# Modeling Customer Equity: A Stochastic Modeling Approach for Arrival and Departure of Customers

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*Customer Equity (CE) is an important concept for marketers in managing their customers. It allows them to better evaluate the contribution of existing customers and the potential purchases of new customers to their overall value. Such valuation provides better segmentation schemes that eventually lead to better financial performance. Various models in the literature aim at modeling CE with different structural forms and assumptions but most commonly are based on evaluating the value of current, active customers. This paper introduces a different approach for evaluating CE that consider current and future (potential) customers by developing a two-stage model that is based on stochastic and actuarial calculations. In the first stage, we consider the value of current customers and relate it to their remaining potential consumption lifetime. In the second stage, we use stochastic arrival and departure processes to account for potential new customers using stochastic calculations of remaining purchasing lifetime. As a result, the model can better predict current and future value of customers compared with current approaches.*

Business executives are well aware of the fundamental role customers play in a company's success. As a result, customer management as a tool to improve company performance has become increasingly common. There has been steady growth in attention to this issue in both the academic world and the business world (e.g., Chang & Wildt, 1994; DeSarbo, Jedidi, & Sinha, 2001). This growing interest has led scholars to seek new models for calculating the value of customers to a company. The current models of Customer Equity (CE) can best be categorized into three types (Jain &

Singh, 2002). The first category is primarily concerned with achieving optimal resource allocation to maximize CE. The second category includes models that estimate the value of customers based on their past purchasing behavior. The third category includes a macro-type approach for CE, to gain deeper insight into policy-related issues.

In their extensive review of current customer life time value (CLV) and CE models, Jain & Singh (2002) examine the various structures of the current models for calculating CE. The comprehensive comparison of the various models in that study allows evaluating their strengths and weaknesses. As a result, several limitations of the current models can be inferred from this review, with regard to the accuracy of estimation and calculation. In particular, current models are based on restrictive assumptions about customer behavior with respect to loyalty, purchasing patterns, and future behavior. These models include only current customers in their calculations and ignore potential customers or the acquisition of new customers. These models also fail to consider the retention of current customers. Furthermore, they ignore the stochastic nature of the purchasing process and assume a particular time structure for future cash flows.

Another important issue that is generally not factored into the assessment of CE is the remaining lifetime of current and future customers. For example, consumers might change their purchasing patterns as they grow older (e.g., regular soft drinks to diet soft drinks, two-door sports car to four-door sedan, etc.). In such cases, there is usually a time frame in which consumers use a certain product type in a particular category and later move to another product type in the same category. In other words, the lifetime of such consumers is limited to a specific time frame in their total consumption lifetime. A consumer that is close to the end of his/her consumption stage of a certain product category is probably worth less to the firm than a consumer that is at the beginning of that stage since the former will remain an active customer for less time than the latter. Thus, consideration of a consumer's potential remaining lifetime is an essential building block in CE assessment.

Therefore, current CE models are static and cannot contend with the dynamic nature and inherent uncertainty of future customer activities. Recently, Kumar, Ramani, & Bohling (2004) introduced a different modeling approach that address the issue of active and non-active consumers by employing a probabilistic approach to identify customers' activity in future time periods. Thus, alleviating some of the limitations mentioned earlier.

In light of these limitations, this paper introduces a different CE model that resolves some of the shortcomings inherent in the current approaches. The proposed model resolves the restrictive nature of current models with respect to the dynamics of customer purchasing behavior. A two-stage model is used to enable differentiation between the value of existing customers and the value of future customers. In the first stage we introduce an actuarial model which takes into account the effect of the remaining lifetime of each consumer (i.e., the probability of current customers to stay active in the next time period) and enabled us to calculate the CE value of a firm. In the second stage we use a stochastic modeling approach that enables us to get a more realistic CE valuation that accounts for future customers' acquisition and retention.

We employed the $M / G / \infty$ model that assumes customers' arrival as a Poisson process, a random level of a firm effort (e.g., marketing communications) to attract a customer that is distributed according to a general distribution, G, and that a firm can accommodate all potential customers immediately at their arrival time. Such modeling approach improves current commonly used approaches by allowing us to evaluate the overall CE at the time of customer departure. Note that at that time the number of all remaining customers and their remaining active lifetimes with the firm is known and, therefore, a close form solution for the model can be obtained. This advantage is attributed to the assumption that the customers' arrivals follow the Markov arrival Poisson process, also termed as Poisson Arrivals See Time Averages (PASTA). The outcome of this modeling approach is the ability to better predict the value of current and future customers compared with current approaches.

