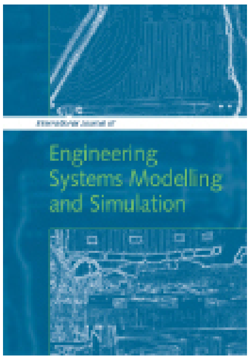

International Journal of Engineering Systems Modelling and Simulation

ISSN online: 1755-9766 - ISSN print: 1755-9758
https://www.inderscience.com/ijesms

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DOI: 10.1504/IJESMS.2025.10067657

Article History:

Received:	18 June 2023
Last revised:	11 September 2023
Accepted:	01 October 2023
Published online:	02 December 2024

Dynamic modelling of CNTs under multiple moving nanoparticles based on non-local strain gradient theory

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Abstract: This paper presents an investigation on the dynamic behaviour of single-walled carbon nano-tubes (SWCNT) crossed by multiple moving nano-particles modelled as mobile loads based on Timoshenko's non-local beam theory, including rotational inertia. The governing equations are derived using Hamilton's principle combined with Galerkin's method. The eigen-frequencies are determined by solving analytically the system of equations which govern the problem of the eigenvalues. Temporal integration of the Newmark's method is adopted to find the dynamic response of single-walled carbon nanotubes (SWCNT). The obtained eigen-frequencies are validated by the results of previous published studies. A detailed parametric study is carried out to analyse the effects of non-local parameters, the slenderness ratio, the displacement speed and the number of moving loads on the vibration characteristics of the model. Type or paste your abstract here as prescribed by the journal's instructions for authors. The obtained results show that above mentioned effects play an important role and have a significant effect on the transitional dynamic behaviour of single-walled carbon nanotubes (SWCNT).

Keywords: dynamic behaviour; SWCNT; multiple moving nano-particles; Timoshenko's non-local beam theory; displacement speed; eigen-frequencies; dynamic response lower.

Reference to this paper should be made as follows: Elmeiche, A., Bouamama, M., Elhannani, A. and Lousdad, A. (2025) 'Dynamic modelling of CNTs under multiple moving nanoparticles based on non-local strain gradient theory', *Int. J. Engineering Systems Modelling and Simulation*, Vol. 16, No. 1, pp.23–33.

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1 Introduction

Nano-technologies and nano-sciences are a study domain of systems measuring less than forty nanometres. The nano-technology is not limited only to the study of the systems but also aims to manufacture and manipulate nano-metric objects and nano-materials. In fact the applications of the nano-technology are multiple in various domains (Dequesnes, 2019).

During these last years the structures at nano-metric scale has brought a considerable interest for future applications of nano-electro-mechanical systems (NEMS) and microscopy of atomic force (MAF) (Dai et al., 1996; Lourie and Wagner, 1998; Ahouel et al., 2016; Wang, 2022; Agarwal et al., 2022). Conventional models of structures based on classical continuous media theories were unable to describe the effects due to the lack of the material scale parameters (Khetir et al., 2016). One of the most known models is the Eringen's non-local elasticity theory. Works based on this non-local elasticity theory had been published such as Eringen and Wegner (2003), and Aydogdu (2009) Thai (2012). In 1983 Eringen (1983) suggested that the integral constitutive law can be simplified under the form of differential equations when the type of the function core is specified.

On the contrary, to the local theories which assume that the stress at a point is a function of the deformation at the same point, the non-local elasticity theory supposes that the stress at a point depends on the deformations of all points of the continuous media. The non-local elasticity theory has been extensively applied to analysing the static and dynamic response of nano-beams (Şimşek, 2014; Karličić et al., 2016; Li et al., 2018; Baroudi et al., 2018; Eptaimeros et al., 2018; Noroozi et al., 2019; Ghafarian and Ariaei, 2016), nano-plates (Pugno, 2005; Murmu and Adhikari, 2011; Hedayatian et al., 2020; Xu et al., 2023), nano-shells (Brito-Silva et al., 2013; Ahmadi and Arami, 2013; Shi et al., 2020 as well as nano-tubes (Dequesnes et al., 2004; Pataki et al., 2007; Adali, 2023; Shokouhifard et al., 2023; Borjalilou et al., 2019).

Many studies had been focused on the forced vibration of nanobeams subjected to moving loads. Şimşek (2011) made a dynamic analysis of a single walled carbon nano-tube (SWCNT) traversed by a single moving load using the non-local Timoshenko beams theory. Togun (2016) studied the non-linear vibration of a nano-beam with a mass attached at its extremity using the non-local elasticity theory. Abouelregal and Zenkour (2017) had determined the dynamic response of a nano-beam induced by ramped heating and under a moving load. Ghadiri et al. (2017) studied the non-linear forced vibrations of Euler-Bernoulli nano-beams under a concentrated moving load supported on a viscoelastic foundation taking into consideration the thermal and surface effects. Rahmani et al. (2018) analysed the forced vibration of a nano-tube under excitation of a moving harmonic force with considering modified non-local elasticity. Elmeiche et al. (2018) presented a forced dynamic behaviour of functionally graded nano-beams excited by a unique concentrated moving load using the non-local high order beams theory. Malikan et al. (2018) makes an analysis of a damped forced vibration of a SWCNT by means of a new beams shear deformation theory; SWCNT are modelled as a flexible beam resting on a viscoelastic foundation drown in thermal environment. Barati et al. (2018) examined the dynamic response of a nano-beam subjected to moving load under hygrothermal environments based on the gradient theory of non-local deformation.

