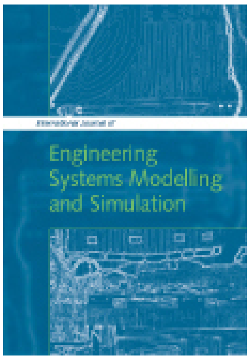

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Influence of geometric properties on the natural frequency of bending vibration of polymethacrylamide sandwich composite

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Abstract: The influences of fibre orientation and thicknesses of the skin and the core on the natural frequencies of cantilever polymethacrylamide rohacell sandwich composites (PMRCs) were numerically studied in this work, using the FEA approach. The study established that maximum natural frequency is obtained for the sandwich when the fibre orientation of its GFRP skin is either lateral, (i.e., 0°) or longitudinal (90°) due to the comparative higher bending stiffness of the skin at these angles. It was also found that the natural frequency increases with increasing core thickness for all bending modes, increases with increasing skin thickness for the fundamental bending mode, and decreases with increasing skin thickness for higher bending modes. It was further found that higher frequencies of bending vibration are obtained in the PMRC sandwich when the PMRF core has a superior thickness ratio of the entire sandwich thickness compared to the GFRP skin.

Keywords: vibration analysis; finite element analysis; FEA; natural frequency; sandwich laminate; honeycomb structure.

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1 Introduction

Composite structures have been increasingly used in the automobile, aerospace, and other engineering industries in recent years due to their comparative mechanical advantages (Apalowo et al., 2018). Compared to monolithic structures, these structures significantly improve stiffness, weight, and acoustic properties (Apalowo et al., 2019). However, they are sensitive to vibration and sound transmission (Crane and Santiago, 1993), governed by the structures' geometry and physical properties. Obviously, the variation of the geometry and/or the material properties, such as the thickness or the fibre orientation of composite structures, will eventually alter the dynamic response and, hence, the structures' natural frequency and mode shapes. Therefore, evaluating the effects of these changes is a field of extensive study.

The increased usage of the structures has necessitated the development of solution models and methods for predicting the dynamic response and optimising the design parameters of composite structures. The finite element approach (FEA) has been the most widely used numerical method for studying the dynamic behaviour of composite beams, plates, and shells. Ochoa and Reddy (1992) and Reddy (2003) presented different FEA models for determining the dynamic response of laminated composite structures. Ahmed (1971) applied the FEA method to evaluate the influence of different physical and geometric parameters of a curved sandwich beam on the resonant frequency of the beam. Various FEA models have also been developed to study composite structures' vibrational parameters, such as the modal and damping properties. These include a transverse shear deformation model for filamentary composite plates developed by Dun-xiang et al. (1986), modified shear deformation theory for free vibration analysis of composite laminates developed by Shimpi and Ainapure (2001), and exact solution techniques for free vibration analysis of laminated composites developed by Chandrashekhara et al. (1990). Advanced FEA methods, such as the super finite element model, have also been implemented by Vaziri et al. (1996) for transient analysis of laminated composite plates and shells and Jiang and Olson (1993) for nonlinear dynamic analysis of stiffened laminated plates and shells. The wave FEA has also been extensively applied for predicting composite's vibroacoustic and dynamic properties (Chronopoulos et al., 2017; Apalowo et al., 2017; Apalowo and Chronopoulos, 2021).

Experimental measurements have also been implemented to determine the dynamic response of laminated composites. Liu and Zhao (2002) developed an experimental technique to evaluate the roles of anisotropic core on the vibration properties of honeycomb sandwich panels. Baba et al. (2011) measured the dynamic response of composite sandwich beams with curvature and debonds. Crawley (1979) measured the natural frequencies and mode shapes of graphite fibre-reinforced polymer composite plates and shells of different aspect ratios. The dynamic characteristics of sandwich plates with holes and cut-outs were experimentally investigated by Mondal et al. (2015). Nagasankar et al. (2015) experimentally studied the roles of geometric parameters and fibre orientation on the transverse shear damping properties of polypropylene honeycomb sandwich plates. Anderson and Nayfeh (1996) tested the influence of fibre orientation and boundary conditions on the dynamic properties of different graphite-epoxy plates. All the listed experimental studies compared the experimental measurements with FEA-based numerical, theoretical, and simulation results, and excellent agreements were observed. This validates the application or implementation of the existing FEA-based analytical, numerical, and simulation techniques in predicting the dynamic response of laminated composite structures.

The geometric parameters, such as the thickness of the core or skin laminae and fibre orientation of the plies, significantly affect the strength and stiffness of sandwich structures due to changes in bending and shear transfer between skin and core layers at different thickness and ply angles. Yim et al. (2003) studied the effect of the aspect ratio of sandwich composite on the loss factor of the structure. Havaldar et al. (2012) studied the influence of the cell size of a honeycomb core on the natural frequency of fibre-reinforced honeycomb sandwich plates, and it was established that the frequency decreases with increasing cell size of the core. Sahoo et al. (2015) studied the influence of geometric and material characteristics, such as the thickness, aspect ratio, and boundary conditions, on the natural frequency and mode shapes of laminated glass-fibre plates.

