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Abstract: In light of the drastic exhaustion of natural resources and increased environmental pollution, to promote the use of green products, the government subsidises them and levies taxes on non-green ones. This paper considers a dual-channel supply chain with a manufacturer selling a green product online and a substitutable non-green product offline using a retail channel. The price differential splits the market into two segments. The stochastic linear demand is modelled as a function of prices, green quality level, and sales effort level, considering government tariffs and demand leakage. A centralised decision model is investigated for the case of uniform distribution and distribution-free demand. It is shown that the green quality level and the total supply chain profit increase with an increase in demand leakage. Also, it reveals that with an increment in government subsidy and tax, the total supply chain profit and green quality level are higher in uniform distribution. [Received: 20 April 2023; Accepted: 9 October 2023]

Keywords: dual-channel supply chain; government tariffs; cannibalisation; uniform distribution; distribution-free.

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1 Introduction

Due to the proliferation of internet facilities, customers prefer to purchase products online instead of offline. Online channels are convenient for customers because many product choices exist in online channels. In 2021, there were approximately 185 million online customers in India. It will increase to 425 million by 2027 (https://www.statista.com/statistics/1191958/india-number-of-annual-online-shoppers/). As a result, most manufacturers are expanding their online outlets in addition to their retail channels. Such kind of channel structure is known as a dual-channel supply chain (DCSC). It creates more of the total supply chain (SC) profit as well as its members. According to Forrester Report 2000, an online channel is added by 68% of manufacturers (Allen et al., 2000). For example, Nike, Adidas, Dell, and many others sell their products through both channels. This dual-channel approach encompasses not only direct sales from manufacturers to consumers but also indirect distribution through retailers. Hence, the pricing strategy is the most important aspect that influences the SC profitability in the case of DCSC. Zhang et al. (2012) examined the pricing strategies under different power models in DCSC. Nowadays, the greening strategy has become a crucial research topic for manufacturers from the viewpoint of the environment.

In the last few decades, the preservation of the environment has been marked as the primary issue for manufacturers along with customers. The purpose of this study is to achieve a low-carbon economy with an aim to manufacture a green product (GP). For example, manufacturers produce LED lights instead of conventional bulbs. An LED bulb consumes 8 to 10 times less energy and 25 to 50 times longer life than a normal bulb (https://climaterealtalk.org/what-are-green-product-examples-16common-green-products-around-you/). In real life, there exist online platforms such as Greenhandle, Earthhero, and Ecohoy that offer green products (GPs) through online channels. The aim of such firms is to provide sustainable products whose substitutes are already present in the market. They want responsible consumerism and to educate and encourage people to use green products. Customers are also accepting such platforms and are even ready to pay more for sustainable products. In a report by Business Wire (2019), 50% of customers rank sustainability as the top five value drivers, and around one-third of consumers are willing to pay a premium for sustainable products. There are many more real-life examples that support online sales of GPs. So, in our model, we have a setting of offering a green product through an online channel and a non-green product through an offline channel. A DCSC provides products to customers with greater accessibility. This accessibility is crucial for green product sales because it ensures that environmentally conscious customers can easily access and purchase eco-friendly products. Moreover, a DCSC allows manufacturers to reduce carbon emissions, optimise logistics, reduce transportation emissions, and implement eco-friendly packaging practices. This is consistent with the eco-friendly nature of green products.

To encourage the manufacturing of a green product, the government may offer subsidies for those who purchase an eco-friendly product. Both tariffs are linear, and the amount of the incentive increases and decreases in proportion to the level of greenness of the product. For example, moving from a highly polluting car to a moderately polluting one may significantly impact emissions reduction, but moving from a moderately polluting car to a slightly less polluting one may yield a smaller environmental benefit. It can be costly to achieve the highest levels of environmental quality. Therefore, policies that provide incentives or subsidies for very high levels of green quality may have diminishing returns in terms of environmental benefits compared to focusing resources on more moderate improvements. The quadratic effect recognises that not all improvements in green quality are equally valuable to consumers and society, and it accounts for the fact that customers may have a diminishing willingness to pay for increasingly green products. The manufacturer pays the additional cost for the greening innovation. For example, the government subsidises manufacturers producing biodegradable packaging materials or recycling facilities. This promotes the use of eco-friendly packaging and results in a reduction of plastic waste. In contrast, government impose taxes on single-use plastic products or non-recyclable materials, discouraging the use of environmentally harmful packaging. In 2009, the Japanese Government invested 100,000 yen subsidy and tax reductions and exemptions were offered in 2009 to encourage the use of green vehicles (Hao et al., 2014). In June 2020, Accenture conducted a survey all over the world to investigate the impact of GPs on customers and found that approximately 80% of customers readily adopt GPs. Many companies like IBM, Toyota and Ford manufacture GPs.

In addition to government subsidy and dual-channel, we address price differentiation and demand leakage (or cannibalisation) in this study (Phillips, 2005; Raza, 2014; Raza and Govindaluri, 2019a, 2019b). Price differentiation is a principal tool of revenue management that brings extra profit. Price differentiation divides market demand into two or more price-differentiated segments by utilising customers' willingness to pay (Fiebig et al., 2010; Talluri and Van Ryzin, 2006; Phillips, 2005). Price fencing is used to make sure that consumers have the same level of willingness to pay, usually for products from the same segment. Still, the best-price fencing plans often fail to include consumers from purchasing from the lower-priced segments. This creates demand leakage for the higher-cost product. Utilising a price differential, segmentation is accomplished. Some recent studies have examined the impact of demand leakage on a company's revenue management decisions under joint pricing and inventory control policy (Zhang and Bell, 2010; Zhang et al., 2010).

The novelty of the current research lies in the fact that it considers a DCSC with a dedicated online channel for a GP independently managed by a manufacturer and an offline retail channel for a substitutable non-green product (NGP). Raza and Govindaluri (2019b) considered an online channel for the standard green product and an offline channel for the green product. Also, Raza and Govindaluri (2019a, 2019b) did not examine the effect of governmental tariffs in their studies. Moreover, this research explores how price differentiation affects market segmentation due to demand leakage, governmental subsidy and taxation policy for environmental purposes, product greenness, and the sales effort for the NGP. So, this study attempts to answer the following research questions:

1 How do government subsidies, taxes, and cannibalisation affect the pricing strategies of the GP and NGP in a DCSC under stochastic demand?

2 How do demand leakage, government subsidies, and taxes affect the optimal green quality level (GQL), sales effort level (SEL), sales prices, price differentiation, order quantity, and expected profits?

To address the aforementioned issues, we study a DCSC with the governmental financial intervention for subsidy and tax on the manufacturing of GP and NGP, respectively. Demand for each channel is stochastic and price-greening-sales-effort-dependent. The retailer selects the selling price and SEL for the NGP, whereas the GP's sales price and GQL are set by the manufacturer. In this study, two analytical decision models are studied:

1 centralised DCSC with uniform demand distribution

2 centralised DCSC with distribution-free demand.

To find the optimal solutions to the problem, a joint optimisation technique is employed. The worst-case analysis is considered in the case of the SC coordination problem (Gallego and Moon, 1993; Moon and Gallego, 1994), though not for the exact same problem studied in this paper.

The remaining sections of the paper are structured as follows. The relevant review of the literature is discussed in Section 2. Section 3 presents the model development. Section 4 introduces the model analysis and solution approach. The results and numerical examples are presented in Section 5. At last, in Section 6, we provide our concluding remarks and suggestions for future research work.

2 Literature review

The literature review has been summarised in three categories: SC models with government tariffs, price differentiation and cannibalisation, and the distribution-free approach.

2.1 SC models with government tariffs

The government's regulatory requirements and incentives for production and use are powerful motivators for promoting GPs. Government subsidies affect manufacturers' ability to produce GPs. According to Ghosh and Shah (2012), governments must offer manufacturers a subsidy to be green and increase product demand through lower prices. In a closed DCSC, Ma et al. (2013) describe how government consumption subsidies could enhance the SC's performance. Li et al. (2016) study a collaboration model in a green DCSC and discuss pricing and greening strategy in centralised and decentralised models, considering the greening level of a product as the manufacturer's decision. Liu et al. (2016) explain the effect of government subsidy on pricing strategies for the WEEE recycling market in a DCSC. Madani and Rasti-Barzoki (2017) investigate government tariff intervention in a SC. They show that the government subsidy raises the profits of the SC and government and the product's greenness. Zhu and He (2017) investigate the effect of the product categories with high green marginal costs and greenness competition on the decision variables in centralised and decentralised SC models. Yang and Xiao (2017) investigated the pricing and GQL of a GP considering government intervention under fuzzy uncertain demand. Hafezalkotob (2018) evaluates six alternative energy-saving programs and utilises them to study the two forms of centralised and decentralised green SCs. Zhang et al. (2019) examine the impact of subsidies and taxes on producing GP for an environmentally conscious market. He and Chen (2021) investigate the impact of government subsidy programs to find the effect of subsidies on consumers. Gao et al. (2021) investigate the effects of manufacturer subsidies and eco-label laws on a DCSC where two different types of GPs are manufactured using various green technologies. Barman et al. (2021) explored the pricing and greening strategies for two manufacturers and one retailer to maximise the SC profit under government subsidy and tax. Mondal and Giri (2022) study the effects of various pricing strategies, product collection rates, marketing initiatives, and green innovation in a two-period CLSC having a single manufacturer and a single retailer. Guo et al. (2021) study the subsidy issue for SC disruption. Government subsidies for manufacturers and retailers were analysed, and it was determined that manufacturer subsidies are more advantageous than retailer subsidies. In a DCSC, stochastic demand was studied by Fander and Yaghoubi (2021). Vehicles as green and non-green items were taken into account in their analysis of the issue. Chen et al. (2022) investigate two strategies of government subsidy for maximising government utility along with social welfare. They concluded that the consumer would benefit more from a policy that maximises government utility. Li et al. (2022) investigate the influence of government subsidies on the manufacturer's productivity. They found that the government subsidy enhances the productivity of the manufacturer. Pal et al. (2023) investigated the promotional effort in a DCSC considering greening innovation levels. Li and Shan (2023) analysed the pricing and greening strategies in a DCSC considering free-riding. Das et al. (2023) considered the SC coordination problem in a DCSC to study the pricing and greening strategies. Zhao et al. (2023) examined the effect of greenness in a DCSC consisting of a single retailer and multiple manufacturers.

These studies have concentrated on greening government-tariffed items within a SC. Ma et al. (2013) and Li et al. (2016) studied a single new and GP, respectively. Liu et al. (2016) and Gao et al. (2021) considered two GPs. Liu et al. (2016) and Fander and Yaghoubi (2021) did not include any direct online channel in their study. The previous studies have not examined the combined effect of the pricing and government tariffs in DCSC. We considered selling GP through an online channel and NGP through an offline channel. As we know, a manufacturer has to increase its investment to produce more GP, so we are considering selling GP through an online channel. Here, the manufacturer reduces the overall cost of GP by managing transportation, inventory, and service costs.

Also, the manufacturer tries to increase the customers' awareness towards GP to improve future sales, which further leads to the preservation of the environment. In contrast, in the previous studies, they incorporated offline channels for the same. The offline channel results in carbon emissions, leading to the decay of the environment. Also, they have to pay an extra tax imposed by the government for not supporting GP. In addition, there is a paucity of relevant literature in the DCSC considering the impact on pricing strategies of governmental tariffs (subsidy and tax rate) for substitute products (GP and NGP). In this way, our work proposes dual benefits of government subsidies along with environmental sustainability. This helps to reduce the selling price of GP.

2.2 Price differentiation and cannibalisation

In this study, the impact of cannibalisation or demand leakage is incorporated. Due to price differences between products, market segmentation is generated. Consequently, customers switch to lower price channels from higher price channels to purchase products. This type of customer behaviour generates demand leakage. In order to manage the expected market share of various market categories, Phillips (2005) used price differentiation as a technique. Differentiating prices in different market segments has been shown to increase profits (Talluri and Van Ryzin, 2006; Phillips, 2005). Zhang and Bell (2007) and Zhang et al. (2010) contributed to the research by investigating the consequences of demand leakages across segments and presenting fencing strategies to reduce leakages. They also considered risk-averse SC, cost-sharing and revenue-sharing contracts for green coordination. Raza (2014) generalised Phillips' (2005) method for stochastic demand under both full and partial demand distribution information. Raza and Govindaluri (2019b) explored the coordination of a DCSC with revenue and cost-sharing contracts for standard and regular products. They investigated the effect of risk aversion, leakage in demand, and uncertainty in the market. Also, Raza and Govindaluri (2022) investigated the effect of demand leakage for a single product under sequentially observed stochastic demand.

