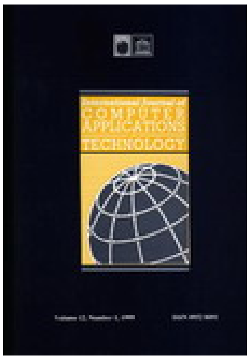

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Numerical simulation of financial fluctuation period based on non-linear equation of motion

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Abstract: The traditional numerical simulation method of financial fluctuation cycle does not focus on the study of non-linear financial fluctuation but has problems such as high numerical simulation error and long time. To solve this problem, this paper introduces the non-linear equation of motion to optimise the numerical simulation method of financial fluctuation cycle. A comprehensive analysis of the components of the financial market, the establishment of a financial market network model and the acquisition of relevant financial data under the support of the model. Based on the collection of financial data, set up financial volatility index, measuring cycle, the financial wobbles, to establish the non-linear equations of motion, the financial wobbles, the influence factors of the financial volatility cycle as variables in the equation of motion, through the analysis of different influence factors under the action of financial volatility cycle change rule, it is concluded that the final financial fluctuation cycle, the results of numerical simulation. The simulation results show that, compared with the traditional method, the numerical simulation of the proposed method has high precision, low error and short time, which provides relatively accurate reference data for the stable development of regional economy.

Keywords: non-linear equation of motion; financial fluctuation; fluctuation period; numerical simulation.

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1 Introduction

Finance refers to the economic activities in which banks, securities or insurance companies raise funds from one or more market entities and lend them to other market entities. In a broad sense, all capital flows generated by market entities such as governments, individuals and organisations through raising, allocating and using funds can be called finance (Wang et al., 2019). Finance can be regarded as three kinds of economic behaviours of fund-raising allocation, investment and financing. Its essence is capital circulation. Financial market, like our natural world, is a complex non-linear system, which mainly shows that there are many factors influencing the price of financial market, such as natural person, legal person, government policy, interest rate, exchange rate, etc., which interact and influence each other, resulting in the non-linear characteristics of financial system (Ge and Wu, 2019). In general, demand and availability influence the prices on the financial markets. In order to accurately represent the interactions and dynamics inside the market, it might be quite useful to build a responsive network model of the financial market while maintaining the fundamental circulation concept. The fundamental tenet of the financial market is circulation, which emphasises the movement of money, information and assets among different players and creates a web of connections. Because of this, the financial sector is comparable to other capital sectors. A purchaser and a vendor trade money for shareholdings whenever a stock is being sold. When a stock is bought, the price that was paid for it becomes the new market price value. If non-linearities and instability define stock market returns so the market is ineffective. This implies that stock prices may be excessive or undervalued since they do not take all available data into consideration. Shareholders can report excess returns or loss as a result. Many different economic conditions could lead to non-linearities. The development of methodologies for incorporating non-linearities into macro models has advanced to an astounding degree, which will help with economic development policymaking.

Financial physics is an interdisciplinary subject of non-linear dynamics, statistical physics and financial mathematics. The main idea is to conduct research on the operation mechanism of financial markets through the above interdisciplinary subjects (Guan et al., 2018). Influenced by many factors, the financial market will fluctuate. In a narrow sense, financial fluctuation refers to the unstable state of the assets and reputation of financial institutions in the financing process due to changes in the objective environment, decision-making errors or other reasons (Zhu et al., 2019). Financial volatility is an inevitable phenomenon in the process of economic development, which has the characteristics of objectivity, diffusion, non-linearity, complexity, controllability and so on. Financial volatility is composed of four kinds of sub-volatility: long-term volatility trend, seasonal volatility, random volatility and periodic volatility. Long-term patterns, which are impacted by alterations in the economy and market mood, illustrate the direction of overall volatility over time. Regular patterns connected to occasions like earnings seasons or vacations are shown by seasonal volatility. The unpredictable emergence of random volatility from news and events needs noise filtering for precise interpretation. Regular cycles that are driven by predetermined events, market openings, or algorithmic trading are referred to as periodic volatility. The specific forms of financial fluctuation include seasonal fluctuation, random fluctuation and periodic fluctuation. Among them, the financial cycle fluctuation refers to the law of financial fluctuation after eliminating the long-term fluctuation trend, seasonal fluctuation and random fluctuation (Gao et al., 2017). The financial cycle can be conceptualised as economic variations that the financial system either amplifies or directly causes. It often shows up as a co-movement among asset values and credit accumulation, with a potential effect on actual economic development as well. The difference between periodic fluctuation and long-term fluctuation is that it does not move continuously in a single direction, but it is a cyclical fluctuation with fluctuation alternating or expansion and contraction alternating. Long-term bonds are particularly susceptible to fluctuations in interest rates. The rationale is that bonds include a fixed income component, so when an investor buys a financial product, e.g., they are essentially buying a portion of the loan held by the corporation. This is to ensure that longer-term bonds, which are closer to maturity and have fewer outstanding dividend payments, have a longer duration than shorter-term bonds. The difference between seasonal fluctuation and seasonal fluctuation is that its regularity is significantly lower than seasonal fluctuation because it only has free law, while seasonal fluctuation has fixed law; secondly, periodic fluctuation only has recognisable model, seasonal fluctuation not only has recognisable model, but also has its own fixed or constant cycle.

The financial cycle and fluctuation can directly reflect economic growth and judge whether there is a financial crisis in a country or a region. Therefore, it is necessary to study a feasible numerical simulation method of financial fluctuation cycle. The variables include market sentiment, technology, regulations, liquidity, corporate earnings, speculation, external shocks, monetary and fiscal policies, economic indicators, global events and market structure and investor behaviour. The traditional numerical simulation methods of financial fluctuation period include normal distribution, Brownian motion and partial differential equation. A mathematical framework called the Brownian model is used to explain how prices in the financial markets fluctuate erratically over time. It is a cornerstone of financial mathematics and frequently serves as the foundation for more intricate models in the discipline of quantitative finance. In economics and business, Brownian motion was a straightforward continuously stochastic process frequently used to simulate irrational behaviour that changes over time. More significantly, all financial asset prices and derivative pricing schemes are built on the fundamentals of the statistical equations used to explain Brownian motion. The model has continuous volatility, no discernible patterns and no historical price memory to affect changes in the future. The work being carried out on product markets and risk assessment relies heavily on such models. To highlight just a few of its uses, it is fundamental to the study of stochastic differentiation equations, economic mathematics and filtration. The financial assessment system will take advantage of prospective possibilities due to the utilisation of partial differential equations in valuing for some goods and services, and the positive buffer income acts as a security net against unanticipated business crises, failure or managerial errors. The link between an item or service's demand and supply determines how much it will cost. Prices rise when demand exceeds supply, and prices also tend to rise when supply is constrained. A price equilibrium is attained when supply and demand are equal. However, after a long time of research and analysis, it is found that the traditional numerical simulation method cannot get more accurate analysis results of financial fluctuation cycle and cannot play an effective reference role in the actual financial work. Therefore, based on the traditional method, the concept of non-linear equation of motion is introduced and a numerical simulation method of financial fluctuation period based on non-linear equation of motion is proposed. The non-linear equation is the equation that the relationship between dependent variable and independent variable is not linear. There are many such equations, such as square relationship, logarithmic relationship, exponential relationship, trigonometric function relationship, etc. The non-linear motion equation can be divided into the non-linear motion algebraic equation and the non-linear motion differential equation. Combined with the above two equations, the analysis of the financial market volatility cycle is realised from the non-linear dynamics level of the financial market, and a more accurate numerical simulation result of the financial volatility cycle is obtained, to realise the optimal design of the numerical simulation method of the financial volatility cycle.

