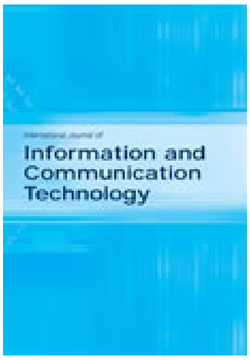

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SA-enhanced PSO Newton algorithm for fractal art graphic design

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Abstract: Fractal art graphic design has been very popular in recent years. Specifically, there are many researches on computational graphics and scientific visualisation. As a result, we propose a SA-enhanced PSO Newton interactive algorithm for efficient fractal art generation. The method integrates simulated annealing (SA) with the PSO-enhanced Newton iterative algorithm together, improving the exploration ability. This combination also prevents early convergence, resulting in high-quality fractal art generation. The entire process of our algorithm is: firstly, the SA is used to enhance the optimisation ability of particle swarm optimisation (PSO) algorithm; subsequently, we use the Newton-Raphson method to produce high-quality fractal images with the SA-PSO-optimised parameters. According to our experimental results, the proposed method achieves significant enhancement in computational efficiency and generates high-quality images. This proves that our method can be used as an efficient tool for fractal art design in computer graphics.

Keywords: fractal art; swarm optimisation; Newton iterative algorithm; computational graphics.

Reference to this paper should be made as follows: Ma, N. (2025) 'SA-enhanced PSO Newton algorithm for fractal art graphic design', *Int. J. Information and Communication Technology*, Vol. 26, No. 18, pp.86–99.

Biographical notes: Ning Ma received her Bachelor degree form Anyang Normal University in June 2008. She is currently a Lecturer at the Shangqiu University. Her research interests include machine learning, art design and machine vision.

1 Introduction

Fractal art graphic design is a combination of mathematics, computer science and visual creativity. There are a lot researches on utilising mathematics equations and computer algorithms to design fractal graphics. Among all of the computational techniques, the Newton iterative algorithm has gained the attention because of its efficiency. This method is primarily designed to determine the roots of nonlinear equations. However, according to studies, researchers find the root-exploration ability of Newton method is beneficial to create visually appealing fractal patterns (Alves et al., 2020; Gdawiec et al., 2021). Nevertheless, this method still faces the disadvantages such as early convergence and sub-optimal exploration abilities. To solve these problems, researchers have tried to combine the Newton iterative method with PSO algorithm, which can enhance the Newton's root searching ability (Kannan and Diwekar, 2024).

Meanwhile, the SA is another famous method for searching best particles. It introduces controlled perturbations to escape the local optima and explore the solution space more effectively (Morales-Castañeda et al., 2019; Alkhateeb and Abed-Alguni, 2019; Hajji et al., 2024). According to current research, SA has been successfully applied to image processing and fractal generation (Wang and Chen, 2023; Ahmad et al., 2022). There are also researches about the integration of SA with other optimisation techniques. This combination has been proved that it improves the efficiency and robustness of optimisation processes (Gad, 2022; Zhang et al., 2015; Jahandideh-Tehrani et al., 2020). As a result, we aim to implement the SA to improve the PSO's optimisation ability.

In this paper, we propose a novel hybrid approach which name SA-enhanced PSO algorithm. This hybrid method is then used to optimise the parameters of Newton iterative method. Through this strategy, the whole method can avoid being stuck into local optima and find more possible high-quality solutions. With these solutions, it is quite easy to generate better fractal art. Meanwhile, it also enhances the overall effectiveness of the algorithm and achieves a balance between exploration and exploitation. The contributions of this work are threefold:

1 we propose the SA-driven PSO algorithm, an adaptive optimisation algorithm that automatically adjusts swarm parameters and cooling schedules for optimising the algorithm's performance

2 we use the proposed hybrid algorithm to optimise the parameters of the Newton iterative method to overcome the local optima and solution diversity issues in fractal art generation

3 we demonstrate the effectiveness of the proposed method through carrying out extensive experiments, proving its ability to generate visually stunning fractal images with improved efficiency and diversity.