In the previous literature, no more research exists on the dual-channel SC under price differentiation, cannibalisation effect, and government intervention in the case of green and regular products, except for Raza and Govindaluri (2019b). But, they assumed that the GP and NGP are sold through the retailer and the manufacturer, respectively. The purpose of consideration of price differentiation is to manage the demand leakage of GP. Also, the consideration of DCSC with the same setting where the GP and NGP are sold through the manufacturer and the retailer is completely absent in the literature. In our study, price differentiation has been incorporated to mitigate the demand leakage of GP from the online channel to the offline channel. This reduction leads to an increment in the demand for GP, which automatically triggers a reduction in the sales of NGP. Furthermore, this helps in the preservation of our environment.

2.3 Distribution-free approach

In practice, the stochastic demand does not depend on known distribution functions with no available complete information regarding the distribution function. The distribution-free approach is used for worst-case analysis in such cases. For example, Scarf (1958) addressed the newsvendor problem with complete information on mean and variance. Using the minimax approach, he tackled the challenge by attempting to minimise the maximum cost caused by the worst conceivable demand distribution. After a long time, Gallego and Moon (1993) and Moon and Gallego (1994) investigated the aspects of Scarf's rule to extend the idea for the four cases, including recourse, fixed ordering cost, random yields, and multiproduct. Moon and Choi (1994, 1995) extended the model of Gallego and Moon (1993) to develop inventory models based on unknown distribution under stochastic and deterministic demands. Liao et al. (2011) examined a single-period newsvendor model for combined baulking behaviour, shortage penalty and fixed ordering costs. They assumed the demand distribution's mean and standard deviation were known without assuming a specific distributional form. Lee and Hsu (2011) investigated the impact of advertising by developing a distribution-free newsboy model in which the demand's mean and variance are only known. Raza and Govindaluri (2019a) described the channel coordination that integrates price differentiation and cannibalisation effect for the two products (green and regular). They considered the stochastic demand and distribution-free approach.

To date, the distribution-free approach has been the subject of very limited research in green DCSCs. In light of this examination of the relevant literature, the subsequent research gaps are found:

1. There is a dearth of literature on the topic of two substitutable goods subject to government tariffs in a DCSC.
2. Previous studies examined price differentiation and cannibalisation in a single-channel SC, except Raza and Govindaluri (2019b). No similar research has been conducted in a DCSC that handles both GP and NGP offered through a retailer and a manufacturer.
3. The effect of price differentiation and cannibalisation has not been investigated using a distribution-free approach in a DCSC.

3 Model development

In this work, we consider a DCSC having a retailer and a manufacturer where the production of a green substitutable product and a traditional NGP is done by a manufacturer who uses an offline retail channel for the NGP and a direct online channel for the GP. Figure 1 shows the crucial structure of the DCSC and the apparent movement of the products with subsidy on the GP and tax on the NGP. Here, the manufacturer decides the selling price, GQL and order quantity of the GP, whereas the retailer decides the selling price, order quantity and the SEL for NGP, along with price differentiation. The government is responsible for propelling the production of green sustainable products. Usually, the government supports GPs by providing subsidies to lower the final price of products, encouraging customers to buy GPs. In contrast, the government penalises by taxing NGPs in order to make up for environmental damages, which lessens the incentives for consumers to purchase NGPs. The notation for the parameters, superscripts, subscripts and the decision variables in the model is shown in Table 1.

Figure 1 Dual-channel SC framework

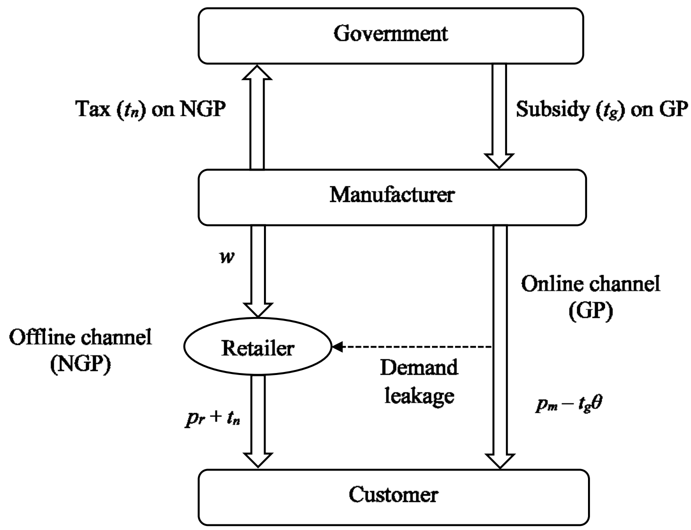


Table 1 List of parameters, superscripts, subscripts and decision variables

Notation	*Description*
a	Basic market demand
b_1	Coefficient of price elasticity of demand
α	Demand leakage rate, $0 \leq \alpha \leq 1$
c_1	Manufacturing cost per unit of the green product
c_2	Manufacturing cost per unit of the non-green product
w	Wholesale price per unit, assumed the same for green and non-green products
τ	Elastic coefficient of sales effort, $0 \leq \tau \leq 1$
β	Expansion effectiveness coefficient of green quality level per unit of green product, $0 \leq \beta \leq 1$
η	Cost coefficient of the green quality level per unit
k	Cost coefficient of sales effort level per unit
t_g	Subsidy rate for the green degree for a unit of green product
t_t	Tax rate for a unit of non-green product
h_i	Holding cost for each unsold product in channel i, $i = \{m, r\}$
s_i	Shortage cost for each unmet demand in channel i, $i = \{m, r\}$
μ_i	Mean of stochastic demand factor ε_i, where $\varepsilon_i \in [\underline{\varepsilon}_i, \overline{\varepsilon}_i]$, $\forall i = \{m, r\}$
σ_i	Standard deviation of stochastic demand factor, ε_i, $\sigma_i \geq 0$, $i = \{m, r\}$
y_m	Online channel's deterministic demand
y_r	Offline channel's deterministic demand

Table 1 List of parameters, superscripts, subscripts and decision variables (continued)

Notation	*Description*
D_m	Online channel's stochastic demand
D_r	Offline channel's stochastic demand
$f_i(\cdot)$	Probability distribution function for stochastic demand, $i = \{m, r\}$
$F_i(\cdot)$	Cumulative distribution function for stochastic demand, $i = \{m, r\}$
$E(.)$	Expected profit function
π_i	Profit of manufacturer and retailer, $i = m$ and r
$\pi = \pi_m + \pi_r$	Total profit of the centralised dual-channel supply chain
Superscripts	
$\sim$	Centralised dual-channel supply chain for distribution-free approach
$\frown$	Centralised dual-channel supply chain for distribution-free approach while using a worst case analysis
Subscripts	
m	Manufacturer
r	Retailer
Decision variables	
p_m	Selling price of the green product (manufacturer's decision)
p_r	Selling price of the non-green product (retailer's decision)
θ	Green quality level of the green product (manufacturer's decision)
s	Sales-effort level of the retailer (retailer's decision)
Q_m	Order quantity of green product (manufacturer's decision)
Q_r	Order quantity of non-green product (retailer's decision)
v	Price differentiation, $v \geq 0$ (retailer's decision)

We develop a mathematical model for a DCSC subject to stochastic demand under demand leakage (cannibalisation), government subsidies for GP, and taxes on NGP. The online and offline channel's demand functions for GP and NGP are given by equations (1) and (2), respectively

$$y_m(p_m) = (1-\alpha)(a - b_1 p_m) \tag{1}$$

$$y_r(p_m, p_r, v) = \alpha(a - b_1 p_m) + b_1(v - p_r) \tag{2}$$

Here, a represents the basic demand, b_1 represents the coefficient of price elasticity of demand, and α $(0 \leq \alpha \leq 1)$ denotes demand leakage between two products. Considering all, the linear demand function is expressed as given in equations (1) and (2) (Phillips, 2005; Zhang et al., 2010). In equation (1), $a - b_1 p_m$ represents the deterministic demand component dependent on online selling price p_m and GQL. The term $b_1 p_m$ represents the price elasticity for the demand in the online channel. Moreover, in equation (2), v is used to denote the price differentiation which divides the market demand into two parts such that $p_m \geq v \geq p_r$ and $v \geq 0$ (Phillips, 2005; Raza and Rathinam, 2017; Raza et al., 2018). In addition, $b_1(v - p_r)$ is non-cannibalised demand experienced by the offline channel.

After considering the product's GQL, θ, and SEL, s of the retailer, the demand for each channel can be written as

$$y_m(p_m, \theta) = (1-\alpha)(a - b_1 p_m + \beta\theta) \tag{3}$$

and

$$y_r(p_m, \theta, v, p_r, s) = \alpha(a - b_1 p_m + \beta\theta) + b_1(v - p_r) + \tau s \tag{4}$$

To promote GP, the government pays $t_g\theta$ per unit subsidy to the manufacturer, and customers have to effectively pay only $p_m - t_g\theta$ to purchase the GP. Also, for compensating the financial loss and social cost of NGP, government charges a penalty of t_n per unit on the NGP as a tax, and customers are made to pay $p_r + t_n$ to purchase the NGP. It can be noted that the government's income from imposing taxes on NGP would be devoted to recompensating damages to nature by subsidising GP (Ghosh and Shah, 2012; Madani and Rasti-Barzoki, 2017). Thus, after considering government intervention for online and offline channels, the demand functions can be expressed as

$$y_m(p_m, \theta) = (1-\alpha)\left[a - b_1(p_m - t_g\theta) + \beta\theta\right] \tag{5}$$

and

$$y_r(p_m, \theta, v, p_r, s) = \alpha\left[a - b_1(p_m - t_g\theta) + \beta\theta\right] + b_1\left[v - (p_r + t_n)\right] + \tau s \tag{6}$$

Accordingly, we have

$$p_m - t_g\theta \geq v \tag{7}$$

and

$$v \geq p_r + t_n \tag{8}$$

In the literature, the randomness in demand is modelled in two ways, additive and multiplicative (Mills, 1959; Petruzzi and Dada, 1999). In this study, the exogenous random variables of additive type are considered for both online and offline demands due to their inherent simplicity. Thus, the stochastic demand functions can be obtained as

$$D_i = y_i + \varepsilon_i, \qquad \forall i \in \{m, r\} \tag{9}$$

where $\varepsilon_i \in [\underline{\varepsilon}_i, \overline{\varepsilon}_i]$ is a continuous twice differentiable random variable with probability distribution function $f(\varepsilon_i)$, cumulative distribution function $F(\varepsilon_i)$, the expected value $E(\varepsilon_i) = 0$ and the standard deviation σ_i. We consider h_i as the holding cost for each unsold product and s_i as the shortage penalty for each unmet demand in channel i. Thus, the manufacturer's expected profit function can be formulated as

$$\begin{aligned} E\left[\pi_m(p_m, \theta, Q_m)\right] &= (p_m - t_g\theta)\min(Q_m, D_m) - c_1 Q_m - h_m E[Q_m - D_m]^+ \\ &\quad - s_m E[D_m - Q_m]^+ + (w - c_2)Q_r - \frac{1}{2}\eta\theta^2 \end{aligned} \tag{10}$$

The manufacturer pays the extra amount for green quality to produce the product is $\frac{1}{2}\eta\theta^2$, where η the price coefficient of green quality per unit (Ghosh and Shah, 2012; Swami and Shah, 2013).