2 Design of numerical simulation method for financial fluctuation cycle

2.1 Building a financial market network model

With the development of information technology, it has been able to monitor every transaction in the financial market in an all-round way. All kinds of financial data can be obtained more easily than in the past, which provides an opportunity for the study of numerical simulation of financial volatility cycle. The financial volatility technique has benefits for forecasting, trading methods, discovering inefficiencies, risk measurement, option pricing, portfolio diversification, asset allocation, model validation and market sentiment analysis. On the other hand, since physicists have introduced the methods and tools of statistical physics and theoretical physics to the research of financial market, the hidden statistical laws in financial time series have been revealed more and more, making this research field an important part of financial econometrics (Bulygin and Zemlyanov, 2018).

Following the basic circulation principle of financial market, the corresponding network model of financial market is constructed. It alludes to the ongoing movement of money, goods and knowledge inside the market. It is a fundamental idea that captures the interaction and exchange of financial instruments and resources amongst many parties, including investors, borrowers, intermediaries and regulators. The network model is an abstract 'image' of a small part or several aspects of the real world, starting from the concept of network. Therefore, the constructed network model needs to reflect the input, output, state variables and their functional relationships (Long et al., 2017). To do this, the problem must be defined, the criteria and sub-criteria must be identified, a hierarchy must be established, pairwise comparisons must be made to evaluate relative relevance, weights must be calculated, consistency must be maintained, weights must be aggregated, alternatives must be ranked and sensitivity analysis must be carried out. In this paper, the analytic hierarchy process is used to build the financial market network model. The analytic hierarchy process is a multi-level decision-making method that decomposes complex problems into multiple factors, forms an orderly increasing hierarchy according to the dominant relationship, weighs the influence of various aspects and synthesises individual judgment to obtain the importance order of the influencing factors. An approach to decision-making characterised as that of the Analytic Hierarchy Process (AHP), which includes coordinating multiple factors in detail into a hierarchy, evaluating the relative relevance of these criteria and contrasting options for each criterion and evaluating the possibilities overall according to cost, advantages and risk. A network model of the financial markets is built using a systematic decision-making AHP process. In order to help with risk assessment, portfolio management and strategy planning within the financial system, AHP systematically assesses the significance of many aspects. Despite the fact that decisions are dependent on the values and interests of the decision makers, a set of guidelines or precise objectives can be employed when ranking projects and figuring out what an ideal connection among benefits and expenses actually means. AHP is a technique that is frequently used in the field of finance, particularly for the evaluation of financial success, the evaluation of credit, the forecast of financial disaster, the assessment of exchange rates and the selection of projects. By incorporating personal preferences and judgements into the decision-making process in an organised manner, AHP offers a methodical and rigorous approach. It guarantees openness, uniformity and thorough examination of several factors, enabling more informed and efficient judgements. Prior to making financial selections, it is tough to evaluate and pick initiatives. The method of white box/black box is used. A black box model is useful in the financial markets, analyses market data and generates buying and selling strategies focused on that research. Black box includes watching inputs and outputs while depending on data and forecasts without comprehending internal workings. Understanding a system's fundamental workings completely entails employing analytical techniques to explain behaviour and forecast consequences. It provides serious issues since they are opaque and difficult to understand. The models' comprehension, validation and modification challenges, as well as potential biases, their limited flexibility, ethical issues and conflicts with regulatory compliance, are all problems. Although the outcomes can be understood by the black box user, the reasoning that underlies them cannot be. Owing the black-box models' capacity to capture significant non-linearity and variable interactions, they frequently outperform white-box equivalents in simpler settings. Because most engineering systems are white boxes with clear internal structure and characteristics, the system model can be derived by analysis and deduction based on some known basic laws. A key idea in systems analysis connected to white box modelling is the principle of causation. It places a strong emphasis on identifying and comprehending the causal connections between system parts, allowing for precise predictions and well-informed judgements on system behaviour. For grey boxes and black boxes with unclear or unclear internal structure characteristics, if direct experimental observation is allowed, the hypothesis model can be constructed for those non-engineering systems that belong to the black box but are not allowed to carry out experimental observation directly, the method of data collection and statistical induction is generally used to build the hypothesis model (Wei et al., 2019; Chen et al., 2022). Grey box, a sophisticated investment technique or model, is renamed as black box. A black box is typically a computer that uses intricate calculations to provide returns in the desired manner. A black box can result in unexpected issues since an investor might not comprehend the model. Unexpected problems may occur in a 'black box' system because of the lack of transparency and knowledge of its operations. As a result, there may be issues with scalability, compatibility, restricted control, unexpected behaviour, erroneous outputs, debugging difficulty and vulnerability to outside assaults. Black box systems, especially in fields like algorithmic trading or pattern recognition, can successfully employ complex computations to produce desired results.

These systems do, however, include some dangers connected to openness, prejudices, mistakes, flexibility and ethical issues.

Based on the above analysis, combined with the non-linear characteristics of the financial market, the following principles must be followed when building the financial market network model: first, we must ensure that the dynamic characteristics of the market are not distorted; second, we must maintain the dynamic characteristics of the financial system that have a greater impact on the dynamic characteristics of the system, and at the same time, we must ensure that the financial model has a clear practical significance. Economic development, job opportunities, income distribution, policy responses, relationships between nations, innovation and human behaviour are just a few of the areas that the financial system's dynamic features have a wide range of effects on outside of finance. The distinctive traits and behaviours of the financial realm, such as volatility, risk and innovation, are explicitly addressed by the dynamic characteristics of the financial system. However, a generic system's dynamic properties cover wider ideas that are relevant to many kinds of systems, such as feedback loops, emergence, adaptation and equilibrium. People can help address financial difficulties owing to the dynamic properties of financial products, investment period with low transaction costs, which improves operational efficiency and timeously financial documents that cause market prices to properly represent the information available. As a direct consequence, prices tend to fluctuate more in in-line with changes in underlying value than with liquidity needs. Taking financial institutions as network nodes, such as banks, stock markets, various companies, etc., the connection lines between nodes in the network are used to express the mutual relations among these companies, stocks and banks, such as credit relationship, risk exposure, mutual capital flow in interbank payment system, etc. However, there is a lag in the spread of some information in the financial market, which will have a certain impact on the whole financial market, so it is necessary and practical to discuss the lag in the financial market (Zhang et al., 2017; Kovalnogov et al., 2021). Therefore, based on the above analysis, a financial market network model with time delay is constructed, which is described as follows:

$$\dot{K}(t) = Ax_i(t) + Bf(x_i(t)) + \sum_{j=1}^{N} c_{ij}\Gamma_1 x_j(t) + \sum_{j=1}^{N} d_{ij}\Gamma_2 x_j(t-\tau) + I, t > 0 \quad (1)$$

In the formula, A and B are coefficient matrix of linear term and non-linear term respectively, x_i is financial institution, $f(x_i(t))$ is non-linear function of financial market, x_j is financial flow function, t is time interval of financial data fluctuation, c_{ij} and d_{ij} are non-time lag internal coupling matrix and time lag internal coupling matrix between different financial institutions, Γ_1 and Γ_2 are non-time lag internal coupling matrix of different financial institutions Lag internal connection weight and lag internal connection weight, τ is the lag of information transmission between financial institutions, I is the external interference parameter of financial institutions (Sripana and Chatanin, 2017). Non-time-lagged matrices concentrate on immediate interactions and are simpler and easier to analyse, whereas time-lagged matrices take into account delays in the impacts of variables on one another, making them excellent for capturing dynamic behaviours. In formula (1), $Ax_i(t)$ represents the linear relationship between some financial institutions, while $Bf(x_i(t))$ represents the non-linear relationship between some financial institutions.