The paper structure is as follows: Section 2 reviews the background work on fractal art generation and optimisation algorithms. Section 3 introduces the background on the Newton iterative method, PSO, and SA. Besides, we also describe the design and implementation of the novel SA-based PSO Newton algorithm. Section 4 gives the experiment environment, revealing the evaluation results and analysis. Section 5 discusses our future work. Finally, Section 6 concludes the paper.

2 Related work

2.1 Fractal art generation

Recently, fractal art graphic design has gained significant attention. Researchers have explored various computational methods to enhance design efficiency and artistic quality.

Chao (2020) studied the application of interactive genetic algorithms (IGA) in fractal art design. It enhanced the fundamental theory through implementing genetic algorithm to improve the colour design approach. This study showed that IGA can combine user preferences with mathematical algorithms to create personalised fractal images, making the creative process more efficient. Similarly, Lv et al. (2019) also used IGA when implementing a fractal pattern design system, where users interact with a simple interface to generate complex fractal patterns. After the evaluation, they found the system succeeded in using genetic algorithms to optimise the parameters to improve the design quality and efficiency.

Bouteraa and Khishe (2025) present a novel chaotic map and fractal-enhanced grey wolf optimiser (CF-GWO) for optimising deep convolutional neural networks (DCNNs). It demonstrates significant improvements in image classification tasks, outpacing 23 classifiers with 87.37% accuracy across nine datasets, while highlighting the potential of chaotic and fractal techniques in architecture design.

Deep learning technique has also been applied to fractal art generation. Zhang and Jia (2023) proposed a fractal art graphic generation model based on convolutional neural networks (CNN), which is utilised to extract the features from images. They tried to improve the math-method-based images through minimising the total loss. After the training process, the model can generate high-quality images with both visual information and texture information. Their method achieves 96.21% in F1-score, which is 12% better than AlexNet and 7% better than ResNet. The fractal art generation suffers from unreasonable design and low-level evaluation. As a result, a CAD-based fractal pattern design and evaluation method is introduced by Liu and Zhu (2022). They divided the fractal images into two parts based on the feature trees, which helps to recognise the symmetrical relationships. What is more, this division allows parallel computation of the algorithm, reducing the computational cost.

2.2 Newton's iterative algorithm

Apart from fractal art generation, there are also studies on using Newton's iterative algorithm to computational optimisation and system modelling. Xu (2015) implemented the Newton method to determine the parameters of dynamic systems. Different for other researches, they separate the searching process to two stages. In the first stage, they estimated two parameters (gain and pole), while in the second stage, they found the values of another three parameters (gain and two poles). Tian et al. (2022) proposed a novel Newton iterative algorithm which computes the increment rather than the iterative solution. In the meantime, they developed the Newton method under the Fréchet derivative framework. Their method showed better convergence ability compared with existing methods.

Zhang et al. (2020) combined stochastic parallel gradient descent (SPGD) with Newton iteration in order to design a hybrid optimisation approach. The combination helped to get adaptive interferometry in freeform surface metrology, without requiring computer-generated holograms or null lenses. This hybrid method beat the baseline methods, which proves its effectiveness.

3 Methodology

3.1 Background

Fractal art is composed with the points which are generated with mathematical transformations. Usually, the iterative techniques are necessary to transform the points repeatedly. One of the most popular equation is the Newton iterative method, which can be written in the form $f(x) = 0$. The Newton Iterative method equation is shown as follows:

$$x_{n+1} = x_n \frac{f(x_n)}{f'(x_n)} \tag{1}$$

Although the Newton-Raphson method is effective for finding roots, it shows high sensitivity to initial settings. This disadvantage will lead to the convergence to local optima. Meanwhile, this also limits its ability to produce diverse fractal patterns. What is more, the computational complexity increases with the complexity of the fractal equation, making it difficult to generate real-time patterns.