## Conceptual Background

The concept of customer lifetime value has long been accepted in almost all business literature (e.g., Porter, 1985). The concept entails that a company can gain a competitive advantage over its rivals by creating value for its customers. Using this value, a company can better communicate the benefits to consumers who purchase the product. Once consumers acknowledge the value the company creates for a particular product, the company can assess the long-term benefits in retaining these consumers. That is, the company can estimate the value of its customers at any given time through their purchases, or aggregate it over time to obtain their lifetime value.

CE models have multiple applications in various types of business organizations. These include, for example, models that aid companies in making both strategic and tactical decisions; strategic decisions identifying customers, their characteristics and the customers that should be pursued in the long run. Tactical business decisions might include resource allocation among marketing mix variables, in the short run. CLV models have also been employed in helping companies identify their profitable customers, as opposed to their non-profitable ones. Such an analysis enables companies to better allocate their marketing resources to relevant consumers and market segments. In their extensive review of CLV models and their application, Jain & Singh (2002), present three basic model types.

These types include: A) Models for calculation of CE—this category includes models that are specifically formulated to calculate CE and/or extend the calculation to obtain optimal methods of resource allocation to maximize CE. These are applied models and are more relevant to those who wish to use CE as a basis for making strategic or tactical decisions. B) The second type includes models of customer-based analysis. Such models take into account the past purchase behavior of the entire customer base to calculate probability of purchase in the next time period. These models consider the stochastic behavior of customers in making purchases; therefore, these models look at each customer individually to compute the probability of purchase in the next time period. C) The third type includes normative models of CE. Such models provide valuable insight for policy-making. This category comprises two typical models: a customer equity model and a dynamic pricing model based on CE.

For more information, see Jain & Singh (2002), in which they also identified directions for future research.

As discussed, the current CE models are limited in several respects. To begin with, treatment of consumer retention over time is inadequate. This means it is necessary to better capture the departure of customers over time from the company's list of active customers. In addition, insufficient attention is paid to the attraction of new customers to the company. This refers to the effect of the company's reputation in acquiring new business. Furthermore, the retention of customers over time is dependent not only on the quality of the company's products and services, but also on the remaining lifetime of the customer in a specific product category. That is, consumers might continue to purchase a certain product type for a certain period and move to another product type as they grow older. This change is independent of the company's efforts to retain the customer through its offerings.

Another aspect that might affect the attraction of future customers is the firm's reputation. That is, the strength of a firm's position might increase the inflow of potential future customers and, therefore, might increase potential future cash flows. The two main schools of thought aiming at estimating the worth of this reputation, or goodwill, are accountancy calculations and economic theory. The accountancy approach is based on the idea that the difference between a firm market value and the value of its tangible assets as appeared in its balance sheet represent the value of its intangible assets. One of these intangible assets is the strength of the firm reputation to attract new customers in the future. The other common approach to evaluate this type of firm reputation is based on game theory techniques that are derived from economic theory (e.g., Fudenberg & Kreps, 1987; Fudenberg & Levine, 1989; Kreps, 1990; Kreps, Milgrom, Roberts, & Wilson, 1992; & Hart, 1995). Kreps (1990), for example, used the Folk theorem to develop a theory of a firm as a bearer of reputation and showed that reputation can become a tradable asset.

This paper aims to develop a dynamic stochastic model to evaluate the worth of the future stream of cash flow resulting from future potential customers. Stochastic calculation provides a more realistic means of addressing the issue of customer retention. Under such an approach, the future of a customer with the company is modeled by assigning both a positive probability that the customer will continue to buy the company's product in the next period, and a positive probability that the customer will not. These probabilities are linked to the customer's potential remaining lifetime.

Using these probabilities, we can calculate the present value of the financial stream, namely the creation of CE. Note that the classic deterministic calculations of CE do not take these probabilities into consideration, and assume that the customer will continue to purchase the company's product at a probability of 1. Our calculation, using the stochastic models, makes the value obtained from CE assessment more realistic and in line with the reality of the market.

## The Model

In order to resolve the issue of customer acquisition, we introduce a two-stage model. In the first stage, we calculate the value of current customers using actuarial calculations. This model provides a more realistic approach of calculating current customers' lifetime value, as it accounts for the stochastic nature of customer purchasing patterns. This characteristic of the model affects the discounting of future cash flows through a different time structure. This stage of the model development is similar in nature to the one proposed by Kumar et al. (2004) and Gupta, Lehamnn, & Stuart (2004).