2.2 Collecting financial data information

The elements of finance include five parts: financial object, financial way, financial institution, financial place, system and regulatory mechanism. The financial object is money, and the financial way is represented by loan-based financing, including direct financing and indirect financing. While indirect finance includes middlemen like banks enabling the movement of money between lenders and borrowers via mechanisms like loans and leases, direct financing entails getting cash directly from investors through strategies like issuing stocks and bonds. A lender can be obtained from a financial institution, financial firm, investment company, pension system, health insurer or from any other business that primarily engages in providing financial money and making investments in loans, except for general populace or quasi-public organisations and international bodies. Loans based financing is an innovative funding model that avoids conventional banking. Financial institutions are usually divided into banks and non-bank financial institutions, and financial place is financial market Including capital market, money market, foreign exchange market, insurance market and derivative financial instrument market (He et al., 2017; Sun et al., 2022). A financial instrument would be any asset that can hold money and be exchanged on the open market. The following are some examples of financial instruments checks, shares, notes, futures markets. In addition, the system and regulation mechanism refer to the supervision and regulation of financial activities. The financial elements are relatively independent and interrelated. A variety of economic factors, investor behaviour, information flow, market structure, outside events and other factors contribute to the relative independence and interdependence of financial elements within a system. While investor behaviour, mood, and information transmission are what contribute to interrelation, economic fundamentals like supply and demand are what drive independence. The financial object and financial place are the hardware elements of the financial system, the financial mode, system and regulation mechanism are the software elements of the financial system, and the financial institutions are the comprehensive elements (Chen et al., 2019; Liu et al., 2022). Specifically, financial activities generally take credit instruments as the carrier, and realise the transfer of the right to use monetary funds through the transaction of credit instruments in the financial market. With the support of the above theory and the financial market network model, aiming at the elements of finance and the quantitative parameters of financial fluctuation cycle, the data collection and statistics are realised from two aspects of financial price data and financial fluctuation duration. In order to support technical analysis and investment decisions, financial price data includes the price levels and price alterations of financial instruments. The amount of time that prices vary, both up and down, is referred to as the financial fluctuation duration.

2.2.1 Statistics of financial price data

Suppose that the investors' investment attitude to the financial market is the main cause of stock price fluctuation, and the investors' investment attitude can be expressed by the interactive particle state in the election system (Long and Thuyet, 2017). Mathematical models for networks having numerous elements that randomly communicate with one another are known as interacting particle systems. While the behaviour of each component is regulated by straightforward principles, the interplay between the elements can make the behaviour of the entire system rather complex. Investment attitudes can be divided into buying stocks, selling stocks and holding a neutral view, which can be divided into three categories. Suppose that each investor can trade shares more than once a day, but only one unit at a time. Let l represent the length of tradable time of each day, and $P_N(\tau)$ represent the stock price at time τ of the N-th trading day, where the value range of τ is $[0, l]$. Suppose there are $M+1$ investors in the market, they are arranged on one-dimensional integer points, the specific arrangement is as follows:

$$\left[-\frac{M}{2}, ..., -1, 0, 1, ..., \frac{M}{2}\right] \subset Z \tag{2}$$

At the beginning of every day's financial trading, a certain proportion of investors are randomly selected from the constructed financial market network model and assumed to have obtained market trading information (Chéron, 2018). We define a random variable with values of 1, –1 and 0 and use p_{+1}, p_{-1} and $1-(p_{+1}+p_{-1})$ to express the investors' buying view, selling view and neutral view, respectively. Then, spread bullish, bearish or neutral information to their limited distance neighbours. Because investors can interact with each other, this can achieve the spread of investment information. The share price on the N-th trading day at time τ is:

$$R_N(\tau) = \eta_t \cdot \frac{|\xi_\tau^\theta|}{M+1} \tag{3}$$

In the formula, η_t is a random variable, θ is the proportion of selected investors, ξ_τ^θ is the buying view held by an investor at any time, and ξ_τ^θ is taken as follows:

$$|\xi_\tau^\theta| = \sum_{w=-\frac{M}{2}}^{\frac{M}{2}} \xi_\tau^\theta(w) \tag{4}$$

A framework known as financial pricing theory is employed to comprehend and clarify how prices for different financial assets, including stocks, bonds, options and derivatives, are established. According to the financial pricing theory, the connection here between good or service's demand and availability anywhere at time determines the cost for that item or service. As M may depend on the length of trading day, according to the above definition and financial price theory, the discrete stock price formula of the N-th trading day is as follows:

$$\begin{cases} P_N(\tau) = P_{N-1}(\tau)\exp\{\beta R_N(\tau)\} \\ P_N(\tau) = P_D \exp\left\{\beta \sum_{N=1}^{t} R_N(\tau)\right\} \end{cases} \tag{5}$$

In the formula, β represents the fluctuation parameter of market trading, and P_D represents the stock price at D.

According to the theory of election interaction system, if the system strength parameter is $\lambda > \lambda_c$, it means that the market information can spread quickly, which ultimately affects the stock price volatility [19]. Through emergence, the interplay of system components produces complex behaviours. This complexity is a result of non-linear connections, feedback loops, network effects and emergent features. Targeted messaging, policy alignment, wooing swing voters, micro-targeting, modifying campaign tactics, highlighting problems, coalition building, persuasion and candidate presentation are some of the techniques. Uncertainty can be impacted by lending rates, new taxes, inflation expectations as well as other financial policies, while also being influenced by changing market and local, national and international events, is a major contributor to stock market volatility. Lending rates that fluctuate or are unpredictable can have an impact on borrowing, investment, consumer spending, as well as the housing market. They may also have an effect on financial planning, foreign exchange rates and company expenses. Based on the above theory, the price data of all stock, currency and other components in the financial market network model are counted, and the data collection results about financial prices are obtained. The calculation formula is as follows:

$$r(t) = \frac{\lambda\left(InP_N(\tau) - InP_{N-1}(\tau)\right)}{\lambda_c \cdot K(t)} \tag{6}$$

2.2.2 Statistical analysis of fluctuation duration

Financial volatility is reflected in many aspects, such as monetary value volatility and financial market return rate. Financial asset price swings are measured by monetary value volatility, which reveals investor, market and risk emotion. It helps with risk assessment, portfolio management and analysing prospective price fluctuations because it is quantified using annual standard deviation and variance. The regularity and magnitude of fluctuations in the value of a particular currency are known as monetary value volatility. The dispersal of exchange rate fluctuations all around mean, represented in terms of day, week, monthly or yearly standard deviation and variance, is the method used to evaluate it. The analysis of financial market return rate volatility is very important for asset price evaluation and actual financial management. It helps to quantify investment risk, optimise investment portfolio and provide key input parameters for option pricing model relying on asset volatility estimation (Mohamed and Ali, 2017; Chen and Deng, 2017). A volatility duration series $\{D(t), t = 1, 2, ..., T\}$ is generated from a normalised volatility time series. Since $P_N(\tau)$ represents the stock price of the *N*-th trading day at time τ, the financial volatility series is defined as:

$$V(\tau) = \left|\log P_N(\tau) - \log P_N(\tau - 1)\right| \tag{7}$$

Then the normalised wave sequence *v*(*f*) can be defined as:

