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A copula-based analysis for examining and enhancing performance of serial system

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Abstract: This study investigates the performance of a serial system consisting of five dissimilar subsystems A, B, C, D and E employing Copula's attributes for estimation. Subsystem A has six units working as 4-out-of-6, subsystems B, C and E possess each has one unit while subsystem D has three units working in active parallel. The system experiences partial and complete malfunction during operation: partial malfunction leads the system into reduced capacity mode, and a complete malfunction resulting in system failure. The research employs the technique of supplementary variable and transforms of Laplace to establish and solve the governing differential equations corresponding to schematic system's diagram. Performance models for the system such as profit, MTTF, availability, sensitivity of MTTF and reliability are numerically validated and presented through tables and graphs. The research findings are highly valuable for evaluating performance, identifying ideal system designs, and developing workable maintenance plans. The research contributes to enhancing system performance, increasing production output, and improving revenue mobilisation in various application domains.

Keywords: performance; profit; reliability; mean time to failure; MTTF; availability.

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1 Introduction

Reliability and performance examination constitute essential components of assessing and maximising the operational efficiency of systems in a variety of areas, ranging from computer networks and software applications to mechanical and electrical systems. These two critical areas contribute to ensuring that systems achieve their stated objectives while running efficiently and consistently. Both reliability and performance analysis are essential components of system engineering, and they are frequently intertwined. A highly reliable system is not always efficient, and a high-performance system is not always reliable. As a result, striking a balance between reliability and performance is critical for developing robust, efficient systems that meet the needs of users and stakeholders. These analyses assist in identifying flaws, making informed design decisions, and ultimately improving the overall quality of systems in a variety of applications.

System failures can have disastrous consequences in the aerospace, healthcare, transportation, and energy industries. Keeping systems reliable and performing well prevents accidents and saves lives. Downtime, inefficiencies, and maintenance costs can

have a significant economic impact on organisations. Researchers look into ways to improve system performance and reliability in order to reduce operational costs and increase productivity.

Indeed, due to the substantial effect on the quality, safety, and efficiency of systems that play critical roles in our daily lives, the reliability and performance examination of systems has attracted significant attention from researchers across various fields and industries. To cite few, Jia et al. (2020), for example, proposed a multi-state selection diagram technique for evaluating the reliability of power systems. This approach includes a multi-state performance sharing procedure as well as warm standby units. Jia et al. (2021) created a power system model that included warm standby units and energy storage, alongside reliability assessed using the multi-valued decision diagram approach.

Kumar et al. (2020) investigated the fuzzy reliability of series, parallel, and linear sequential k-out-of-n: F systems. In this study, they used hesitant fuzzy sets and triangular fuzzy numbers to figure out the reliability of systems such as series and parallel, while the Weibull distribution was used for assessing the fuzzy reliability of linear consecutive k-out-of-n: F systems. Lin et al. (2021) introduced a copula-based Bayesian reliability model to quantify the interdependence among parallel system components and estimate the system's failure rate in a separate study.

Mellal and Zio (2020) investigated the design and implementation of a novel cuckoo optimisation algorithm for reliability redundancy allocation, with a particular emphasis on a cold-standby strategy. Mellal et al. (2020) also proposed a hosted cuckoo optimisation algorithm to address the problem of reliability-redundancy allocation in a heterogeneous system. Okafor et al. (2018) investigated the reliability of a multi-state parallel system using an approach based on Archimedean copulas. Pundir et al. (2021) investigated reliability metrics for two non-identical cold standby unit systems using various types of prior assumptions for unknown parameters. Sharifi et al. (2019) investigated a redundancy allocation problem with the goal of optimising both the reliability and the cost of weighted-k-of-n parallel systems. For assessing the reliability and cost of each subsystem, they used an amalgamation of a universal generating function, a non-dominated sorting genetic algorithm, and a non-dominated ranked genetic algorithm. Xie et al. (2021a) investigated the efficiency of a safety system that is prone to cascading failures, resulting in a series of subsequent failures.

Xie et al. (2021b) focused their research on measuring the efficiency of a k-out-of-n redundant system in the presence of consecutive failures. The paper also describes a novel method for minimising and avoiding cascading failures. Xie et al. (2020) offered a systematic method for gathering and analysing the effects of catastrophic breakdowns on the reliability of series-parallel systems. The technique is centred on improved reliability block diagram designs. Furthermore, Xie et al. (2019) put forward a method for evaluating efficiency and estimating the probability of medium-frequency hazardous failures in safety instrumental systems prone to cascading failures. Yemane and Colledani (2019) propose a framework for assessing the efficiency of unreliable manufacturing processes that adopts unidentified machine reliability forecasts into account.

Kumar et al. (2019) presented a framework based on the multi objective grey wolf optimiser (MOGWO) algorithm, a modern optimisation technique inspired by nature. In order to improve the technical specifications optimisation of the residual heat removal system (RHRS) in a nuclear power plant safety system, this algorithm mimics the hunting and hierarchical behaviour seen in grey wolves (*Canis lupus*). Singh et al. (2022a) focus on performance assessment of n-identical units repairable complex system through

copula linguistic. Singh et al. (2022b) focus on performance assessment repairable complex system having consecutive k degraded states through copula linguistic.

Singh et al. (2016) delve into performance analysis of a system made up of two separate subsystems connected in series. They obtain and numerically analyse the performance measures associated with the system's efficiency using the copula approach. Lado et al. (2018) explored cost estimation based on system performance for a configuration involving two consecutive subsystems. They employed supplementary variables and gave precedence to repairing the initially failing unit. The copula repair technique was used by Yusuf et al. (2020) to demonstrate the efficient functioning of a multi-computer system made up of three sequential subsystems. In order to analyse a system made up of two sequentially arranged subsystems that experienced various types of failures, Lado and Singh (2019) used the copula technique. Additionally, Gahlot et al. (2020) studied the performance characteristics of repairable systems operating in a series configuration, considering multiple failure types and two distinct repair approaches, utilising the Copula technique. Poonia (2021) conducted a study utilising supplementary techniques and Laplace transforms on a multistate computer network featuring a series arrangement of three database servers and five web servers. This network was also considered for potential copula-based repairs. Poonia (2021) delves into investigation of some performance measures of distributed computer network composing of load balancers, web servers and database servers for replication under the redundancy scheme of k -out-of- n . Sundari et al. (2020) analysed a non-Markovian queuing framework, the clients who arrive in groups adhere to a Poisson distribution. Each of the clients who arrive has the option of selecting from any of the N varieties of services provided. The duration of the service is governed by a general distribution. Prajapati et al. (2021) delineates the different variables pertaining to environmental management systems (EMS) that contribute to the efficacious establishment of such systems in the context of small and medium enterprises (SMEs) situated within India. Pai et al. (2023) critically examines the tribological factors pertaining to the upkeep, dependability, and strategic management of assets. Bahrami et al. (2023) simulate two series and parallel configurations and assess the efficacy of these configurations in comparison to each other as well as their respective singular models. Jain et al. (2023) ascertain the primary elements that influence the responsiveness of personnel in the manufacturing industry with respect to the standard of their work-life. Isa et al. (2022) explore on reliability analysis of computer network which comprises of three subsystems: router, workstation and hub. Yusuf et al. (2021) consider a distributed system with five standby subsystems A (the clients), B (two load balancers), C (two distributed database servers), D (two mirrored distributed database servers) and E (centralised database server) is considered arranged as series-parallel system. Wang et al. (2020) introduces a pioneering framework for a diversified production system capable of accommodating multiple forms of productivity, Bao et al. (2021) a novel iteration of the conventional MSS framework has been devised to proficiently manage scenarios where the incorporation of non-immediate state-transition attributes is imperative.