Rohacell sandwich structures are advanced composites with superior material properties, such as high bending stiffness-to-weight ratio, than ordinary fibre-reinforced composites. Despite the remarkable mechanical properties, the dynamic properties of the sandwich have not been fully investigated. It is also a known fact that the core thickness of sandwich composites significantly affects the bending stiffness of the structure. However, the effect of this property on the bending vibrational parameters of the structure has not been fully investigated. Hence, this study considers the dynamic analysis of polymethacrylamide rohacell composites (PMRC), which comprises a polymethacrylamide rohacell foam (PMRF) core sandwiched between two glass fibre-reinforced polymer (GFRP) skins. The study evaluates the influence of fibre orientation and the thicknesses of the GFRP skin and the PMRF core on the natural frequency of bending vibration of the PMRC sandwich.

2 Finite element analysis (FEA)

FEA is a tool for modelling the performance of a design under certain conditions. ANSYS mechanical APDL is a commercial FEA software capable of performing finite element simulation in line with the conventional FEA methods in a fraction of the time. The user-programmable capability of ANSYS also greatly reduces the number of physical tests required while allowing the designer to experiment with a wide range of design options to select the best option for the final design.

2.1 Finite element model

2.1.1 Governing equations

The governing equation for composite laminate in bending is expressed as:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = -\rho h \frac{\partial^2 w}{\partial t^2} \tag{1}$$

where h is the laminate thickness, $H = D_1 + 2D_{xy}$, and D_x, D_y, D_{xy}, D_1 are the constituents of the laminate bending rigidities defined as:

$$D_x = \frac{E_x h^3}{12(1 - v v_m)}; \; D_y = \frac{E_y h^3}{12(1 - v v_m)}; \; D_{xy} = \frac{G_{xy} h^3}{12}; \quad D_1 = \frac{E_y h^3}{12(1 - v v_m)} \tag{2}$$

where v and v_m are major and minor Poisson's ratios, respectively.

2.1.2 Finite element formulation

Four node elements having six degrees of freedom at each node are employed in this study. The DoF includes three displacements (u, v, w) and three rotations (ϕ_x, ϕ_y, ϕ_z) along x, y, z directions, respectively. The variations of the displacements and rotations are expressed as follows:

$$u = \sum_{i=0}^{4} N_i u_i;\ v = \sum_{i=0}^{4} N_i v_i;\ w = \sum_{i=0}^{4} N_i w_i; \\ \phi_x = \sum_{i=0}^{4} N_i \phi_{xi};\ \phi_y = \sum_{i=0}^{4} N_i \phi_{yi};\ \phi_z = \sum_{i=0}^{4} N_i \phi_{zi}; \tag{3}$$

where xi, yi, zi are nodal coordinates and N_i is the shape functions expressed as:

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{Bmatrix} = \frac{1}{4} \begin{Bmatrix} (1-\xi)(1-\eta)(-\xi-\eta-1) \\ (1+\xi)(1-\eta)(\xi-\eta-1) \\ (1+\xi)(1+\eta)(\xi-\eta-1) \\ (1-\xi)(1+\eta)(-\xi+\eta-1) \end{Bmatrix} \tag{4}$$

The element stiffness and mass matrices are given as:

$$[K_e] = \iint [B]^T [D][B] dx\, dy \tag{5}$$

$$[M_e] = \iint [N]^T [P][N] dx\, dy \tag{6}$$

where $[B]$ is the strain displacement matrix, $[D]$ is the constitutive matrix of the laminate's elastic properties and $[P]$ is a function of laminate mass density. The stiffness $[K_e]$ and mass $[M_e]$ matrices are evaluated by expressing the integrals in the local isoparametric coordinates η and ξ of the element before performing the numerical integration. For more details on the elemental formulation, see Reddy (2003). The element matrices are assembled to obtain the entire laminate matrices $[K]$ and mass $[M]$. The free vibration eigenvalue equation of the composite laminate is obtained as:

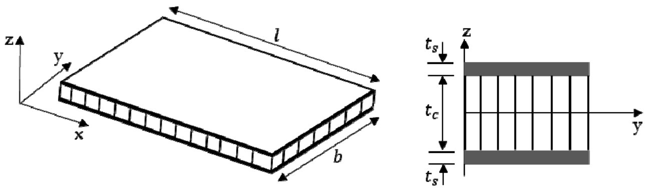
$$\left| [K] - \omega_{mn}^2 [M] \right| = 0 \tag{7}$$

where ω_{mn} is the natural frequency of vibration of the composite laminate.

2.2 Description and boundary conditions of the model

A cantilever PMRC beam of dimensions l =254 mm and b = 25.4 mm is considered, as shown in Figure 1. The sandwich beam comprises a PMRF core sandwiched between the top and bottom skins. Each skin laminate consists of four orthotropic layers of glass/epoxy (GFRP). The polymer composite beam is laid up symmetrically, with each skin laminate having a symmetric lay-up of [$+\theta/-\theta/-\theta/+\theta$] and a thickness of t_s. The ply angle for the PMRF core is 0° with a thickness of t_c. An adhesive coupling is applied in the FEA framework to bond the upper and lower skins to the foam core. The sets of coincident nodes between the interfaces of the core and skins are coupled together using the contact module of ANSYS. In the multibody analysis of the sandwich structure, bonded contacts are applied at the skin/core interfaces to ensure that the sandwich beam acts like one body and no debond exists during the modal analysis. Also, the skins and core cannot move (slide or separate) or rotate between each other during the analysis. The structural segments coexist as one body. The material properties of the sandwich beam constituents are presented in Table 1. The beam is cantilevered on its left edge.