The objective of this research consists in studying the forced dynamic behaviour of a SWCNT using Eringen's non-local elasticity theory. The development of the model is founded on Timoshenko beams theory under passing of many crossing moving loads at constant speeds. The global equation of the movement is derived via a combination of Hamilton's principle and Galerkin's method taking into consideration the effect of the rotational inertia. The mathematical solutions are treated analytically by modal analysis and the forced dynamic responses are computed numerically using the Newmark's direct integral. A computational numerical code is developed under MATLAB software in order to determine the eigen-frequencies and modal responses of the vibratory system. A parametric study is made in order to analyse the different physical and geometric criteria that react on the transient dynamic behaviour of the SWCNT.

2 Non-local elasticity theory

The response of structures at a nano-metric scale is different from that of the classical theory. According to the non-elasticity theory, the stress field on an arbitrary point 'x' in an elastic continuum not only depends on the stress field at the same point but also on the stresses at all other points of the body Eringen and Edelen (1972).

This hypothesis can be expressed as:

$$\left[1-\mu\nabla^2\right]\bar{\sigma}=\bar{C}:\bar{\varepsilon} \tag{1}$$

where $\mu = (e_0a)^2$ is the non-local parameter, 'e_0' is an appropriate constant for each material, 'a' is the internal characteristic length Şimşek and Reddy (2013). ∇^2 is the Laplace operator. $\bar{\sigma}$ is the stress tensor indicates the product at a double point. $\bar{C}$ is the Hookean's elasticity tensor, $\bar{\varepsilon}$ is the deformation tensor. For a homogeneous isotropic Timoshenko beam the non-local constitutive relation is written as:

$$\begin{aligned}\sigma_{xx}-\mu\frac{\partial^2\sigma_{xx}}{\partial x^2}&=E.\varepsilon_{xx}\\ \sigma_{xz}-\mu\frac{\partial^2\sigma_{xz}}{\partial x^2}&=G.\gamma_{xz}\end{aligned} \tag{2}$$

where E is the modulus of elasticity. $G = E/2(1 + v)$ is the shear modulus (where v is Poisson's coefficient, σ_{xx} is the axial stress, σ_{xz} is the shear stress, ε_{xx} is the axial strain and is the shear deformation. When 'γ_{xz}' is null we can derive the constitutive relation of the classical theory.

3 Dynamic model of the SWCNT

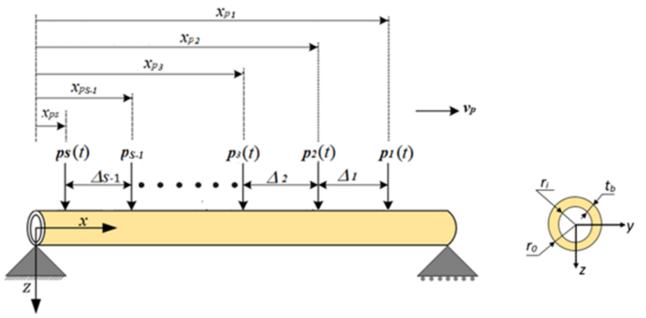
A simply supported SWCNT with a length (L), a diameter (d) and a thickness (t_b) is shown in Figure 1. The SWCNT is subjected to several moving loads $Ps(t)$ which move in the axial direction of the nano-tubes with a constant speed vps.

On the basis on the Timoshenko beam theory, the displacement field of any point of the nano-beam is given as follow:

$$\begin{cases}U(x,z,t)=u_0(x,t)+z.\psi_0(x,t)\\ \mathrm{W}(x,z,t)=w_0(x,t)\end{cases} \tag{3}$$

where u_0 (x, t) and $w_0(x, t)$ are displacement components in the median plane, ψ_0 is the rotation of the nano-beam section and 't' is the time.

Figure 1 Model of the simply supported single-walled carbon nano-tube (see online version for colours)



In the assumption of small perturbations the deformation tensor is expressed as:

$$\begin{cases}\varepsilon_{xx}=\dfrac{\partial u_0}{\partial x}+z\dfrac{\partial\psi_0}{\partial x}\\ \gamma_{xz}=\dfrac{\partial w_0}{\partial x}+\psi_0\end{cases} \tag{4}$$

The energy of deformation U_{int} is given by:

$$U_{int}=\frac{1}{2}\int_V\left(\sigma_{xx}\varepsilon_{xx}+\sigma_{xz}\gamma_{xz}\right)dV \tag{5}$$

$$U_{int}=\frac{1}{2}\int_0^L\left[N^x\frac{\partial u_0}{\partial x}+Q^x\left(\frac{\partial w_0}{\partial x}+\psi_0\right)+M^x\frac{\partial\psi_0}{\partial x}\right]dx \tag{6}$$

Nx and Mx are the results of the stresses defined:

$$N^x=\int_A\sigma_{xx}dA, M^x=\int_A\sigma_{xx}zdA, Q^x=\int_A\sigma_{xz}dA \tag{7}$$

The kinetic energy K can be calculated from:

$$K=\int_0^L\left[I_1\left(\frac{\partial u_0}{\partial t}\right)^2+2I_2\left(\frac{\partial u_0}{\partial t}\right)\left(\frac{\partial\psi_0}{\partial t}\right)+I_3\left(\frac{\partial\psi_0}{\partial t}\right)^2+I_1\left(\frac{\partial w_0}{\partial t}\right)^2\right]dx \tag{8}$$