The previously mentioned literature above laid out its findings concerning the examination of performance and reliability in certain sequential systems, indicating improvements in system efficiency. Nonetheless, there is a significant gap in studies using copula-based techniques, particularly when it comes to an in-depth examination of reliability and performance in solar powered irrigation systems. To address this research

gap, this paper employs copula techniques to investigate the reliability and performance, dependability and efficiency of a solar-powered irrigation system composed of multiple photovoltaic (PV) panels, a controller, a submersible pump, storage tanks, and a watering tank. This study attempts are extremely important in addressing ongoing and emerging challenges in a rapidly changing technological framework. Executing precise performance assessments is critical in preventing disasters and protecting valuable resources, both time and money. As a result, there is an increasing need for innovative methodologies and models to improve the dependability, longevity, reliability, and seamless functionality of solar irrigation systems. As a result, a consistent and efficient water supply for agricultural purposes is ensured, addressing a critical need in sustainable agriculture.

The paper is structured into several sections. Section 2 encompasses the notation employed for analysing the proposed model. Section 3 encompasses the state description the proposed system. Section 4 is dedicated to presenting reliability and performance models of the system, with specific cases discussed. Finally, the study concludes with the results illustrated in Section 5.

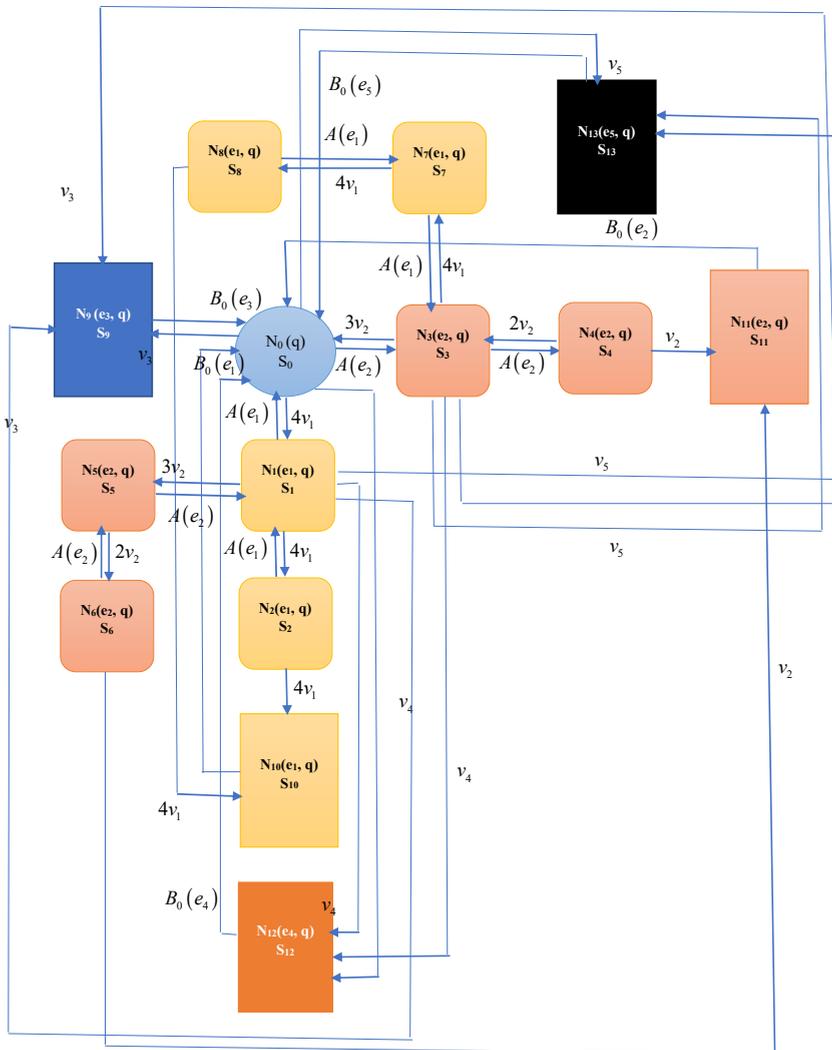
2 Notations

- q : representing time variable.
- s : representing variable of transformation of Laplace.
- ν_1 : rate of failure representation of panel.
- ν_2 : rate of failure representation of storage tank.
- ν_3 : rate of failure representation of controller.
- ν_4 : rate of failure representation of submersible pump.
- ν_5 : rate of failure representation of watering tank.
- $A(e_1)/B_0(e_1)$: rate of repair representation for diminish performance/complete breakdown of panel.
- $A(e_2)/B_0(e_2)$: rate of repair representation for diminish performance/complete breakdown failure of storage tank.
- $B_0(e_3)$: rate of repair representation for complete breakdown of controller.
- $B_0(e_4)$: rate of repair representation for complete breakdown of submersible pump.
- $B_0(e_5)$: rate of repair representation for complete breakdown of watering tank.
- $N_i(q)$: stand for chance of the equipment staying in any state at instants for $i = 0$ to 13.
- $\bar{N}_0(s)$: representation for transformation of Laplace with probability $N(q)$.
- $N_k(e_1, q)$: representation for chance of the system staying in any state with service duration is (e_1, q) with service variable e_1 and time q .

- $N_k(e_2, q)$: representation for chance of the equipment sojourning in any state for $i = 1 \dots 13$, the with service duration is (e_2, q) with service e_2 and time q .
- $E_p(q)$: profit anticipation profit in $[0, q)$.
- T_1, T_2 : income and cost of service cost per unit time, respectively.
- $B_0(e)$: representation of joint probability according to Gumbel-Hougaard family copula definition is given as (failed state S_i to good state S_0).

3 Description of solar powered system

Figure 1 Schematic diagram of the system (see online version for colours)



- S₀ Represent initial state of the system, which is free from any failures.
- S₁ In PV panel subsystem, represent the initial malfunction of a panel.
- S₂ In PV pane subsystem, depict the initial malfunction of second panel, previously the first panel is in malfunctioning state.
- S₃ Illustrate in storage tank subsystem, the initial malfunction of first tank.
- S₄ Represent the state where in storage tank subsystem experiences the second malfunction of another tank, previously the first panel is in malfunctioning state.
- S₅ Illustrate in storage tank subsystem, the initial malfunction of first tank, previously the first panel is in malfunctioning condition.
- S₆ Represent the state where in storage tank subsystem experiences the second malfunction of another tank, previously the first panel and tank are in malfunctioning state.
- S₇ In PV panel subsystem, represent the initial malfunction of a panel, previously the first tank is in malfunctioning condition.
- S₈ In PV pane subsystem, depict the initial malfunction of second panel, previously the first tank and panel are in malfunctioning condition.
- S₉ Specify the complete breakdown state of controller.
- S₁₀ Specify the complete breakdown state of PV panel subsystem due breakdown of all panels.
- S₁₁ Specify the complete breakdown state of the system due breakdown of all storage tanks.
- S₁₂ Specify the complete breakdown state of the system due breakdown of submersible pump.
- S₁₃ Specify the complete breakdown state of the system due breakdown of watering tank.

Table 1 System configuration

<i>Subsystem</i>	<i>Total units in the subsystem</i>	<i>Mode of operation</i>
A	6	4-out-of-6
B	1	1-out-of-1
C	1	1-out-of-1
D	3	1-out-of-3
E	1	1-out-of-1

4 Preparation and presentation of performance models

To developed the probabilistic models for systems, the supplementary variable technique and Laplace transforms are frequently used in reliability modelling and analysis. Utilising a supplementary variable to represent the system's state at a specific moment is known as

the supplementary variable technique. The differential equations that describe the system can be made simpler using this method, which makes them simpler and easier to solve. Differential equations can be converted into algebraic equations that are simple to solve using Laplace transforms. This method enables the system's probability distribution function's Laplace transform to be derived. After obtaining the probability distribution function, initial and boundary conditions can be used to calculate the steady state probabilities. These probabilities form the foundation for the creation of reliability models, which can be used to assess the system's performance and pinpoint possible failure modes.