$$v(f) = \frac{V(f)}{\left[V^2(f) - V^2(f)^2\right]^{\frac{1}{2}}} \tag{8}$$

The time series of volatility duration can be obtained by the following process: for period f, consider the next day's normalised volatility $v(f+1)$, if $v(f+1) < v(f)$, it means that the financial volatility is locally declining in time f. If the waiting time for the first time that $v(f+1)$ exceeds $v(f)$ in time f is σ, the shortest time that the future fluctuation of stock price exceeds the current fluctuation is expressed as follows:

$$I_1(f) = \max\{\sigma : v(f+1) < v(f)\} \tag{9}$$

Similarly, if $v(f+1) < v(f)$, the corresponding expression is:

$$I_2(f) = \min\{\sigma : v(f+1) < v(f)\} \tag{10}$$

The $I(f)$ result obtained through statistics is the duration of financial fluctuation in the financial market environment, and the calculation formula is as follows:

$$I(f) = \frac{I_1(f) + I_2(f)}{2} \tag{11}$$

The statistical results of the financial volatility duration under the financial market network model can be obtained by combining the shortest time and the longest time.

2.3 Identify financial fluctuation cycle

Under the financial market network model, based on the collected financial data and information, the cycle of financial fluctuation is identified according to the law of financial market movement and change (Vorotnikova and Golovashkin, 2017; Dinh and Long, 2017). Political unrest, rate of interest, contemporary issues, exchange rate movements, environmental disasters and several other factors all have an impact on the stock market. These variables may impact your returns, but if you have a firm grasp of the market, you can choose when is the optimum to purchase or sell equities. Macroeconomic variables including rates of unemployment, per capita income and the gross domestic product are measured to assess the variations. Exchange rates can affect international trade in goods, economic expansion, foreign investment, hyperinflation and borrowing costs. A currency's price will rise if there is a great demand for it or a lack of that currency. We must keep in mind that crises frequently spread by contagion from one country to other countries while evaluating the consequences of global economic volatility. The country where the crisis first began is limited financially due to the loss of jobs and labour income that accompany crises (Xu et al., 2022). The movement of financial markets is shown in Figure 1.

Figure 1 Schematic diagram of financial market cycle

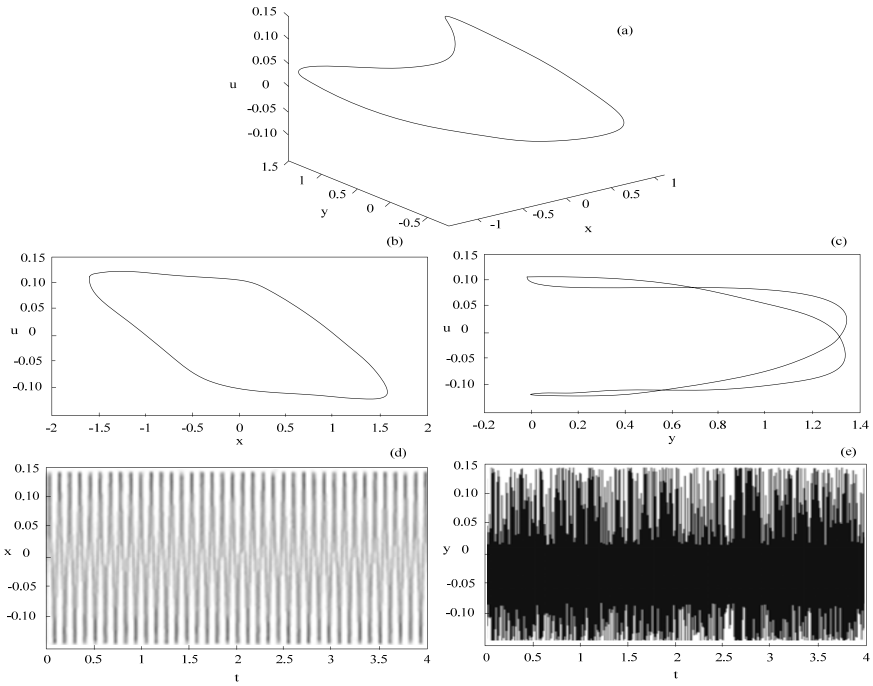


Figure 1(a) represents the basic phase diagram of the financial market network model, Figure 1(b) represents the phase diagram of the financial market network model in the steady state, Figure 1(c) represents the phase diagram of the financial market network model in the unsteady state. Figures 1(d) and 1(e) respectively correspond to the time course of the movement of the financial market network model in the steady state and the unsteady state.

2.3.1 Setting financial fluctuation indicators

On the basis of financial theory and prior knowledge, five indexes are selected as financial fluctuation indexes from four aspects of credit, asset price, money market and capital flow: total amount of domestic credit, sales volume of commercial housing of real estate development enterprises, Shanghai Composite Index, money supply M2/GDP, capital and financial project balance/GDP. It is possible to select financial fluctuation indexes depending on factors of credit, asset price, money market and capital flow. CDS spreads, asset price indices, money market circumstances and capital flow patterns are examples of credit risk insights. Index selection should take research objectives and data accessibility into account. The reasons are as follows: credit plays an important role in investment and saving; the real estate price and stock price in asset price have a great influence on financial market, and are very representative, so this paper takes the real-estate price and stock price as the proxy index of asset price, and puts them into the financial fluctuation index system; the openness of capital account will produce the implementation of monetary policy in a country Influence, and then affect the financial cycle (Zitouni and Khaled, 2017). Therefore, when setting up the financial fluctuation index, the agency indexes related to money market and capital flow are included in the financial fluctuation index system. The data of total domestic credit, Shanghai Composite Index, capital and financial project balance/GDP are from Wande information, while the data of real estate development enterprises' commercial housing sales and money supply M2/GDP are from the statistical database of China economic network. In order to affect M2, monetary authorities can also raise or lower their interest rates. Lower rates of interest will probably increase loans, that will boost the amount of money available. In contrast, if interest rates increase, borrowing would become more expensive, which will discourage consumers from obtaining loans. In general, the value of currency in circulation rises whenever the rate of GDP growth indicates increased economic output. This is so that every unit of money can be later used to purchase higher expensive services and products. Take the insurance part of the financial market as an example, the setting of the fluctuation index is shown in Table 1.

Among them, the data of property insurance premium income, life insurance premium income, accident insurance premium income, health insurance premium income and other indicators are from the established financial market network model. The insurance depth C_5 and insurance density C_6 are calculated by the following formula:

$$\begin{cases} C_5 = \dfrac{I_n}{D} \times 100\% \\ C_6 = \dfrac{I_n}{n_p} \end{cases} \tag{12}$$

where I_n is the premium income in the financial insurance market, D is the total GDP and n_p is the population.

Table 1 Setting of some fluctuation indexes in financial market

Market class	*Financial fluctuation index*	*Index code*
Insurance market	Property insurance premium income	C_1
	Life insurance premium income	C_2
	Accident insurance premium income	C_3
	Insurance premium income	C_4
	Insurance depth	C_5
	Insurance density	C_6

2.3.2 Measuring financial fluctuation cycle

Financial volatility is mainly used to measure the volatility of a financial variable in the sample range, generally expressed by its 5 years moving standard deviation. The advantage of this method lies in that on the one hand, it can show the volatility of the financial variable in a certain stage, on the other hand, it can also show the change of its time trend.