The second stage involves an assessment of future acquisition of customers. This is predominantly attributed to the company's brand recognition or goodwill. This second stage consists of consumers' stochastic arrival and departure processes to and from the company, using a stochastic approach of remaining purchasing lifetime. This feature of our modeling approach is a way to evaluate the value of future customers. The summation of the two stages, therefore, provides a more accurate CE. For better illustration, we begin with the current, common model for CE calculation. We then present the first stage and conclude with the second stage.

*Basic Structural Model of CE*

We start with a basic structural model of CE in the form of:

$$CE = \sum_{k=1}^{n} \frac{R_k - C_k}{(1+d)^{k-0.5}} \qquad (1)$$

where

$K$ – The time period of cash flow from customer transactions;

$R_k$ – The revenue from customer at time period $k$;

$C_k$ – The total accrued cost of generating revenue $R_k$ at time period $k$;

$d$ – Annual discount rate;

$n$ – Total number of time periods of the projected life time of customers under consideration;

The *0.5* in equation (1) reflects the approximation that all expenditures incurred in the middle of each purchase cycle. In this model, it is assumed that all cash flows take place at the end of a period. It identifies a class of different CE models that are based on the net present value (NPV) of the future cash flows from customers. Such a model is limited. As noted earlier, these limitations include an assumption about a particular time structure of cash flow, consideration for current customers (not future), ignoring acquisition costs, no consideration for the stochastic nature of the purchase process and timing of cash flows, and no provisions for customer variations.

The first stage in our two-stage modeling approach is based on actuarial calculations in which neither the future periods of cash flows nor the probability that current customers will remain active in the future are known. In the second stage, we use a stochastic model based on queuing theory that can address the stochastic nature of the purchasing process and timing of cash flows. In a nutshell, queuing theory is based on theories of random walks and stochastic processes to optimize a queuing system. In the current study, each customer is treated as a single customer that arrives at a random time in the future and remains an active customer in the system for a random time period. Between time periods, there is a probability that a customer will remain in the same state or change his/her behavior (i.e., random walk). The overall behavior of all customers over time is considered a stochastic process.

The model is based on three components: a Markov process that represents the arrival of new customers at the company, a general process (i.e., an unknown distribution) of the active lifetime of customers (i.e., duration of purchasing the company's products) and no limitation on the number of potential customers that can be served by the company. The Markov process is used because of its property of independence between the arrivals of potential customers. Furthermore, under certain conditions, the Markov process allows the assumption of exponential distribution of the time between customer arrivals. This property allows for the inclusion of previously active customers who are currently non-active and may become active again in the future. We therefore use the notation of $M / G / \infty$ , as in queuing theory, to denote the Markov process (i.e., M), the general distribution (i.e., G) and the unlimited number of potential customers (i.e., $\infty$). In the following section we show how to evaluate the current net worth of future purchasing stream from new customers using the general distribution, G. We also use specific distribution to show the evaluation of such future cash flow stream. Since our model is not restrictive in its structure (i.e., a general G distribution), it can accommodate a wide range of specific distributions as might be needed for specific real-life scenarios.

We assume a Poisson distribution of the Markov arrival process for customers buying company products. With the above assumptions and model components, we give explicit expressions for the future CE resulting from the activities of both current and future customers. Unlike current CE models, the proposed model considers the reputation of the company and its potential to attract new customers. This customer acquisition will lead to several outcomes. To begin with, some customers will develop loyalty and will continue to purchase company products. Some customers, on the other hand, will stop purchasing the company's products in the future (due to changing needs, dissatisfaction, etc.). As all these customers contribute to the NPV of the company, it is important to include them all (i.e., current, future, and inactive customers). The proposed model will account for these types of customers and improve the accuracy of CE calculations. The first stage of our model accounts for the value of current customers; the second stage accounts for the value of future customers.