According to Gallego and Moon (1993) and AlFares and Elmorra (2005), we have

$$\min(Q_i, D_i) = Q_i - E[Q_i - D_i]^+, \qquad \forall i \in \{m, r\} \tag{11}$$

and

$$E[D_i - Q_i]^+ = y_i - Q_i + E[Q_i - D_i]^+, \qquad \forall i \in \{m, r\} \tag{12}$$

Using the above relationships in equation (10), we get

$$\begin{aligned} E\left[\pi_m\left(p_m, \theta, Q_m\right)\right] = &\left(p_m - t_g\theta - c_1 + s_m\right)Q_m \\ &-\left(p_m - t_g\theta + h_m + s_m\right)E\left[Q_m - D_m\right]^+ \\ &+\left(w - c_2\right)Q_r - s_m y_m - \frac{1}{2}\eta\theta^2 \end{aligned} \tag{13}$$

The retailer's expected profit function can then be obtained as

$$\begin{aligned} E\left[\pi_r\left(p_r, s, v, Q_r\right)\right] = &\left(p_r + t_n\right)\min\left(Q_r, D_r\right) - wQ_r - h_r E\left[Q_r - D_r\right]^+ \\ &-s_r E\left[D_r - Q_r\right]^+ - \frac{1}{2}sk^2 \end{aligned} \tag{14}$$

Using equations (11) and (12) in equation (14), the retailer's expected profit function can be obtained as

$$\begin{aligned} E\left[\pi_r\left(p_r, s, v, Q_r\right)\right] = &\left(p_r + t_n - w + s_r\right)Q_r - \left(p_r + t_n + h_r + s_r\right)E\left[Q_r - D_r\right]^+ \\ &-s_r y_r - \frac{1}{2}ks^2 \end{aligned} \tag{15}$$

As a result, the DCSC's total expected profit equals the sum of the manufacturer's and retailer's expected profit, that is

$$E\left[\pi\left(p_m, p_r, \theta, s, v, Q_r, Q_m\right)\right] = E\left[\pi_m\left(p_m, \theta, Q_m\right)\right] + E\left[\pi_r\left(p_r, s, v, Q_r\right)\right].$$

Using equations (13) and (15), the total expected profit of the DCSC can be written as

$$\begin{aligned} E\left[\pi\left(p_m, p_r, \theta, s, v, Q_r, Q_m\right)\right] = &\left(p_m - t_g\theta - c_1 + s_m\right)Q_m \\ &-\left(p_m - t_g\theta + h_m + s_m\right)E\left[Q_m - D_m\right]^+ \\ &+\left(p_r + t_n - c_2 + s_r\right)Q_r \\ &-\left(p_r + t_n + h_r + s_r\right)E\left[Q_r - D_r\right]^+ \\ &-s_m y_m - \frac{1}{2}\eta\theta^2 - s_r y_r - \frac{1}{2}ks^2 \end{aligned} \tag{16}$$

4 Model analysis and solution approach

In this section, a vertically integrated and centralised DCSC is considered, with one decision maker ensuring an optimal value for every decision variable by maximising the DCSC's total profit. We analyse the DCSC to derive optimal solutions with two types of distributions:

1 uniform distribution

2 distribution-free.

4.1 Model analysis with uniform distribution

In this section, we assume that the demand follows uniform distribution to derive the optimal solutions for the DCSC. Uniform distribution is an appropriate distribution if there is insufficient information to adequately characterise the demand distribution (Lodree and Taskin, 2008). Also, uniform distribution makes the mathematical model tractable and easy to analyse (Nikkhoo et al., 2018; Liu et al., 2022). The present mathematical model is a nonlinear constrained optimisation problem. By using the joint optimisation technique, the stochastic problem has been solved. In order to find the optimal decisions for prices (p_m^* and p_r^*), GQL (Q^*), sales-effort (s^*), and order quantities (Q_m^* and Q_r^*), a system of equations can be obtained using Karush-Kuhn-Tucker (KKT) conditions, which can be further solved using numerical methods. A closed-form solution can also be derived for uniformly distributed price and greenness-dependent stochastic demand. In equation (16), $E[Q_i - D_i]^+$ can be derived as follows.

$$E[Q_i - D_i]^+ = \int_{\underline{\varepsilon}_i}^{Q_i - y_i} (Q_i - D_i) f_i(\varepsilon_i) d\varepsilon_i, \qquad \forall i \in \{m, r\}.$$

Putting $D_i = y_i + \varepsilon_i$ and applying integration by parts (Chen et al., 2004, 2006), $E[Q_i - D_i]^+$ can be written as

$$\int_{\underline{\varepsilon}_i}^{Q_i - y_i} (Q_i - y_i - \varepsilon_i) \frac{dF_i(\varepsilon_i)}{d\varepsilon_i} d\varepsilon_i = \left[(Q_i - y_i - \varepsilon_i) F_i(\varepsilon_i) \right]_{\underline{\varepsilon}_i}^{Q_i - y_i} - \int_{\underline{\varepsilon}_i}^{Q_i - y_i} \frac{d(Q_i - y_i - \varepsilon_i)}{d\varepsilon_i} F_i(\varepsilon_i) d\varepsilon_i.$$

As $F_i(\underline{\varepsilon}_i) = 0,$ thus

$$E[Q_i - D_i]^+ = \int_{\underline{\varepsilon}_i}^{Q_i - y_i} F_i(\varepsilon_i) d\varepsilon_i, \qquad \forall i \in \{m, r\}. \tag{17}$$

Using equation (17) for $E[Q_m - D_m]^+$ in equation (16), the total expected profit can be expressed as

$$\begin{aligned} E\left[\pi\left(p_m, p_r, \theta, s, v, Q_m, Q_r\right)\right] = &\left(p_m - t_g\theta - c_1 + s_m\right)Q_m \\ &-\left(p_m - t_g\theta + h_m + s_m\right)\int_{\underline{\varepsilon}_m}^{Q_m - y_m} F_m\left(\varepsilon_m\right)d\varepsilon_m \\ &+\left(p_r + t_n - c_2 + s_r\right)Q_r \\ &-\left(p_r + t_n + h_r + s_r\right)\int_{\underline{\varepsilon}_r}^{Q_r - y_r} F_r\left(\varepsilon_r\right)d\varepsilon_r \\ &-s_m y_m - \frac{1}{2}\eta\theta^2 - s_r y_r - \frac{1}{2}ks^2 \end{aligned} \tag{18}$$

To decide the optimal results of order quantities of GP and NGP (Q_m and Q_r), we set the first-order-derivatives of equation (18) with respect to Q_m and Q_r to zero and get

$$\frac{dE(\pi)}{dQ_m} = \left(p_m - t_g\theta - c_1 + s_m\right) - \left(p_m - t_g\theta + h_m + s_m\right)F_m\left(Q_m - y_m\right) = 0 \tag{19}$$

and

$$\frac{dE(\pi)}{dQ_r} = \left(p_r + t_n - c_2 + s_r\right) - \left(p_r + t_n + h_r + s_r\right)F_r\left(Q_r - y_r\right) = 0. \tag{20}$$

From equation (19), we have

$$F_m\left(Q_m - y_m\right) = \frac{p_m - t_g\theta - c_1 + s_m}{p_m - t_g\theta + h_m + s_m}. \tag{21}$$

Therefore, the expression for Q_m can be written as

$$Q_m = y_m + F_m^{-1}\left(\frac{p_m - t_g\theta - c_1 + s_m}{p_m - t_g\theta + h_m + s_m}\right). \tag{22}$$

Let

$$\phi_m = \frac{p_m - t_g\theta - c_1 + s_m}{p_m - t_g\theta + h_m + s_m}. \tag{23}$$

Thus,

$$Q_m = y_m + F_m^{-1}\left(\phi_m\right). \tag{24}$$

Similarly, from equation (20)

$$F_r\left(Q_r - y_r\right) = \frac{p_r + t_n - c_2 + s_r}{p_r + t_n + h_r + s_r}. \tag{25}$$

Therefore, the expression for Q_r can be written as

$$Q_r = y_r + F_r^{-1}\left(\frac{p_r + t_n - c_2 + s_r}{p_r + t_n + h_r + s_r}\right) \tag{26}$$

Let

$$\phi_r = \frac{p_r + t_n - c_2 + s_r}{p_r + t_n + h_r + s_r} \tag{27}$$

Thus,

$$Q_r = y_r + F_r^{-1}\left(\phi_r\right). \tag{28}$$

$E[\pi]$ will be concave in Q_m and Q_r, provided the following Hessian matrix is negative definite.

$$H = \begin{bmatrix} \dfrac{d^2E(\pi)}{dQ_m^2} & \dfrac{d^2E(\pi)}{dQ_m dQ_r} \\ \dfrac{d^2E(\pi)}{dQ_r dQ_m} & \dfrac{d^2E(\pi)}{dQ_r^2} \end{bmatrix}.$$

Substituting the expressions for each element, we get

$$H\left[\pi\left(Q_m, Q_r\right)\right] = \begin{bmatrix} -\left(p_m - t_g\theta + h_m + s_m\right) f_m\left(Q_m - y_m\right) & 0 \\ 0 & -\left(p_r + t_n + h_r + s_r\right) f_r\left(Q_r - y_r\right) \end{bmatrix}.$$

Thus, the total expected profit $E[\pi]$ will be concave with respect to Q_m and Q_r if

$$\left(p_m - t_g\theta + h_m + s_m\right)\left(p_r + t_n + h_r + s_r\right) f_m\left(Q_m - y_m\right) f_r\left(Q_r - y_r\right) > 0. \tag{29}$$

Now, to calculate optimal decision variable values, we consider the problem

Max $E\left[\pi\left(p_m, p_r, \theta, s, v, Q_r, Q_m\right)\right]$ in equation (18)

Subject to: constraints (7) and (8).

First, we have to convert the above constrained nonlinear optimisation problem into a Lagrange function as follows.

$$\begin{aligned} L\left(p_m, p_r, Q_m, Q_r, \theta, s, v, \lambda_m, \lambda_r\right) = E\left[\pi\left(p_m, p_r, \theta, s, v, Q_r, Q_m\right)\right] \\ + \lambda_m\left(p_m - t_g\theta - v\right) + \lambda_r\left(v - p_r - t_n\right). \end{aligned} \tag{30}$$

Now, using KKT conditions (Bertsekas, 1999), we obtain the following equations.

$$\begin{aligned} \frac{dL}{dp_m} = Q_m - \int_{\varepsilon_m}^{Q_m - y_m} F_m\left(\varepsilon_m\right) d\varepsilon_m + \left(p_m - t_g\theta + h_m + s_m\right) F_m\left(Q_m - y_m\right)\frac{dy_m}{dp_m} \\ - s_m\frac{dy_m}{dp_m} + \left(p_r + t_n + h_r + s_r\right) F_r\left(Q_r - y_r\right) - s_r\frac{dy_r}{dp_m} + \lambda_m \\ = 0, \end{aligned} \tag{31}$$

$$\frac{dL}{dp_r} = Q_r - \int_{\underline{\varepsilon}_r}^{Q_r - y_r} F_r(\varepsilon_r)\,d\varepsilon_r + (p_r + t_n + h_r + s_r) F_r(Q_r - y_r)\frac{dy_m}{dp_m} - s_r \frac{dy_r}{dp_r} - \lambda_r = 0, \tag{32}$$

$$\frac{dL}{d\theta} = -t_g Q_m + t_g \int_{\underline{\varepsilon}_m}^{Q_m - y_m} F_m(\varepsilon_m)\,d\varepsilon_m + (p_m - t_g\theta + h_m + s_m) F_m(Q_m - y_m)\frac{dy_m}{d\theta} - s_m \frac{dy_m}{d\theta} - \eta\theta + (p_r + t_n + h_r + s_r) F_r(Q_r - y_r)\frac{dy_r}{d\theta} - s_r \frac{dy_r}{d\theta} - \lambda_m t_g = 0, \tag{33}$$

$$\frac{dL}{ds} = (p_r + t_n + h_r + s_r) F_r(Q_r - y_r)\frac{dy_r}{ds} - s_r \frac{dy_r}{ds} - ks = 0, \tag{34}$$

$$\frac{dL}{dQ_m} = (p_m - t_g\theta - c_1 + s_m) - (p_m - t_g\theta + h_m + s_m) F_m(Q_m - y_m) = 0, \tag{35}$$

$$\frac{dL}{dQ_r} = (p_r + t_n - c_2 + s_r) - (p_r + t_n + h_r + s_r) F_r(Q_r - y_r) = 0, \tag{36}$$

$$\frac{dL}{dv} = (p_r + t_n + h_r + s_r) F_r(Q_r - y_r)\frac{dy_r}{dv} - s_r \frac{dy_r}{dv} - \lambda_m + \lambda_r = 0, \tag{37}$$

$$\lambda_m [p_m - t_g\theta - v] = 0, \tag{38}$$

and

$$\lambda_r [v - p_r - t_n] = 0, \tag{39}$$

where $\lambda_m \geq 0$, $\lambda_r \geq 0$.