The following partial differential equations are obtained via Figure 2.

$$\begin{aligned}
 & \left(\frac{\partial}{\partial q} + 4v_1 + 3v_2 + v_3 + v_4 + v_5 \right) N_0(q) \\
 &= \int_0^\infty A(e_1) N_1(e_1, q) de_1 + \int_0^\infty A(e_2) N_3(e_2, q) de_2 \\
 &+ \int_0^\infty B_0(e_1) N_{10}(e_1, q) de_1 + \int_0^\infty B_0(e_2) N_{11}(e_2, q) de_2 \\
 &+ \int_0^\infty B_0(e_3) N_9(e_3, q) de_3 + \int_0^\infty B_0(e_4) N_{12}(e_4, q) de_4 \\
 &+ \int_0^\infty B_0(e_5) N_{13}(e_5, q) de_5
 \end{aligned} \tag{1}$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial e_1} + 4v_1 + 3v_2 + v_3 + v_4 + v_5 + A(e_1) \right) N_1(e_1, q) = 0 \tag{2}$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial e_1} + 4v_1 + A(e_1) \right) N_2(e_1, q) = 0 \tag{3}$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial e_2} + 4v_1 + 2v_2 + v_3 + v_4 + v_5 + A(e_2) \right) N_3(e_2, q) = 0 \tag{4}$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial e_2} + v_2 + A(e_2) \right) N_4(e_2, q) = 0 \tag{5}$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial e_2} + 2v_2 + A(e_2) \right) N_5(e_2, q) = 0 \tag{6}$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial e_2} + v_2 + A(e_2) \right) N_6(e_2, q) = 0 \tag{7}$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial e_1} + 4v_1 + A(e_1) \right) N_7(e_1, q) = 0 \tag{8}$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial e_1} + 4v_1 + A(e_1) \right) N_8(e_1, q) = 0 \tag{9}$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial e_3} + B_0(e_3) \right) N_9(e_3, q) = 0 \quad (10)$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial e_1} + B_0(e_1) \right) N_{10}(e_1, q) = 0 \quad (11)$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial e_2} + B_0(e_2) \right) N_{11}(e_2, q) = 0 \quad (12)$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial e_4} + B_0(e_4) \right) N_{12}(e_4, q) = 0 \quad (13)$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial e_5} + B_0(e_5) \right) N_{13}(e_5, q) = 0 \quad (14)$$

with boundary conditions below:

$$N_1(0, q) = 3v_1 N_0(q) \quad (15)$$

$$N_2(0, q) = 4v_1 N_1(0, q) \quad (16)$$

$$N_3(0, q) = 3v_2 N_0(q) \quad (17)$$

$$N_4(0, q) = 2v_2 N_3(0, q) \quad (18)$$

$$N_5(0, q) = 3v_2 N_1(0, q) \quad (19)$$

$$N_6(0, q) = 2v_2 N_5(0, q) \quad (20)$$

$$N_7(0, q) = 4v_1 N_3(0, q) \quad (21)$$

$$N_8(0, q) = 4v_1 N_7(0, q) \quad (22)$$

$$N_9(0, q) = v_3 (N_0(q) + N_1(0, q) + N_3(0, q)) \quad (23)$$

$$N_{10}(0, q) = 4v_1 (N_2(0, q) + N_8(0, q)) \quad (24)$$

$$N_{11}(0, q) = v_2 (N_4(0, q) + N_6(0, q)) \quad (25)$$

$$N_{12}(0, q) = v_4 (N_0(q) + N_1(0, q) + N_3(0, q)) \quad (26)$$

$$N_{13}(0, q) = v_5 (N_0(q) + N_1(0, q) + N_3(0, q)) \quad (27)$$

with initial conditions below:

$$N_0(0) = 1, \text{ other states probabilities at } q = 0, \text{ are zero} \quad (28)$$

Applying Laplace transformations of equation (1) – (27) the following relations are obtained:

$$\begin{aligned}
 & (s + 4v_1 + 3v_2 + v_3 + v_4 + v_5) \bar{N}_0(s) \\
 &= 1 + \int_0^\infty A(e_1) \bar{N}_1(e_1, s) de_1 + \int_0^\infty A(e_2) \bar{N}_3(e_2, s) de_2 \\
 &+ \int_0^\infty B_0(e_1) \bar{N}_{10}(e_1, s) de_1 + \int_0^\infty B_0(e_2) \bar{N}_{11}(e_2, s) de_2 \\
 &+ \int_0^\infty B_0(e_3) \bar{N}_9(e_3, s) de_3 + \int_0^\infty B_0(e_4) \bar{N}_{12}(e_4, s) de_4 \\
 &+ \int_0^\infty B_0(e_5) \bar{N}_{13}(e_5, s) de_5
 \end{aligned} \tag{29}$$

$$\left(s + \frac{\partial}{\partial e_1} + 4v_1 + 3v_2 + v_3 + v_4 + v_5 + A(e_1) \right) \bar{N}_1(e_1, s) = 0 \tag{30}$$

$$\left(s + \frac{\partial}{\partial e_1} + 4v_1 + A(e_1) \right) \bar{N}_2(e_1, s) = 0 \tag{31}$$

$$\left(s + \frac{\partial}{\partial e_2} + 4v_1 + 2v_2 + v_3 + v_4 + v_5 + A(e_2) \right) \bar{N}_3(e_2, s) = 0 \tag{32}$$

$$\left(s + \frac{\partial}{\partial e_2} + v_2 + A(e_2) \right) \bar{N}_4(e_2, s) = 0 \tag{33}$$

$$\left(s + \frac{\partial}{\partial e_2} + 2v_2 + A(e_2) \right) \bar{N}_5(e_2, s) = 0 \tag{34}$$

$$\left(s + \frac{\partial}{\partial e_2} + v_2 + A(e_2) \right) \bar{N}_6(e_2, s) = 0 \tag{35}$$

$$\left(s + \frac{\partial}{\partial e_1} + 4v_1 + A(e_1) \right) \bar{N}_7(e_1, s) = 0 \tag{36}$$

$$\left(s + \frac{\partial}{\partial e_1} + 4v_1 + A(e_1) \right) \bar{N}_8(e_1, s) = 0 \tag{37}$$

$$\left(s + \frac{\partial}{\partial e_3} + B_0(e_3) \right) \bar{N}_9(e_3, s) = 0 \tag{38}$$

$$\left(s + \frac{\partial}{\partial e_1} + B_0(e_1) \right) \bar{N}_{10}(e_1, s) = 0 \tag{39}$$

$$\left(s + \frac{\partial}{\partial e_2} + B_0(e_2) \right) \bar{N}_{11}(e_2, s) = 0 \tag{40}$$

$$\left(s + \frac{\partial}{\partial e_4} + B_0(e_4) \right) \bar{N}_{12}(e_4, s) = 0 \tag{41}$$

$$\left(s + \frac{\partial}{\partial e_5} + B_0(e_5) \right) \bar{N}_{13}(e_5, s) = 0 \tag{42}$$

with boundary conditions as follows:

$$\bar{N}_1(0, s) = 3v_1 \bar{N}_0(s) \tag{43}$$

$$\bar{N}_2(0, s) = 4v_1 \bar{N}_1(0, s) \tag{44}$$

$$\bar{N}_3(0, s) = 3v_2 \bar{N}_0(s) \tag{45}$$

$$\bar{N}_4(0, s) = 2v_2 \bar{N}_3(0, s) \tag{46}$$

$$\bar{N}_5(0, s) = 3v_2 \bar{N}_1(0, s) \tag{47}$$

$$\bar{N}_6(0, s) = 2v_2 \bar{N}_5(0, s) \tag{48}$$

$$\bar{N}_7(0, s) = 4v_1 \bar{N}_3(0, s) \tag{49}$$

$$\bar{N}_8(0, s) = 4v_1 \bar{N}_7(0, s) \tag{50}$$

$$\bar{N}_9(0, s) = v_3 (\bar{N}_0(s) + \bar{N}_1(0, s) + \bar{N}_3(0, s)) \tag{51}$$

$$\bar{N}_{10}(0, s) = 4v_1 (\bar{N}_2(0, s) + \bar{N}_8(0, s)) \tag{52}$$

$$\bar{N}_{11}(0, s) = v_2 (\bar{N}_4(0, s) + \bar{N}_6(0, s)) \tag{53}$$

$$\bar{N}_{12}(0, s) = v_4 (\bar{N}_0(s) + \bar{N}_1(0, s) + \bar{N}_3(0, s)) \tag{54}$$

$$\bar{N}_{13}(0, s) = v_5 (\bar{N}_0(s) + \bar{N}_1(0, s) + \bar{N}_3(0, s)) \tag{55}$$