## Stage I – CE of Current Customers

We begin by calculating the value of current customers. The main feature of our model for stage I is the probability, p, that a current customer at time t=1 will continue to purchase the company's products during the next time period (i.e., t=2), and another probability that this customer will leave the company during the next time period, $(1-p)$. Let ${}_tP_k$ be the conditional probability that a customer who is at stage I in the potential lifetime of a certain product category and bought the company's products during that time period (i.e., $k$) will continue to buy from the company for $t$ time periods thereafter. That is, we link the purchasing probability from the company's products to the remaining potential lifetime of a customer. Let $l_k$ be the number of customers who are at stage I in the potential lifetime of a certain product category and bought the company's products at the beginning of that period (i.e., $k$). That is, $l_{k-0.5}$

is the number of customers who buy the company's products during the middle of period $k$ for $k=1, 2, 3\ldots$ From an empirical perspective, the variable $l_k$ can be estimated from the company's past purchasing history for each period $k$, to represent the number of customers who buy the company's products. Therefore the probability that a customer will continue to purchase in the future is:

$$ {}_t p_k = \frac{l_{k+t}}{l_k} \tag{2} $$

We assume, without loss of generality, that customers' purchases are made at the middle of each time period. The probability that a customer who bought the company's products during period k will continue to buy from the company for t periods thereafter becomes:

$$ ({}_{t-0.5}\, p_k) = \frac{l_{k+t-0.5}}{l_k}\ . \tag{3} $$

We can, therefore, define CE using (3) by:

$$ CE = \sum_{k=1}^{n}\left[({}_t\, p_{k-0.5})\left(\frac{R_t - C_t}{(1+d)^{k-0.5}}\right)\right]. \tag{4} $$

where
CE – The NPV of all purchases from period 1 to n;
$R_t$ – The revenue from each customer during period t;
$C_k$ – The cost of serving a customer during period t;
$d$ – The discounting rate.

An important feature of this CE formulation is the consideration in each period of the probability that the customer will buy the company's products. Thus, we account for customer retention over time and resolve the limitation, inherent in most current CE models, of treating customers as active in all periods. Note that if we use ${}_t\, p_k=1$ (the probability that an active customer during time k will continue to purchase the company's products $t$ time units from time $k$ is equal to 1 in equation (4), we get the result in equation (1) – the basic CE model (i.e., no uncertainty). That is, this result is the one that is primarily used by current models that assume certainty with respect to future purchasing by customers. As can be seen, the current model allows for uncertainty about future purposes.

This model is similar in structure to that proposed by Berger & Nasr (1998). Our model is different, however, in the sense that we use an actuarial approach to determine the probability of a customer remaining active. The Berger-Nasr model uses a Markov chain with transition probabilities of a customer being active in the next period. The actuarial approach allows us to link the probability of a customer being active to his/her potential total consumption life span. For example, when considering future purchasing probabilities for a car, it is more accurate to relate this probability to the buyer's age, since the older the potential customer, the less likely he/she will remain active with the company.

To demonstrate the usefulness of this model, we use a numerical example from the insurance industry.

## Numerical Example - Model I

When an insurance company calculates the present value on life annuity of one dollar per year until the death of an insured, it must estimate the person's future lifetime. This is generally done by using demographic methods in which insurance companies track a certain age group, such as age x, in the population and calculate the number of individuals who will survive within the next t periods from age x. Then, by calculating the average number of individuals that will survive each year, it is possible to calculate the probability that individuals from this age group will survive within the next t periods. These calculations for different age groups are called "actuarial life tables" in the insurance industry. We can use these tables to calculate the present value of this life annuity.

We calculate the net present profit from an individual customer based on time periods of three, five, ten and thirty years to illustrate the variations in CE. We use the English lifetime table (Neill, 1977), also known as table A (67-70), which specifies the survival probabilities for each age group. This life table is commonly used in actuarial calculations. In general, for insurance or other business purposes, managers need to estimate the probability that a current active customer will continue to be active in the next time period.

For expository purposes, we assume the customer is 30 years old and that the profit margin, $(R_k- C_k)/ R_k$, in this industry is 30 percent ($R_k$ – the revenue from the customer in period $k$, $C_k$ – the total cost of generating revenue $R_k$ in period $k$). We also assume a yearly discount rate of 15 percent). This estimate is in line with other marketing studies that used discount rates in the range of 12 percent to 20 percent (e.g., Berger & Nasr 1998; Kim, Mahajan, & Srivastava, 1995).

In Table 1, we present the assessment of the CE resulting from the use of our actuarial model (i.e., equation 4) and the CE of the basic model (i.e., equation 1) using the A(67-70) lifetime table, a monetary value of 1 for revenues, and yearly time periods.