Figure 1 ROHACELL sandwich beam configurations



The role of the fibre orientation angle θ on the bending vibration response of the PMRC beam is examined in this work. The influences of the PMRF core's thickness and that of the GFRP skin are also studied.

2.3 Solution procedure

The sandwich beam is modelled in ANSYS APDL 19.2 and meshed using the SHELL 181 element. The element type is chosen due to its capability of modelling thin to moderately thick shell structures. Considering a thickness not exceeding 20 mm in the 254 mm × 25.4 mm beam, the span-thickness ratio is within the working limit of the SHELL 181 element.

The element has four nodes with six degrees of freedom at each node, translations in the x, y, and z directions, and rotations about the x, y, and z axes. These are suited to adequately capture the bending modes propagating within the sandwich beam. The element solution is governed by the Mindlin-Reissner (or the first-order shear-deformation) beam theory, in which the transverse shear strain of the sandwich shell is assumed to be constant through its thickness (ANSYS, 2019).

The clamped-free (cantilever) boundary condition of the beam is modelled by constraining all the degrees of freedom on the left edge of the beam. A mesh sensitivity analysis is conducted to determine an appropriate element size for the finite element model.

The entire solution procedure is implemented in an APDL parametric code for creating the model geometry, defining material properties, meshing, defining boundary conditions, and solver settings to conduct multiple analyses with different structural design parameters. Finite element analyses were conducted to evaluate the influence of varying the fibre orientation, the thickness of the skin laminae, the thickness of the core lamina, and the core/skin thickness ratio.

A typical APDL parametric code for computing the natural frequencies of the sandwich shell at a particular fibre orientation and thicknesses of the skin and the core is presented in Appendix.

Table 1 The material properties of the PMRC sandwich beam

Material	E_1 *(GPa)*	E_2 *(GPa)*	E_3 *(GPa)*	G_{12} *(GPa)*	G_{13} *(GPa)*	G_{23} *(GPa)*	v_{12}	v_{13}	v_{23}	ρ *(kg/m³)*
GFRP	54	54	48	3.16	1.78	1.78	0.06	0.313	0.313	2,000
PMRF	0.18	-	-	-	-	-	0.286	-	-	110

Table 2 The material properties of the carbon/epoxy-polyurethane foam sandwich composite

Material	E_1 *(MPa)*	E_2 *(MPa)*	E_3 *(MPa)*	G_{12} *(MPa)*	G_{13} *(MPa)*	G_{23} *(MPa)*	v_{12}	v_{13}	v_{23}	ρ *(kg/m³)*
CFRP	10,658	10,658	10,658	4,000	4,000	4,000	0.26	0.26	0.26	1,446
PUHF	115	-	-	-	-	-	0.3	-	-	119

3 Numerical results and discussion

3.1 Validation study

The FEA procedure is hereby validated for computing the natural frequency of the sandwich beam. To validate the accuracy of the solution procedure for predicting the bending vibration response of sandwich structures, a validation case study is conducted using a sandwich structure similar to the one considered in Baba et al. (2011). The experimental results of Baba et al. (2011) are compared with the numerical predictions obtained through the FEA scheme presented in this study. To ensure uniformity and a proper comparison of results, the constructions of the sandwich beams, including geometry, material properties, boundary conditions, element types, etc. used in the validation study are precisely similar to the ones used in Baba et al. (2011).

The structure is a carbon/epoxy-polyurethane foam sandwich composite consisting of a polyurethane honeycomb foam (PUHF) sandwich between layers of woven carbon fibre reinforced polymer (CFRP) fabric. The fibre orientation of the CFRP woven fabric is ±45°. The material and geometric properties applied for the CFRP skin and PUHF core are similar to the ones used in vibration measurement presented by Baba et al. (2011), as shown in Table 2. An adhesive resin is used in boding the CFRP skins to the polyurethane foam in the experimental work of Baba et al. (2011). In the present work, being a finite element numerical approach, the skin/core interfaces are bonded using the interface nodes coupling technique, in which the sets of coincident nodes between the interfaces of the core and skins are coupled using the contact module of ANSYS. This technique models similar physical and mechanical contact characteristics typical of adhesive resins (Chronopoulos et al., 2013; Baba et al., 2011).

Three FE models were developed: the CFRP skin laminate model with dimension 254 × 25.4 × 2.54, the PUHF core laminate model with dimension 254 × 25.4 × 12.7 and the carbon/epoxy-polyurethane foam sandwich model having a PUHF core lamina 230 × 25.4 × 12.7 sandwich between two CFRP skin lamina 230 × 25.4 × 0.762 each. All dimensions are in millimetres. The ply orientation for the CFRP skin layer is $[(45°)/(-45°)]_s$ and that of the PUHF core layer is [0°]. All dimensions and material orientations are similar to the ones presented in Baba et al. (2011).

The natural frequencies under the free-free boundary condition for the CFRP laminate and the PUHF laminate are presented in Table 3. The FEA-computed natural frequencies are compared with the experimental results of Baba et al. (2011). With an observed maximum difference of 6.58 % and 1.29 % for the CFRP skin and the PUHF core models, respectively, it can be stated that an excellent agreement was observed between the results obtained through the numerical approach of the present work and the experimental results presented in Baba et al. (2011).