The following equations are obtained with assist of boundary conditions

$$\bar{N}_0(s) = \frac{1}{L(s)} \tag{56}$$

$$\bar{N}_1(s) = \frac{3v_1}{L(s)} \left(\frac{1 - \bar{s}_A (s + 4v_1 + 3v_2 + v_3 + v_4 + v_5)}{s + 4v_1 + 3v_2 + v_3 + v_4 + v_5} \right) \tag{57}$$

$$\bar{N}_2(s) = \frac{12v_1^2}{L(s)} \left(\frac{1 - \bar{s}_A (s + 4v_1)}{s + 4v_1} \right) \tag{58}$$

$$\bar{N}_3(s) = \frac{3v_2}{L(s)} \left(\frac{1 - \bar{s}_A (s + 4v_1 + 2v_2 + v_3 + v_4 + v_5)}{s + 4v_1 + 2v_2 + v_3 + v_4 + v_5} \right) \tag{59}$$

$$\bar{N}_4(s) = \frac{6v_2^2}{L(s)} \left(\frac{1 - \bar{s}_A(s + v_2)}{s + v_2} \right) \tag{60}$$

$$\bar{N}_5(s) = \frac{9v_1v_2}{L(s)} \left(\frac{1 - \bar{s}_A(s + 2v_2)}{s + 2v_2} \right) \tag{61}$$

$$\bar{N}_6(s) = \frac{18v_1v_2^2}{L(s)} \left(\frac{1 - \bar{s}_A(s + v_2)}{s + v_2} \right) \tag{62}$$

$$\bar{N}_8(s) = \frac{48v_1^2v_2}{L(s)} \left(\frac{1 - \bar{s}_A(s + 4v_1)}{s + 4v_1} \right) \tag{63}$$

$$\bar{N}_9(s) = \frac{(v_3 + 3v_1v_3 + 3v_2v_3)}{L(s)} \left(\frac{1 - \bar{s}_{B_0}(s)}{s} \right) \tag{64}$$

$$\bar{N}_{10}(s) = \frac{(48v_1^3 + 192v_1^3v_2)}{L(s)} \left(\frac{1 - \bar{s}_{B_0}(s)}{s} \right) \tag{65}$$

$$\bar{N}_{11}(s) = \frac{(6v_3^3 + 18v_1v_3^3)}{L(s)} \left(\frac{1 - \bar{s}_{B_0}(s)}{s} \right) \tag{66}$$

$$\bar{N}_{12}(s) = \frac{(v_4 + v_1v_4 + 3v_2v_4)}{L(s)} \left(\frac{1 - \bar{s}_{B_0}(s)}{s} \right) \tag{67}$$

$$\bar{N}_{13}(s) = \frac{(v_5 + 3v_1v_5 + 3v_2v_5)}{L(s)} \left(\frac{1 - \bar{s}_{B_0}(s)}{s} \right) \tag{68}$$

where $L(s)$ is defined as:

$$L(s) = \left(\begin{array}{c} s + 4v_1 + 3v_2 + v_3 + v_4 + v_5 \\ - \left(\begin{array}{c} 3v_1\bar{s}_A(s + 4v_1 + 3v_2 + v_3 + v_4 + v_5) \\ + 3v_2\bar{s}_A(s + 4v_1 + 2v_2 + v_3 + v_4 + v_5) \\ + \left(\begin{array}{c} v_3 + 3v_1v_3 + 3v_2v_3 + 48v_1^3 + 192v_1^3v_2 \\ + 6v_3^3 + 18v_1v_3^3 + v_4 + 3v_1v_4 + 3v_2v_4 \\ + v_5 + 3v_1v_5 + 3v_2v_5 \end{array} \right) \bar{s}_{B_0}(s) \end{array} \right) \end{array} \right) \tag{69}$$

4.1 Probability of running state and down state

The entire working probability of the system is

$$\bar{N}_{up}(s) = \frac{1}{L(s)} \left(\begin{aligned} &1 + 3v_1 \left(\frac{1 - \bar{s}_A (s + 4v_1 + 3v_2 + v_3 + v_4 + v_5)}{s + 4v_1 + 3v_2 + v_3 + v_4 + v_5} \right) \\ &\quad + 12v_1^2 \left(\frac{1 - \bar{s}_A (s + 4v_1)}{s + 4v_1} \right) \\ &+ 3v_2 \left(\frac{1 - \bar{s}_A (s + 4v_1 + 2v_2 + v_3 + v_4 + v_5)}{s + 4v_1 + 2v_2 + v_3 + v_4 + v_5} \right) \\ &\quad + 6v_2^2 \left(\frac{1 - \bar{s}_A (s + 4v_2)}{s + 4v_2} \right) \\ &+ 9v_1v_2 \left(\frac{1 - \bar{s}_A (s + 2v_2)}{s + 2v_2} \right) + 18v_1v_2^2 \left(\frac{1 - \bar{s}_A (s + v_2)}{s + v_2} \right) \\ &+ 12v_1v_2 \left(\frac{1 - \bar{s}_A (s + 4v_1)}{s + 4v_1} \right) + 48v_1^2v_2 \left(\frac{1 - \bar{s}_A (s + 4v_1)}{s + 4v_1} \right) \end{aligned} \right) \tag{70}$$

$$\bar{N}_{down}(s) = 1 - \bar{N}_{up}(s) \tag{71}$$

4.2 Presentation and examination of the models

4.2.1 Presentation and analysis of availability through copula repair

From (70), considering

$$S_{v_0}(s) = \bar{S}_{\exp[y^\theta + \{\log \varphi(y)\}^\theta]^{1/\theta}}(s) = \frac{\exp[y^\theta + \{\log \varphi(y)\}^\theta]^{1/\theta}}{s + \exp[y^\theta + \{\log \varphi(y)\}^\theta]^{1/\theta}},$$

$$\bar{S}_{v_1}(s) = \frac{v_1}{s + v_1},$$

and $v_1 = 0.0021$, $v_2 = 0.0022$, $v_3 = 0.0023$, $v_4 = 0.0024$, $v_5 = 0.0025$ rates of repair as $A(e_1) = A(e_2) = 1$, and inverting the Laplace to have:

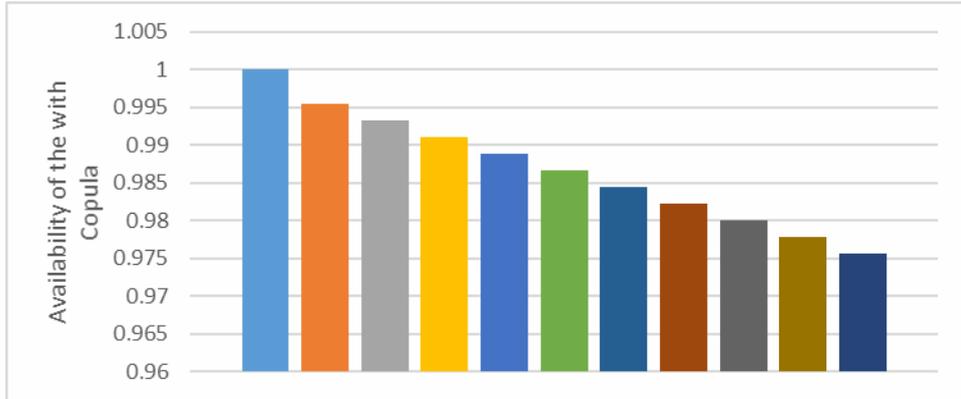
$$\bar{N}_{up}(q) = \left(\begin{aligned} &0.002699e^{-2.72565q} - 0.000362e^{-1.03377q} - 0.000002e^{-1.02103q} \\ &+ 0.997759e^{-0.00223q} - 0.000053e^{-1.00840q} - 0.000017e^{-1.00220q} \\ &\quad - 0.000023e^{-1.00440q} \end{aligned} \right) \tag{72}$$

For availability examination, considering passage of time $q \in [0, 10]$ to have Table 2 using equation (72).