The inertia coefficients of equation (8) are defined as:

$$[I_1,I_2,I_3]=\int_A\rho\left[1,z,z^2\right]dA$$

The potential energy of the moving forces is given by:

$$U_{ext}=\int_0^L F_{ext}w_0dx=-\sum_{s=1}^{nload}\int_0^L\left[P_s\delta\left(x-\upsilon_{ps}t_s\right)w_0\right]dx \tag{9}$$

where (δ.) is the Dirac-delta function, v_{ps} is the speed of the moving loads, P_s is the module of the punctual force for the s^{th} order load, defined by:

$$p_s=\frac{Q_0}{s}$$

Using the Hamilton's principle:

$$\int_0^t(\delta U_{int}-\delta K+\delta U_{ext})dt=0 \tag{10}$$

By replacing equations (6), (8) and (9) in equation (10) and performing partial integration the following equilibrium equations are obtained:

$$\begin{cases} \dfrac{\partial N^c}{\partial x} = I_1\left(\dfrac{\partial^2 u_0}{\partial t^2}\right) + I_2\left(\dfrac{\partial^2 \psi_0}{\partial t^2}\right) \\ \dfrac{\partial Q^x}{\partial x} + F_{ext} = I_1\left(\dfrac{\partial^2 w_0}{\partial t^2}\right) \\ \dfrac{\partial M^x}{\partial x} - Q^x = I_2\left(\dfrac{\partial^2 u_0}{\partial t^2}\right) + I_3\left(\dfrac{\partial^2 \psi_0}{\partial t^2}\right) \end{cases} \tag{11}$$

By introducing the equilibrium equations (11) in Eringen non-local constitutive relation (2), the following differential equations are obtained:

$$\begin{cases} N^x = A_{xx}\dfrac{\partial u_0}{\partial x} + B_{xx}\dfrac{\partial \psi_0}{\partial x} + \mu\left[I_1\left(\dfrac{\partial^3 u_0}{\partial x \partial t^2}\right) + I_2\left(\dfrac{\partial^3 \psi_0}{\partial x \partial t^2}\right)\right] \\ M^x = B_{xx}\dfrac{\partial u_0}{\partial x} + D_{xx}\dfrac{\partial \psi_0}{\partial x} + \mu\left[I_2\left(\dfrac{\partial^3 u_0}{\partial x \partial t^2}\right) + I_3\left(\dfrac{\partial^3 \psi_0}{\partial x \partial t^2}\right) + I_1\left(\dfrac{\partial^2 w_0}{\partial t^2}\right) - F_{ext}\right] \\ Q^x = A_{xz}\left(\dfrac{\partial w_0}{\partial x} + \psi_0\right) + \mu\left[I_1\left(\dfrac{\partial^3 w_0}{\partial x \partial t^2}\right) - \dfrac{\partial F_{ext}}{\partial x}\right] \end{cases} \tag{12}$$

Such as:

$$[A_{xx}, B_{xx}, D_{xx}] = \int_A E.[1, z, z^2].dA \tag{13}$$

$$A_{xz} = k_S\int_A G.dA; \quad k_S = \frac{6(1+\upsilon)(1+e^2)^2}{(7+6\upsilon)(1+e^2)^2 + (20+12\upsilon)e^2} \tag{14}$$

$$e = \frac{r_{inner}}{R_{outer}}$$

Using equations (7), (9) and (12) the governing equations in term of displacement can be found and expressed as:

$$\begin{cases} A_{xx}\dfrac{\partial^2 u_0}{\partial x^2} + B_{xx}\dfrac{\partial^2 \psi_0}{\partial x^2} - \left(1-\mu\dfrac{\partial^2}{\partial x^2}\right)\left[I_1\left(\dfrac{\partial^2 u_0}{\partial t^2}\right) + I_2\left(\dfrac{\partial^2 \psi_0}{\partial t^2}\right)\right] = 0 \\ A_{xz}\left(\dfrac{\partial^2 w_0}{\partial x^2} + \dfrac{\partial \psi_0}{\partial x}\right) - \left(1-\mu\dfrac{\partial^2}{\partial x^2}\right)\left[I_1\left(\dfrac{\partial^2 w_0}{\partial t^2}\right)\right] = \left(1-\mu\dfrac{\partial^2}{\partial x^2}\right)\sum_{s=1}^{nload}\left[P_s\delta(x - v_{p_s}.t_s)\right] \\ B_{xx}\dfrac{\partial^2 u_0}{\partial x^2} + D_{xx}\dfrac{\partial^2 \psi_0}{\partial x^2} - A_{xz}\left(\dfrac{\partial w_0}{\partial x} + \psi_0\right) - \left(1-\mu\dfrac{\partial^2}{\partial x^2}\right)\left[I_2\left(\dfrac{\partial^2 u_0}{\partial t^2}\right) + I_3\left(\dfrac{\partial^2 \psi_0}{\partial t^2}\right)\right] = 0 \end{cases} \tag{15}$$

4 Mathematical solutions

To study the dynamic behaviour of the SWCNT, the components $u_0(x, t)$, $w_0(x, t)$ and $\psi_0(x, t)$ are expressed in modal expansions as the following:

$$u_0(x,t) = \sum_{j=1}^{n}\varphi_j(x).u_j(t); \quad w_0(x,t) = \sum_{k=1}^{n}\xi_k(x).\mathrm{w}_k(t); \quad \psi_0(x,t) = \sum_{p=1}^{n}\phi_p(x).\psi_p(t) \tag{16}$$

where $u_j(t)$, $w_k(t)$ and $\psi_p(t)$ are the general unknown coordinates which depend on time and $\phi_p(t)$ is the eigen-mode function of the nano-tubes.