The results of this empirical illustration highlight some of the issues addressed by our model. The CE of the basic model is higher than the stochastic model, as the latter includes a certain probability that the customer will cease to be active. This probability increases as the active lifetime increases and the remaining potential lifetime of the customer decreases. That is, if the active period is 5 (30), the remaining potential period is large (small) and the probability that a customer will survive the next period is larger (smaller). This is reflected by the trend of increasing difference between the CE of the basic model, which is constant and does not take this issue into account, and stochastic CE as the active lifetime increases. These differences increase when the customer enters at a later stage in his life (i.e., 40 years) and the customer lifetime value is smaller compared with the same active time of an earlier entrance (i.e., 30 years).

In summary, the actuarial model accounts for the decreasing probability of a customer remaining active as the active lifetime increases and potential remaining lifetime decreases. Managerial implications that can be drawn from these results are that managers should be cautious in their evaluations of future income as current approaches inflate this value. Thus, an effective managing and monitoring of customer relationship can provide better understanding of the potential decrease in current customer activities that is independent of the firm activities (e.g., aging of customers). Furthermore, effective customer relationships can find new ways (e.g., offering new more suitable products) to maintain the current purchasing level of current customers in the future.

**Table 1:** *CE of 1 Monetary Unit for Different Ages and Remaining Lifetimes*

| | | Time | (years) | |
|---|---|---|---|---|
| Model Type | 3 | 5 | 10 | 30 |
| Basic CE Model | 0.79 | 1.16 | 1.73 | 2.27 |
| Stochastic CE Model (30 years) | 0.74 | 1.09 | 1.62 | 2.11 |
| Difference (Basic - Stochastic) | 6.8% | 6.4% | 6.8% | 7.6% |
| Stochastic CE Model (40 years) | 0.73 | 1.08 | 1.61 | 2.08 |
| Difference (Basic - Stochastic) | 6.8% | 7.4% | 7.5% | 9.1% |

Key:
Basic CE Model – based on equation 1.
Stochastic CE Model (30 years) – based on equation 4, starting age 30, and an active life span of 3, 5, 10, and 30 time periods.
Stochastic CE Model (40 years) – based on equation 4, starting age 40, and an active life span of 3, 5, 10, and 30 time periods.
Difference (Basic – Stochastic) - the percentage difference between the basic CE model and the stochastic model.

## Stage II – CE Model of Future Customers

In the second stage of our model, we capture the value of potential new customers. For instance, potential new customers might be attracted to the company because of its reputation. The better the company's reputation, the higher the arrival rate of new customers. The attraction of new customers can be described by a stochastic arrival process. These new customers, upon purchasing the company's products (i.e., arrival process), may develop loyalty and continue to purchase the company's products in the future, or for various reasons they may cease to be active customers (i.e., departure process). We are interested in modeling the steady state of these arrival and departure processes, since they capture the total value of the company's reputation as reflected in the CE. Recall that the value of the CE is the sum of the two stages we present: Stage I and stage II.

We use the $M / G / \infty$ model described earlier. In this general framework, we use a special feature that results from the Markov arrival Poisson process. Specifically, we use the Poisson Arrivals See Time Averages (PASTA) feature of the Poisson process, which allows us to evaluate overall CE at the time of customer departure. Given that

at this time we know the number of all remaining customers and their remaining active lifetimes with the firm, we can obtain a close form solution for the model. We therefore make the following assumptions:
(i) Poisson arrival process for new customers who purchase the company's products;
(ii) The remaining lifetime of the new customers has a general distribution (hence the symbol G after the first slash).

For the sake of simplicity, we consider a continuous formulation for this stage and not a discrete formulation, as in the first stage. We start with the basic structural model of CE (equation 1) and define the model's parameters in a continuous form to generate continuous cash flows (for further information on such a formulation see Berger & Nasr (1998), case 4).