Table 2 Time passage with the corresponding availability using copula repair

Q	0	1	2	3	4	5
Availability	1.0000	0.9955	0.9932	0.9910	0.9888	0.9866
Q	6	7	8	9	10	
Availability	0.9844	0.9822	0.9800	0.9778	0.9756	

Figure 2 Time passage with the corresponding availability under copula repair (see online version for colours)



4.2.2 Presentation and analysis of availability through general repair

For availability examination, considering passage of time $q \in [0, 10]$ to have Table 3 using equation (73).

Table 3 Time passage with the corresponding availability using general repair

q	0	1	2	3	4	5
Availability	1.0000	0.9934	0.9896	0.9869	0.9845	0.9822
q	6	7	8	9	10	
Availability	0.9800	0.9778	0.9756	0.9734	0.9713	

Figure 3 Time passage with the corresponding availability under general repair (see online version for colours)

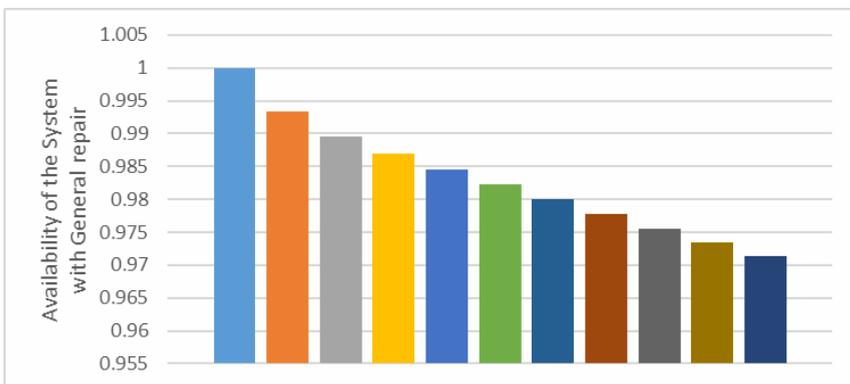
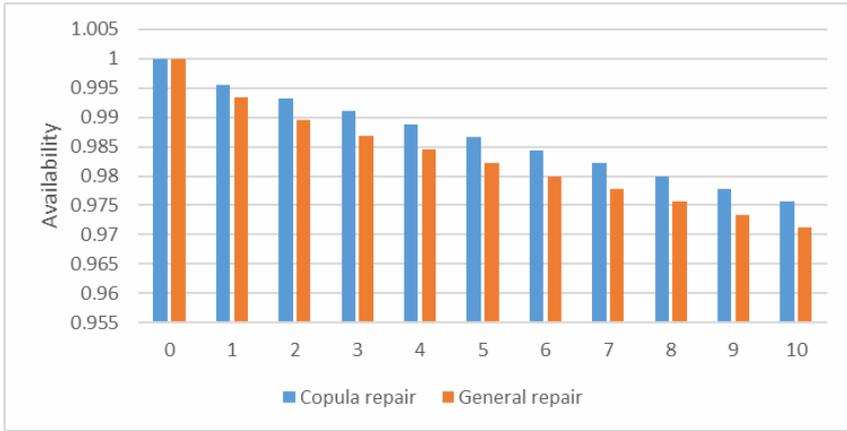


Figure 4 Comparison of availability of the system under copula and general repair with passage of time (see online version for colours)



4.2.3 Presentation and analysis of reliability model

Following the similar procedure in Section 4.2, $A(e_1)$ and $A(e_2)$ assigned to zero, and inverting the Laplace in (70), the reliability model of the system is

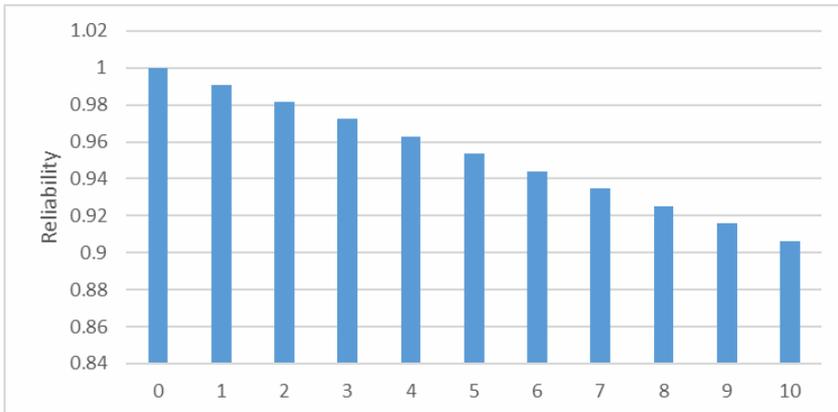
$$R(q) = \left(\begin{array}{l} 0.002335e^{-0.00440q} + 0.007885e^{-0.00840q} + 0.001461e^{-0.00220q} \\ + 3e^{-0.02000q} + 4.494382e^{-0.02220q} \end{array} \right) \quad (74)$$

For reliability examination, considering passage of time $q \in [0, 30]$ to have Table 4.

Table 4 Time passage with the corresponding reliability

Q	0	1	2	3	4	5
$R(q)$	1.0000	0.9908	0.9816	0.9723	0.9630	0.9536
Q	6	7	8	9	10	
$R(q)$	0.9442	0.9348	0.9253	0.9158	0.9063	

Figure 5 Time passage with the corresponding reliability (see online version for colours)



4.2.4 Presentation and analysis of MTTF and sensitivity

By considering $A(e_1)$, $A(e_2)$ and B_0 to zero in equation (70) to have the MTTF is produced as:

$$\begin{aligned}
 MTTF &= \lim_{s \rightarrow 0} \bar{N}_{up}(s) \\
 &= \frac{1}{4v_1 + 3v_2 + v_3 + v_4 + v_5} \left(\begin{aligned} &1 + \frac{3v_1}{4v_1 + 3v_2 + v_3 + v_4 + v_5} \\ &+ \frac{12v_1^2}{4v_1} + \frac{3v_2}{4v_1 + 2v_2 + v_3 + v_4 + v_5} \\ &+ \frac{6v_2^2}{v_2} + \frac{9v_1v_2}{2v_2} + \frac{18v_1v_2^2}{v_2} + \frac{12v_1v_2}{4v_1} + \frac{18v_2v_1^2}{4v_1} \end{aligned} \right) \quad (75)
 \end{aligned}$$

Considering $v_1 = 0.0021$, $v_2 = 0.0022$, $v_3 = 0.0023$, $v_4 = 0.0024$, $v_5 = 0.0025$ in (75), MTTF of the system is computed and presented in Table 5.

Table 5 Effect of rate of failure on MTTF

Failure rate	MTTF v_1	MTTF v_2	MTTF v_3	MTTF v_4	MTTF v_5
0.0001	112.0903	90.0547	86.0737	86.6910	87.3166
0.0002	109.2237	89.1328	85.4645	86.0737	86.6810
0.0003	106.5129	88.2328	84.8631	85.4645	86.0737
0.0004	103.9447	87.3539	84.2696	84.8631	85.4645
0.0005	101.5072	86.4949	83.6837	84.2696	84.8631
0.0006	99.1900	85.6551	83.1054	83.6837	84.2616
0.0007	96.9840	84.8336	82.5344	83.1054	83.6837
0.0008	94.8808	84.0297	81.9706	82.5344	83.1054
0.0009	92.8731	83.2426	81.4140	81.9706	82.5344

Figure 6 Impact of rate of failures on MTTF (see online version for colours)