By performing partial integration on equation (15) with the moderated functions respectively $\varphi_i(x)$, $\xi_k(x)$ and $\phi p(x)$; ($i = k = p = 1, 2, \ldots n$), which must satisfy the boundary conditions, the weak forms of the governing equation of the movement which is also equivalent to the ordinary differential equation can be expressed in the following form:

$$\begin{aligned} &A_{xx}\int_0^L \varphi_i'(x)\left(\sum_{j=1}^n \varphi_j'(x).u_j(t)\right).dx + B_{xx}\int_0^L \varphi_i'(x)\left(\sum_{p=1}^n \phi_p'(x).\psi_p(t)\right).dx \\ &+I_1\int_0^L \varphi_i(x)\left(\sum_{j=1}^n \varphi_j(x).\ddot{u}_j(t)\right).dx + I_2\int_0^L \varphi_i(x)\left(\sum_{p=1}^n \phi_p(x).\ddot{\psi}_p(t)\right).dx \\ &-\mu\left[I_1\int_0^L \varphi_i(x)\left(\sum_{j=1}^n \varphi_j''(x).\ddot{u}_j(t)\right).dx + I_2\int_0^L \varphi_i(x)\left(\sum_{j=1}^n \varphi_p''(x).\ddot{\psi}_p(t)\right).dx\right] \\ &-\left[N^x.\varphi_i(x)\right]_0^L = 0 \end{aligned} \tag{17}$$

$$\begin{aligned} &A_{xz}\int_0^L \xi_i'(x).\left(\sum_{k=1}^n \xi_k'(x).w_k(t)\right).dx + A_{xz}\int_0^L \xi_i'(x)\left(\sum_{p=1}^n \phi_p(x).\psi_p(t)\right).dx \\ &-I_1\int_0^L \xi_i(x).\left(\sum_{j=1}^n \xi_k(x).\ddot{w}_k(t)\right).dx + \mu\left[I_1\int_0^L \xi_i''(x)\left(\sum_{j=1}^n \xi_i''(x).\ddot{w}_k(t)\right).dx\right] \\ &+\left[Q^x.\xi_i(x)\right]_0^L = \int_0^L \xi_i(x).\left(\sum_{s=1}^{nload} p_s\delta\left(x - \upsilon_{p_s}.t_s\right)\right).dx \\ &-\mu\left[\int_0^L \xi_i''(x)\left(\sum_{s=1}^{nload} p_s\delta\left(x - \upsilon_{p_s}.t_s\right)\right).dx\right] \end{aligned} \tag{18}$$

$$\begin{aligned} &B_{xx}\int_0^L \phi_i'(x).\left(\sum_{j=1}^n \varphi_j'(x).u_j(t)\right).dx + D_{xx}\int_0^L \phi_i'(x)\left(\sum_{p=1}^n \phi_p'(x).\psi_p(t)\right).dx \\ &+A_{xz}\int_0^L \phi_i(x).\left(\sum_{k=1}^n \xi_i'(x).w_k(t)\right).dx + A_{xz}\int_0^L \phi_i(x)\left(\sum_{p=1}^n \phi_p(x).\psi_p(t)\right).dx \\ &+I_2\int_0^L \phi_i(x).\left(\sum_{j=1}^n \varphi_j(x).\ddot{u}_j(t)\right).dx + I_3\int_0^L \phi_i(x).\left(\sum_{p=1}^n \phi_p(x).\ddot{\psi}_j(t)\right).dx \\ &-\mu\left[I_2\int_0^L \varphi_i(x)\left(\sum_{j=1}^n \varphi_j''(x).\ddot{u}_j(t)\right).dx + I_3\int_0^L \varphi_i(x)\left(\sum_{j=1}^n \varphi_p''(x).\ddot{\psi}_p(t)\right).dx\right] \\ &+\left[M^x.\phi_i(x)\right]_0^L = 0 \end{aligned} \tag{19}$$

The Dirac-delta function of the transient forces is defined by Fryba (1972) as:

$$\int_{x_1}^{x_2}\xi(\mathrm{x}).\delta^{(\mathrm{n})}(x - x_p).dx = \begin{cases} -(1)^n.\xi^n(x_p) & si\; x_1 \prec x_p \prec x_2 \\ 0 & Autrement \end{cases} \tag{20}$$

where $\delta(n)$ represents the nth derivative of Dirac-delta function. The coefficients of the test functions in the limited integrals are called secondary variables. Their specifications constitute the boundary conditions. In this investigation, the analytical solutions are proposed for an isotropic simply supported nano-beam defined as:

$$\varphi_n(\mathrm{x}) = \cos\left(\frac{n\pi}{L}x\right), \xi_n(x) = \sin\left(\frac{n\pi}{L}x\right), \phi_n(x) = cos\left(\frac{n\pi}{L}x\right) \tag{21}$$

The secondary are eliminated through the virtual displacement principle. The global system of equations of the SWCNT crossed by multiple mobile forces is written under the matrix form as:

$$[K]^{Lc}\{q(\mathrm{t})\}+<[M]^{Lc}-\mu[\bar{M}]^{NLc}>\{\ddot{q}(\mathrm{t})\}=\{F(t)\}^{Lc}-\mu\overline{\{F(\mathrm{t})\}}^{NLc} \quad (22)$$

The terms $[K]$ and $[M]^{Lc}$ are the global stiffness and mass matrices in the classical local elasticity theory. Their order is $[3n \times 3n]$. $[M]^{NLc}$ is a $[3n \times 3n]$ supplementary global mass matrix due to the non-local effect. $\{q(t)\}$ is the global column vector of temporary unknown coefficients of order $[3n \times 1]$, $\{\ddot{q}(t)\}$ is the acceleration. $\{F(t)\}^{Lc}$ and $\{\overline{F(t)}\}^{NLc}$ are respectively the global local and non-local vectors produced by the mobile transversal forces of order $[3n \times 1]$.