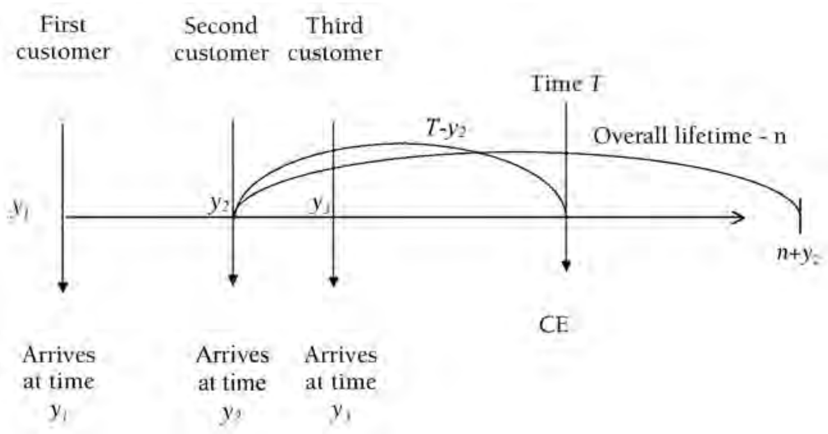
Model Assumptions

1) The arrival process $N_1 = \{N_1(t), t \geq 0\}$ is a Poisson process with a $\lambda$ rate.
   The parameter $\lambda$ is the average arrival number of customers purchasing the company's products; it is a deterministic parameter that is determined exogenously. From an empirical perspective, this parameter can be estimated using historical purchase data. Although this parameter can be modeled as a time-dependent variable, $\lambda$(t), which better captures the average arrival of customers, we use the deterministic formulation for parsimonious reasons.
2) The retaining process (i.e., the time span between the arrival and the departure of a potential customer) has a general distribution $G$, having with a mean expected value of $\frac{1}{\mu}$. The departure process can be defined by $N^2 = \{N_2(t), t \geq 0\}$ (i.e., the number of customers that has left the firm until time $t$). This is a non-homogeneous Poisson process with an intensity function of $\lambda(t) = \lambda G(t)$, as we do not know the distribution of customer departure, and therefore assumed a general distribution. (see, for example, Ross 1996, pp.78-82). Note that this type of departure process formulation ensures that each customer will remain active with the company for a different period.
3) To consider only the lifetime value of potential new customers, we assume that the system is empty at time $0$ (i.e., there are no customers who purchase the company's products).

We analyze this stochastic model by enabling the system to start acquiring customers. That is, new customers begin to purchase the company's products and remain active for a random period that is no greater than $n$ periods. One way to view the analysis of such a stochastic model is to consider the case of an inspector that samples the system randomly at time $T$ in the future. This sampling can result in observation of two types of consumers: those who purchased from the company before time T$T$ and ceased to be active (i.e., left the company) before the sampling took place, and those who purchased from the company before time $T$ and are still active with the company at time $T$. Since this stochastic process is used to analyze an established company, we are not interested in the first type of customer (i.e., not active at time $T$, as they will not contribute to the future cash flow of the company and hence will not contribute to the CE).

As aforementioned, one important feature of the PASTA $M / G / \infty$ model is its ability to evaluate the remaining customer lifetime with the company following the sampling time period. Once this remaining lifetime is established, we can calculate the CE by applying a discount factor to calculate the NPV at time *0*. In Figure 1, we illustrate this sampling process and the lifetime duration, or the retention process, of future customers.

**Figure 1:** *Stochastic retention process*




**Key:**
*y* – Random arrival time of a new customer.
*T* – Random time of the evaluation of the system (inspection) or equilibrium time.
*n* – Random remaining lifetime of a customer.

At time *T* at which the equilibrium is achieved, the second customer, for example, has actively purchased for a time frame of $T$- $y_2$ out of a total lifetime of *n*. We can obtain *n* through the properties of the equilibrium of the system. Recall that if *G* is the distribution of inter-arrival times, then the asymptotic (equilibrium) distribution of the remaining lifetime $G_e$ is given by:

$$P\left(X_e \leq x\right) = G_e^{(x)} = \mu \int_0^x \overline{G}(u)du,\ t \geq 0 . \qquad (5)$$

where

$P\left(X_e \leq x\right)$– The cumulative probability distribution that a customer will remain active until time *x*.

$G_e^{(x)}$ – The general distribution of a customer remaining active in equilibrium until time *t*.

$\mu = \dfrac{1}{E(G)}$ – The rate of customer departure

The PASTA model guarantees that when the sampling occurs at time $T=t$, where enough time has passed and equilibrium has been attained, the distribution of the remaining lifetime of a customer sampled randomly is $G_e$ . In other words, when the sampling occurs, the system is in a steady state (i.e., customer shopping behavior) that can either be busy (i.e., purchasing) or idle (i.e., not purchasing). The probability that a specific state will be sampled is equal to the fraction of time for which this process has been in this state. For further explanation, see Wolff (1989, p.78). In summary, the basic premise is that a new customer will remain active for a random lifetime, which resolves some of the limitations inherent in current CE models (i.e., it is based on a more realistic assumption than the use of a deterministic lifetime).

