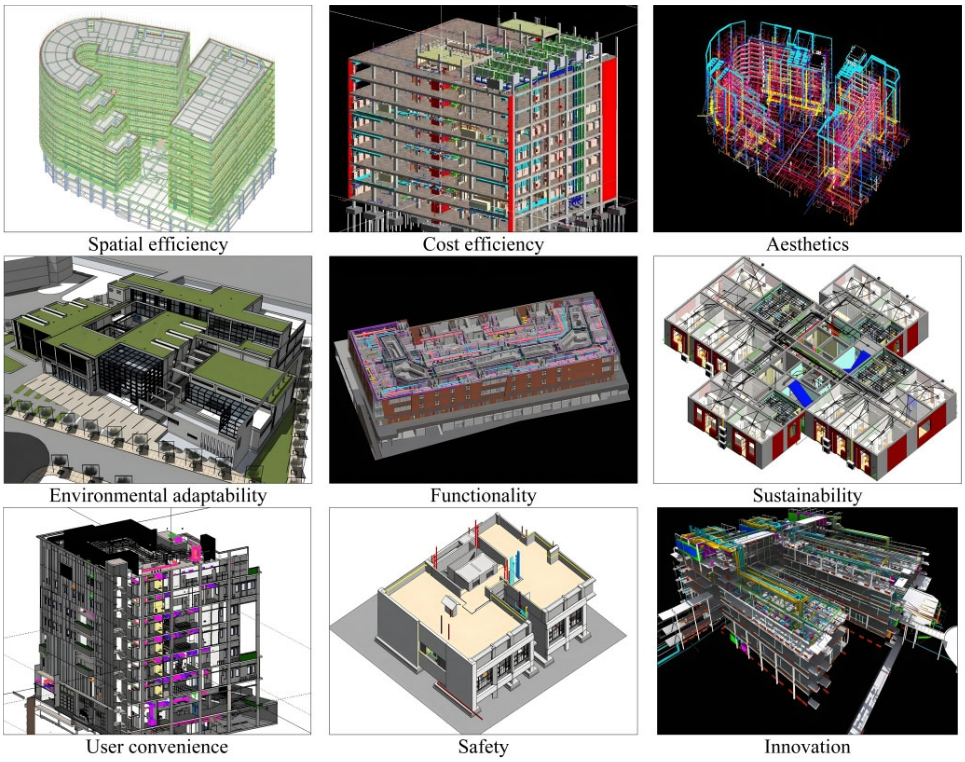
Parameter indicator	*Observed value*	*Numerical solution result*	*Relative error*	*Stability score*	*Reliability score*
Building height	120 m	118.8 m	1.00%	0.98	0.97
Building shape conformance	0.95	0.94	1.05%	0.97	0.96
Building size ratio	1.5	1.49	0.67%	0.99	0.98
Contour linearity	0.90	0.91	1.11%	0.96	0.95
Material reflectivity	0.3	0.305	1.67%	0.98	0.97
Colour saturation	0.7	0.698	0.29%	0.99	0.99
Natural light efficiency	0.8	0.79	1.25%	0.97	0.96
Artificial light uniformity	0.85	0.84	1.18%	0.96	0.95
Spatial organisation flow	0.9	0.89	1.11%	0.97	0.97
Visual focus prominence	0.95	0.945	0.53%	0.99	0.98
Functional focus adequacy	0.9	0.905	0.56%	0.98	0.97
Surface texture consistency	0.8	0.805	0.63%	0.98	0.96
Pattern clarity	0.85	0.847	0.35%	0.99	0.98

In light of the results presented in Table 1, despite the high accuracy and reliability demonstrated by the model, there are still instances of relatively large errors, such as inconsistencies in contour line orientations and the utilisation of material reflectance. These deviations stem from various factors, including the selection of model parameters, the establishment of boundary conditions, and the approximation errors during numerical solving. In simulating the spatial structures of architectural landscapes, precise modelling of specific material properties and light reflection characteristics is crucial, as it directly influences the appearance and visual effects of buildings. Further enhancements to the model necessitate a more precise consideration of these factors. This study aims to improve simulation accuracy through finer mesh division, more accurate material parameters, and more sophisticated lighting models.