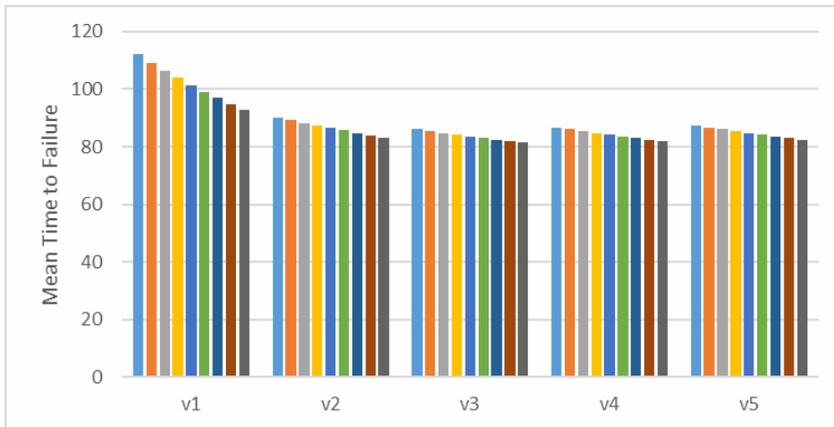
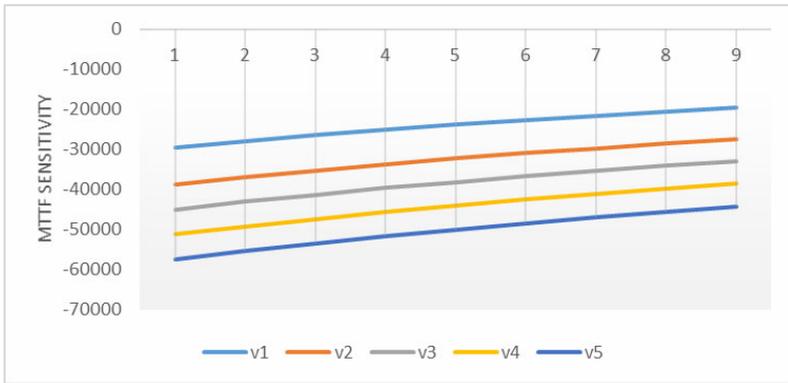


Table 6 Effect of rate of failure on Sensitivity of the system

Failure rate	$\frac{\partial(MTTF)}{v_1}$	$\frac{\partial(MTTF)}{v_2}$	$\frac{\partial(MTTF)}{v_3}$	$\frac{\partial(MTTF)}{v_4}$	$\frac{\partial(MTTF)}{v_5}$
0.0001	-29,493.8336	-9,332.7613	-6,132.7229	-6,214.3985	-6,297.6411
0.0002	-27,863.3532	-9,107.3694	-6,052.5763	-6,132.7229	-6,214.3985
0.0003	-26,374.5835	-8,892.7681	-5,973.9219	-6,052.5763	-6,132.7229
0.0004	-25,010.5378	-8,688.0648	-5,896.7240	-5,973.9219	-6,052.5763
0.0005	-23,756.8084	-8,492.4686	-5,820.9478	-5,896.7240	-5,973.9219
0.0006	-22,601.1019	-8,305.2764	-5,746.5597	-5,820.9478	-5,896.7240
0.0007	-21,532.8663	-8,125.8612	-5,673.5269	-5,746.5597	-5,820.9478
0.0008	-20,542.9951	-7,953.6627	-5,601.8177	-5,673.5269	-5,746.5597
0.0009	-19,623.5888	-7,788.1780	-5,531.4012	-5,601.8177	-5,673.5269

Figure 7 MTTF sensitivity of the system with respect to failure (see online version for colours)



4.2.5 Presentation and analysis of cost using copula repair

The expression for the profit anticipation accumulated in $[0, q]$

$$E_p(q) = T_1 \int_0^q N_{up}(r) dr - T_2 r \tag{76}$$

From (70), the subsequent equation (77) follows:

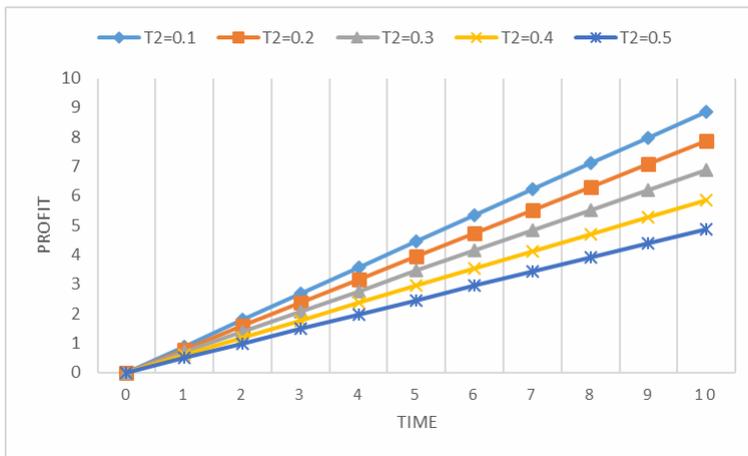
$$E_p(q) = T_1 \left\{ \begin{array}{l} -0.000990e^{-2.72565q} + 0.000350e^{-1.03377q} \\ +0.000002e^{-1.02103q} - 445.616229e^{-0.00223q} \\ +0.000053e^{-1.00840q} + 0.00017e^{-1.00223q} \\ +0.000023e^{-1.00440q} + 445.6167733 \end{array} \right\} - T_2(q) \tag{77}$$

The anticipated profit is calculated using equation (77) as displayed in Table 7.

Table 7 Passage of time with the corresponding profit using copula

q	$E_p(q)$ for $T_1 = 1$ $T_2 = 0.1$	$E_p(q)$ for $T_1 = 1$ $T_2 = 0.2$	$E_p(q)$ for $T_1 = 1$ $T_2 = 0.3$	$E_p(q)$ for $T_1 = 1$ $T_2 = 0.4$	$E_p(q)$ for $T_1 = 1$ $T_2 = 0.5$
0	0	0	0	0	0
1	0.8972	0.7972	0.6972	0.5972	0.4972
2	1.7916	1.5916	1.3916	1.1916	0.9916
3	2.6838	2.3838	2.0838	1.7838	1.4838
4	3.5737	3.1737	2.7737	2.3737	1.9737
5	4.4615	3.9615	3.4615	2.9615	2.4615
6	5.3470	4.7470	4.1470	3.5470	2.9470
7	6.2364	5.5304	4.8304	4.1304	3.4304
8	7.1115	6.3115	5.5115	4.7115	3.9115
9	7.9905	7.0905	6.1905	5.2905	4.3905
10	8.8672	7.8672	6.8672	5.8672	4.8672

Figure 8 Time passage with the corresponding profit under copula repair (see online version for colours)



4.2.6 Presentation and analysis of cost using general repair

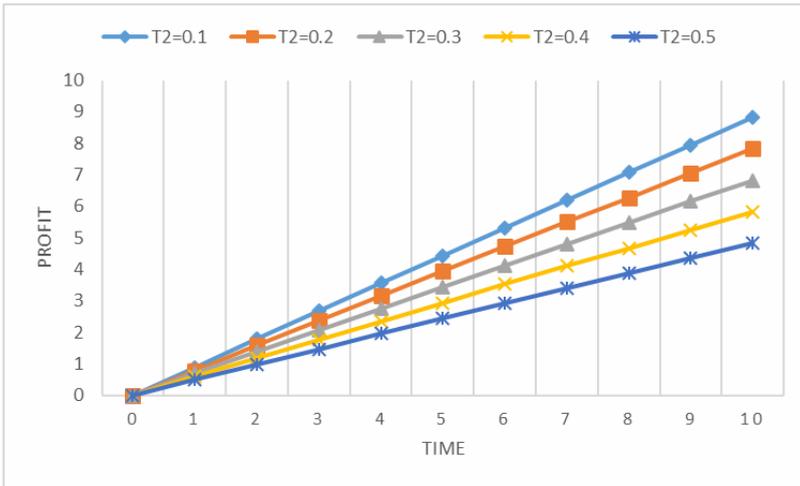
$$E_p(q) = T_1 \left\{ \begin{array}{l} 0.002920e^{-1.03693q} - 0.000030e^{-1.02106q} \\ -0.004188e^{-1.00417q} - 44.610117e^{-0.00222q} \\ +0.000094e^{-1.00840q} + 0.000017e^{-1.00220q} \\ +0.000407e^{-1.00440q} + 445.6167732 \end{array} \right\} - T_2(q) \tag{78}$$

The expected profit is determined using equation (78) as indicated in Table 8.