In the process of identifying financial market volatility, it is not only necessary to adjust or eliminate the influence of seasonal factors on time series data, but also necessary to decompose the trend and periodic characteristic components in the series. Suppose $\{Y_t\}$ is a time series with both trend and fluctuation characteristics, $\{Y_t^T\}$ is the trend characteristic component of the series and $\{Y_t^\varepsilon\}$ is the fluctuation characteristic component of the series. The long-term, slow movement in the data, indicating underlying economic or structural alterations, is represented by the time series' trend characteristic component. While, the fluctuation characteristic component reflects transient changes and irregularities surrounding the trend that are brought on by things like seasonality, noise and transient occurrences. The relationship among them can be expressed as follows:

$$Y_t = Y_t^T + Y_t^\varepsilon \tag{13}$$

where the value range of t is $[1, T]$. Using H-P filter, $\{Y_t^T\}$ is filtered out from $\{Y_t^\varepsilon\}$ and $\{Y_t\}$ is decomposed by minimisation two multiplication, and the trend characteristic component $\{Y_t^T\}$ is obtained. Its expression is as follows:

$$\{Y_t^T\} = \min \sum_{t=1}^{T} \left\{ \left(Y_t - Y_t^T\right)^2 + \lambda \left[c(L) Y_t^T \right]^2 \right\} \tag{14}$$

In the formula, $c(L)$ represents the polynomial of delay operator, substituting the polynomial corresponding to $c(L)$ into formula (14), then the filtering problem will be converted into the minimisation problem of loss function and the change of $\left[c(L)Y_t^T\right]^2$ adjustment $\left\{Y_t^T\right\}$ will be used to solve the minimisation problem, and the financial cycle calculation result will be obtained, as shown in Figure 2.

2.4 Non-linear dynamic equation of financial market volatility

The financial system consists of four sub-modules: production, capital, stock and labour. They are represented by three first-order differential equations. The system describes the time variation of three state variables: interest rate x, investment demand y and price index z. From the point of view of economics, the factors influencing interest rate are not only related to investment demand and price index, but also to average profit rate, which is in direct proportion to interest rate. The non-linear dynamic behaviour of financial system shows different periodic motion and chaotic motion. Harmonic oscillations are an example of periodic motion because they exhibit regular and predictable recurrence, stable patterns and definite frequencies. Unexpected chaotic motion displays sensitive beginning condition dependency and complicated, unexpected behaviour. Therefore, the non-linear dynamic equation of financial market fluctuation is as follows:

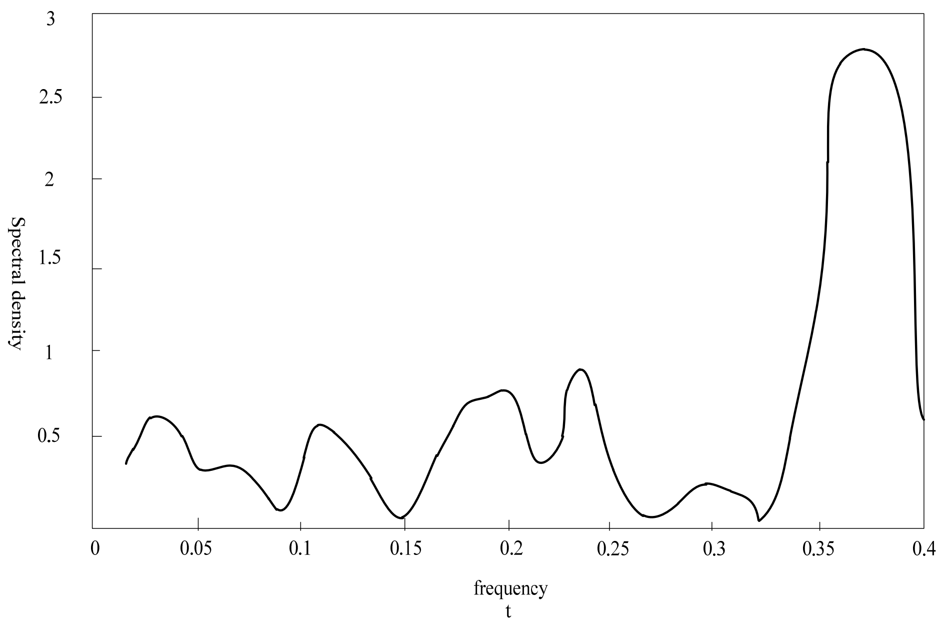
$$f(x) = \begin{cases} x' = z + (y - a)x + u \\ y' = 1 - by - x^2 \\ z' = -x - cz \\ u' = -dxy - ku \end{cases} \tag{15}$$

where u is the average profit margin, a is the capital storage, b is the investment growth rate, c is the supply and demand coefficient and parameters d and k are positive constants.

2.5 Analyse the influencing factors of financial fluctuation cycle

In the network model of financial market, the financial fluctuation cycle presents a non-linear change mode. The financial market's complexity, unpredictable nature and openness to numerous psychological, economic and outside forces are all reflected in its non-linear properties. However, when there is an external force in the financial market, it will cause the financial data to fluctuate which does not conform to the cycle law, resulting in the error of numerical simulation results. In order to solve the problem of high-error rate in the traditional numerical simulation method, based on the non-linear motion equation, several factors that may affect the financial fluctuation cycle are fully considered, and the relevant influencing factors are substituted into the non-linear motion program constructed in this paper, and the calculation method of the financial fluctuation cycle is adjusted in time, so as to ensure the accuracy of the calculation results. There are a number of issues with it, such as oversimplifications, insufficient representation of real-world complexity, an inability to capture dynamic market changes, assumptions of rational behaviour, problems with rare events, feedback loop complexity, model risk and so on.

Figure 2 Spectral density curve of financial market fluctuation cycle



Money supply is derived from the base money on the basis of money multiplier, and affected by both, so money supply is directly related to the stability of money multiplier. The money-multiplier process illustrates how well a rise in the monetary base results in an increase in the supply of money that is doubled. The money supply is decided by it, and this has an influence on interest rates. Because it affects monetary and fiscal policy and the stability of the financial industry, it is also significant in the banking industry. It affects government expenditures, exchange rates, inflation control and interest rates. Banking stability and financial crises are impacted by the money supply's impact on liquidity, credit availability and interbank activities. But money multiplier is restricted by many factors such as cash leakage rate, deposit multiple, legal deposit reserve ratio and Commercial Bank excess reserve ratio. Because of the instability of capital market, these factors are constantly changing, which makes it more difficult to make accurate expectation of money multiplier. It can be seen from this that the state's monetary regulation is one of the factors that affect the financial fluctuation cycle. Taking the dynamic model of macroeconomic regulation and control strategy which uses monetary policy tools to adjust market money supply under the influence of price index fluctuation as an example, the impulse dynamic model of monetary regulation and control is established.