In order to deal with these kinds of problems, we employ the global optimisation techniques such as PSO and SA. Swarm parameter optimisation refers to adjusting the settings (such as swarm size and the influence of each particle) in PSO to improve how particles (representing potential solutions) move and interact during the search process, helping the algorithm converge more efficiently to a good solution. The PSO algorithm is inspired by the social behaviour of bird flocking. It uses a population of particles to explore the solution space. Specifically, each particle updates its position and velocity based on its own experience and the global experience from the swarm. The velocity and position updates are shown as below:

$$\mathbf{v}_i^{k+1} = w\mathbf{v}_i^k + c_1 r_1 \left(\mathbf{p}_i^k - \mathbf{x}_i^k\right) + c_2 r_2 \left(\mathbf{g}^k - \mathbf{x}_i^k\right) \tag{2}$$

$$\mathbf{x}_i^{k+1} = \mathbf{x}_i^k + \mathbf{v}_i^{k+1} \tag{3}$$

where $\mathbf{v}_i^k$ and $\mathbf{x}_i^k$ represent the velocity and position of the i^{th} particle at iteration k. $\mathbf{p}_i^k$ is the particle's best-known position and $\mathbf{g}^k$ represents the global best position. In spite of its global search capabilities, PSO also has disadvantages such as converging too early and arriving at sub-optimal positions. These disadvantages indicate that it is necessary to implement some extra mechanisms to further enhance the exploration process.

SA is a probabilistic optimisation technique inspired by the annealing process in metallurgy. It implements controlled perturbations to the searching space and helps the algorithm to escape local optima. Meanwhile, the temperature parameter is used to determine how likely it is to accept the sub-optimal solutions. The temperature decreases according to a cooling equation:

$$T_{k+1} = \alpha T_k \tag{4}$$

where α is the cooling rate ($0 < \alpha < 1$). The algorithm stops when the temperature T_k reaches the set threshold or when the fitness value of the best solution remains stable. This convergence criterion ensures that the algorithm will not stop too early and explores the solution space until a satisfactory solution is found. Temperature-controlled perturbation mechanism refers to a process in simulated annealing (SA) where the 'temperature' is gradually reduced during optimisation. At higher temperatures, the algorithm allows more randomness or 'perturbations' in the search for solutions, and as the temperature decreases, it becomes more focused on finding the optimal solution by accepting fewer suboptimal ones. The perturbation magnitude Δx is drawn from a Gaussian distribution with a variance proportional to the current temperature T_k:

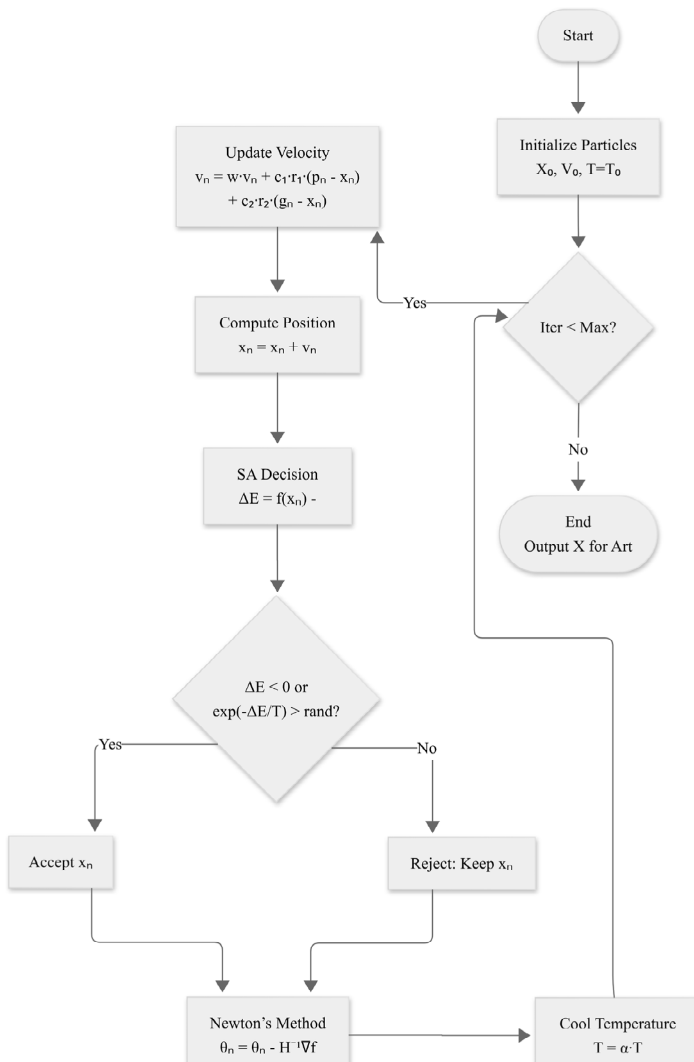
$$\Delta x \sim \mathcal{N}\left(0, \sigma^2\right) \tag{5}$$