Further, based on the values of λ_m and λ_r, the following four cases are possible:

- Case 1: if $\lambda_m > 0$, $\lambda_r > 0$

  In this condition, from equations (38) and (39), we can conclude that $v = p_m - t_g\theta$ and $v = p_r + t_n$, which fails to achieve price differentiation between GP and NGP, therefore, this case is discarded.

- Case 2: if $\lambda_m = 0$, $\lambda_r = 0$

  In this case, it is clearly observed that from equations (38) and (39), the pricing-related parameters are unrestricted, and this may lead to an infeasible solution. That is why it is not preferred.

- Case 3: if $\lambda_m > 0$, $\lambda_r = 0$

  Using $F_r(Q_r - y_r)$ from equation (25) and $\frac{dy_r}{dv} = b_1$ from equation (6) in equation (37), we have

$$\lambda_m = b_1\left(p_r + t_n - c_2\right) > 0. \tag{40}$$

And if $v > p_r + t_n$, then from equation 39, $\lambda_r = 0$.

So, in this case, there is a unique solution.

- Case 4: if $\lambda_m = 0$, $\lambda_r > 0$

  Similar to case 3, using $F_r(Q_r - y_r)$ from equation (25) and $\frac{dy_r}{dv} = b_1$ from equation (6) in equation (37), we get

$$\lambda_r = -b_1\left(p_r + t_n - c_2\right) \tag{41}$$

  It fails to meet the condition $\lambda_r > 0$. Therefore, this case is also discarded.

Now, with $\mu_i = 0$ and $\underline{\varepsilon}_i = 0, \forall i \in \{m, r\}$, the expression $\int_{\underline{\varepsilon}_i}^{Q_i - y_i} F_i(\varepsilon_i)d\varepsilon_i$ for uniform distribution can be derived as

$$\int_0^{Q_i - y_i} F_i\left(\varepsilon_i\right)d\varepsilon_i = \frac{Q_i - y_i}{2\sqrt{3}\sigma_i}\left(\frac{Q_i - y_i}{2}\right) + \frac{Q_i - y_i}{2}, \qquad \forall i \in \{m, r\} \tag{42}$$

Using equations (24) and (28) for Q_m and Q_r, respectively, in equation (42), we obtain

$$\int_0^{Q_i - y_i} F_i(\varepsilon_i)d\varepsilon_i = \frac{F_i^{-1}\left(\phi_i\right)}{2}\left(\frac{F_i^{-1}\left(\phi_i\right)}{2\sqrt{3}\sigma_i} + 1\right), \qquad \forall i \in \{m, r\}. \tag{43}$$

From equations (5) and (6), we have

$$\begin{aligned} &\frac{dy_m}{dp_m} = -b_1(1-\alpha), \frac{dy_m}{d\theta} = (1-\alpha)\left(b_1 t_g + \beta\right), \\ &\frac{dy_r}{dp_m} = -b_1\alpha, \frac{dy_r}{dp_r} = -b_1, \frac{dy_r}{d\theta} = \alpha\left(\beta + b_1 t_g\right), \frac{dy_r}{ds} = \tau \end{aligned} \tag{44}$$

Thus, using $\lambda_r = 0$ and $\lambda_m = b_1(p_r + t_n - c_2)$ from case 3, $F_i(Q_i - y_i)$, $\forall i \in \{m, r\}$ from equations (24) and (28), and equations (43) and (44) in equations (31) to (34), we get

$$\begin{aligned} &y_m + F_m^{-1}\left(\phi_m\right) - \frac{F_m^{-1}\left(\phi_m\right)}{2}\left(\frac{F_m^{-1}\left(\phi_m\right)}{2\sqrt{3}\sigma_m} + 1\right) - b_1(1-\alpha)\left(p_m - t_g\theta + h_m + s_m\right)\phi_m \\ &\quad + b_1 s_m(1-\alpha) + \left(p_r + t_n + h_r + s_r\right)\phi_r + b_1\alpha s_r + b_1\left(p_r + t_n - c_2\right) = 0, \end{aligned} \tag{45}$$

$$\begin{aligned} &y_r + F_r^{-1}\left(\phi_r\right) - \frac{F_r^{-1}\left(\phi_r\right)}{2}\left(\frac{F_r^{-1}\left(\phi_r\right)}{2\sqrt{3}\sigma_r} + 1\right) - b_1(1-\alpha)\left(p_r + t_n + h_r + s_r\right)\phi_r + b_1\alpha s_r \\ &\quad = 0, \end{aligned} \tag{46}$$

$$
\begin{aligned}
&-t_g\left[y_m+F_m^{-1}(\phi_m)\right]+\frac{F_m^{-1}(\phi_m)}{2}\left(\frac{F_m^{-1}(\phi_m)}{2\sqrt{3}\sigma_m}+1\right)t_g\\
&\quad+(1-\alpha)(b_1t_g+\beta)(p_m-t_g\theta-c_1+s_m)\phi_m-s_m(1-\alpha)(b_1t_g+\beta)-\eta\theta\\
&\quad+\alpha(\beta+b_1t_g)(p_r+t_n+h_r+s_r)\phi_r-\alpha(\beta+b_1t_g)s_r-b_1(p_r+t_n-c_2)t_g=0
\end{aligned}
\tag{47}
$$

and

$$
\tau(p_r+t_n+h_r+s_r)\phi_r-s_r\tau-ks=0. \tag{48}
$$

It is challenging to derive a closed-form solution due to the inherent complexity. Therefore, to find the optimal selling prices of GP and NGP (p_m^* and p_r^*), GQL (θ^*), and sales-effort (s^*), we solve the simultaneous equations (45) to (48). Then, the optimal order quantity of GP and NGP is obtained using the following equation.

$$
Q_i^*=y_i^*+F^{-1}(\phi_i^*),\qquad \forall i\in\{m,r\},
$$

where

$$
\phi_m^*=\frac{p_m^*-t_g\theta^*-c+s_m}{p_m^*-t_g\theta^*+h_m+s_m},
$$

$$
\phi_r^*=\frac{p_r^*+t_n-c+s_r}{p_r^*+t_n+h_r+s_r},
$$

$$
y_m^*(p_m^*,\theta^*)=(1-\alpha)(a-b_1p_m^*+\beta\theta^*)
$$

and

$$
y_r(p_m^*,\theta^*,v^*,p_r^*,s^*)=\alpha(a-b_1p_m^*+\beta\theta^*)+b_1(v^*-p_r^*)+\tau s^*.
$$

Finally, the optimal differentiation price (v^*) for market segmentation is calculated as $p_m^*-t_g\theta^*$.

The mathematical model analysis is introduced for the stochastic demand using the distribution-free approach in the next section.

4.2 Distribution-free approach

In this section, stochastic demand is considered with partial information. There is minimal access to demand parameters, including the mean and standard deviation. If the demand distribution is not known precisely, a distribution-free approach using Scarf's rule (1958) is utilised to find an upper bound on $E[Q_i-D_i]^+$, $\forall i\in\{m,r\}$. With $\mu_i=0$, $\forall i\in\{m,r\}$, utilising the distribution-free approach (AlFares and Elmorra, 2005; Gallego and Moon, 1993), $E[Q_i-D_i]^+$ can be written as

$$
E[Q_i-D_i]^+\le\frac{\sqrt{\sigma_i^2+(Q_i-y_i)^2}+(Q_i-y_i)}{2},\qquad \forall i\in\{m,r\}. \tag{50}
$$

Now, on substituting $E[Q_i - D_i]^+$, $\forall i \in \{m, r\}$ from equation (50) in equation (16), the expected profit of DCSC can be obtained as

$$
\begin{aligned}
&E\left[\tilde{\pi}\left(\tilde{p}_m, \tilde{p}_r, \tilde{\theta}, \tilde{s}, \tilde{v}, \tilde{Q}_m, \tilde{Q}_r\right)\right] \\
&\quad = \left(\tilde{p}_m - t_g\tilde{\theta} - c_1 + s_m\right)\tilde{Q}_m \\
&\quad - \left(\tilde{p}_m - t_g\tilde{\theta} + h_m + s_m\right)\frac{\sqrt{\sigma_m^2 + \left(\tilde{Q}_m - \tilde{y}_m\right)^2} + \left(\tilde{Q}_m - \tilde{y}_m\right)}{2} \\
&\quad - s_m\tilde{y}_m - \frac{1}{2}\eta\tilde{\theta}^2 + \left(\tilde{p}_r + t_n - c_2 + s_r\right)\tilde{Q}_r \\
&\quad - \left(\tilde{p}_r + t_n + h_r + s_r\right)\frac{\sqrt{\sigma_r^2 + \left(\tilde{Q}_r - \tilde{y}_r\right)^2} + \left(\tilde{Q}_r - \tilde{y}_r\right)}{2} - s_r\tilde{y}_r - \frac{1}{2}k\tilde{s}^2
\end{aligned}
\tag{51}
$$

The profit function expressed in equation (51) does not need distribution information, and it gives an estimate of a lower bound on the expected profit function in equation (16). The profit can be realised using this lower bound when demand is unknown or when members of the SC face the worst possible demand distribution.

To decide the optimal results of order quantities of GP and NGP ($\tilde{Q}_m$ and $\tilde{Q}_r$), we set the first order derivatives with respect to $\tilde{Q}_m$ and $\tilde{Q}_r$ to zero for fixed ($\tilde{p}_m$, $\tilde{p}_r$, $\tilde{\theta}$, $\tilde{s}$) and get

$$
\frac{dE(\tilde{\pi})}{d\tilde{Q}_m} = \left(\tilde{p}_m - t_g\tilde{\theta} - c_1 + s_m\right) - \frac{\tilde{p}_m - t_g\tilde{\theta} + h_m + s_m}{2}\left(1 + \frac{\tilde{Q}_m - \tilde{y}_m}{\sqrt{\sigma_m^2 + \left(\tilde{Q}_m - \tilde{y}_m\right)^2}}\right) = 0 \tag{52}
$$

and

$$
\frac{dE(\tilde{\pi})}{d\tilde{Q}_r} = \left(\tilde{p}_r + t_n - c_2 + s_r\right) - \frac{\left(\tilde{p}_r + t_n + h_r + s_r\right)}{2}\left(1 + \frac{\tilde{Q}_r - \tilde{y}_r}{\sqrt{\sigma_r^2 + \left(\tilde{Q}_r - \tilde{y}_r\right)^2}}\right) = 0. \tag{53}
$$

Let

$$
\frac{\tilde{p}_m - t_g\tilde{\theta} - c_1 + s_m}{\tilde{p}_m - t_g\tilde{\theta} + h_m + s_m} = \tilde{\phi}_m \tag{54}
$$

and

$$
\frac{\tilde{p}_r + t_n - c_2 + s_r}{\tilde{p}_r + t_n + h_r + s_r} = \tilde{\phi}_r \tag{55}
$$

On solving equation (52) for $\tilde{Q}_m$ and equation (53) for $\tilde{Q}_r$, we get

$$
\tilde{Q}_m = \tilde{y}_m + \frac{\sigma_m\left(2\tilde{\phi}_m - 1\right)}{2\sqrt{\tilde{\phi}_m\left(1 - \tilde{\phi}_m\right)}} \tag{56}
$$

and

$$\tilde{Q}_r = \tilde{y}_r + \frac{\sigma_r\left(2\tilde{\phi}_r - 1\right)}{2\sqrt{\tilde{\phi}_r\left(1-\tilde{\phi}_r\right)}}. \tag{57}$$

In order to prove that $E[\tilde{\pi}]$ is concave in $\tilde{Q}_m$ and $\tilde{Q}_r$, the Hessian matrix of $E[\tilde{\pi}]$ has to be negative definite.