Table 3 Natural frequencies of the CFRP and the PUHF laminates under the free-free boundary condition

Laminate	*Bending mode*	*Baba et al. (2011)*	*Present work*	*% Diff**
		Experiment (Hz)	*FEA (Hz)*	
CFRP laminate	1	116	109.7	5.43
	2	327	305.5	6.58
	3	659	617.3	6.33
	4	1,113	1,063.9	4.41
	5	1,715	1,675.9	2.28
PUHF laminate	1	152	152.6	–0.39
	2	420	414.6	1.29
	3	797	797.0	0.00
	4	1,276	1,285.5	–0.75
	5	1,876	1,866.4	0.51

Note: *Experiment as reference.

The natural frequencies under clamped-clamped boundary condition for the carbon/epoxy-polyurethane foam sandwich are presented in Table 4. Similarly, with an observed maximum difference of 6.93 %, it can be concluded that the finite element solution procedure computes natural frequency results, which are in excellent agreement with experimentally measured results.

3.2 Parametric study

Having validated the finite element solution procedure for calculating the natural frequency of bending vibrational response of sandwich composite structures, the FEA solution procedure is hereby applied to study the influence of structural parameters such as the fibre orientation and the thicknesses of the skin and the core on the natural frequency of bending vibration of the PMRC sandwich composite presented in Section 2.1.

Table 4 Natural frequencies of the carbon/epoxy-polyurethane foam sandwich under clamped-clamped boundary condition

Laminate	*Bending mode*	*Baba et al. (2011)*	*Present work*	*% Diff**
		Experiment (Hz)	*FEA (Hz)*	
CFRP/ PUHF/ CFRP sandwich	1	541	578.5	–6.93
	2	1,225	1,252.0	–2.20
	3	2,038	2,032.1	0.29
	4	2,848	2,847.2	0.03
	5	3,688	3,675.3	0.34

Note: *Experiment as reference.

3.2.1 *Influence of fibre orientation*

The effect of fibre orientation on the natural frequency of bending vibration of the PMRC sandwich is investigated by considering fibre orientations θ varying from 0° to 90° for the skin laminae with a symmetric lay-up of $[\theta/(-\theta/-\theta/\theta)]$ each. The PMRC sandwich structure has a core thickness of 10 mm, with the top and bottom skin laminae having a thickness of 4 mm each. The natural frequencies and mode shapes of bending vibration at varying fibre orientations under clamped-free boundary conditions are presented in Table 5 and Figure 2.

It is observed that a maximum natural frequency is obtained for the PMRC sandwich when the fibre direction of its GFRP skins is oriented laterally, (i.e., 0°) or longitudinally (90°), and minimum frequency is observed at 45° fibre orientation. It is due to the comparative higher bending stiffness (and hence lower bending deformation) of the GFRP skin at 0°. The bending stiffness and, therefore, the natural frequency reduces as the fibre orientation either reduces from 90° towards 45° or increases from 0° towards 45°, as shown in Figure 3.

It is also observed that similar natural frequency values are obtained at fibre orientations with an angular difference of 90° (e.g., 30°and 60°). This is due to the transversely-isotropic GFRP skin exhibiting a unidirectional property in the in-plane direction, in which the moduli properties E_1 and G_{13} are equal to E_2 and G_{23} respectively. Therefore, fibre orientations of an angular difference of 90° yield a similar bending stiffness property, bending deformation, and hence natural frequency.

An analysis of the stress state sensitivity to a change in the PMRC fibre orientation is also conducted by subjecting the PMRC sandwich of different fibre angles to a transverse displacement of 0.02 m. As shown in Figure 3, the sensitivity of the equivalent stress to fibre orientation is similar to that of the natural frequency, with the equivalent stress exhibiting a maximum value at 0° and 90° and a minimum at 45°.

In the remaining studies, the 90° fibre orientation, having given the optimal result, will be used for the skin laminae in a lay-up of [(90°)/(–90°/–(90°)/(90°))].

3.2.2 *Influence of skin thickness*

The influence of the GFRP skin thickness on the natural frequency of bending vibration of the PMRC sandwich is investigated by considering PMRC laminates of constant core thickness and varying skin thicknesses. Sandwich laminates, with each skin lamina having thicknesses ranging from 1.0 mm to 5.0 mm, are considered. These are considered at a constant core thickness of 10 mm, with the total thicknesses of the sandwich laminates ranging from 12 mm to 20 mm.

The natural frequencies and mode shapes of the first five modes of bending vibration of the PMRC sandwich with different skin thicknesses are presented in Table 6 and Figure 4. The fundamental frequency mode increases with increasing skin thickness. This is due to the increase in the bending stiffness of the sandwich with respect to skin thickness in this mode.

The variation differs at higher vibration modes, as shown in Figure 5. At higher modes, there is a monotonic decrease in the natural frequency of bending vibration as the thickness of the skin laminae increases. This can be attributed to a decrease in the bending stiffness of the sandwich as the skin thickness increases. This is due to an increase in the bending load applied by the heavier skin laminae on the softer core lamina as the thickness (and hence the weight) of the skin laminae increases.