5 Results and discussions

The transient vibrations of the SWCNT simply supported acting by several moving loads are investigated for different physical and geometrical parameters. The choice of the effective wall thickness and the elasticity modulus was for a long time a problem in nano-mechanics of SWCNT (Wang and Zhang, 2008). However this problem has been recently treated and solved in Gupta et al. (2010). The mechanical properties are expressed as: the Young's modulus $E = 1Tpa$, the density ρ = 2,300 kg/m^3, the Poisson's coefficient υ = 0.3. The internal and external diameters of the nano-tube are respectively d_{inner} = 0.3 nm and d_{outer} = 1 nm.

The forced modal responses are numerically solved using the Newmark's temporal integral Newmark (1959), with β = 0.25 and γ = 0.5. For a better satisfaction in Newmark schema, the time increment is fixed at dt = Tf/500. Tf is the necessary time to make move the last punctual load (Ps) to the right extremity of the nano-tube. The transient forces are spaced from each other by a uniform distance Δ = L/4. For practical reasons the numerical results are presented in terms of a standard form:

$$\varpi=\omega L\sqrt{\frac{\rho A}{EI}};D(t)=\max\frac{w\left(L/2,t\right)}{w_s\left(L/2\right)};D_{\max}=\max\frac{w\left(L/2,t\right)}{w_s\left(L/2\right)};\alpha=\frac{v_p}{v_{cr}};T=\frac{T_s}{T_l} \quad (23)$$

ws (L/2) is the deflection at mid-span of the SWCNT under the static load Q_0. vcr is critical value defined by Fryba (1972).

In order to validate the mathematical model the first three fundamental frequencies of the simply supported SWCNT are compared in Table 2. This is accomplished by varying the values of the slenderness ratio (L/d) and the non-local scale factor (μ). The Table 1 shows that there is a good agreement between the analytical results of the present study with the numerical solutions of reference Esen (2020). It is worth mentioning that the eigen-frequencies have been obtained using the same geometrical and material parameters of reference Esen (2020) with the shear modulus was Ks = 5/6.

From the analysis of the dynamic results, it can be noticed that the non-local effect soften the values of the fundamental frequencies which create a decrease the stiffness of the nano-tubes systems. The scale factor has a significant effect on the higher frequencies. For a non-local parameter (μ) which varies in the interval from [0 to 4] the first frequency (λ_1) is reduced by an average of 15.33% and the second frequency (λ_2) by 37.73% while the third frequency (λ_3) is reduced by 53.14%. Moreover, the increase of the geometrical ratio (L/d) also has a significant effect on the higher eigen-frequencies than that of lower eigen-frequencies. For a change from 5 to 100 of the aspect ratio, the change increment increases of 6.40% for the first frequency (λ_1), 22.65% for the second frequency (λ_2) and 44.30% for the third frequency (λ_3).

Table 2 illustrates the maximum normalised dynamic responses (Dmax) obtained from different thickness aspect ratios (L/h) taking into account the three modes of solicitations of the transient loads of a SWCNT. The numerical results are determined by varying the speed of the displacement load (α) and the non-local elasticity parameter (μ). We notice that the non-local parameter (μ) is related to Dmax. This tendency is quasi-linear to all the excitation modes whatever the applied displacement speeds. It is also observed a decreasing slope when the aspect ratio (L/h) increases. We also notice that the geometric ratio is inversely proportional to the maximum dynamic responses.

The solicitation of a single mobile load has an additional effect on the dynamic flexions compared to the other excitation modes. For a geometrical shrink from 100 to 5, the average Dmax ratio increases by 58.09% for a solicitation of a single moving load, by 54.05% for two moving loads and by 53.02% for three moving loads.

Figure 2 describes the forms of the transversal non dimensional displacements with respect to the normalised time at the mid-span of the simply supported SWCNT using different non-local parameters. The normalised dynamic deflections are plotted under several numbers of punctual moving loads with variable displacement speeds (α). The geometrical ratio is taken in the critical state with a value of L/d = 5 (see Table 2). We notice that the dynamic flexions are proportional to the non-local parameter (μ). The rate of deformation becomes minimum when the SWCNT is modelled without taking into consideration the scale effect (μ). It is maximum when the non-local parameter (μ) takes extreme values. This is due to the flexibility given to the vibratory system which generates instability under the form of dynamic responses.

Moreover the displacement speed f the moving loads plays an essential role in the study of forced vibration since the structure become more stable when the speed (α) overcomes the critical speed ($vp \geq vcr$). For the low crossing speeds, the dynamic behaviour of the SWCNT reacts in a remarkable flexibility which provokes disequilibrium in the form of the transversal dynamic responses. The solicitation to several moving loads induces a damping of the movement in the SWCNT which leads to a stabilised forced dynamic behaviour.

This stability is reflected in the free dynamic vibratory of the system which allows optimising the harmonic dynamic responses.

Figures 3(a) and 3(b) show the variation of the normalised maximum dynamic responses (Dmax) at the mid-span of the simply supported SWCNT with respect to the speed parameter (α) with and without taking into account the non-local effect. The study is based on different numbers of moving loads for a critical aspect ratio (L/d = 5). It is observed that the effect of the interatomic bonding which is taken into account by the small scale parameter (μ) leads to a more important modal responses with respect to those of the classical local theory which gives insignificant and erroneous results. We note that the maximum dynamic flexion increases up to a certain value of the speed (vp = [0.5 to 0.7].Vcr). After this critical value the increase in moving speed generate a quasi progressive reduction. Moreover, the modelisation for a single concentrated moving load on the SWCNT gives non acceptable results in term of the dynamic stability with respect to other models for two or three moving loads, i.e., the transversal dynamic responses are reduced with the increase of the number of moving loads.