$$H\left[\tilde{\pi}\left(\tilde{Q}_m, \tilde{Q}_r\right)\right] = \begin{bmatrix} \frac{d^2E[\tilde{\pi}]}{d\tilde{Q}_m^2} & \frac{d^2E[\tilde{\pi}]}{d\tilde{Q}_m d\tilde{Q}_r} \\ \frac{d^2E[\tilde{\pi}]}{d\tilde{Q}_r d\tilde{Q}_m} & \frac{d^2E[\tilde{\pi}]}{d\tilde{Q}_r^2} \end{bmatrix}.$$

Since

$$\frac{d^2E[\tilde{\pi}]}{d\tilde{Q}_m^2} = -\frac{\sigma_m^2\left(p_m - t_g\theta + h_m + s_m\right)}{2\left[\sigma_m^2 + \left(\tilde{Q}_m - \tilde{y}_m\right)^2\right]^{\frac{3}{2}}} < 0,$$

$$\frac{d^2E[\tilde{\pi}]}{d\tilde{Q}_r^2} = -\frac{\sigma_r^2\left(p_r + t_n + h_r + s_r\right)}{2\left[\sigma_r^2 + \left(\tilde{Q}_r - \tilde{y}_r\right)^2\right]^{\frac{3}{2}}} < 0$$

and

$$\frac{d^2E[\tilde{\pi}]}{d\tilde{Q}_m^2} \times \frac{d^2E[\tilde{\pi}]}{d\tilde{Q}_r^2} - \left(\frac{d^2E[\tilde{\pi}]}{d\tilde{Q}_m d\tilde{Q}_r}\right)^2 = \frac{\sigma_m^2\sigma_r^2\left(p_m - t_g\theta + h_m + s_m\right)\left(p_r + t_n + h_r + s_r\right)}{4\left[\left(\sigma_m^2 + \left(\tilde{Q}_m - \tilde{y}_m\right)^2\right)\left(\sigma_r^2 + \left(\tilde{Q}_r - \tilde{y}_r\right)^2\right)\right]^{\frac{3}{2}}} > 0,$$

the Hessian matrix $H[\tilde{\pi}(\tilde{Q}_m, \tilde{Q}_r)]$ is positive definite. Thus, $E[\tilde{\pi}]$ is concave with respect to $\tilde{Q}_m$ and $\tilde{Q}_r$.

On substituting $\tilde{Q}_m$ from equation (56), $\tilde{Q}_r$ from equation (57), $(\tilde{p}_m - t_g\tilde{\theta} - c_1 + s_m)$ with $\tilde{\phi}_m(\tilde{p}_m - t_g\tilde{\theta} + h_m + s_m)$ from equation (54) and $(\tilde{p}_r + t_n - c_2 + s_r)$ with $\tilde{\phi}_r(\tilde{p}_r + t_n + h_r + s_r)$ from equation (55) into equation (51), we obtain

$$\begin{aligned} E\left[\tilde{\pi}\left(\tilde{p}_m, \tilde{p}_r, \tilde{\theta}, \tilde{s}, \tilde{Q}_m, \tilde{Q}_r\right)\right] = {} & \left(\tilde{p}_m - t_g\tilde{\theta} - c_1\right)\tilde{y}_m \\ & -\left(\tilde{p}_m - t_g\tilde{\theta} + h_m + s_m\right)\sigma_m\sqrt{\tilde{\phi}_m\left(1-\tilde{\phi}_m\right)} \\ & -\frac{1}{2}\eta\tilde{\theta}^2 + \left(\tilde{p}_r + t_n - c_2\right)\tilde{y}_r \\ & -\left(\tilde{p}_r + t_n + h_r + s_r\right)\sigma_r\sqrt{\tilde{\phi}_r\left(1-\tilde{\phi}_r\right)} - \frac{1}{2}k\tilde{s}^2. \end{aligned} \tag{58}$$

To carry out the worst case analysis for $E[\tilde{\pi}]$ in equation (58), consider the maximum value of $\sqrt{\tilde{\phi}_i(1-\tilde{\phi}_i)}$, $\forall i \in \{m, r\}$ which occurs at $\tilde{\phi}_i = \frac{1}{2}$, and we get a lower bound on $E[\tilde{\pi}]$ as

$$E[\tilde{\pi}] \geq \left(\tilde{p}_m - t_g\tilde{\theta} - c_1\right)\tilde{y}_m - \frac{1}{2}\eta\tilde{\theta}^2 - \frac{1}{2}\left(\tilde{p}_m - t_g\tilde{\theta} + h_m + s_m\right)\sigma_m + \left(\tilde{p}_r + t_n - c_2\right)\tilde{y}_r \\ - \frac{1}{2}k\tilde{s}^2 - \frac{1}{2}\left(\tilde{p}_r + t_n + h_r + s_r\right)\sigma_r. \tag{59}$$

Now, to find the optimal results of the all decision variables, we consider the following problem.

$$\text{Max } E\left[\tilde{\pi}\left(\tilde{p}_m, \tilde{p}_r, \tilde{\theta}, \tilde{s}\right)\right] = \left(\tilde{p}_m - t_g\tilde{\theta} - c_1\right)\tilde{y}_m - \frac{1}{2}\eta\tilde{\theta}^2 \\ - \frac{1}{2}\left(\tilde{p}_m - t_g\tilde{\theta} + h_m + s_m\right)\sigma_m + \left(\tilde{p}_r + t_n - c_2\right)\tilde{y}_r \\ - \frac{1}{2}k\tilde{s}^2 - \frac{1}{2}\left(\tilde{p}_r + t_n + h_r + s_r\right)\sigma_r, \tag{60}$$

Subject to

$$\tilde{p}_m - t_g\tilde{\theta} \geq \tilde{v}, \tag{61}$$

$$\tilde{v} \geq \tilde{p}_r + t_n. \tag{62}$$

To solve the above constrained nonlinear optimisation problem, we apply the KKT conditions with the following Lagrange function.

$$L\left(\widehat{p}_m, \widehat{p}_r, \widehat{\theta}, \widehat{s}, \widehat{v}, \widehat{\lambda}_m, \widehat{\lambda}_r\right) = E\left[\widehat{\pi}\left(\widehat{p}_m, \widehat{p}_r, \widehat{\theta}, \widehat{s}, \widehat{v}, \widehat{Q}_r, \widehat{Q}_m\right)\right] \\ + \widehat{\lambda}_m\left(\widehat{p}_m - t_g\widehat{\theta} - \widehat{v}\right) + \widehat{\lambda}_r\left(\widehat{v} - \widehat{p}_r - t_n\right), \tag{63}$$

where

$$\widehat{y}_m\left(\widehat{p}_m, \widehat{\theta}\right) = (1-\alpha)\left[a - b_1\left(\widehat{p}_m - t_g\widehat{\theta}\right) + \beta\widehat{\theta}\right]$$

and

$$\widehat{y}_r\left(\widehat{p}_m, \widehat{\theta}, \widehat{v}, \widehat{p}_r, \widehat{s}\right) = \alpha\left[a - b_1\left(\widehat{p}_m - t_g\widehat{\theta}\right) + \beta\widehat{\theta}\right] + b_1\left[\widehat{v} - \left(\widehat{p}_r + t_n\right)\right] + \tau\widehat{s}.$$

Utilising the KKT optimality conditions, we obtain the following equations.

$$\frac{dL}{d\widehat{p}_m} = \widehat{y}_m - b_1(1-\alpha)\left(\widehat{p}_m - t_g\widehat{\theta} - c_1\right) - \frac{\sigma_m}{2} - b_1\alpha\left(\widehat{p}_r + t_n - c_2\right) + \widehat{\lambda}_m = 0, \tag{64}$$

$$\frac{dL}{d\widehat{p}_r} = \widehat{y}_r - b_1\left(\widehat{p}_r + t_n - c_2\right) - \frac{\sigma_r}{2} - \widehat{\lambda}_r = 0, \tag{65}$$

$$\frac{dL}{d\widehat{\theta}} = -t_g \widehat{y}_m + (1-\alpha)\left(b_1 t_g + \beta\right)\left(\widehat{p}_m - t_g\widehat{\theta} - c_1\right) + \frac{\sigma_m t_g}{2} - \eta\widehat{\theta}$$
$$+\alpha\left(\beta + b_1 t_g\right)\left(\widehat{p}_r + t_n - c_2\right) - \widehat{\lambda}_m t_g \tag{66}$$
$$= 0,$$

$$\frac{dL}{d\widehat{s}} = \tau\left(\widehat{p}_r + t_n - c_2\right) - k\widehat{s} = 0, \tag{67}$$

$$\frac{dL}{d\widehat{v}} = b_1\left(\widehat{p}_r + t_n - c_2\right) - \widehat{\lambda}_m + \widehat{\lambda}_r = 0, \tag{68}$$

$$\widehat{\lambda}_m\left(\widehat{p}_m - t_g\widehat{\theta} - \widehat{v}\right) = 0, \tag{69}$$

and

$$\widehat{\lambda}_r\left(\widehat{v} - \widehat{p}_r - t_n\right) = 0, \tag{70}$$

where $\widehat{\lambda}_m \geq 0$, $\widehat{\lambda}_r \geq 0$.

Further, depending upon the values of $\widehat{\lambda}_m$ and $\widehat{\lambda}_r$. There could be four possible cases.

- Case 1: if $\widehat{\lambda}_m > 0$, $\widehat{\lambda}_r > 0$

  In this condition, from equations (69) and (70), we derive the possible solutions as $\widehat{v} = \widehat{p}_m - t_g\widehat{\theta}$ and $\widehat{v} = \widehat{p}_r + t_n$, which fails to achieve price differentiation between GP and NGP, therefore, this case is discarded.

- Case 2: if $\widehat{\lambda}_m = 0$, $\widehat{\lambda}_r = 0$

  In this case, it is clearly observed that from equations (69) and (70) the pricing related parameters are unrestricted which may generate an infeasible solution. Hence we do not prefer it.

- Case 3: if $\widehat{\lambda}_m > 0$, $\widehat{\lambda}_r = 0$

  From equation (68), we have

  $$\widehat{\lambda}_m = b_1\left(\widehat{p}_r + t_n - c_2\right) > 0. \tag{71}$$

  and if $\widehat{v} > \widehat{p}_r + t_n$, then from equation (70), $\widehat{\lambda}_r = 0$.

  So, in this case, there is a unique solution.

- Case 4: if $\widehat{\lambda}_m = 0$, $\widehat{\lambda}_r > 0$

  From equation (68), we get

  $$\widehat{\lambda}_r = -b_1\left(\widehat{p}_r + t_n - c_2\right) < 0. \tag{72}$$

  It fails to meet the condition $\widehat{\lambda}_r > 0$. Therefore, this case is also discarded.