The frequency of the fundamental mode exhibits a different (incremental) trend compared to the higher modes due to the presence of nodal regions at the fundamental mode, exhibiting a slight frequency change irrespective of the weight of the structural system. As the change is independent of the weight, it increases bending stiffness as the thickness increases. This phenomenon can be attributed to the fundamental mode being a global mode where the up and down motions of the transverse vibration occur simultaneously when the structure is excited in its resonance frequency, leading to a reduced natural frequency. The lesser frequency change can be inferred to be due to the presence of nodal or precisely zero displacement regions at the fundamental frequency. The nodal regions are known to possess a lower or passive vibration region.

3.2.3 *Influence of core thickness*

The influence of the PMRF core thickness on the natural frequency of bending vibration of the PMRC sandwich is investigated by considering laminates of constant skin thickness with varying core thicknesses. Sandwich laminates with core thicknesses ranging from 5 mm to 12 mm, with a constant thickness of 4 mm each for the skin laminae, are considered. The total thicknesses of the sandwich laminate range from 13 mm to 20 mm.

Table 5 Natural frequencies of the PMRC sandwich for different fibre orientations

Mode	*Natural frequency (Hz)*						
	Fibre orientation θ						
	0°	*15°*	*30°*	*45°*	*60°*	*75°*	*90°*
1	211.42	204.62	175.54	130.76	175.54	204.62	211.42
2	700.43	688.94	639.60	554.29	639.60	688.94	700.43
3	1,339.69	1,330.04	1,281.33	1,170.45	1,281.33	1,330.04	1,339.69
4	1,921.88	1,914.14	1,875.05	1,777.55	1,875.05	1,914.14	1,921.88
5	2,511.65	2,505.25	2,472.51	2,386.13	2,472.51	2,505.25	2,511.65

Figure 2 Bending vibration mode shapes of the PMRC sandwich composite with different fibre orientations (see online version for colours)

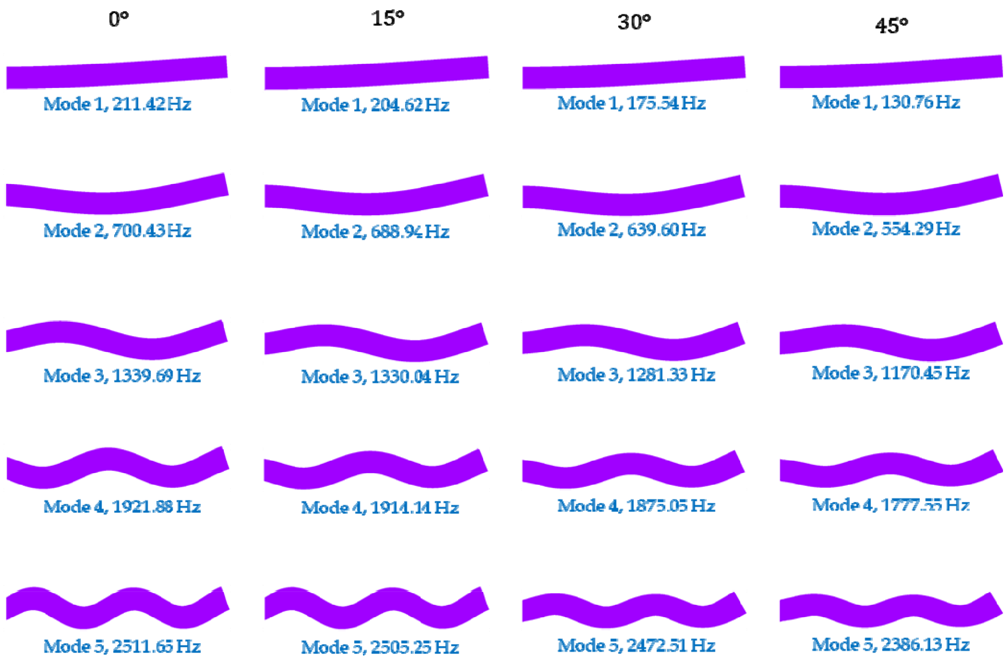


Figure 3 Natural frequencies (mode I) and equivalent stress for different fibre orientations