Table 1 Fundamental frequencies (ϖ) for simply supported SWCNT

L/d	*μ (nm²)*	λ_1		λ_2		λ_3	
		Present	*Esen (2020)*	*Present*	*Esen (2020)*	*Present*	*Esen (2020)*
5	0	9.2740	9.2740	32.1665	32.1948	61.4581	61.4592
	1	8.8477	8.8476	27.2364	27.2654	44.7247	44.7847
	2	8.4752	8.4753	24.0453	24.0555	36.8831	36.8941
	3	8.1461	8.1461	21.7642	21.7842	32.1036	32.1536
	4	7.8526	7.8527	20.0293	20.0495	28.8023	28.8523
10	0	9.7075	9.7075	37.0962	37.0999	78.1547	78.1855
	1	9.2612	9.2613	31.4105	31.4148	56.8753	56.8953
	2	8.8713	8.8714	27.7303	27.7340	46.9034	46.9220
	3	8.5269	8.5270	25.0996	25.1035	40.8254	40.8414
	4	8.2196	8.2197	23.0989	23.1025	36.6272	36.6413
20	0	9.8281	9.8281	38.8299	38.8309	85.6619	85.6672
	1	9.3763	9.3763	32.8786	32.8796	62.3385	62.3429
	2	8.9816	8.9816	29.0263	29.0271	51.4087	51.4137
	3	8.6328	8.6328	26.2727	26.2733	44.7469	44.7490
	4	8.3218	8.3219	24.1785	24.1790	40.1454	40.1472
100	0	9.8679	9.8679	39.4517	39.4517	88.6914	88.6915
	1	9.4143	9.4144	33.4051	33.4052	64.5431	64.5435
	2	9.0180	9.0181	29.4911	29.4911	53.2268	53.2268
	3	8.6678	8.6679	26.6934	26.6935	46.3294	46.3294
	4	8.3555	8.3556	24.5657	24.5658	41.5652	41.5655

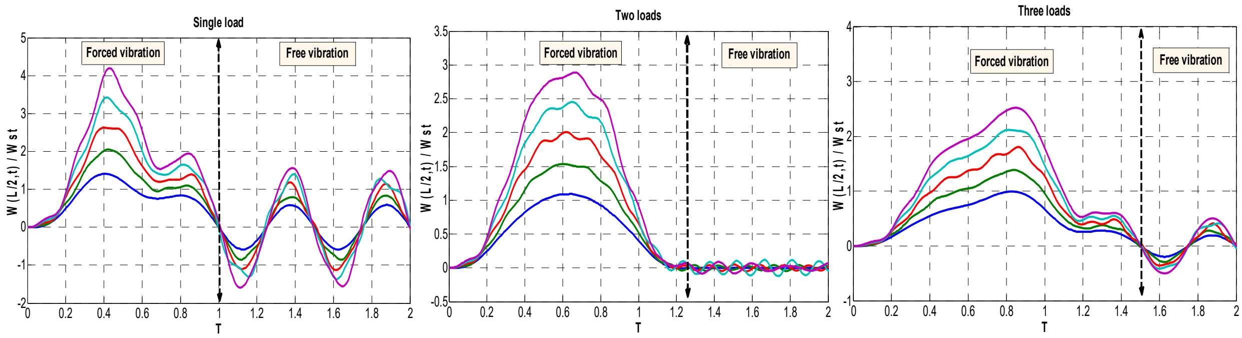
Table 2 Maximum dynamic deflections (Dmax) for simply supported SWCNT

L/d	*μ (nm²)*	*α = 0.1*			*α = 0.5*			*α = 1*		
		Single load	*Two loads*	*Three loads*	*Single load*	*Two loads*	*Three loads*	*Single load*	*Two loads*	*Three loads*
5	0	1.2357	1.1016	0.9192	1.9066	1.6327	1.2655	1.7445	1.6106	1.4159
	1	1.8557	1.5452	1.2675	2.7219	2.2966	1.8282	3.2603	2.3192	2.0411
	2	2.4556	1.9851	1.6171	3.3150	3.1264	2.3381	3.2747	2.5771	2.7209
	3	3.0935	2.4511	1.9576	4.4546	4.0468	2.7396	4.9127	3.7786	3.2198
	4	3.7291	2.8707	2.3489	5.7490	4.8841	3.3325	6.1055	4.4823	3.6861

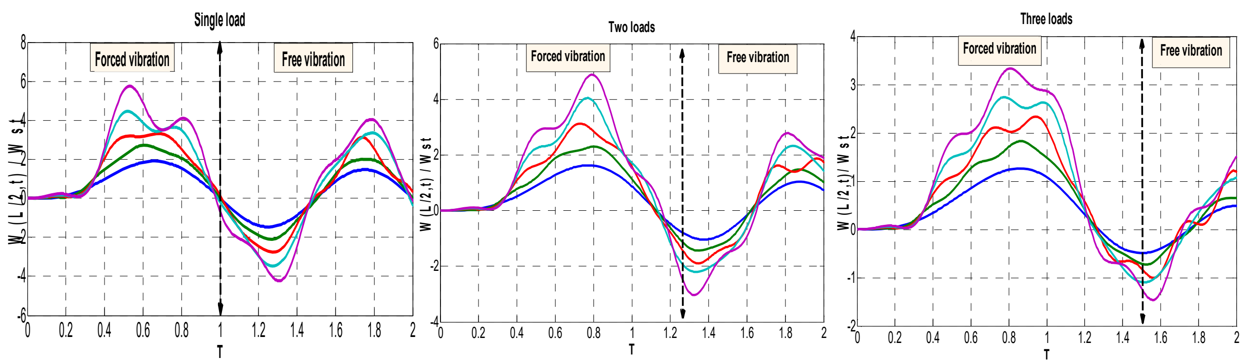
Table 2 Maximum dynamic deflections (Dmax) for simply supported SWCNT (continued)