By substituting the results of $\hat{\lambda}_m$ and $\hat{\lambda}_r$ obtained in case 3 in equations (64) to (67), we get

$$\begin{aligned}&(1-\alpha)\left[a-b_1\left(\hat{p}_m-t_g\hat{\theta}\right)+\beta\hat{\theta}\right]-b_1(1-\alpha)\left(\hat{p}_m-t_g\hat{\theta}-c_1\right)-\frac{\sigma_m}{2}\\&\quad-b_1\alpha\left(\hat{p}_r+t_n-c_2\right)+\hat{\lambda}_m=0,\end{aligned}\tag{73}$$

$$\alpha\left[a-b_1\left(\hat{p}_m-t_g\hat{\theta}\right)+\beta\hat{\theta}\right]+b_1\left[\hat{v}-\left(\hat{p}_r+t_n\right)\right]+\tau\hat{s}-b_1\left(\hat{p}_r+t_n-c_2\right)-\frac{\sigma_r}{2}=0,\tag{74}$$

$$\begin{aligned}&-t_g\left[(1-\alpha)\left\{a-b_1\left(\hat{p}_m-t_g\hat{\theta}\right)+\beta\hat{\theta}\right\}\right]+(1-\alpha)\left(b_1t_g+\beta\right)\left(\hat{p}_m-t_g\hat{\theta}-c_1\right)\\&\quad+\frac{\sigma_m t_g}{2}-\eta\hat{\theta}+\alpha\left(\beta+b_1t_g\right)\left(\hat{p}_r+t_n-c_2\right)-\hat{\lambda}_m t_g=0,\end{aligned}\tag{75}$$

$$\left(\hat{p}_r+t_n-c_2\right)\tau-k\hat{s}=0.\tag{76}$$

On solving equation (73)–(76), we get

$$\hat{\theta}^*=\frac{\beta\begin{bmatrix}2kb_1^2\{(1-\alpha)c_1+(1-\alpha)c_2\}+\tau^2\{2a(1-\alpha)-\sigma_m\}\\-b_1\{2(1-\alpha)\tau^2c_1+k\{4a-(2+\alpha)\sigma_m-(1+\alpha)\sigma_r\}\}\end{bmatrix}}{4b_1\left(k\beta^2+\eta\tau^2\right)-2k(3+\alpha)\eta b_1^2-2(1-\alpha)\beta^2\tau^2}\tag{77}$$

$$\hat{p}_m^*=\frac{\begin{array}{c}\tau^2\left[2(1-\alpha)\left(a+\beta\hat{\theta}^*\right)-\sigma_m\right]\\+b_1\left[2k\sigma_m+k(1-\alpha)\sigma_r-2(1-\alpha)\left\{k(2+\alpha)\left(a+\beta\hat{\theta}^*\right)-\tau^2\left(2\hat{\theta}^*t_g+c_1\right)\right\}\right]\\-2kb_1^2(1-\alpha)\left[2c_1-c_2+(3+\alpha)t_g\hat{\theta}^*\right]\end{array}}{2b_1(1-\alpha)\left[2\tau^2-b_1k(3+\alpha)\right]},\tag{78}$$

$$\hat{p}_r^*=\frac{\begin{array}{c}4\tau^2\left(c_2-t_n\right)+2kb_1\left[(3+\alpha)t_n-(1-\alpha)c_2\right]\\+k\left[\sigma_m+2\sigma_r-2(1+\alpha)\left(a+\beta\hat{\theta}^*\right)\right]\end{array}}{4\tau^2-2kb_1(3+\alpha)}\tag{79}$$

and

$$\hat{s}^*=\frac{\tau\left[2b_1\left(2c_2-(1-\alpha)c_1\right)\right]+\sigma_m-2\left[(1-\alpha)\left(a+\beta\hat{\theta}^*\right)-\sigma_r\right]}{4\tau^2-2kb_1(3+\alpha)}.\tag{80}$$

The optimal differentiation price v^* is given by $\hat{v}^*=\hat{p}_m^*-t_g\hat{\theta}^*$, thus

$$\hat{v}^*=\frac{\begin{array}{c}\tau^2\left[2(1-\alpha)\left(a+\beta\hat{\theta}^*\right)-\sigma_m\right]\\+b_1\left[2(1-\alpha)\tau^2c_1+k\left\{2\sigma_m-(1-\alpha)\left\{2(2+\alpha)(a+\beta\hat{\theta}^*)-\sigma_r\right\}\right\}\right]\\-2kb_1^2(1-\alpha)\left(2c_1-c_2\right)\end{array}}{2b_1(1-\alpha)\left[2\tau^2-k(3+\alpha)b_1\right]}.\tag{81}$$

We first find the value of optimal GQL $\widehat{\theta}^*$ using equation (77). Then, utilising the value of $\widehat{\theta}^*$, the optimal decisions for prices $\widehat{p}_m^*$ and $\widehat{p}_r^*$, SEL $\widehat{s}^*$, and price differentiation $\widehat{v}^*$ are calculated using equations (78)–(81), respectively. Finally, the optimal order quantities $\tilde{Q}_m^*$ and $\tilde{Q}_r^*$ are found using equations (56) and (57), respectively.

5 Results and discussions

In this section, a numerical analysis of the proposed model is presented. In addition, we analyse the impact of key model parameters on the obtained optimal solutions.

5.1 Numerical illustrations

In this section, we conduct a numerical analysis of our model using the three datasets in Table 2 taken from the literature (Modak and Kelle, 2019; Ranjan and Jha, 2019; Madani and Rasti-Barzoki, 2017; Raza, 2015). The manufacturing costs of GP and NGP are considered equal (i.e., $c_1 = c_2 = c$). Also, the holding and shortage costs of GP and NGP are considered the same (i.e., $h_m = h_r$ and $s_m = s_r$). Further, the stochastic demand factor, ε_i, $\forall i \in \{m, r\}$ is bounded such that $\varepsilon_i \in [-\sqrt{3}\sigma_i, \sqrt{3}\sigma_i]$, $\forall i \in \{m, r\}$ (Mostard et al., 2005), with $\mu_i = 0$, $\forall i \in \{m, r\}$.

Table 2 Three numerical datasets

Dataset	α	a	b_l	w	τ	c	η	k	β	t_g	t_n	h_m	s_m	h_r	s_r	σ_m	σ_r
1	0.25	60	0.75	31.5	0.55	4.0	3.5	4.0	0.65	1.3	27.14	3	30	3	30	5	4
2	0.55	100	1.95	17.0	0.65	5.5	4.0	4.5	0.55	2.0	20.65	5	32	5	32	7	6
3	0.75	150	2.00	34.0	1.90	6.0	4.5	5.0	0.95	2.5	43.82	7	34	7	34	9	8

The numerical analysis is performed for the uniform distribution and worst-case analysis of the distribution-free approach, as shown in Tables 3 and 4, respectively.

5.1.1 Comparison between uniform distribution and distribution-free approach

The results obtained with uniform distribution and distribution-free approaches for the datasets in Table 2 are shown in Tables 3 and 4. In all datasets, the total profit of the DCSC and the manufacturer's profit decrease, while the retailer's profit increases by shifting from the distribution-free approach to the uniform distribution. For dataset 1, the order quantity of the GP increases, but it decreases for the other two datasets by moving from uniform distribution to the distribution-free approach. In contrast, the order quantity of the NGP increases in datasets 1 and 2, while it decreases in dataset 3 by shifting from the uniform to distribution-free approach. For all datasets, the price differentiation increases by shifting from distribution-free to the uniform distribution. The values of decision variables (selling prices, GQL, and SEL) decrease by moving from the uniform to distribution-free approach.

Table 3 Optimal results of a uniform distribution

Dataset	p_m^*	p_r^*	θ^*	s^*	Q_m^*	Q_r^*	v^*	y_m^*	y_r^*	π_m^*	π_r^*	π^*
1	74.31	11.94	9.67	4.82	22.37	30.23	61.74	14.99	24.64	1,681.78	67.82	1,749.60
2	48.33	6.47	5.00	3.12	21.79	46.83	38.33	13.05	39.84	1,035.81	376.31	1,412.12
3	85.11	5.20	9.71	16.35	20.99	92.71	60.84	9.38	82.86	3,118.78	458.14	3,576.92

Table 4 Optimal results for a distribution-free approach

Dataset	$\hat{p}_m^*$	$\hat{p}_r^*$	$\hat{\theta}^*$	$\hat{s}^*$	$\hat{Q}_m^*$	$\hat{Q}_r^*$	$\hat{v}^*$	$\hat{y}_m^*$	$\hat{y}_r^*$	$\hat{\pi}_m^*$	$\hat{\pi}_r^*$	$\hat{\pi}^*$
1	71.53	8.45	9.21	4.34	24.01	30.95	59.56	15.98	25.69	508.37	636.86	1,145.23
2	46.76	4.80	4.74	2.88	21.09	47.24	37.29	13.88	41.91	136.31	630.24	766.55
3	81.20	2.50	9.15	15.32	20.36	92.54	58.33	10.51	84.65	–85.48	24,77.15	2,391.68

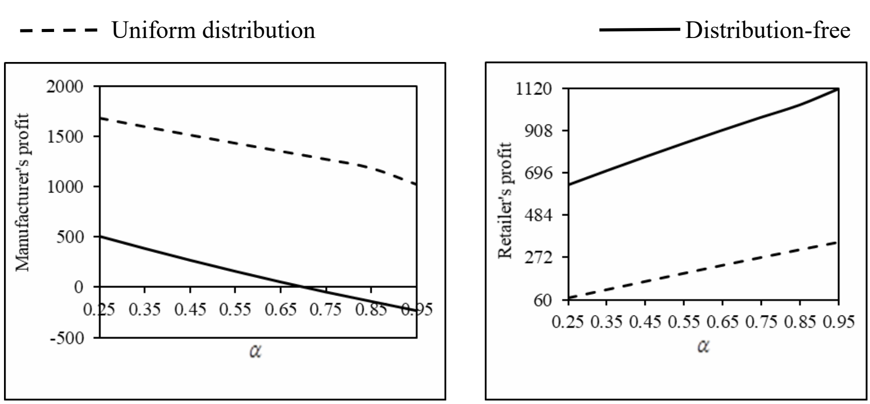
5.2 Sensitivity analysis

We perform sensitivity analysis to study the impact of demand leakage (α), government subsidy rate (t_g) and government tax rate (t_n). For the purpose, the base value of the parameters is considered as $\alpha = 0.25$, $t_g = 1.3$, $t_n = 27.14$, $a = 60$, $b_1 = 0.75$, $w = 31.5$, $\tau = 0.55$, $c_1 = c_2 = 4$, $\beta = 0.65$, $\eta = 3.5$, $k = 4$, $h_m = h_r = 3$, $s_m = s_r = 30$, $\sigma_m = \sigma_r = 4$.

5.2.1 Effects of demand leakage (α)

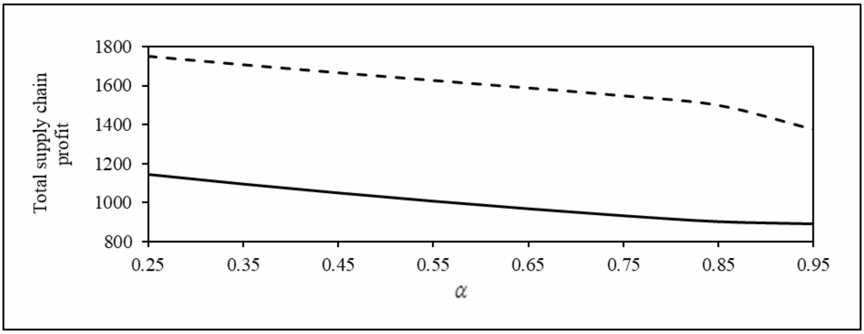
The impact of the demand leakage (α) on the profit of the manufacturer, the retailer and the total profit of the SC are shown in Figure 2. For the given values of parameters, the retailer always has a positive profit. Whereas in the case of the manufacturer, an increment in α, profit decreases and becomes negative after a certain value of α. The manufacturer's profit and the total SC profit are higher in uniform distribution. However, the retailer's profit is higher in the distribution-free approach.