The perturbed solution $x_n' = x_n + \Delta x$ is evaluated and accepted with a probability based on the Metropolis criterion:

$$P(\Delta E) = \exp\left(-\frac{\Delta E}{T_k}\right) \tag{6}$$

where ΔE represents the change in the fitness value between the perturbed and current solutions. This mechanism allows the algorithm to explore suboptimal regions, enhancing solution diversity and enabling escape from local optima.

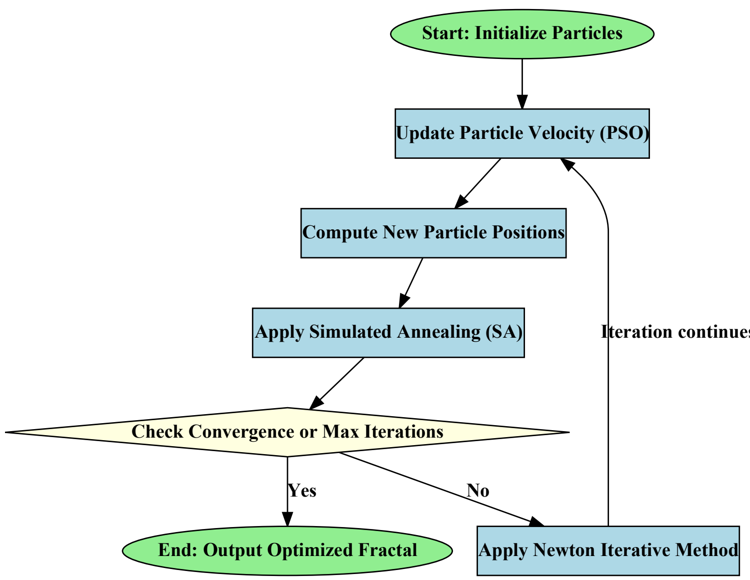
Figure 1 The flowchart of the SA-enhanced PSO Newton Iterative algorithm



3.2 Proposed SA-enhanced PSO-Newton iterative algorithm

The whole process of the algorithm is given as below: the algorithm starts with a group of particles, each of them representing a potential solution to the fractal equation. The PSO algorithm will update its position if there is a more suitable fitness value. However, the SA introduces controlled perturbations to enhance this process. It will randomly keep the worse position without changing it to a better area. After several epochs, this SA-enhanced PSO will find a group of optimal parameters, including initial guesses, max iterations, tolerance and escape radius. These parameters will influence the convergence route of the Newton iterative algorithm. Finally, the convergence behaviour is mapped to different colours and creates fractal images with intricate patterns.

Figure 2 The entire process of our proposed method (see online version for colours)



4 Evaluation

4.1 Experimental setup

In this section, we will introduce the baseline fractal equations, algorithms and evaluation which are used in our experiments. Meanwhile, we also show our experimental environment.

4.1.1 Fractal equations

We conducted the experiments with two other baseline fractal equations: Mandelbrot set and Julia set.

Mandelbrot set is defined as below:

$$z_{(n+1)} = z_n^2 + c \tag{7}$$

where z starts at 0, while c is a complex number representing a point in the complex plane.

The Julia set uses a similar format as Mandelbrot set but with a constant parameter C:

$$z_{(n+1)} = z_n^2 + C \tag{8}$$

These two methods are utilised to evaluate the generation of Newton method. The formula of Newton Iterative method is:

$$z_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)} \tag{9}$$

z_n is the current approximation of the root and $f(z_n)$ represents the functions. If we set $f(z)$ as:

$$f(z) = z^3 - 1 \tag{10}$$

We get the formula:

$$z_{n+1} = z_n - \frac{z_n^3 - 1}{3z_n^2} \tag{11}$$

Table 1 The key parameters and convergence condition of three methods

Fractal type	*Key parameters*	*Convergence condition*
Mandelbrot set	Complex plane range, max iterations	-
Julia set	Choice of c, complex plane range, max iterations	-
Newton fractal	Polynomial $f(z)$, complex plane range, max iterations	Convergence to root of f(z)

These equations provide a diverse method for evaluating the proposed algorithm's performance in fractal art graphic design (Nishonov, 2022; Antal et al., 2021).