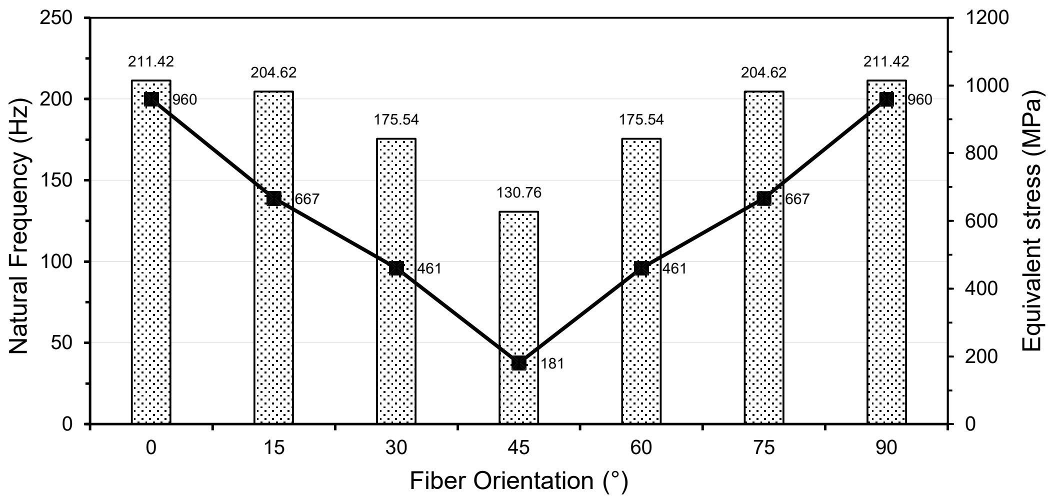


Table 6 Natural frequencies of the PMRC sandwich for different skin thicknesses

Skin thickness (mm)	*Natural frequency (Hz)*				
	Bending mode				
	1st mode	*2nd mode*	*3rd mode*	*4th mode*	*5th mode*
1.0	194.37	809.07	1,692.79	2,549.43	3,405.24
1.5	200.19	770.34	1,569.67	2,321.17	3,074.10
2.0	203.39	743.30	1,486.97	2,174.97	2,866.68
2.5	205.68	725.05	1,429.78	2,076.13	2,727.80
3.0	207.65	712.91	1,389.32	2,006.92	2,630.92
3.5	209.53	705.10	1,360.39	1,957.49	2,561.69
4.0	211.42	700.43	1,339.69	1,921.88	2,511.65
4.5	213.35	698.08	1,325.08	1,896.33	2,475.46
5.0	215.33	697.50	1,315.07	1,878.28	2,449.59

Figure 4 Bending vibration mode shapes of the PMRC sandwich composite with different skin thicknesses (see online version for colours)

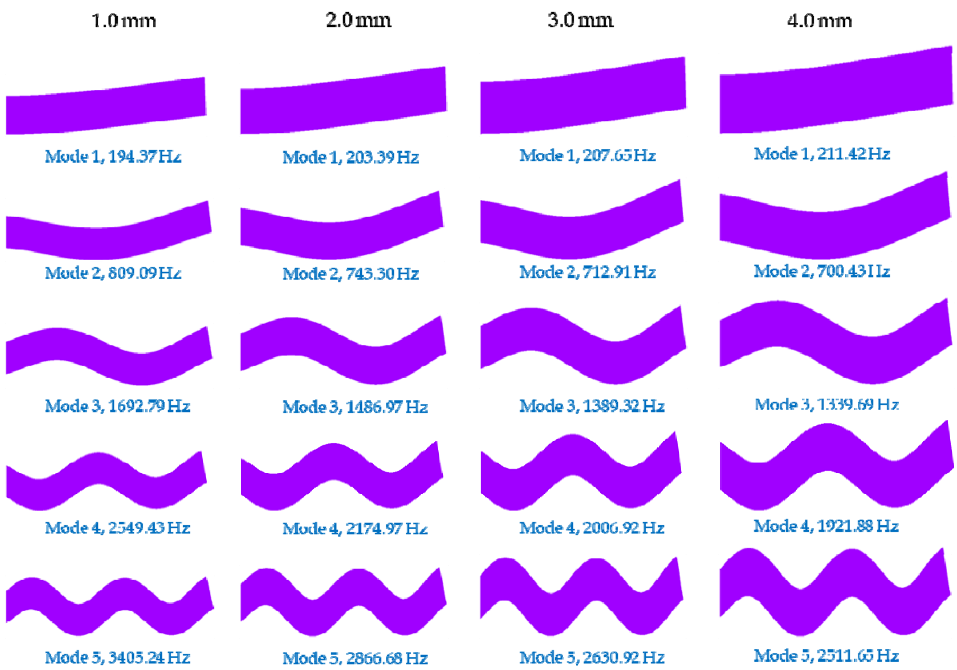


Figure 5 Variation of the natural frequency of the PMRC sandwich with the skin thickness

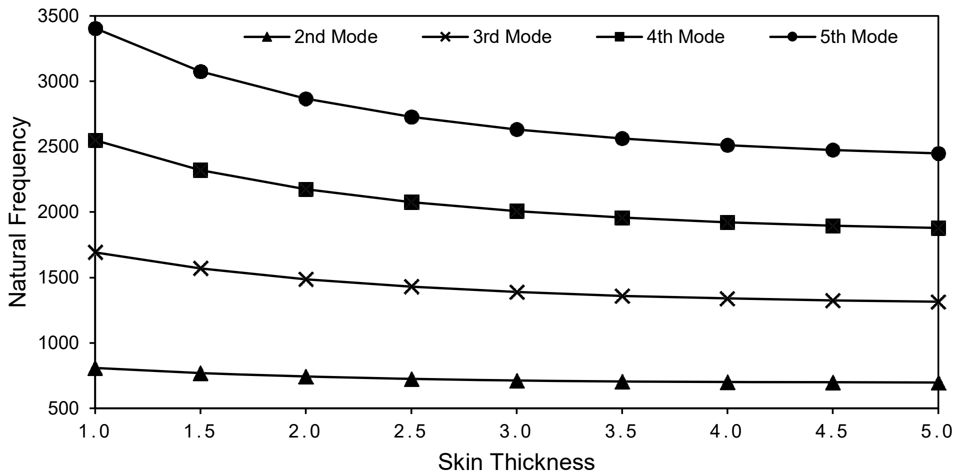


Figure 6 Variation of the natural frequency of the PMRC sandwich with the core thickness