Under normal conditions, when money supply $M(t)$ equals money demand $\bar{M}(t)$, the price index will be stable in equilibrium $\bar{P}(t)$. When there is a large deviation between the two, the price index will be far away from the equilibrium state and generally speaking, when the deviation is relatively large, the price index will rise more, otherwise the decline will be larger. Therefore, it can be assumed that the relative change rate of the price index is directly proportional to the relative deviation of money supply and demand, which satisfies the following formula:

$$\frac{dP}{dt} = \alpha P(t)\left[\frac{M(t)}{\bar{M}(t)} - 1\right] \tag{16}$$

In the above formula, α represents the elasticity coefficient of financial currency price index, whose value is greater than 0. Combined with the influence of time delay in monetary regulation, the non-linear motion equation corresponding to the financial fluctuation cycle can be adjusted to:

$$\left(M(t-\kappa)\right) = M''(t) + \bar{f}\left(t, M(t), M'(t), \theta\right) \tag{17}$$

In the formula, $M'(t)$ and $M''(t)$ represent the result of one-time and multiple adjustment of money supply respectively, κ is the delay of money regulation and θ is the control parameter. Considering the influence of external impulsive disturbance, together with the impulsive function term in the original non-linear motion equation of financial fluctuation cycle, the monetary policy regulation equation with impulsive influence can be established under the financial system oscillation and fluctuation:

$$\begin{cases} M''(t) + \bar{f}\left(t, M(t), M'(t), \theta\right), t \neq \varphi(M, M') \\ M(t^+) = h\left(M(t)\right), t = \varphi(M, M') \\ M'(t^+) = g\left(M(t), M'(t)\right), t = \varphi(M, M') \end{cases} \tag{18}$$

In the formula, $\bar{f}\left(t, M(t), M'(t), \theta\right)$ is the function of money supply and demand, h and g are the corresponding pulse functions, and $\varphi(M, M')$ is the limiting condition for the stimulation of external interference factors on the relative deviation of money adjustment.

In the same way, in addition to monetary control, artificial holding of financial stock market, talent flow of financial market and other factors may affect the change of financial fluctuation cycle. Therefore, in the actual numerical simulation process, we need to integrate the above factors and adjust the non-linear equation of motion according to the above adjustment method.

2.6 Numerical simulation of financial fluctuation period based on non-linear equation of motion

The law of programming refers to the macroscopic change rule of financial market. The general regularities discovered are: the peak and fat tail characteristics of price distribution, the short-range correlation of price returns, the long-range correlation and clustering effect, the leverage effect, the bubble and crash, intermittence and multifractal characteristics. This paper analyses the correlation and aggregation of price fluctuation cycle in financial fluctuation cycle. The uncorrelation between relative price changes is an important feature of Brownian motion model of financial market.

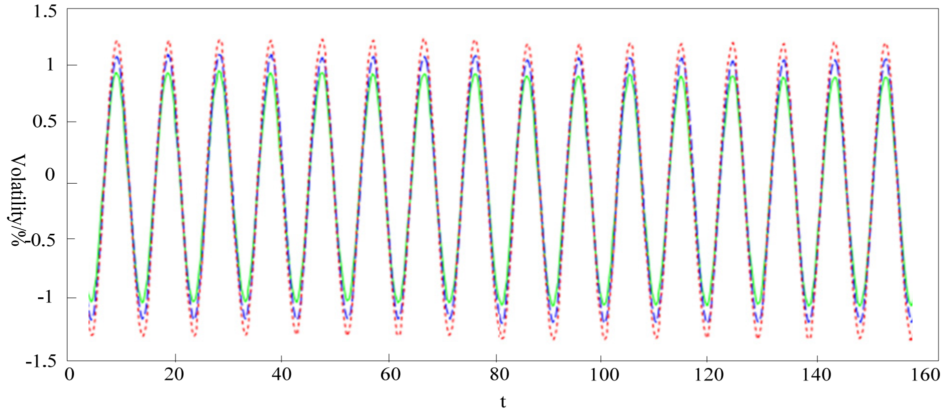
Based on the non-linear motion equation and the influencing factors of the financial fluctuation cycle, the corresponding non-linear equation format of the financial fluctuation cycle numerical simulation is adjusted and the specific assignment of each variable and parameter in the equation is carried out, then the numerical simulation results of the financial fluctuation cycle under different conditions are obtained. Economic indicators, interest rates, market volatility, liquidity, investor mood, regulations, technology, world events, market structure, behavioural biases, policy responses, systemic risk, credit dynamics and time delays are just a few examples of the elements that influence financial markets. The calculation formula is as follows:

$$c_t(\Delta t) = \frac{\left[f\left(M(t-\kappa)\right)\right]\left[r_\tau(t) r_\tau(t+\Delta t) - r_\tau(t)^2\right]}{\left[r_\tau(t) r_\tau(t+\Delta t)^2 - r_\tau(t)^2\right]} \tag{19}$$

where r_τ is the exponential attenuation coefficient.

After the research, we found that the earnings only have autocorrelation in a short time, and the disappearance of earnings correlation satisfies the exponential decay. Only from the perspective of linear correlation analysis, it is impossible to draw the conclusion that the income function is independent and identically distributed and because of the existence of higher-order correlation, the variables in the model meet the random walk and there is no correlation. The autocorrelation function of price fluctuation has a positive correlation with the delay time, and the magnitude of the fluctuation range appears in different time periods, so the financial fluctuation cycle has certain aggregation. The simulation results of financial fluctuation cycle without external force are shown in Figure 3.

Figure 3 Numerical simulation results of financial fluctuation cycle without external force (see online version for colours)



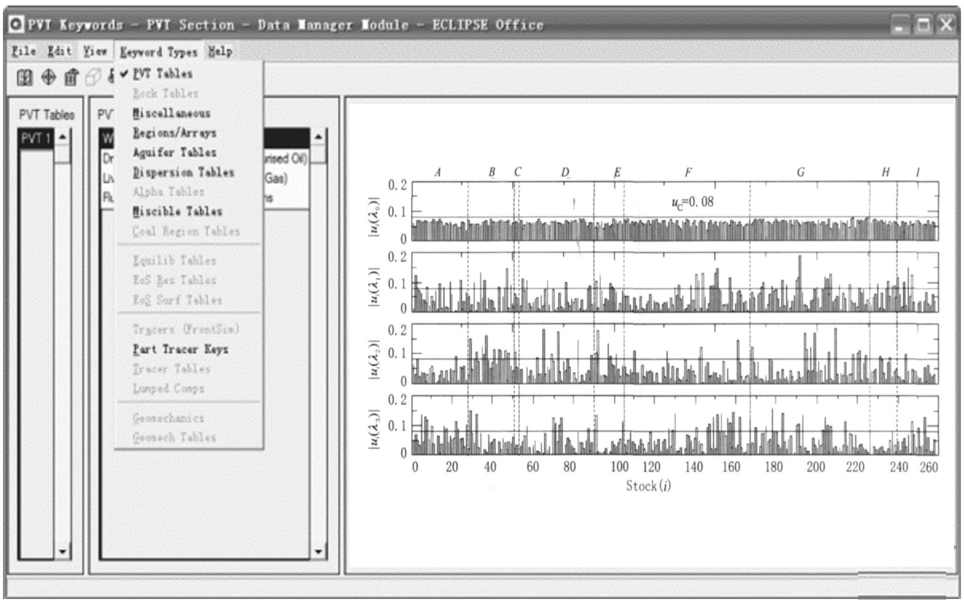
3 Analysis of numerical simulation experiment

In order to prove the simulation function and application effect of the numerical simulation method based on non-linear motion equation, the simulation experiment is designed. In this experiment, to test the accuracy of the simulation results and the effectiveness of the application effect, we start from the function and performance of the numerical simulation method. In order to highlight the performance of the designed numerical simulation method of financial fluctuation period based on non-linear equations of motion, in this numerical simulation experiment, in addition to setting the design method as the experimental method, we also need to take the numerical simulation method of financial fluctuation period based on normal distribution and the numerical simulation method of financial fluctuation period based on brown motion as the comparison method. The initial experimental data of this method are the same, so as to ensure the uniformity of experimental variables. The experimental tool selected in the numerical simulation experiment is SAS simulation software. In this experimental environment, the program codes corresponding to the methods in this paper and the experimental comparison methods are imported into the experimental environment, and a call link is formed to ensure that the three methods are independent in the operation process, so as to improve the authenticity of the simulation experiment

3.1 Select financial samples

In the simulation experiment, the simulation data comes from wind database and the time span is financial data from 1995 to 2019. Using econometrics related theories, this paper takes GDP, material capital stock, material capital stock of financial industry, financial development index, gross national consumption and other corresponding income tax income and related expenditures as research variables, all of which are converted by the constant price in 2005, with the unit of billion yuan. In addition, the daily closing data of the representative stock index is also required as the data sample of the stock part in the financial market. The frequency spectrum of the financial data sample used in the experiment is obtained by synthesising various data in the financial market, as shown in Figure 4.