4.1.2 Baseline methods

We compared our proposed algorithm with three other baseline methods: standard Newton iterative algorithm, PSO-enhanced Newton iterative algorithm and simulated annealing with Newton iterative algorithm.

The standard Newton iterative algorithm follows the standard Newton-Raphson update equation without any optimisation process, which aims to evaluate the effectiveness of PSO and SA. The PSO-enhanced Newton iterative algorithm integrates the PSO algorithm only, which is used to evaluate the benefits of adding SA algorithm. Finally, the simulated annealing with Newton iterative algorithm applies SA to the method without PSO, aiming to evaluate the SA's ability of improving solution diversity.

4.1.3 Evaluation metrics

In this study, we implement three metrics to evaluate the performance of different algorithms. Above all, in order to indicate the accuracy of the fractal equation's root, we use the solution quality, which is measured by the fitness value calculated from the best solution. Meanwhile, we measure the algorithm's exploration ability of the solution space through the metric diversity of solutions. This metric is influenced by the entropy of the solution distribution. Computational efficiency is assessed by the runtime of generating fractal images. It includes the time for both iterative updating and perturbation processes. At last, we evaluated the visual quality of the generated fractal images with visual appeal, which focuses on the images' complexity, symmetry, and aesthetic appeal.

4.1.4 Implementation details

The proposed algorithm was implemented in Python. We used the NumPy library for numerical computations and the PyCUDA library for GPU acceleration. The PSO algorithm was implemented with a size of 50 particles, while the SA algorithm was set with an initial temperature $T_0 = 100$ and a cooling rate $\alpha = 0.95$. What is more, the Newton-Raphson method was applied with a convergence threshold of 10^{-6}. All experiments were conducted with an NVIDIA RTX 3090 GPU and an AMD Ryzen 9 5950X CPU. For the fractal generation, the Mandelbrot and Julia sets were generated with a maximum of 500 iterations, a complex plane range of [–2, 2], and a resolution of 1,000 × 1,000 pixels. The Newton fractal used 100 iterations and a convergence tolerance of 0.000006. Newton's method was tuned with an appropriate number of iterations to ensure high precision without excessive computational time.

4.2 Experimental results

4.2.1 Comparison between three fractal methods

We compare the Newton iterative method with other two baseline methods: Mandelbrot set and Julia set. We evaluate the convergence speed and computational cost across. The results are in Table 2.

Table 2 The convergence speed and computation time for three methods

Fractal type	*Avg. iterations to converge*	*Computation time (ms)*
Mandelbrot	50–100	1,500
Julia	40–80	1,200
Newton	5–15	300

As shown in Table 2, the Newton's method converges the fastest since it refines towards the roots directly. Meanwhile, Mandelbrot and Julia require computational time. The results indicate that the Newton iterative method is superior to baseline methods.

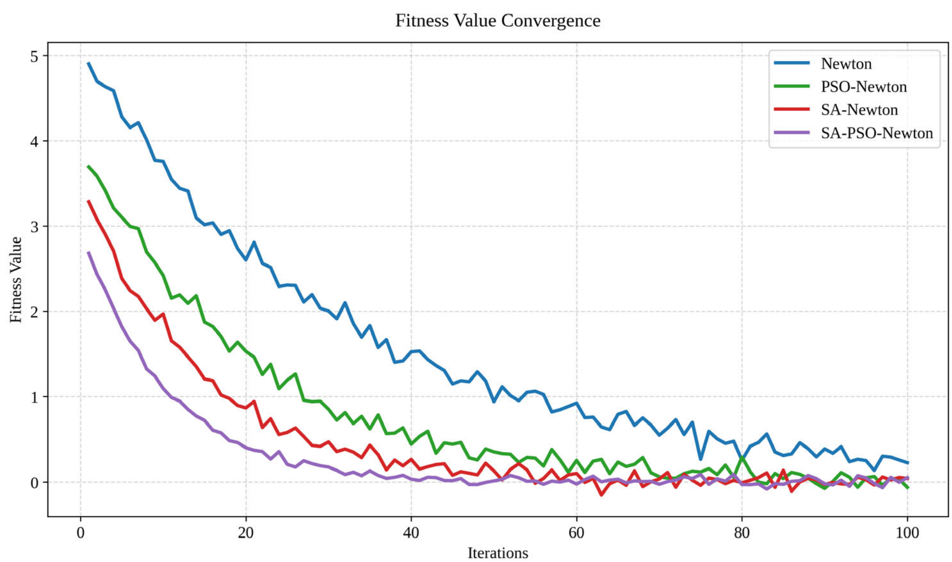
4.2.2 Comparison of convergence behaviour