As noted earlier, we use a continuous formulation for CE in this model. We therefore use a continuous discount rate, $e^{-\delta}$, which corresponds to $\frac{1}{1+d}$ in equation 1, and obtain $\delta = ln(1 + d)$, the instantaneous interest rate (for more details see Berger & Nasr (1998) and Weston & Brigham (1993), p.233).

Let $CE_T y$ denote the conditioned CE of a customer who began to purchase the company's products at time y, calculated at time $T$. Therefore, based on the above discussion, we can compute the conditioned CE at time $T = t$, for an individual customer who became active at time y (i.e., assessment of CE is done $(t-y)$ time units after the customer has joined the system) by:

$$CE_t\, y = \left( \int_{z=0}^{n-t+y} \Big( (R_z - C_z) e^{-\delta z} \overline{G}_e(z) \Big) dz \right), \tag{6}$$

where

$R_z$ – The total revenue from a customer at time $z$;

$C_z$ – The total cost of generating the revenue from the customer at time $z$;

$e^{-\delta z}$ – The continuous discounting factor;

$\overline{G}_e(z)$ – The general distribution of a customer remaining active in equilibrium beyond time z.

By conditioning on the arrival process and considering that the arrival times given in the arrival process are distributed as the order statistic from a uniform distribution on $(0,t)$ (see Ross, 1996, p.67), we obtain the total financial CE of all the customers in the company's lifetime project:

$$\begin{aligned} CE_t &= \sum_{k=0}^{\infty} \left( k \int_{y=0}^{t} (CLV_t | y) \tfrac{1}{t}\, dy \right) \frac{e^{-\lambda t} (\lambda t)^k}{k!} \\ &= \lambda \int_{y=0}^{t} \left( \int_{z=0}^{n-t+y} \Big( (R_z - C_z) e^{-\delta z} \overline{G}_e(z) \Big) dz \right) dy \end{aligned} \tag{7}$$

where

$k$ – An index that accounts for the current number of customers in the system;

$\lambda$ – The rate of customer arrival.

This is an aggregate model, which is the outcome of the specific type of stochastic model used here. The model enables us to obtain the CE of all customers. This is somewhat different from current approaches, which sometimes use an individual-level analysis. The result presented in equation 7 is in the form of the future value of a cash flow stream. That is, it is the CE at a random time $T$ in the future. If, however, interest is in finding the NPV of the future cash flow, the result of equation 7 should be discounted as follows:

$$CE = CLV_T E(e^{-\delta T}) = E(e^{-\delta T})\lambda \int_{y=0}^{t} \left( \int_{z=0}^{n-T+y} \left( (R_z - C_z)e^{-\delta z}\overline{G}_e(z) \right) dz \right) dy \quad (8)$$

where $E$ denotes the expected value of a random variable and $E(e^{-\delta T})$ denotes the discount factor from time $T$ to the time of CE assessment.

This model can result in better understanding of the brand equity component in the customer equity conceptual framework proposed by Rust, Zeithaml & Lemon (2000). The distribution of remaining lifetime of future customers, $G_e$ , in equation 5, provides an assessment of the retention rate of customers due to the company's goodwill.

In practice, as $e^{-T}$ converges quickly to zero, one can safely use $T$ of 10 to 30 years as a base for CE assessment and still obtain a good approximation of true CE. In long-term equilibrium in economic theory, during the examination of a company's lifetime, it is sufficient to examine the system (i.e., equilibrium) after 10, 20, or 30 years.

## Numerical Example – Model 2

We continue with the numerical example from the life insurance industry. Suppose that $R_k$ is the revenue from a customer in period $I$, $C_k$ is the total cost of generating this revenue, and that the company has a 30 percent profit margin, similar to the previous numerical example. We do not consider specific acquisition costs here, but rather assume that these costs are reflected in the profit margin through a higher cost level – C¬k. We also consider three periods for the planning horizon, $T=10$, $T=20$, and $T=30$ in years. The yearly discount rate is again set to 15 percent, and the continuous discount rate is calculated as $e^{-\delta} = 0.13976$, given that $\delta = ln(1 + d)$. The parameter λ, which is the average annual arrival number of customers who purchase from the company per month, is taken to be λ=2000 new customers per year; the ages of the customers are thirty, forty, and fifty years old. The potential lifetime of customers is sixty-five, leaving variations in the entrance and the remaining times in the life cycle.