L/d	*μ (nm²)*	*α = 0.1*			*α = 0.5*			*α = 1*		
		Single load	*Two loads*	*Three loads*	*Single load*	*Two loads*	*Three loads*	*Single load*	*Two loads*	*Three loads*
10	0	1.1300	1.0143	0.8489	1.7577	1.5039	1.1638	1.5993	1.4747	1.3016
	1	1.2591	1.1177	0.9305	1.9245	1.6515	1.2835	1.7771	1.6326	1.4198
	2	1.3871	1.2217	1.0135	2.0978	1.7983	1.3950	1.9197	1.7955	1.5598
	3	1.5167	1.3237	1.0925	2.2807	1.9580	1.5083	2.1331	1.9088	1.6733
	4	1.6433	1.4256	1.1774	2.4058	2.1223	1.6044	2.3181	2.0878	1.8369
20	0	1.1039	0.9927	0.8309	1.7155	1.4686	1.1435	1.5631	1.4405	1.2843
	1	1.1339	1.0173	0.8514	1.7626	1.5082	1.1670	1.6025	1.4803	1.3051
	2	1.1641	1.0418	0.8716	1.7981	1.5399	1.1986	1.6449	1.5199	1.3344
	3	1.1948	1.0677	0.8914	1.8416	1.5799	1.2274	1.6797	1.5457	1.3747
	4	1.2266	1.0926	0.9114	1.8882	1.6198	1.2546	1.7241	1.5880	1.4060
100	0	1.0960	0.9852	0.8256	1.7052	1.4574	1.1346	1.5493	1.4297	1.2746
	1	1.0973	0.9862	0.8264	1.7064	1.4590	1.1359	1.5514	1.4307	1.2762
	2	1.0985	0.9873	0.8273	1.7077	1.4604	1.1372	1.5531	1.4325	1.2775
	3	1.0997	0.9884	0.8281	1.7093	1.4618	1.1384	1.5554	1.4339	1.2787
	4	1.1008	0.9895	0.8288	1.7109	1.4633	1.1396	1.5573	1.4347	1.2799

Figure 2 Time history of the mid-span dynamic deflections d(t): (a) α = 0.25, (b) α = 0.5, (c) α = 0.75, (d) α = 1, (e) α = 1.25, (f) α = 1.5 (see online version for colours)



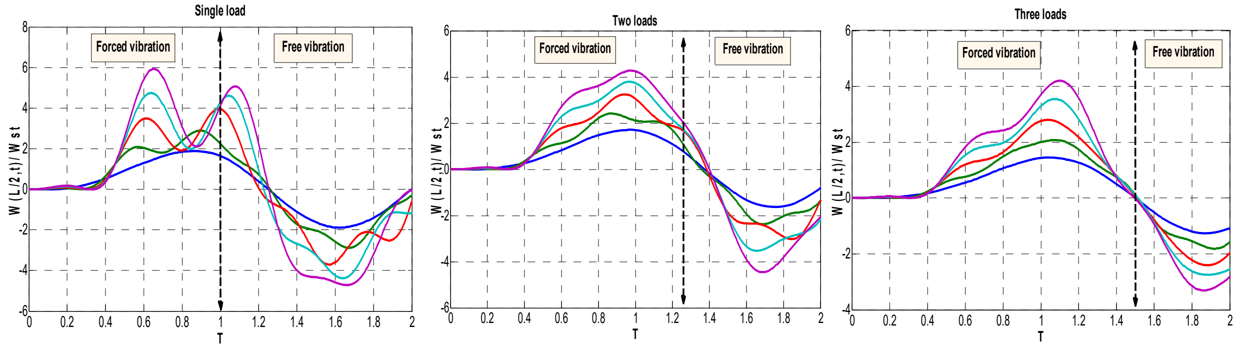
(a)



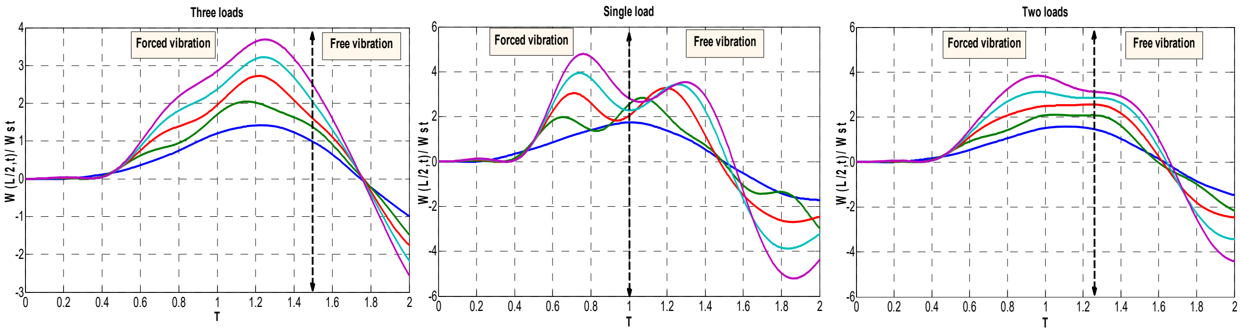
(b)

Note: μ = 0 nm² (——), μ=1 (——), μ = 2 nm² (——), μ = 3 nm² (——), μ = 4 nm² (——).