Figure 4 Sample spectrum of experimental financial data



Quantify the financial initial data in Figure 4 to obtain the descriptive statistical results of the main variables in the numerical simulation experiment, as shown in Table 2.

3.2 Comparison of simulation results and actual values of different methods

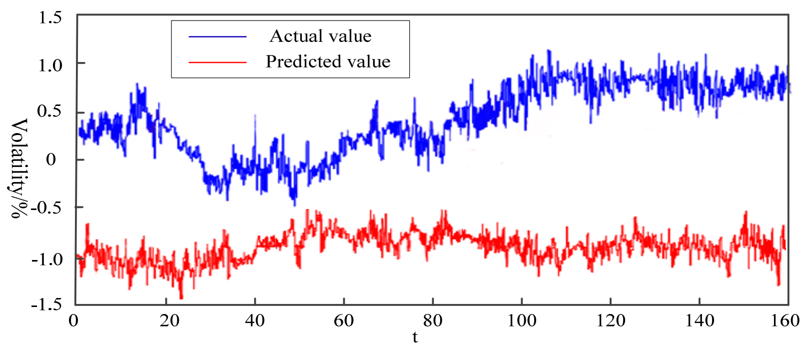
Because there must be many interference factors in the operation of financial market, in order to make the simulation experiment more real, compare the simulation results of this method and the experimental comparison method under the effect of external forces with the actual value, the results are shown in Figure 5.

According to Figure 5, in a certain period, the actual financial volatility changes from –0.5 to 1.0%, the financial volatility simulated by the normal distribution method changes from –0.15 to 0.7%, the financial volatility simulated by the brown motion method changes from –0.12 to 0.5%, the financial volatility in this method changes from –0.3 to 0.7%, and the numerical simulation results are close to the actual values, which shows that the numerical simulation results of this method can accurately reflect the financial fluctuation cycle, with higher accuracy.

Table 2 Descriptive statistics of main experimental variables

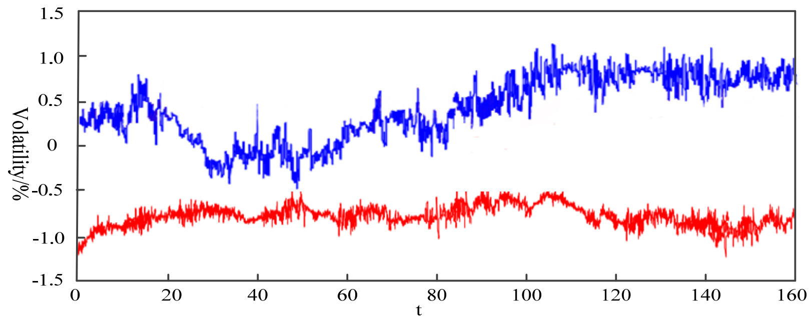
Variable	*Variable name*	*Sample size*	*Average value*	*Standard deviation*
Debt1	Interest bearing load rate	8868	1.119	1.274
Debt2	Net debt to cash ratio	8868	0.111	0.590
Invest1	Investment cash ratio	8868	0.676	1.159
Invest2	New investment stock ratio	8868	0.073	0.208
Hp	Real estate price growth rate	8868	0.112	0.978
Sp	Annual return on shares	8868	0.348	0.872
Hv	Housing market value ratio	8868	1.669	1.216
Net1	Ratio of shareholders' equity	8868	2.269	1.851
Net2	Internal capital ratio	8868	0.318	0.697
Roa	Return on assets	8868	0.045	0.056
Tobin	Book to market ratio	8868	0.421	0.259
Size	Enterprise scale	8868	22.159	1.214
First	Shareholding ratio of the first shareholder	8868	0.354	1.151
State	Shareholding ratio of state-owned shares	8868	0.125	0.200
R	lending rate	8868	0.060	0.007
Cashflow	Cash flow ratio	8868	0.320	0.706
Fn	External financing dependence	8868	0.427	0.257

Figure 5 Comparison of simulation results and actual values of different methods (a) Numerical simulation method of financial fluctuation cycle based on normal distribution (b) Numerical simulation method of financial fluctuation period based on Brownian motion (c) Numerical simulation method of financial fluctuation period based on motion differential equation (see online version for colours)

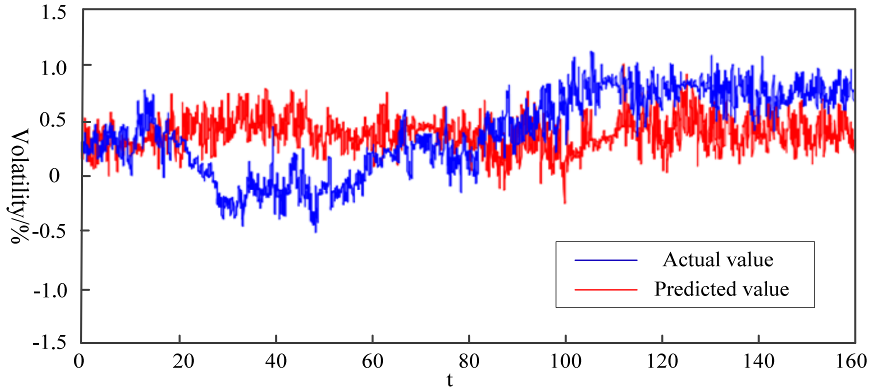


(a)

Figure 5 Comparison of simulation results and actual values of different methods (a) Numerical simulation method of financial fluctuation cycle based on normal distribution (b) Numerical simulation method of financial fluctuation period based on Brownian motion (c) Numerical simulation method of financial fluctuation period based on motion differential equation (see online version for colours) (continued)



(b)



(c)

3.3 Maximum error comparison of different methods

According to the simulation process of the experimental comparison method and the numerical simulation method based on the non-linear motion equation, the financial experiment samples are used as the data base for numerical simulation and the corresponding simulation results are obtained. Call the actual cycle development rule and relevant data in the database as the test comparison standard of simulation results, and finally get the statistical results about the maximum error of numerical simulation, as shown in Table 3.

From the data in the table, it can be seen that both numerical simulation methods use 5 years as the time span to simulate the financial fluctuation cycle, and judge whether there is a financial fluctuation cycle in 5 years. Compared with the numerical results of the standard financial fluctuation period, it is found that the maximum error of the numerical simulation results of the financial fluctuation period based on the non-linear motion equation designed in this paper is far lower than that of the experimental comparison method, which shows that the numerical simulation error of this method is low and the accuracy of the numerical simulation of the financial fluctuation period is high.