6 Effect evaluation

The Pareto optimal solution set of architectural landscape spatial structure can be displayed under multiple feasible solutions, with evaluation criteria including spatial efficiency, cost efficiency, aesthetics, environmental adaptability, functionality, sustainability, user convenience, safety, and innovation. The results are shown in Figure 4.

Figure 4 Pareto optimal solution set of architectural landscape spatial structure under various indicators (see online version for colours)



To better evaluate the Pareto optimal solution set presented in Figure 4, a deeper assessment of the strengths and weaknesses of these solutions is needed to determine the final solution. The Pareto optimal solution set showcases the performance of various feasible solutions across multiple evaluation criteria. Assessing the merits of these solutions requires considering the importance and weighting of each criterion, as well as the adaptability of the solutions to specific scenarios and requirements. When considering spatial efficiency, it is necessary to identify which solutions offer the best space utilisation and layout effectiveness. In terms of cost efficiency, it is crucial to determine which solutions can achieve design objectives at the lowest cost. For aesthetics, it is important to ascertain which solutions best meet aesthetic standards and user preferences. Regarding environmental adaptability, it is essential to identify which solutions best align with the requirements of environmental protection and sustainable development. It can be seen that based on this article's NPDEs, multiple feasible solutions for the spatial structure of architectural landscapes can be effectively obtained, and corresponding architectural structural design schemes can be successfully obtained under various indicators. It can compare the performance of the NPDEs method with Monte Carlo simulation, genetic algorithm (GA), and linear programming (LP) methods in practical applications. The paper collected survey data and feedback from 600 architects and designers, as shown in Table 2.

Figure 2 Feedback from architects and designers

Evaluation criteria	*NPDEs method (%)*	*Monte Carlo simulation (%)*	*Genetic algorithm (%)*	*Linear programming (%)*
Spatial efficiency	92	75	80	70
Cost efficiency	85	70	75	80
Aesthetics	90	65	70	60
Environmental adaptability	88	78	82	70
Functionality	89	73	77	68
Sustainability	87	69	74	65
User convenience	84	72	76	78
Safety	90	80	85	82
Innovation	93	68	79	70

The results showed that the NPDE method achieved a high satisfaction rate of 92% in terms of spatial efficiency, while Monte Carlo simulation, genetic algorithm, and linear programming were only 75%, 80%, and 70%, respectively. In terms of cost efficiency, NPDEs perform slightly better than linear programming (85% compared to 80%), and significantly higher than the other two methods. The high evaluation in terms of aesthetics (90%) and innovation (93%) confirms the ability of NPDEs to promote innovative design and enhance aesthetic value. NPDEs perform better than other methods in terms of environmental adaptability (88%), functionality (89%), and sustainability (87%). It has high efficiency when considering multiple factors in architectural design, and also has high reliability (90%) in terms of safety.

The NPDE method performs remarkably well across various metrics, particularly excelling in spatial efficiency, cost-effectiveness, aesthetic appeal, and innovation. Regarding spatial efficiency, the satisfaction with the NPDE method far surpasses the other three approaches. Furthermore, in terms of cost-effectiveness, the NPDE method slightly outperforms linear programming, demonstrating higher efficiency and reliability. Additionally, the NPDE method significantly outperforms other methods in environmental adaptability, functionality, and sustainability, indicating its effectiveness and reliability in considering multiple factors in architectural design comprehensively. The results fully demonstrate the comprehensiveness and sophistication of the NPDEs method in architectural landscape spatial planning, providing robust mathematical support for innovative design and improving spatial and environmental efficiency.

RMSE, R-squared, and MAPE are important metrics for assessing model performance. RMSE measures the average deviation between model predictions and observed values, with lower values indicating higher prediction accuracy. R-squared quantifies the extent to which the model explains the variance in the observed data, with values closer to 1 indicating a better fit. MAPE evaluates the average percentage error of the model's predictions, with values closer to 0 indicating more accurate predictions.