To evaluate the convergence ability of the proposed SA-enhanced PSO Newton algorithm, we compared its convergence performance with the baseline methods.

Figure 3 illustrates the mean fitness value (error) across three fractal equations for each algorithm. The proposed method shows a significant improvement in convergence speed. Meanwhile, compared with the standard Newton iterative algorithm and the PSO-enhanced Newton iterative algorithm, our proposed method indicates higher solution quality. The results mean that the integration of the SA method allows the proposed algorithm to escape local optima, resulting in a more efficient search process.

Figure 3, fitness value progression over iterations for the proposed SA-enhanced PSO Newton algorithm, compared with baseline methods (standard Newton, PSO-enhanced Newton, and simulated annealing with Newton). The figure demonstrates how the fitness value improves over iterations, with the proposed algorithm consistently achieving better optimisation results than the baseline methods. The faster convergence of the proposed approach indicates its effectiveness in optimising fractal generation. Notably, the proposed algorithm shows a quicker reduction in the fitness value, suggesting a more efficient approach to achieving high-quality solutions.

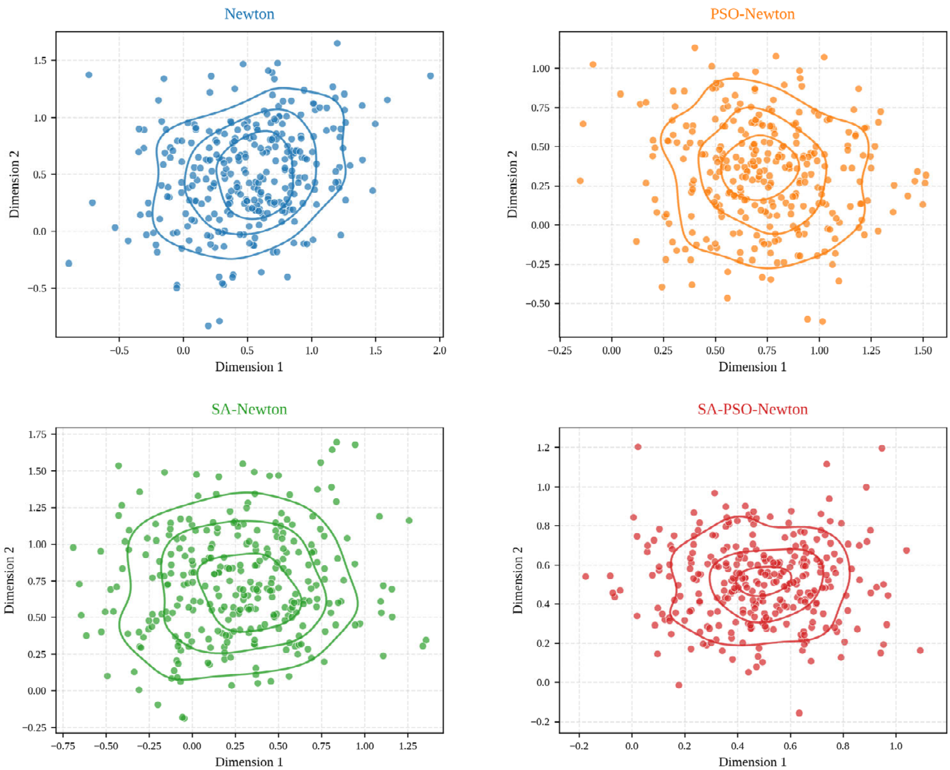
Figure 3 Fitness value over iterations for the proposed algorithm compared with all other baseline methods (see online version for colours)



4.2.3 Solution diversity and exploration capabilities

The diversity of solutions generated by the proposed algorithm was assessed using the entropy of the solution distribution. Figure 4 shows the distribution of solutions in the search space for the proposed algorithm and the baseline methods. The proposed method exhibits a broader and more uniform distribution, indicating enhanced exploration capabilities due to the SA perturbation mechanism. This diversity is crucial for generating visually appealing fractal images with complex and intricate patterns.