Table 8 Passage of time with the corresponding profit using general repair

q	$E_p(q)$ for $T_1 = 1$ $T_2 = 0.1$	$E_p(q)$ for $T_1 = 1$ $T_2 = 0.2$	$E_p(q)$ for $T_1 = 1$ $T_2 = 0.3$	$E_p(q)$ for $T_1 = 1$ $T_2 = 0.4$	$E_p(q)$ for $T_1 = 1$ $T_2 = 0.5$
0	0	0	0	0	0
1	0.8963	0.7963	0.6963	0.5963	0.4963
2	1.7878	1.5878	1.3878	1.1878	0.9878
3	2.6760	2.3760	2.0760	1.7760	1.4760
4	3.5617	3.1617	2.7617	2.3617	1.9617
5	4.4415	3.9415	3.4451	2.9451	2.4451
6	5.3262	4.7262	4.1262	3.5262	2.9262
7	6.2052	5.5052	4.8052	4.1052	3.4052
8	7.0819	6.2819	5.4819	4.6819	3.8819
9	7.9565	7.0565	6.1565	5.2565	4.3565
10	8.8289	7.8289	6.8289	5.8289	4.8289

Figure 9 Time passage with the corresponding profit under general repair (see online version for colours)



4.3 Results and discussion

The system’s performance and reliability measures are evaluated for various failure and repair rates. For fixed rates of repair and failure $v_1 = 0.0021$, $v_2 = 0.0022$, $v_3 = 0.0023$, $v_4 = 0.0024$, $v_5 = 0.0025$, repair rate as $A(e_1) = A(e_2) = 1$.

Tables 2 and 3 and Figures 3 and 4 can be useful tools for tracking system availability under copula and general repair over time and identifying trends or patterns that may require further investigation or action. By regularly monitoring and analysing data on system availability and reliability, organisations can take proactive measures to address issues before they become major problems. From the tables and simulations in figures, it

is evident that availability decreases slightly between the time interval of 0 to 10. From this time interval, it is worthwhile to note that the strength of the system with respect to availability remains strong as a result of the presence of factors which assists the units to with stand any fault that may lead to complete system collapse. The tables and simulations in figures described the availability of the system with passage of time. The figures and tables, exhibit the decrease in availability owing to passage of time. Tables 2 and 3 and Figure 3 and 4 can be useful tools for tracking system availability over time and identifying trends or patterns that may require further investigation or action. By regularly monitoring and analysing data on system availability, organisations can take proactive measures to address issues before they become major problems. Figure 5 delves into comparison of availability of the system using copula and general repair over time. From the figure, it is clear that availability of the system decreases for both copula and general repair with passage of time. It is evident from the figure that copula repair produced better availability over general repair.

Results depicted in Figure 6 and Table 4 have shown that reliability of the system drop slowly as time passes. The table and figure predict system performance in terms of as the system reliability. From the table and figure, it is clear that the system reliability drops over time. As a result, the model's graphical analysis demonstrates that it is simple to forecast the system's performance behaviour using system reliability at any time for any given set of parametric values.

The result of mean time to failure with respect to rates of failure is displayed in Figure 7 and Table 5. The MTTF value change is proportional to the system reliability. MTTF estimations for various failure rates, ν_1 , ν_2 , ν_3 , ν_4 and ν_5 . The fluctuation in the result of MTTF with respect rate of failures is significant. The data in table and figure show that as failure rate rises, MTTF rapidly declines. This suggests that in order to increase the reliability of a product or system, higher failure rates are probably a sign of underlying problems that need to be resolved. Organisations can take proactive steps to identify and address these issues before they become significant issues by routinely monitoring and analysing data on MTTF. Any reliability and maintenance program should track MTTF, and Table 5 and Figure 7 can be helpful resources for processing and deciphering this data. In order to spot any trends or patterns in failure rates that might call for additional research or remedial action, it is critical for organisations to keep track of the mean time to failure (MTTF) of their systems or products. For this purpose, Table 5 and Figure 7 can be helpful resources as they offer a clear illustration of how MTTF varies with respect to various failure rates. The change in MTTF sensitivity against rates of failure is shown in Figure 8 and Table 6 respectively.

Tables 7 and 8 and Figures 9 and 10 show how profit is generated by setting the revenue cost per unit time $T_1 = 1$ and varying the service costs $T_2 \in [0.1, 0.5]$ for copula and general repair respectively. Looking at the tables and Figures closely, one can predict that profit improves for low service costs for both copula and general repair. The tables and figures reflects the impact of service cost and time passage on the expected profit for the system. From the table and figure, it is clear that the expected profit decreases over time passage as the service cost increases from 0.1 to 0.5. From the table and figure, it is evident that with decrease in service cost from 0.5 to 0.1, the expected rises rapidly. In general, when low service is compared to higher service costs, the expected profit is higher, i.e., low service costs yield the highest profit and high service costs yield the lowest profit.

5 Conclusions

The present paper presents Reliability and evaluation of performance of solar powered irrigation system through copula approach. System MTTF, availability, cost function, sensitivity, and reliability explicit expressions are established and statistically validated. The mean time to failure, cost, availability and reliability are some of the requirement that impact the development of solar powered irrigation system. So, in order to get the maximum of operation of the systems, mean time to failure, cost, availability and reliability must be meticulously maintained to lower the rate of failure. Through the cost function analysis, it has been discovered that higher cost of services entailed lower system profit and vice versa. On the basis of MTTF, reliability, sensitivity, availability and cost analysis, repair policy of copula distribution enhanced system performance. These are the contributions of this study.

The study's tables and figures can be helpful tools for tracking system availability, MTTF, sensitivity, reliability, and profit over time, and they can aid farmers/organisations in identifying trends and patterns in system performance that may call for additional research or action. Monitoring and analysing data on system availability, MTTF, sensitivity, reliability, and profit on a regular basis enables farmers to take preventative action before minor issues turn into major ones.

Users or Organisations can gain insights into the overall health of their solar powered irrigation systems and pinpoint areas for improvement by tracking system availability, MTTF, sensitivity, reliability, and profit over time. For instance, if certain times of the day or week consistently see lower levels of system availability, MTTF, sensitivity, reliability, and profit, this may point to specific bottlenecks or resource limitations that need to be addressed. Similar to the previous example, if system metrics like availability, MTTF, sensitivity, reliability, and profit are trending lower over time, this may be a sign that systemic problems need to be resolved in order to keep the system resilient and reliable.

Early detection of these trends and patterns enables users or organisations to take proactive action to address them before they develop into significant problems. To enhance performance and reliability, this may entail making adjustments to the solar powered irrigation system, deploying more resources, or optimising system configurations. Farmers or organisations can ultimately make sure that their irrigation systems are dependable, resilient, and capable of offering the necessary support to meet the demands of the food production by monitoring and analysing system availability data. By adding warm standby components, this work can be expanded.

References

- Ayagi, H.I., Wan, Z. and Yusuf, I. (2023) 'Performance modeling and estimation of serial system exhibiting k-out-of-n: G scheme with controller failure', *JJMIE*, Vol. 17, No. 2, pp.155–181.
- Bahrami, M., Khashei, M. and Amindoust, A. (2023) 'Demand forecasting using modelling and comparison of series and parallel structures of SARIMA and SMLP', *International Journal of Process Management and Benchmarking*, Vol. 13, No. 2, pp.282–301.
- Bao, M., Ding, Y., Yin, X., Shao, C. and Ye, C. (2021) 'Definitions and reliability evaluation of multi-state systems considering state transition process and its application for gas systems', *Reliability Engineering & System Safety*, Vol. 207, p.107387, <https://doi.org/10.1016/j.res.2020.107387>.