The prediction accuracy of the NPDEs method in this article can be compared with the other three types of models, including RMSE, R-squared, and MAPE. The formulas are as follows:

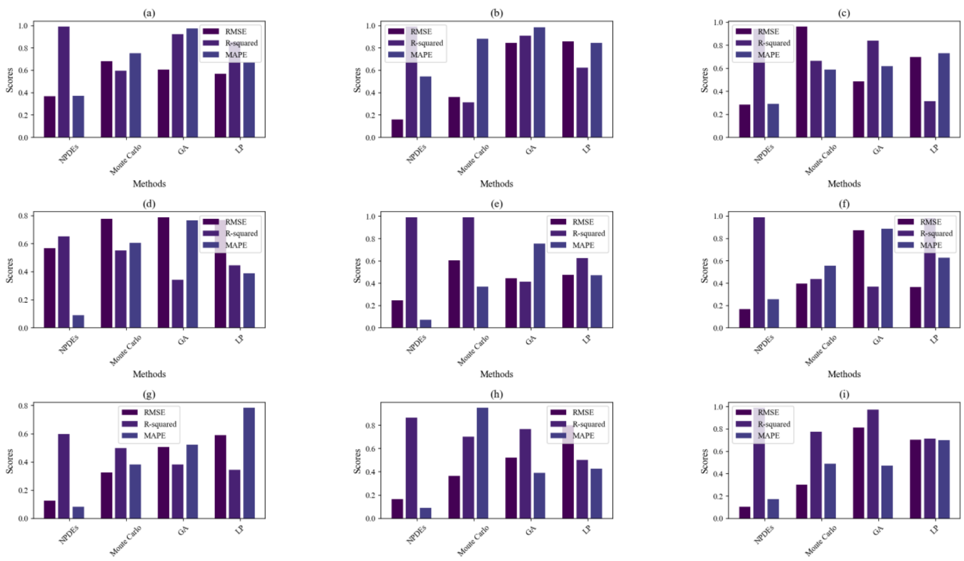
$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2} \tag{8}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2}{\sum_{i=1}^{n}\left(Y_i - \overline{Y}\right)^2} \tag{9}$$

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{Y_i - \hat{Y}_i}{Y_i}\right| \cdot 100\% \tag{10}$$

Among them, Y_i represents the actual value; $\hat{Y}_i$ is the predicted value; $\overline{Y}$ is the mean of the actual value. The calculation results can be displayed as shown in Figure 5.

Figure 5 Comparison of prediction accuracy among different methods, (a) space efficiency (b) cost efficiency (c) aesthetics (d) environmental adaptability (e) functionality (f) sustainability (g) user convenience (h) security (i) innovation (see online version for colours)



For spatial efficiency, the NPDEs method has RMSE, R-squared, and MAPE values of 0.368, 0.99, and 0.370, respectively. In contrast, the values simulated by Monte Carlo are 0.681, 0.597, and 0.752, the values for GA are 0.606, 0.924, and 0.975, and the values for LP are 0.568, 0.854, and 0.670. The data shows that the NPDEs method has lower RMSE and MAPE in spatial efficiency prediction, as well as higher R-squared, which leads to higher prediction accuracy in this area. For other evaluation indicators, the data in the figure also shows the advantages of the NPDE method over other models. In terms of cost efficiency, the NPDEs method has an RMSE of 0.161, R-squared of 0.99, and MAPE of 0.546, showing lower RMSE and MAPE as well as higher R-squared, indicating that it is more accurate and reliable in predicting cost efficiency. This further calculates the length of time required for parameter adjustment of the model before reaching the optimal solution. The average time from initialisation to convergence of the model can be calculated to evaluate:

$$T_{avg} = \frac{1}{m} \sum\nolimits_{j=1}^{m} T_j \tag{11}$$

Here, T_j is the convergence time of the model in the j^{th} experiment, and m is the number of experiments. For complex building planning problems, the efficiency of parameter adjustment directly affects project progress and cost. A shorter T_{avg} indicates higher efficiency. The calculation results are shown in Table 3.