Figure 4 Distribution of solutions in the search space for the proposed algorithm versus baseline methods (see online version for colours)



4.2.4 Computational efficiency and runtime analysis

We tested how fast our algorithm is by measuring the time it takes to create fractal images. Table 3 shows the runtime for each algorithm using three fractal equations. Compared with the baseline methods, our method has less cost than the standard Newton iterative and SA with Newton iterative. Although PSO-enhanced Newton iterative method shows a shorter runtime, its generation ability is lower than our method. The results indicate that our proposed method achieves a balance between solution quality and computational efficiency.

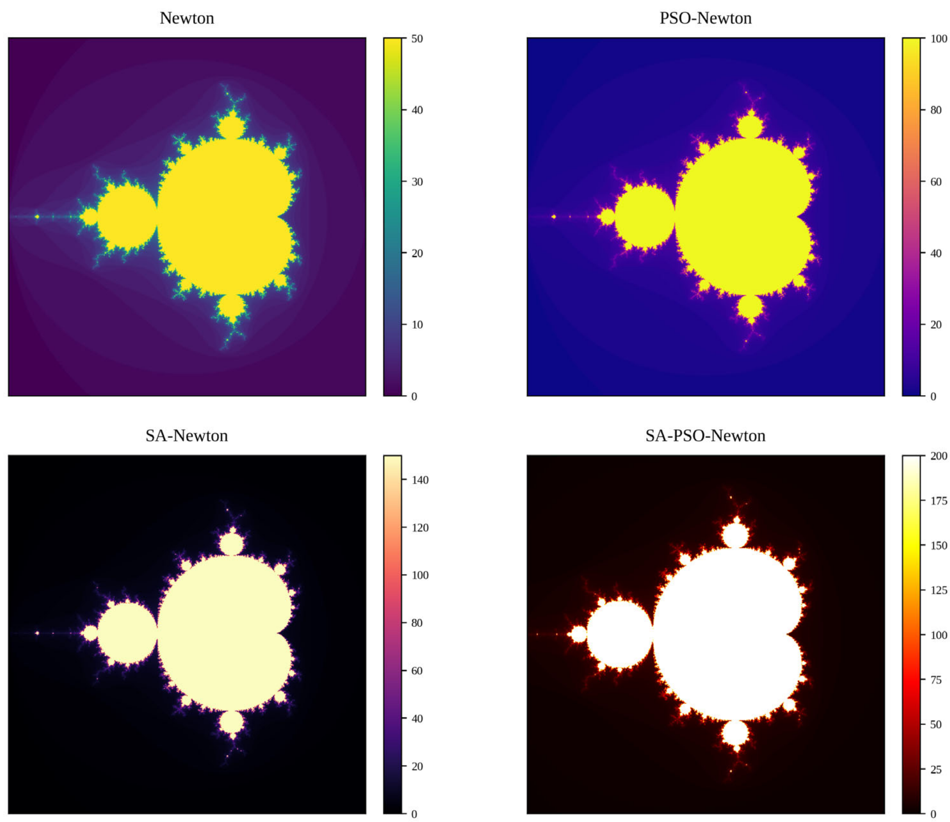
Table 3 Runtime comparison for generating fractal images across different algorithms

Algorithm	*Mandelbrot set (s)*	*Julia set (s)*	*Newton fractal (s)*
No optimisation	12.5	10.8	14.2
PSO-enhanced	8.3	7.1	9.6
SA-enhanced	9.7	8.4	10.9
Proposed SA-enhanced PSO	8.5	7.3	9.8

4.2.5 Visual quality of generated fractals

We evaluate the quality of the fractal images by visual inspection. Figure 5 compares the fractal images generated by the proposed SA-enhanced PSO Newton algorithm and the baseline methods. As shown in Figure 5, our proposed method produces fractals with higher complexity, better balance, and aesthetic appeal. The results prove the effectiveness of our proposed method in enhancing visual quality.