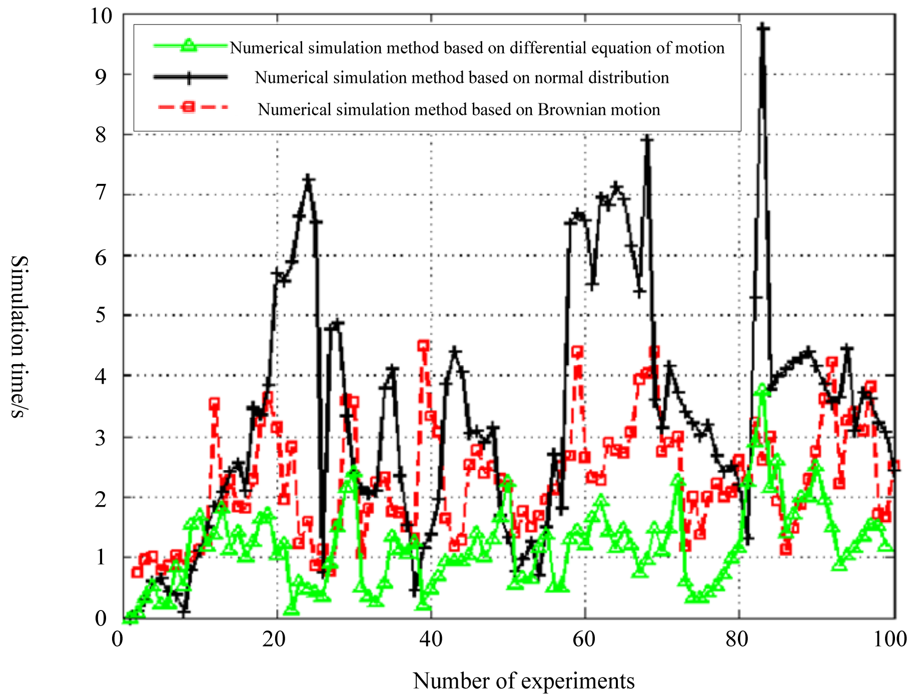
3.4 Comparison of numerical simulation time of different research methods

On the basis of the above experiments, in order to further verify the superiority of the method in this paper, the simulation time comparative experiments of numerical simulation are carried out and the experimental results are shown in Figure 6.

Table 3 Maximum error comparison of simulation results

Particular year	*Numerical simulation method of financial fluctuation cycle based on normal distribution*			*Numerical simulation method of financial fluctuation period based on Brownian motion*			*Numerical simulation method of financial fluctuation period based on non-linear equation of motion*		
	Error of explicit difference scheme/%	*Half hidden difference scheme error/%*	*Implicit difference scheme error/%*	*Error of explicit difference scheme/%*	*Half hidden difference scheme error/%*	*Implicit difference scheme error/%*	*Error of explicit difference scheme/%*	*Half hidden difference scheme error/%*	*Implicit difference scheme error/%*
2004	0.95	0.83	0.89	0.77	0.56	0.73	0.66	0.48	0.57
2009	0.94	0.88	0.91	0.82	0.87	0.79	0.62	0.48	0.58
2014	0.99	0.85	0.82	0.87	0.93	0.88	0.67	0.44	0.57
2019	0.91	0.87	0.93	0.74	0.74	0.78	0.65	0.52	0.61

Figure 6 Comparison of numerical simulation time of different research methods (see online version for colours)

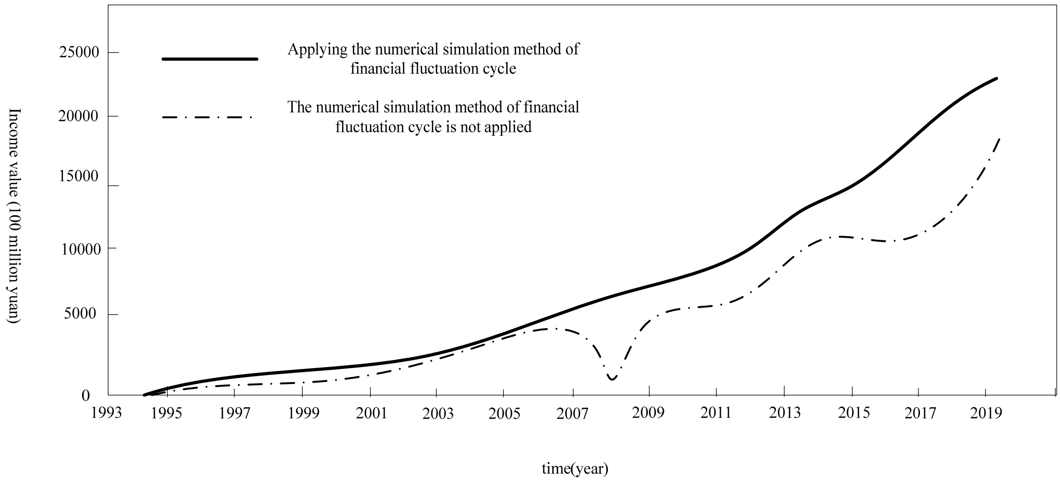


Through the analysis of Figure 6, it can be seen that in many simulation experiments, the simulation time range of numerical simulation based on normal distribution method is 0.1–9.8 s, which is the most simulation time and unstable of the three methods. The simulation time range of numerical simulation based on brown motion method is 0.7–4.6 s, while the simulation time range of numerical simulation in this method is 0.1–3.8 s, which is the shortest. The efficiency of numerical simulation of financial fluctuation cycle is the highest.

3.5 Application performance test of financial fluctuation cycle numerical simulation method

After research, it is found that the financial fluctuation will affect the economic development of the region and even the country. Therefore, the numerical simulation results of the financial fluctuation cycle can be used as a reference. When it is found that there is a huge fluctuation in the financial market, or the fluctuation cycle may be abnormal, immediately start the corresponding control mechanism, so as to make the financial fluctuation cycle return to the normal level and maintain the stability of the financial market Healthy development. In order to test the application performance of this method in the regional economic development, design the test experiment, mainly compare the development of local economy before and after the application of the financial fluctuation cycle numerical simulation method based on the non-linear motion equation, and the specific quantitative comparison index is the income value of local economy. Through simulation and statistical calculation, the application performance comparison curve of this method is obtained, as shown in Figure 7.

Figure 7 Application performance analysis of this method



It can be seen from the curve changes in the figure that the economy of the region is generally in a state of steady improvement. However, when the method in this paper is not applied, due to the impact of the financial crisis, there is a big turning point in the local economy. However, because the method designed in this paper is applied in the experimental group, there is no big fluctuation in the economy. Compared with the economic income in 2019, the economic income value is 15.4% higher after the application of this method. It can be proved that the numerical simulation method of financial fluctuation cycle based on non-linear equation of motion can provide more accurate reference data for regional economic development and has higher application value.

4 Conclusion

With the acceleration of the process of economic globalisation, the integration of the world financial market has become an inevitable trend. Any small change of the world economy may have a huge impact on China's national economy through the world financial market. It is imperative to study the financial fluctuation cycle and realise the quantitative analysis through numerical simulation. Therefore, this paper proposes the financial fluctuation cycle based on the non-linear equation of motion period numerical simulation method, and through simulation experiments to verify the superiority of this method. The proposed method can improve the correctness and scientificity of financial research results, improve the effectiveness of cooperation between banks and enterprises and promote the solution of various financial problems. It is an important way to avoid various financial risks. In the future, relevant researchers should continue to improve the correctness and accuracy of the numerical simulation results of financial volatility cycle, so as to provide scientific basis for the formulation of China's macro-control policies.

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