- Gahlot, M., Singh, V.V., Ayagi, H.I. and Abdullahi, I. (2020) 'Stochastic analysis of a two units' complex repairable system with switch and human failure using copula approach', *Life Cycle Reliability Safety Engineering*, Vol. 9, No. 1, pp.1–11.
- Isa, M.S., Yusuf, I., Ali, U.A., Suleiman, K., Yusuf, B. and Ismail, A.L. (2022) 'Reliability analysis of multi-workstation computer network configured as series-parallel system via Gumbel-Hougaard family copula', *Intern J Operat Res*, Vol. 19, No. 1, pp.13–26.
- Jain, V., Ajmera, P. and Suhaib, M. (2023) 'Modelling the factors affecting the QoWL in the employees of manufacturing organisations in India using ISM approach', *International Journal of Process Management and Benchmarking*, Vol. 14, No. 1, pp.1–22.
- Jia, H., Liu, D., Li, Y., Ding, Y., Liu, M. and Peng, R. (2020) 'Reliability evaluation of power systems with multi-state warm standby and multi-state performance sharing mechanism', *Reliability Engineering & System Safety*, Vol. 204, p.107139, <https://doi.org/10.1016/j.res.2020.107139>.
- Jia, H., Peng, R., Yang, L. and Li, Y. (2021) 'Reliability assessment of power systems with warm standby and energy storage', *2021 Global Reliability and Prognostics and Health Management (PHM-Nanjing)*, October, IEEE, pp.1–5.
- Kumar, A., Pant, S. and Ram, M. (2019) 'Gray wolf optimizer approach to the reliability-cost optimization of residual heat removal system of a nuclear power plant safety system', *Quality and Reliability Engineering International*, Vol. 35, No. 7, pp.2228–2239.
- Kumar, A., Singh, S.B. and Ram, M. (2020) 'Systems reliability assessment using hesitant fuzzy set', *International Journal of Operational Research*, Vol. 38, No. 1, pp.1–18.
- Lado, A. and Singh, V.V. (2019) 'Cost assessment of complex repairable systems in series configuration using Gumbel Hougaard family copula', *Int. J. Qual. Reliab. Manag.*, Vol. 36, No. 10, pp.1683–1698.
- Lado, A., Singh, V.V., Ismail, K.H. and Yusuf, I. (2018) 'Performance and cost assessment of repairable complex system with two subsystems connected in series configuration', *International Journal of Reliability and Application*, Vol. 19, No. 1, pp.27–42.
- Lin, S.W., Matanhire, T.B. and Liu, Y.T. (2021) 'Copula-based Bayesian reliability analysis of a product of a probability and a frequency model for parallel systems when components are dependent', *Applied Sciences*, Vol. 11, No. 4, p.1697.
- Mellal, M.A. and Zio, E. (2020) 'System reliability-redundancy optimization with cold-standby strategy by an enhanced nest cuckoo optimization algorithm', *Reliability Engineering & System Safety*, Vol. 201, p.106973, <https://doi.org/10.1016/j.res.2020.106973>.
- Mellal, M.A., Al-Dahidi, S. and Williams, E.J. (2020) 'System reliability optimization with heterogeneous components using hosted cuckoo optimization algorithm', *Reliability Engineering & System Safety*, Vol. 203, p.107110, <https://doi.org/10.1016/j.res.2020.107110>.
- Okafor, E.G., Ezugwu, E.O., Jemitola, P.O., Sun, Y.C. and Lu, Z. (2018) 'Multistate system reliability modeling using copula function', *2018 3rd International Conference on System Reliability and Safety (ICSRS)*, November, IEEE, pp.135–141.
- Pai, R., Chattopadhyay, G. and Karmakar, G. (2023) 'Maintenance and asset management practices of industrial assets: importance of tribological practices and digital tools', *International Journal of Process Management and Benchmarking*, Vol. 13, No. 2, pp.233–256.
- Poonia, P.K. (2021) 'Performance assessment of a multi-state computer network system in series configuration using copula repair', *International Journal of Reliability and Safety*, Vol. 15, Nos. 1–2, pp.68–88.
- Prajapati, D.R., Singh, P.J. and Tanwer, A.K. (2021) 'Reliability analysis of variables of environmental management in Indian micro scale industries and SMEs', *International Journal of Process Management and Benchmarking*, Vol. 11, No. 1, pp.115–149.
- Pundir, P.S., Patawa, R. and Gupta, P.K. (2021) 'Analysis of two non-identical unit cold standby system in presence of prior information', *American Journal of Mathematical and Management Sciences*, Vol. 40, No. 4, pp.320–335.

- Sharifi, M., Moghaddam, T.A. and Shahriari, M. (2019) 'Multi-objective redundancy allocation problem with weighted-k-out-of-n subsystems', *Heliyon*, Vol. 5, No. 12, <https://doi.org/10.1016/j.heliyon.2019.e02346>.
- Singh, V.V., Gulati, J., Rawal, D.K. and Goel, C.K. (2016) 'Performance analysis of complex system in series configuration under different failure and repair discipline using copula', *Int. J. Reliab. Qual. Saf. Eng.*, Vol. 23, No. 2, pp.812–832.
- Singh, V.V., Mohhammad, A.I., Ibrahim, K.H. and Yusuf, I. (2022a) 'Performance assessment of the complex repairable system with n-identical units under (k-out-of-n: G) scheme and copula linguistic repair approach', *International Journal of Quality & Reliability Management*, Vol. 39, No. 2, pp.367–386.
- Singh, V.V., Ismail, A.L., Chand, U. and Maiti, S.S. (2022b) 'Performance assessment of complex system under the k-out-of-n: G type configuration with k consecutive degraded states through the copula repair approach', *International Journal of Reliability, Quality and Safety Engineering*, Vol. 29, No. 2, p.2150047.
- Sundari, S.M., Murugeswari, N. and Manikandan, P. (2020) 'A study on the performance measures of the non-Markovian model of optional types of service with extended vacation, renege process and service interruption followed by phases of repair', *International Journal of Process Management and Benchmarking*, Vol. 10, No. 4, pp.520–549.
- Wang, W., Lin, M., Fu, Y., Luo, X. and Chen, H. (2020) 'Multi-objective optimization of reliability-redundancy allocation problem for multi-type production systems considering redundancy strategies', *Reliability Engineering & System Safety*, Vol. 193, p.106681, <https://doi.org/10.1016/j.res.2019.106681>.
- Xie, L., Lundteigen, M.A. and Liu, Y. (2020) 'Reliability and barrier assessment of series-parallel systems subject to cascading failures', *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, Vol. 234, No. 3, pp.455–469.
- Xie, L., Lundteigen, M.A. and Liu, Y. (2021a) 'Performance assessment of K-out-of-N safety instrumented systems subject to cascading failures', *ISA Transactions*, Vol. 118, pp.35–43, <https://doi.org/10.1016/j.isatra.2021.02.015>.
- Xie, L., Lundteigen, M.A. and Liu, Y. (2021b) 'Performance analysis of safety instrumented systems against cascading failures during prolonged demands', *Reliability Engineering & System Safety*, Vol. 216, p.107975, <https://doi.org/10.1016/j.res.2021.107975>.
- Xie, L., Lundteigen, M.A., Liu, Y., Kassa, E. and Zhu, S. (2019) 'Performance assessment of safety-instrumented systems subject to cascading failures in high-demand mode', *Proceedings of the 29th European Safety and Reliability Conference (ESREL)*, 22–26 September, Research Publishing Services, Hannover, Germany.
- Yemane, A. and Colledani, M. (2019) 'Performance analysis of unreliable manufacturing systems with uncertain reliability parameters estimated from production data', *International Journal of Computer Integrated Manufacturing*, Vol. 32, No. 9, pp.875–889.
- Yusuf, I., Ismail, A.L. and Ali, U.A. (2020) 'Reliability analysis of communication network with redundant relay station under partial and complete failure', *Journal of Mathematics and Computer Science*, Vol. 10, No. 4, pp.863–880.
- Yusuf, I., Ismail, A.L., Sufi, N.A., Ambursa, F.U., Sanusi, A. and Isa, M.S. (2021) 'Reliability analysis of distributed system for enhancing data replication using Gumbel-Hougaard family copula approach joint probability distribution', *Journal of Industrial Engineering International*, Vol. 17, No. 3, pp.59–78.