Table 3 Time consumption for adjusting model parameters under each experimental project (s)

Experiment	*NPDEs*	*Monte Carlo simulation*	*Genetic algorithm*	*Linear programming*
A	15.04	25.20	30.23	20.30
B	18.83	30.16	35.89	22.21
C	12.46	22.72	28.11	18.96
D	20.58	28.02	32.51	24.62
E	16.54	26.75	29.21	21.87
F	17.56	24.07	31.93	19.20
G	14.40	27.34	33.40	23.52
H	19.04	29.92	34.54	25.56
I	13.05	23.35	27.66	17.70
Average time	16.39	26.39	31.50	21.55

In experiment A, for the architectural planning of a large commercial complex, the NPDEs method and three other traditional methods can be used for parameter adjustment. In the experiment, the multifunctionality and environmental adaptability of the building, as well as user convenience, can be considered. By finely adjusting the spatial layout and functional zoning in the NPDEs method, a shorter parameter adjustment time can be achieved, with an average time of about 15.04 seconds. The adjustment time for Monte Carlo simulation and genetic algorithm is 25.20 seconds and 30.23 seconds respectively, while the adjustment time for linear programming method is 20.30 seconds.

Experiment B involves the planning and design of a residential community, focusing on the aesthetics and sustainability of the building. This article applies different design parameters and constraints to the model, conducts ten experiments, and evaluates the convergence time of the model. In the NPDEs method, an average adjustment time of 18.83 seconds can be achieved by optimising building appearance and material selection, as well as energy utilisation efficiency. The average adjustment time for Monte Carlo simulation and genetic algorithm is 30.16 seconds and 35.89 seconds respectively, while the average adjustment time for linear programming is 22.21 seconds.

In experiment C, the planning problem of a city's public transportation hub can be considered, with a focus on its functionality and user safety. In the NPDEs method, simulations and optimisations were conducted for different traffic demands and pedestrian flows, resulting in an average adjustment time of 12.46 seconds. The adjustment time for Monte Carlo simulation and genetic algorithm is 22.72 seconds and 28.11 seconds respectively, while the adjustment time for linear programming is 18.96 seconds.

Experiments D to I involve different types of architectural planning problems, covering multiple fields such as commerce, housing, and education. Multiple evaluation indicators such as spatial efficiency, cost efficiency, and aesthetics can be comprehensively considered in these experiments, and different model methods can be used for parameter adjustment. The average adjustment time of each model can be obtained through simulation and optimisation of each experiment. In the NPDEs method, a relatively short adjustment time can be achieved through the optimisation of building structure and layout, as well as rational utilisation of materials and resources. The adjustment time for Monte Carlo simulation, genetic algorithm, and linear programming methods is relatively long.

Figure 6 RMSE comparison of different models under different data scales

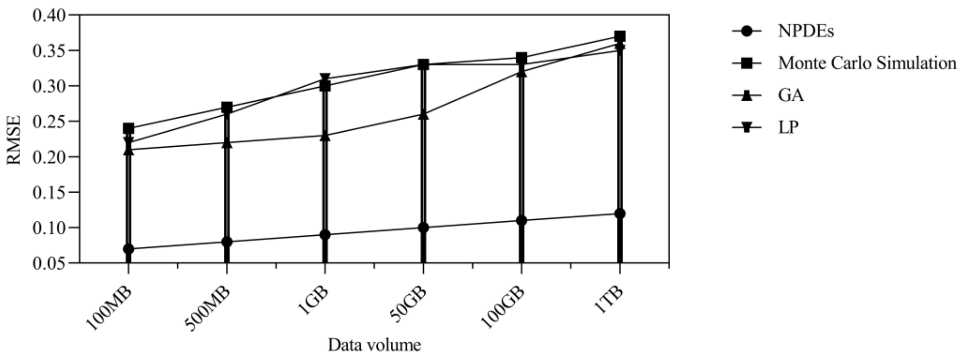
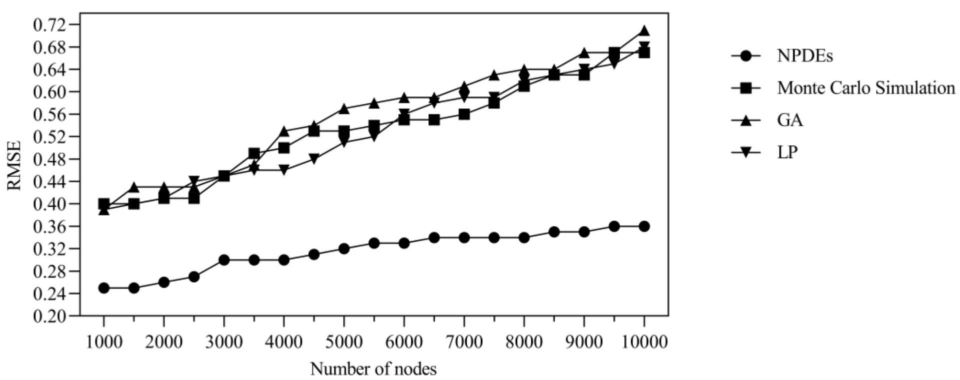