Figure 2 The effects of demand leakage (α) on the optimal results of (a) manufacturer's profit, (b) retailer's profit and (c) SC profit



(a) (b)

Figure 2 The effects of demand leakage (α) on the optimal results of (a) manufacturer's profit, (b) retailer's profit and (c) SC profit (continued)



(c)

Figure 3 The effects of demand leakage (α) on the optimal results of (a) GQL, (b) online selling price, (c) SEL and (d) offline selling price

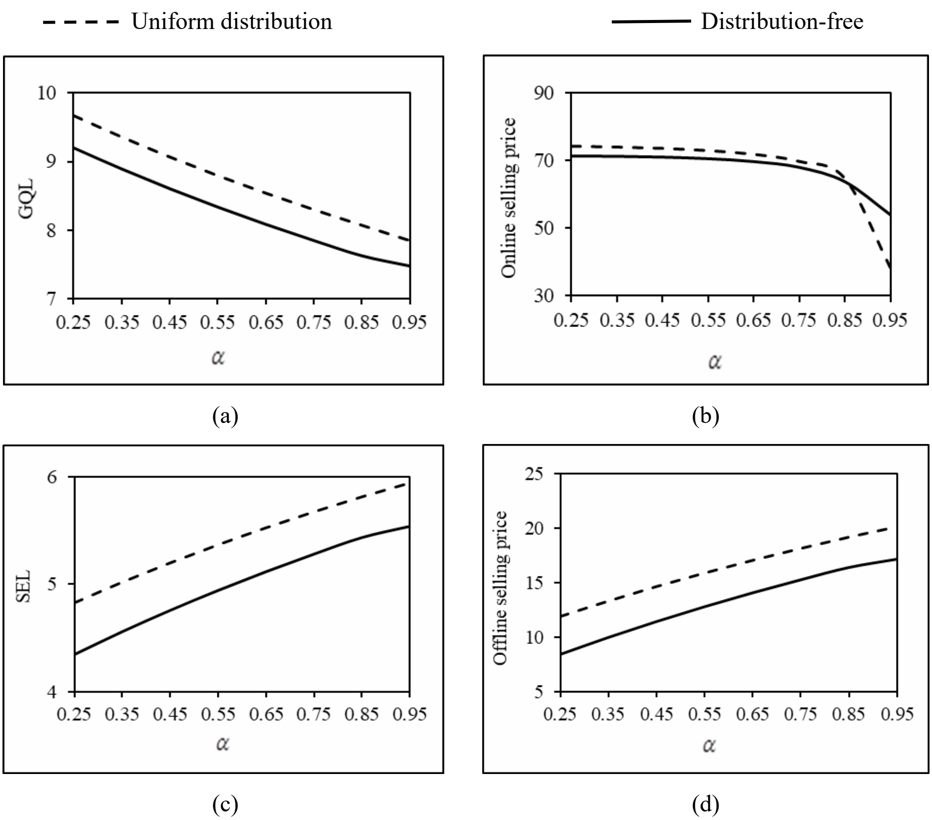
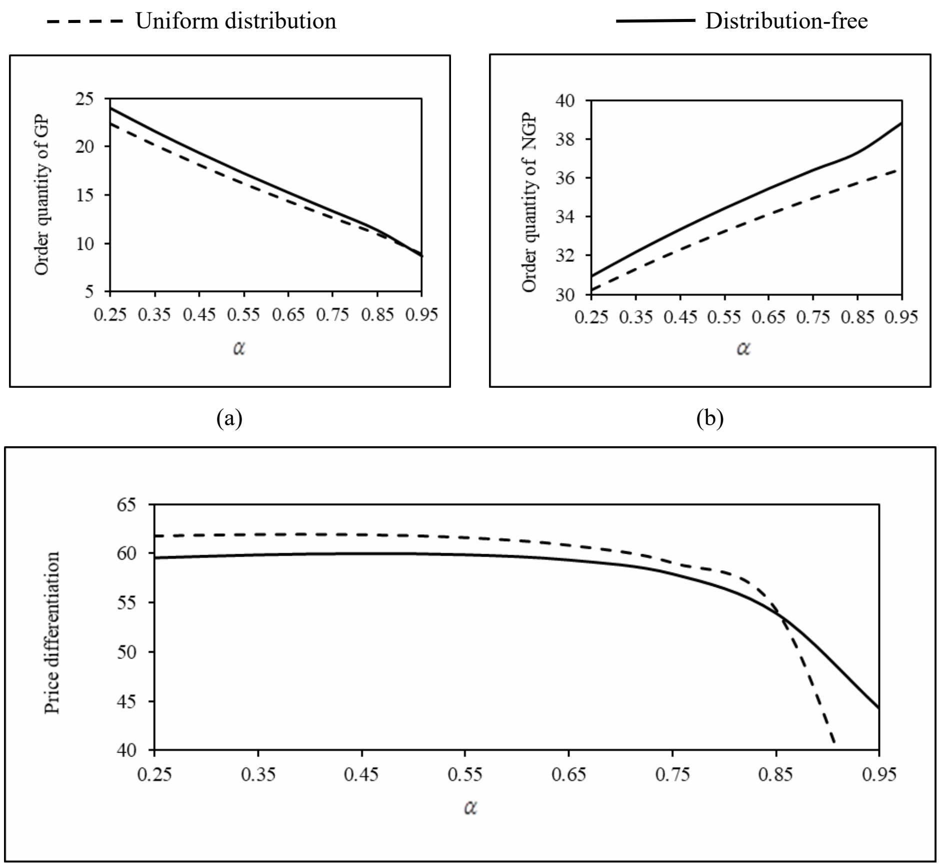


Figure 3 describes the effect of the demand leakage (α) on the GQL, online selling price, SEL, and offline selling price. It can be observed that with an increase in α, the GQL and the online selling price decrease. The GQL has a higher value in the uniform distribution.

The online selling price has a higher value for a larger value of α in the distribution-free approach. The SEL and the selling price of NGP increase with the increase in α and the higher value occurs in uniform distribution.

Figure 4 explains the variation in the order quantity of GP and NGP, and price differentiation with demand leakage (α). With an increase in α, the order quantity of GP decreases, and the order quantity of NGP increases. The order quantity has a larger value in the distribution-free approach for both products. The price differentiation decreases with an increase in α. At a higher value of α, the larger value of price differentiation is observed in the distribution-free approach.

Figure 4 The effects of demand leakage (α) on the optimal results of (a) order quantity of GP (b) order quantity of NGP and (c) price differentiation



(c)

5.2.2 *Effects of government subsidy rate (t_g)*

From Figure 5, it can be observed that the profit of the manufacturer, retailer, and SC are insensitive to t_g. The profits of the SC and the manufacturer have a higher value in uniform distribution. Whereas a larger value of the retailer's profit occurs in the distribution-free approach.

Figure 5 The effects of subsidy (t_g) on the optimal results of (a) manufacturer's profit, (b) retailer's profit and (c) SC profit

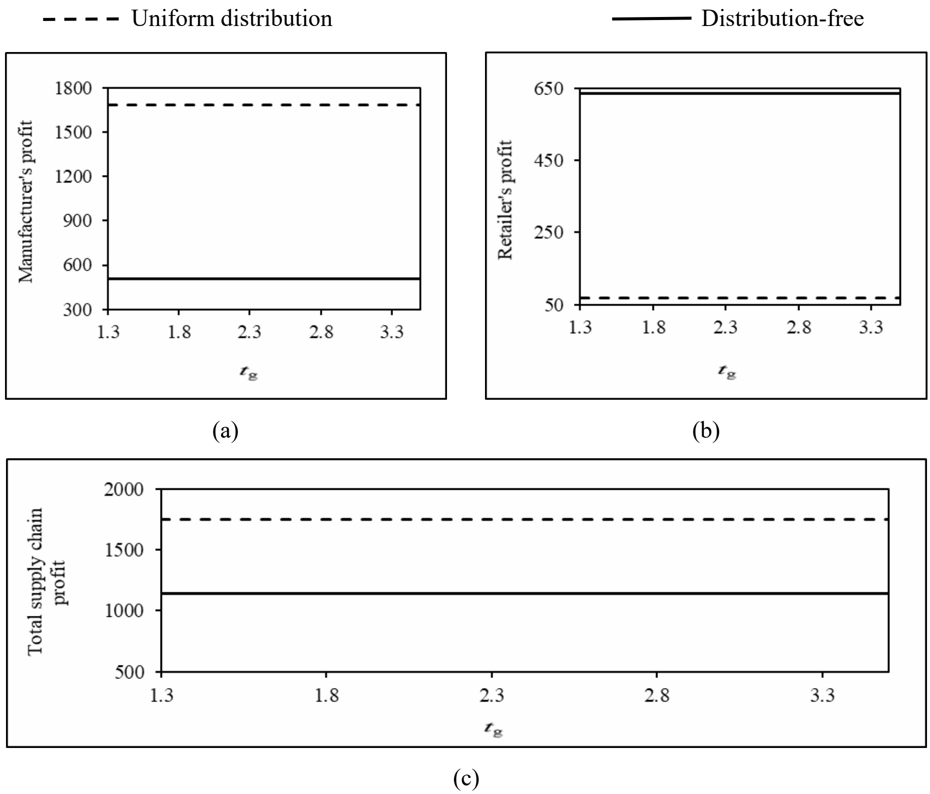
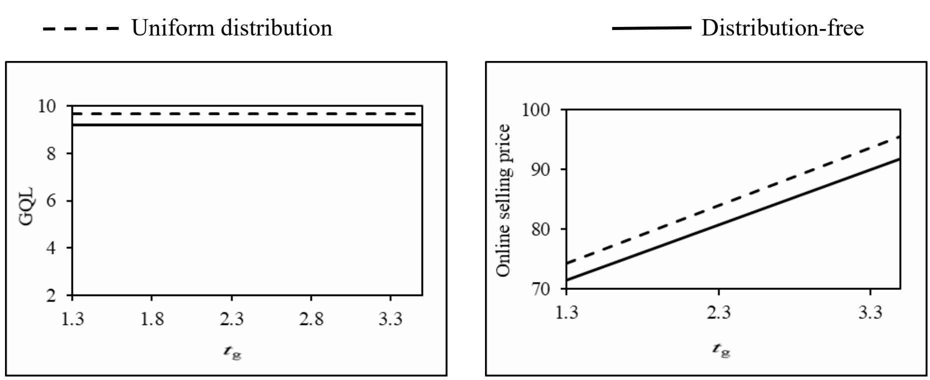


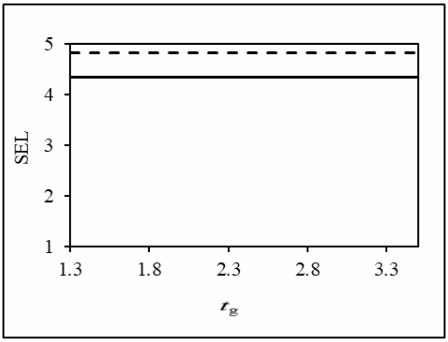
Figure 6 depicts that the selling price of GP increases with an increase in t_g. Also, for all values of t_g, the selling price of GP has a larger value in the uniform distribution than in distribution-free. t_g has no effect on the GQL, SEL, and selling price of NGP.

Figure 6 The effects of subsidy (t_g) on the optimal results of (a) GQL, (b) online selling price, (c) SEL and (d) offline selling price

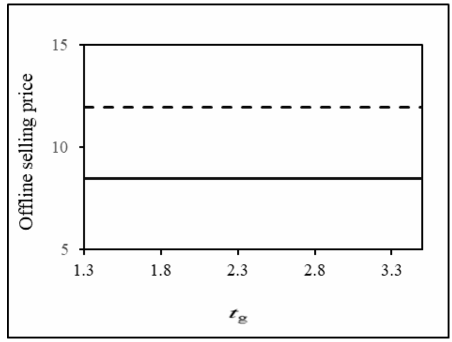


(a) (b)

Figure 6 The effects of subsidy (t_g) on the optimal results of (a) GQL, (b) online selling price, (c) SEL and (d) offline selling price (continued)



(c)

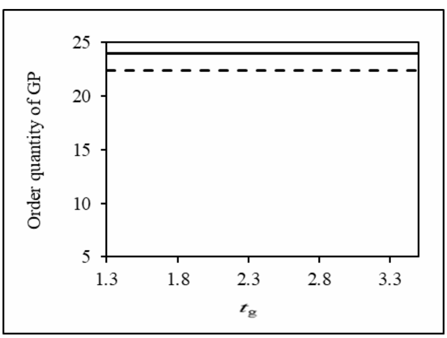


(d)

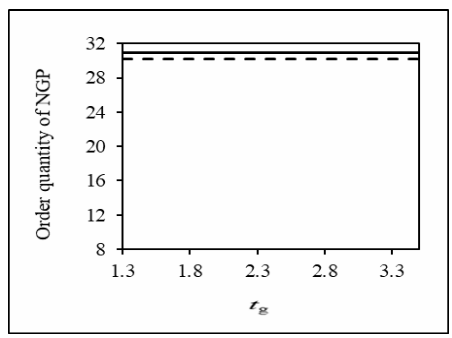
Figure 7 shows that there is no effect of t_g on the order quantities and price differentiation. In the distribution-free approach, the order quantities have a larger value. Whereas price differentiation has a higher value in uniform distribution.