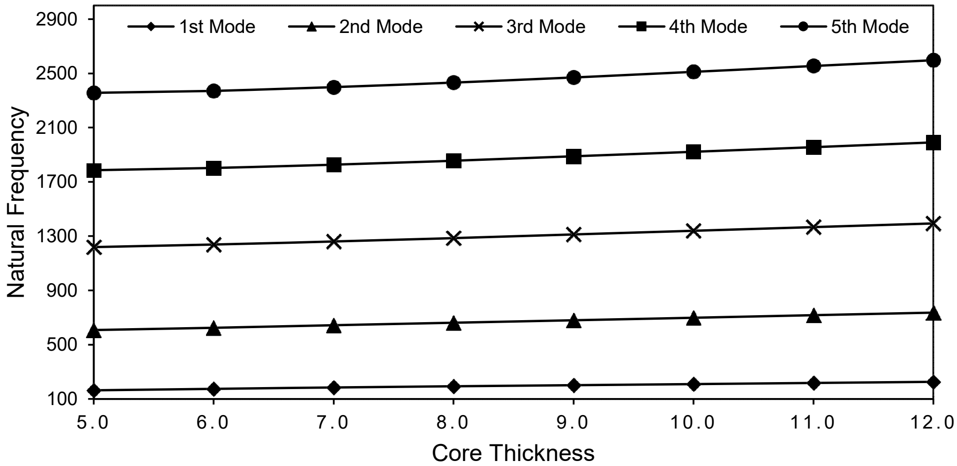
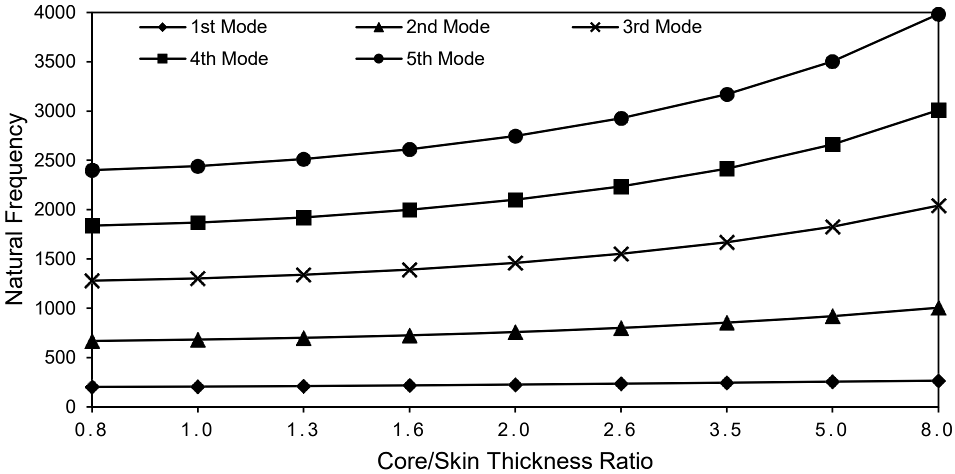


Figure 7 Variation of the natural frequency of the PMRC sandwich with core/skin thickness ratio



The variation of the natural frequency with the core thickness for the first five modes of bending vibration of the PMRC sandwich is presented in Figure 6. The frequency monotonically increases with increasing core thickness for all the bending modes. This is attributed to an increase in the bending stiffness of the sandwich as the thickness of the core increases. The bending stiffness increases due to the rise in the strength of the core to withstand the bending load being applied by the skin laminae.

3.2.4 Influence of core/skin thickness ratio

Based on the studies conducted in the previous sections, it has been established that the bending frequency of the PMRC sandwich is increased by either reducing the thickness of the GFRP skin or increasing the thickness of the PMRF core. However, it is necessary to evaluate the combined thicknesses of the skin and the core, which satisfy a particular frequency band requirement. This section investigates the effect of the combined variations of the skin and core thicknesses on the natural frequency of bending vibration of the PMRC sandwich. The total thickness of the sandwich laminates is 18 mm and constant in all cases. Laminates with core thicknesses ranging from 8 mm to 16 mm and corresponding skin thicknesses ranging from 5 mm to 1 mm for each skin laminae are studied. These are equivalent to core/skin thickness ratios ranging from 0.8 to 8.0.

The natural frequency curves for the first five modes of bending vibration are presented in Figure 7. The natural frequency increases slowly and monotonically with an increase in the core/skin thickness ratio. This is observed until a thickness ratio of 2.0 for all the bending modes. Beyond this ratio, the frequency increases exponentially with increasing core/skin thickness ratio. This is due to an increase in the stiffness of the sandwich to bending deformation when the thickness of the core is in multiples of the thickness of the skin. This suggests that higher frequencies of bending vibration are obtained in the PMRC sandwich when the PMRF core has a superior thickness ratio of the entire sandwich compared to the GFRP skin.