Figure 7 RMSE comparison of different models under different spatial structure complexities



The NPDEs method has shown high efficiency in various experiments, with significantly shorter average adjustment time compared to the other three traditional methods. The NPDE method has better potential for application in complex building planning problems, enabling faster parameter optimisation and model convergence, improving project progress, and reducing costs. Further evaluation can be conducted on the performance of NPDEs in dealing with problems of different scales and complexities, and compared with other three types of models. The results are shown in Figures 6 and 7.

Figures 6 and 7 respectively show the comparison of RMSE values for four types of methods under different data sizes and complexities. The horizontal axis in Figure 6 represents the size of the data, while the horizontal axis in Figure 7 represents the number of nodes in the architectural landscape space. To test the predictive performance of each model, this article sets the data size to 100 MB, 500 MB, 1 GB, 50 GB, 100 GB, and 1 TB in order. The number of spatial nodes can be set to 1,000 to 10,000 in sequence, with an interval of 500 nodes between each test. It can be seen that as the amount of data and the number of spatial nodes increases, the prediction accuracy of all four types of models decreases. Compared to other models, NPDEs always maintain high stability and prediction accuracy. When the data volume reaches 1 TB, the RMSE size is only 0.12, which is about 65.7% higher than the best-performing LP in other models. When the number of nodes in the architectural landscape space reaches 10,000, the RMSE size is only 0.36, which is about 46.3% higher than the best-performing Monte Carlo simulation in other models. When the data size and complexity reach a high level, NPDEs can still perform relatively accurate calculations.

The advantages of the NPDE method lie in its efficiency and flexibility, offering faster parameter adjustment and better model convergence compared to traditional methods. Traditional approaches often face scalability and efficiency issues when dealing with large datasets or complex spatial structures. Meanwhile, the NPDE method, through nonlinear differential equation models, better captures the complexity and dynamic changes of architectural landscape spatial structures. Compared to traditional optimisation models, NPDEs can comprehensively consider various factors and constraints, thereby enhancing the reliability and practicality of planning solutions.

7 Conclusions

This study adopted the NPDEs method to deeply explore the difficulties in model simplification and parameter adjustment in architectural landscape spatial structure engineering. It successfully used partial differential equations to describe the spatial form, functional zoning, and dynamic changes in the pedestrian flow of architectural landscape spatial structure. Key parameters were identified through regression analysis, and a nonlinear regression model was constructed. To meet the needs of different building scenarios, SA can be used for parameter optimisation, effectively improving the applicability of the model. Its application of FEM stably solves nonlinear differential equations, ensuring the accuracy and stability of numerical solutions. The experimental results show that the NPDEs method has significant advantages over traditional models in terms of spatial efficiency, adjustment time, stability, and prediction accuracy. At the same time, there are still certain shortcomings in this study, such as the need for further exploration of its applicability to more complex architectural scenes. Looking ahead, the application of NPDE methods will further expand to more complex architectural scenarios, including considerations such as cultural heritage preservation, climate change adaptation, and community involvement. Additionally, this approach can also be applied in urban transportation planning, ecological restoration projects, and virtual environment design. By being applied across multiple domains, NPDE methods hold the promise of offering new solutions for harmonious interactions between urban development and the environment.

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