Figure 7 The effects of subsidy (t_g) on the optimal results of (a) order quantity of GP (b) order quantity of NGP and (c) price differentiation

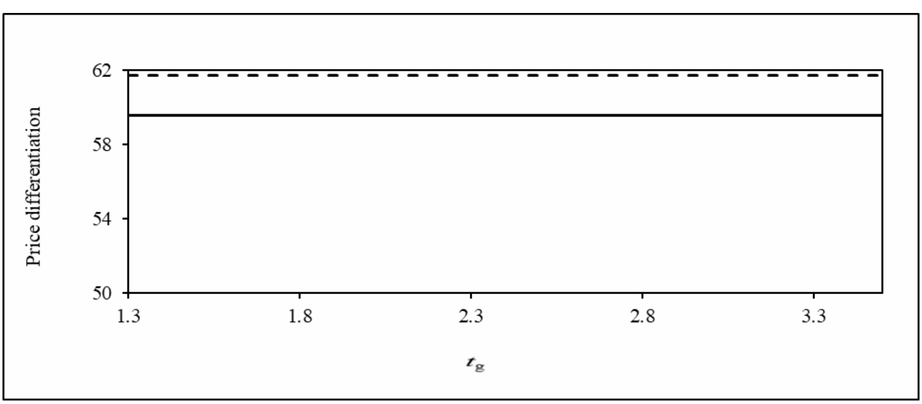
- - - - Uniform distribution ——— Distribution-free



(a)



(b)



(c)

5.2.3 *Effects of the government tax rate (t_n)*

From Figure 8, it appears that the manufacturer's, retailer's, and SC's profits are insensitive to t_n. The profits of the SC and the manufacturer have a higher value in uniform distribution. Whereas the larger value of the retailer's profit occurs in the distribution-free approach.

Figure 8 The effects of tax (t_n) on the optimal results of (a) manufacturer's profit, (b) retailer's profit and (c) SC profit

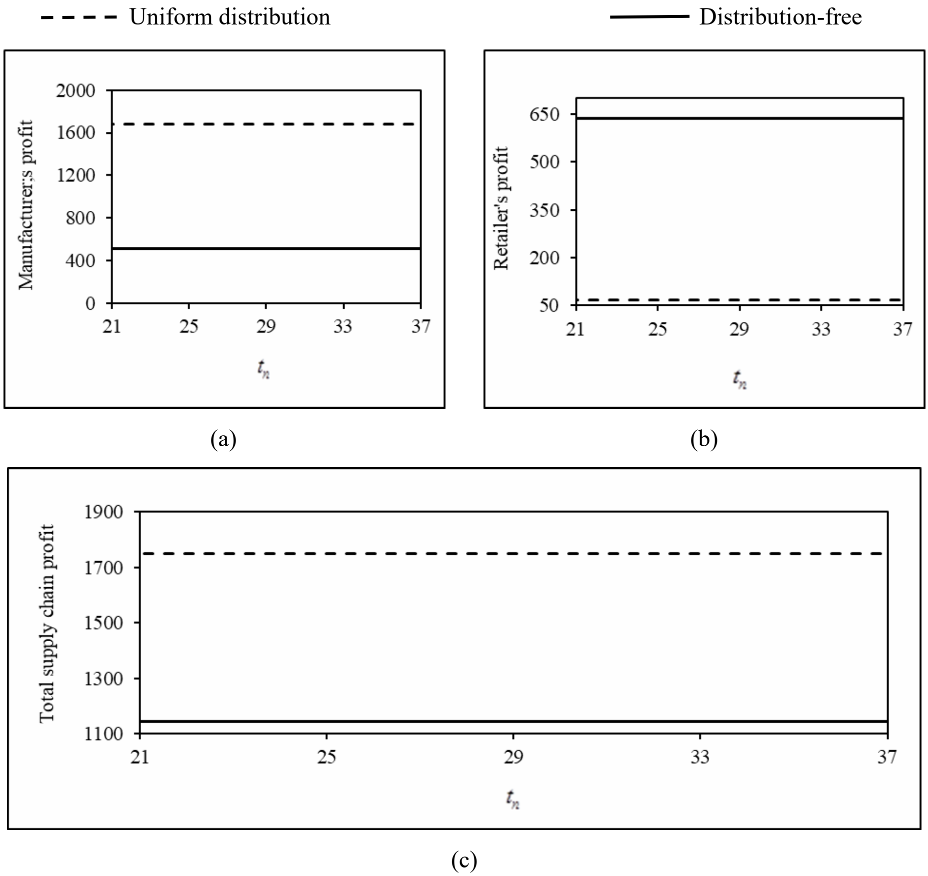
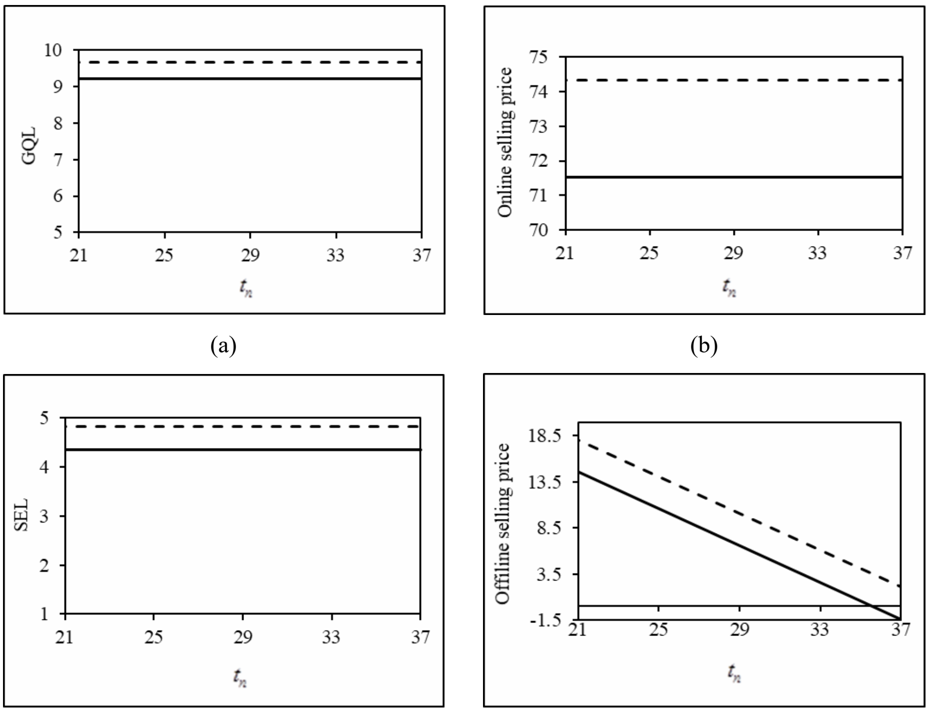


Figure 9(d) illustrates that the selling price of NGP decreases with an increase in t_n. Also, for all values of t_n, the selling price of NGP has a larger value in the uniform distribution than in distribution-free. t_n has no effect on the GQL, SEL, and selling price of GP.

Figure 10 presents that there is no effect of t_n on the order quantities and price differentiation. In the distribution-free approach, the order quantities have a larger value. Whereas the price differentiation has a higher value in uniform distribution.

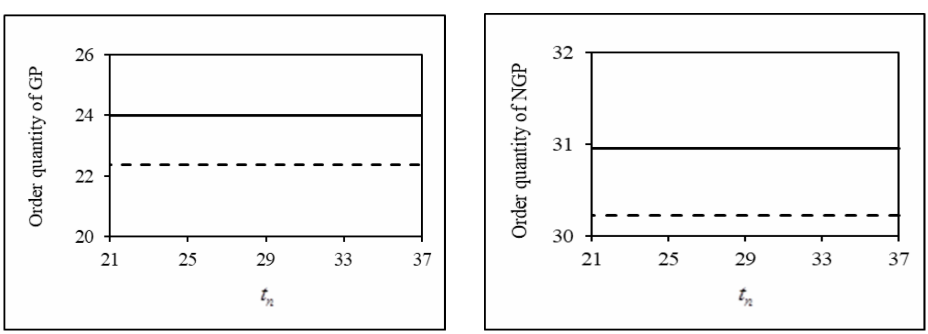
Figure 9 The effects of tax (t_n) on the optimal results of (a) GQL, (b) online selling price, (c) SEL and (d) offline selling price



(a) (b)

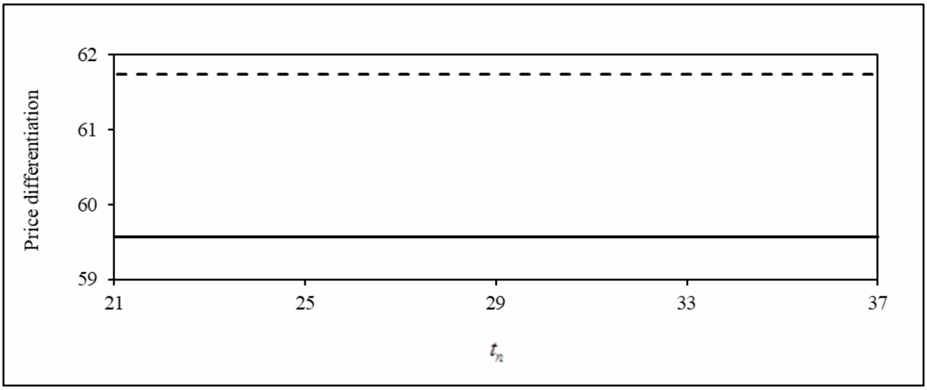
(c) (d)

Figure 10 The effects of tax (t_n) on the optimal results of (a) order quantity of GP (b) order quantity of NGP and (c) price differentiation



(a) (b)

Figure 10 The effects of tax (t_n) on the optimal results of (a) order quantity of GP (b) order quantity of NGP and (c) price differentiation (continued)



(c)

6 Conclusions

This study proposes a centralised model for a DCSC on governmental financial intervention on the degree of greenness, price differentiation, and order quantities of GP and NGP. We consider that the manufacturer experiences leakage of additive type of price-greening-sales-effort-dependent stochastic demand with linear deterministic price-greening-sales-effort-dependent demand. The mathematical model has been developed for profit maximisation. We consider two stochastic demand cases:

1 the uniform distribution

2 the distribution-free approach.

When the demand distribution is not known, the distribution-free approach is employed. In this study, the selling prices of GP and NGP, GQL, SEL, price differentiation, and order quantities of GP and NGP are considered as decision variables. As far as we know, no one has analysed a DCSC considering green quality and sales effort, governmental policies, and price differentiation simultaneously. When the stochastic demand distribution is known, the problem is evaluated employing joint optimisation, and a numerical solution is obtained. Due to mathematical complexity, a closed-form solution could not be derived.

The findings suggest that uniform distribution always provides the highest SC profit and thus aids in achieving the economic goal. The green quality of the product is slightly higher in uniform distribution than in distribution-free, which is advantageous in terms of environmental considerations. Whereas the distribution-free creates a larger order quantity of NGP and increases channel members' profit compared to the uniform distribution. It has been noticed that when the manufacturer offers a high-quality GP, the retailer makes a significant effort to sell the NGP. The manufacturer and retailer both raise prices to account for the higher costs associated with greening and putting in the extra sales effort. Cannibalisation (demand leakage, α) has reduced the competitive advantage that the SC members can achieve by market segmentation using an optimal differentiation price. The impact of the demand leakage shows that the SC profit is higher in uniform distribution than in distribution-free. Also, the result shows that the price differentiation is higher in the uniform distribution, but by increasing the value of demand leakage, the price differentiation is lower in the uniform distribution. Moreover, the price differentiation decreases with an increase in demand leakage, the negative effect of the order quantity of the GP. The government subsidy and tax rates affect online and offline selling prices, respectively. After the above discussion, it is found that uniform distribution is beneficial for the total SC profit and the environmental perspective. While distribution-free is suitable for the profits of the retailer and manufacturer.

The following are some of the restrictions of the proposed study that could be investigated in future. In this paper, the demand function is an additive model of the price-greening-sales-effort-dependent stochastic problem, which can be incorporated as a multiplicative model. In addition, here we focus on a DCSC with just two participants, but examining a DCSC with three players, including the government, would make the analysis interesting under a decentralised scenario. This study can also be investigated for an Omni-channel SC. It would be significant to explore the possibility of selling GP and NGP through both channels of a DCSC by relaxing the assumption that these products have to be sold through separate channels. Moreover, this study is limited to a uniform distribution and a distribution-free. This study can be further investigated using other distributions like normal distribution and exponential distribution. Both the static and dynamic Stackelberg game models could be used to deal with problems like chaos and bifurcation. Finally, this study may be examined for risk-neutral SC members. The problem could be solved by utilising value-at-risk (VaR), conditional-value-at-risk (CVaR) and mean-variance analysis.

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