4 Conclusions

In this work, the modal responses of the bending vibration of cantilever polymethacrylamide rohacell sandwich composites (PMRC sandwich) under different fibre orientations and different thicknesses of the skin and the core were investigated using the commercial FEA software (ANSYS). The PMRC sandwich comprises a bottom carbon fibre-reinforced polymer (GFRP) skin lamina, a PMRF core, and a top GFRP skin lamina. The principal contribution of this work includes the investigation of the impacts of fibre orientation and skin and core thicknesses on the natural frequency of bending vibration of the PMRC sandwich beam. The main outcomes of the study are summarised as thus:

1 Excellent agreement is recorded between the numerical predictions of the FEA scheme and the experimental natural frequency measurements, with a maximum disparity of 6.93%.

2 It was found that the impact of fibre orientation on the natural frequency of vibration of the PMRC sandwich is most significant when the symmetric GFRP skin lay-ups are either laterally, (i.e., 0°) or longitudinally (90°) oriented. This outcome can be attributed to the high bending stiffness and, hence, low bending deformation of the GFRP skins at 0° and 90° fibre orientation.

3 It was established that the frequency of the fundamental mode of bending vibration of the PMRC sandwich increases with the increase of the GFRP skin thickness. Meanwhile, the frequency of the higher modes monotonically decreases with the increase of the skin thickness due to the heavier skin laminae's higher bending deformation of the softer core as the skin thickness increases.

4 It was found that the natural frequency of bending vibration of the PMRC sandwich increases with the increase of the PMRF foam core thickness for all bending modes due to the higher stiffness offered by the core to bending deformation as the core thickness increases.

5 It was also found that the natural frequency of bending vibration of the PMRC sandwich increases monotonically with the increase of the core/skin thickness ratio. The increment rate was found to also increase with the skin/core thickness ratio.

6 It was inferred that a higher natural frequency of bending vibration could be obtained in the PMRC sandwich when the PMRF core thickness is greater than the GFRP skin thickness.

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Appendix

A typical APDL parametric code for computing the natural frequencies of a sandwich composite beam

```
/PMACRO                      ! Enables the input file to call GUI functions
L = 0.254                    ! Length of the beam
b = 0.0254                   ! Width of the beam
/PREP7                       ! Enters the model creation pre-processor
ET, 1, SHELL181
MAT, 1                       ! Inputting the material properties for material
                             ! type 1 (i.e., the skin lamina)
MP, DENS, 1, 2000            ! Density
MP, EX, 1, 54e+9             ! Elastic modulus along x
MP, EY, 1, 54e+9             ! Elastic modulus along y
MP, EZ, 1, 48e+9             ! Elastic modulus along z
MP, GXY, 1, 3.16e+09         ! Shear modulus along xy
MP, GXZ, 1, 1.78e+09         ! Shear modulus along xz
MP, GYZ, 1, 1.78e+09         ! Shear modulus along yz
MP, PRXY, 1, 0.06            ! Poisson's ratio along xy
MP, PRXZ, 1, 0.313           ! Poisson's ratio along xz
MP, PRYZ, 1, 0.313           ! Poisson's ratio along yz
MAT, 2                       ! Inputting the material properties for material
                             ! type 2 (i.e., the core lamina)
MP, DENS, 2, 110
MP, EX, 2, 180e+6
MP, PRXY, 2, 0.286
SECTYPE, 1, SHELL            ! Defining the shell sections
SECDATA, 0.001, 1, 90, 3     ! 1st layer of top skin lamina, Th. 1 mm, Mat. #1, 900
SECDATA, 0.001, 1, –90, 3    ! 2nd layer of top skin lamina, Th. 1 mm, Mat. #1, –900
SECDATA, 0.001, 1, –90, 3    ! 3rd layer of top skin lamina, Th. 1 mm, Mat. #1, –900
SECDATA, 0.001, 1, 90, 3     ! 4th layer of top skin lamina, Th. 1 mm, Mat. #1, 900
SECDATA, 0.01, 2, 0, 3       ! Core lamina, Thickness 10 mm, Mat. #2, 00
SECDATA, 0.001, 1, 90, 3     ! 1st layer of bottom skin lamina, Th. 1 mm, Mat. #1, 900
SECDATA, 0.001, 1, –90, 3    ! 2nd layer of bottom skin lamina, Th. 1 mm, Mat. #1, –900
SECDATA, 0.001, 1, –90, 3    ! 3rd layer of bottom skin lamina, Th. 1 mm, Mat. #1, –900
SECDATA, 0.001, 1, 90, 3     ! 4th layer of bottom skin lamina, Th. 1 mm, Mat. #1, 900
RECTNG, 0, L, 0, b           ! Create a rectangular plate of size L x b
ESIZE, 0, 100                ! Specify element size of 100 line divisions
AMESH, ALL                   ! Mesh the plate area
FINISH                       ! Exits the model creation pre-processor
/SOLU                        ! Enters the solution processor
DL, 4, 1, ALL, 0             ! Impose clamped boundary condition on the
                             ! plate's left edge (cantilever condition)
ANTYPE, 2                    ! Selects modal analysis
MODOPT, LANB, 20, 0, 10000   ! Block Lanczos, 20 modes, 0 to 10,000 Hz
MXPAND, 20
SOLVE                        ! Solve the current load step
FINISH                       ! Exits the solution processor
/POST1                       ! Enters the results postprocessor
SET, LIST                    ! List the computed results for natural frequencies
```