For the sake of convenience, and without loss of generality, suppose that the departure process is a Poisson process with $\mu_{age} = 2.8571 * 10^{-2}$ for all ages, meaning that $G_{e=e} = -2.8571 * 10^{-2z}$(Yosef, 2005). The CE using the stochastic approach is given in Table 2.

The calculations in Table 2 show the CE of potential new customers with probabilities of entering and leaving the company, and consideration of the entrance time (i.e., age) and the remaining time in the customer's life (i.e., 65). That is, a 30-year-old customer has a potential 35 additional periods in which to be active with the company, whereas a 50-year-old customer has a potential of 15 additional periods. As the potential end of the customer's lifetime grows near, the uncertainty of his retention increases and the value becomes lower than that of the 30-year-old customer. This can be seen in Table 2.

**Table 2:** *CE of New Customers Using Stochastic Arrival and Departure Processes for Different Entrance and Remaining Times in the Purchasing Cycle*

| Age | T=10 | T=20 | T=30 |
|---|---|---|---|
| 30 | 35,387 | 69,651 | 97,863 |
| 40 | 34,263 | 62,476 | |
| 50 | 28,213 | | |

## Discussion

The evaluation of CE in marketing is becoming increasingly important. This may be attributed to several factors, including better availability of data due to improvements in information technology, as well as the understanding that the value of a company is directly related to the value of its customers. Thus, there is a clear relationship between effective customer management and a company's financial performance. The literature in this area has attempted to address a variety of related issues that include models of CE calculations. While these models provide both theoretical and practical advancements in better understanding the assessment of CE, they remain somewhat limited. Our model represents a step in resolving some of these limitations. Specifically, we introduce a two-stage model that enables us to calculate the overall value of both current and future customers. The first stage models the value of current customers and allows for customer departure from the company. The second model handles only the value of future customers and allows for random retention rate. That is, we resolve the issue of deterministic lifetime in most current models. Instead of treating a customer as active throughout the planning horizon, we allow for a more realistic structure that considers customers who have ceased to be active. Another feature of our model, which is not fully considered in current models, is the uncertainty associated with the remaining lifetime of current customers. Given that many product types in numerous product categories are manufactured by different companies, and a variety of product types can be used by consumers in the same product category throughout their lives in that category, customers will switch companies because of the availability of a product type and not because of product and service quality. This switching, or non-loyal behavior, will occur due to changing customer needs. These changes will reduce the retention of current customers and produce uncertainty as to the exact time of departure. As the customer approaches some average of the end of such a stage in his/her life, the probability of retention decreases. We use actuarial calculations to account for such decreasing probabilities, resulting in a more realistic model for CE.

We use numerical examples to demonstrate the ease of use of our CE model and compare the results with those of current common CE models. Using this example we show the usefulness of this model in getting predictions of both current and future CE.

Several managerial implications can be inferred from our model. Marketers can identify the value of customers during various stages of the customer life cycle. This evaluation can serve as a basis for several important strategic marketing decisions. First, segmentation schemes based on the variations in customer value across demographic or psychographic variables can be developed to fully maximize a company's allocation of resources. Second, the company can manage its customer base more effectively throughout the customer life cycle by expecting the customer's departure to a different product type. Third, the company can develop a better understanding of the variety and mix of product types in the product category needed to serve customers throughout their lives. Finally, the company can better prepare for the expected changes in the demand for its product types and avoid surplus or shortage of products. Our model also leads to the realization of the value of young customers to the company. Fourth, the value of current customers evaluated under most of current CE modeling approaches tend to be inflated as these models do not adequately account for the dynamic nature of customer purchasing behavior. Furthermore, ignoring the potential increase in future revenues from new potential customers deflates the CE. Thus, careful considerations need to be made by managers to account for these estimations in order to avoid sub-optimal resource allocations. Managers can also identify those customers that might reach a certain stage in their life cycle with the company where they decrease, or even stop, their purchases. Developing targeted marketing mixes for such customers might rejuvenate their activity with the firm.

The current model can be further developed. We assumed independence between customer arrivals (i.e., no word-of-mouth effect). This issue can be further expanded to allow for such effects by developing a functional form for the average number of customers arriving over time.

Another issue that can be further developed is the common use of a deterministic interest rate in CE models, described in Jain & Singh's (2002) review. Acknowledging that future interest rates are important in long-term calculations and the uncertainty surrounding these rates in world markets will have a significant impact on CE calculations. Allowing for stochastic interest rates in the basic structural CE model can resolve such limitations.

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