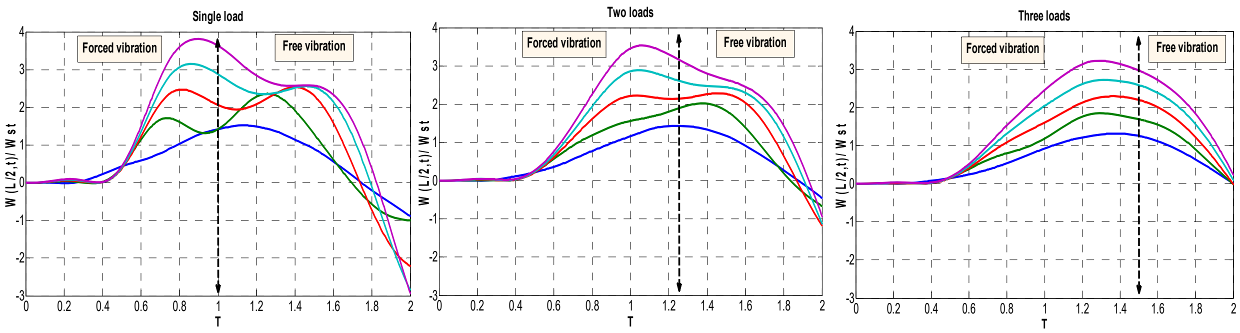
Figure 2 Time history of the mid-span dynamic deflections d(t): (a) α = 0.25, (b) α = 0.5, (c) α = 0.75, (d) α = 1, (e) α = 1.25, (f) α = 1.5 (continued) (see online version for colours)



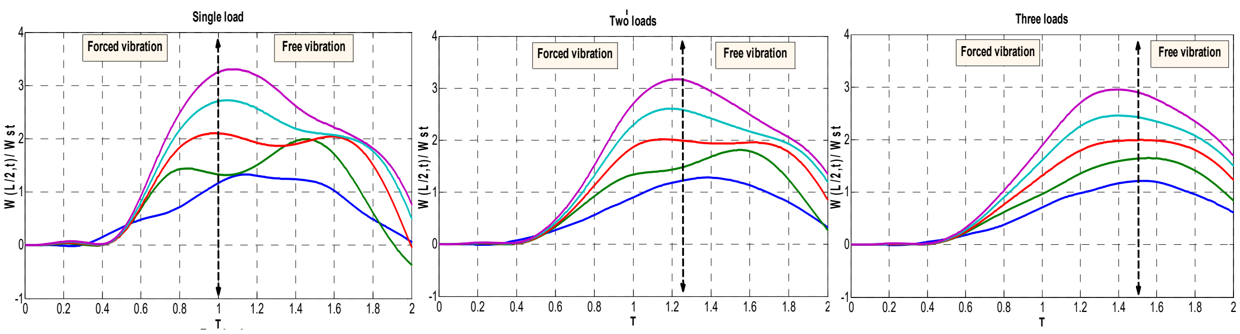
(c)



(d)



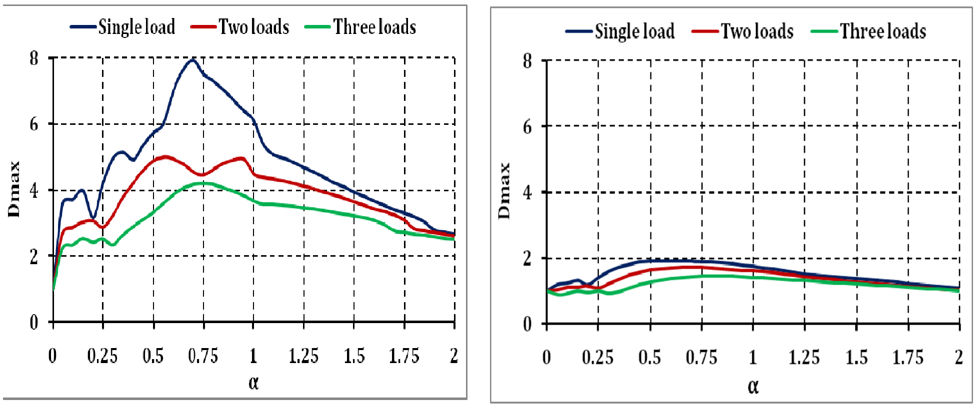
(e)



(f)

Note: μ = 0 nm^2 (——), μ=1 (——), μ = 2 nm^2 (——), μ = 3 nm^2 (——), μ = 4 nm^2 (——).

Figure 3 Variation of the maximum dynamic deflections (Dmax) with moving loads speed (α), (a) with non-local effect, (b) without non-local effect (see online version for colours)



(a) (b)

6 Conclusions

The study of the transient dynamic behaviour of a SWCNT subjected to several moving loads has been carried out based on Eringen's non-local constitutive differential relations. The governing equations of the movement are deduced from the Hamilton's principle combined with Galerkin's method using Timoshenko beam theory. The present model is able to capture simultaneously the scale effect and the shear deformation effect for a SWCNT. The eigen-frequencies have been carried out by a numerical method using the modal approximation and the dynamic responses are computed numerically using Newmark's temporal method.

In this paper, parametric study has been conducted on many physical and geometrical configurations of SWCNT to analysis the impact of the different factors such as: the scale parameter, the geometric aspect ratio, the displacement speed and the number of the crossing loads on the transient dynamic behaviour of SWCNT. From this research the following conclusions have been put into evidence:

- The effect of the interatomic bending affects the stiffness of the system which is directly reflected on the results of the dynamic behaviour of the SWCNTs
- The high frequencies are considerably influenced by the variation of the geometrical aspect ratio compared to low frequencies
- The aspect ratio is inversely proportional to the dynamic responses
- The speed displacement of the moving loads plays an essential role in the analysis of forced dynamic behaviour
- The critical speed has a direct impact on the modulation of the transient modal responses
- The modelling by several moving loads gives better results in terms of dynamic stability with respect to a solicitation by a single concentrated moving load
- The value of the critical speed varies depending on the number of transverse moving loads applied to the SWCNTs.

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