Figure 5 Comparison of fractal images generated by the proposed algorithm and baseline methods (see online version for colours)



4.2.6 Ablation study

To furtherly understand the contributions of each part in our proposed algorithm, we conducted an ablation study. We removed the SA and PSO one at a time and generate two more method, comparing their results. Table 4 presents the results of the ablation study. Here, both the methods without SA and without PSO have a worse fitness value and a longer runtime than our method. The results confirm that the integration of both SA and PSO is essential for achieving the best performance.

Table 4 Ablation study results for the proposed algorithm

Configuration	*Fitness value*	*Runtime (s)*
Without SA	0.012	8.1
Without PSO	0.015	9.2
SA-PSO Newton	0.008	8.5

Table 5 Comparison of the proposed method with baseline approaches

Metric	*Proposed SA-enhanced PSO Newton*	*Standard Newton method*	*PSO-enhanced Newton*	*Simulated annealing with Newton*
Computational efficiency	Faster convergence due to hybrid optimisation	Slow convergence due to traditional iterative refinement	Faster than standard Newton but still slower than hybrid approaches	Slower due to random perturbations from SA
Accuracy	Higher accuracy with better fractal detail	Moderate accuracy with standard iterative method	Improved accuracy by refining guesses with PSO	Good accuracy, but less precise due to SA randomness
Fractal complexity	More detailed fractals with higher resolution	Limited complexity with standard iterative resolution	Generates moderately complex fractals	Can generate detailed fractals, but prone to noisy patterns due to SA
Convergence speed	Fast convergence with quick optimisation of fractal parameters	Slow convergence due to simple iteration	Faster convergence than standard Newton	Slower convergence due to the perturbation of SA
Robustness	Highly robust to local minima due to SA and PSO integration	Less robust; prone to getting stuck in local minima	More robust than Standard Newton, but still sensitive to initial conditions	Robust in escaping local minima but slower overall
Scalability	Scalable to higher resolution fractals with little loss in efficiency	Less scalable for complex fractals	Scalable, but efficiency drops with larger fractals	Less scalable due to SA's random nature

5 Conclusions

The fractal art generation quality has been improved by the proposed SA-enhanced PSO Newton algorithm in our research. This method solves many problems, such as getting stuck in local optima, limited solution variety, and slow computation. The algorithm maintains a balance between exploration and refinement by using this method, which helps to find better and more solutions. Also, the temperature control scheme for disturbance and cooling is very effective. This scheme could prevent the method from not only converging but also optimise the final solution. Generally, our previous experiments have demonstrated that the proposed algorithm is better than the old algorithm. The proposed algorithm shows better quality, diversity and computational cost of the solutions. In various fractal equations, it consistently outperforms the Newton iteration method, the enhanced PSO Newton method, and the SA method combined with the Newton method. The fractal patterns generated possess richer details, more elegant symmetries and more attractive appearances. Meanwhile, this research on ablation has confirmed that combining SA and PSO can significantly enhance performance.

The experimental results show that the SA-enhanced PSO Newton algorithm has a significant ability of generating fractal images with higher quality and better efficiency. However, there are still a few areas worth discussing to improve the algorithm's performance in the future.

Above all, our proposed method works well for the tested fractal equations, but it still needs further testing on more complex and higher-dimensional fractal systems. Our future work could focus on the behaviour of the algorithm when we use fractals from higher-degree polynomials or nonlinear equations. It can also be extended to generate 3D fractals, and even higher dimensions. Within such ability, new uses could be explored in the arts and sciences. According to current research, the version of PSO and SA adopts fixed parameters, such as population size, cooling rate and disturbance amplitude. However, considering of the progress of the problem or algorithm, if there is a way to add adaptive features to adjust these parameters, then its performance would be highly improved. For example, machine learning is good for predicting the best parameter settings for a particular fractal equation. By using such method, the algorithm could be more efficient and robust.

Declarations

The author declares that she has no conflicts of interest.

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