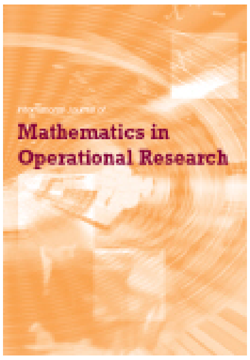

International Journal of Mathematics in Operational Research

ISSN online: 1757-5869 - ISSN print: 1757-5850
https://www.inderscience.com/ijmor

Optimal order quantity and credit period for a time-dependent deteriorated green item under two-level trade credit with reminder cost: a new approach

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DOI: 10.1504/IJMOR.2022.10053351

Article History:

Received:	24 June 2022
Accepted:	04 July 2022
Published online:	23 February 2024

Optimal order quantity and credit period for a time-dependent deteriorated green item under two-level trade credit with reminder cost: a new approach

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Abstract: Nowadays, people prefer to buy green products. In the present competitive market, the wholesalers and retailers offer trade credit to their downstream customers. Against retailers' offers, some customers become defaulters. In rural India, money lenders engage some people to remind the defaulters for payment. Following this idea, we introduce reminder costs in trade-credited systems to reduce default risk, though this system negatively affects the demand. Along with the conventional approach, a new approach for a two-level trade-credited EOQ model with a time-dependent deteriorated green item is presented, solving default risk and reminder cost by using the generalised reduced gradient method through LINGO 19.0. The new approach gives better results than the conventional ones, and in both approaches, the introduction of reminder cost gives more profit. The nature of profit and its dissection concerning decision variables are presented. Some managerial decisions are derived.

Keywords: inventory; two-level trade credit; deterioration; default risk; reminder cost; new approach.

Reference to this paper should be made as follows: Kumar, G., Bera, S., Samanta, G. and Maiti, M. (2024) 'Optimal order quantity and credit period for a time-dependent deteriorated green item under two-level trade credit with reminder cost: a new approach', *Int. J. Mathematics in Operational Research*, Vol. 27, No. 1, pp.75–104.

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1 Introduction

This study is based on green or eco-friendly products used in daily life. These products have a limited lifetime. Green items play an essential role in developing a pollution-free environment. Manufacturing companies in the USA, Japan, and other European countries are increasingly concerned about environmental issues as a result of legislation and preferences from customers (cf. Seman et al., 2012; Ninlawan et al., 2010). A green product is a maintainable product that minimises ecological effects during the whole life cycle and even after no use. Nowadays, worldwide, most businesses start with green products for environmental sustainability.

In traditional economic order quantity, it is assumed that a retailer pays the total purchasing cost at the time of purchase/receipt. This acts as a financial constraint for the retailers. In a trade credited system, if a wholesaler provides a trade credit to a retailer, the retailer does not pay at the time of purchase. He pays the amount at the end of the credit period, which helps the retailer to acquire more business amounts. In this system, if the retailer fails to clear all dues at the end of the credit period, the wholesaler charges a high-interest rate on the dues till it is cleared.

In a two-level trade-credited system, like the wholesaler's credit period, the retailer also gives trade credit to the customer to settle the account, which helps the customers to purchase more. In this case, customer demand increases with the retailer's trade credit period.

There is a disadvantage to the above two-level trade credit system. When the retailer gives the credit period to the customers, some customers enjoy the credit period but do not pay back the due amount. Some customers become defaulters, which brings down the retailer's sales revenue/profit. In the literature, there are several investigations on the two-level trade credited inventory model with default risks for perishable items (Shi and Zhang, 2010; Lee and Rhee, 2011; Lou and Wang, 2013; Wang et al., 2014), etc.

But, till now, none thought of how to reduce the defaulters? Though a retailer's trade credit increases the customer's demand, it also increases the default risk. Now, the question is how to control this risk? In rural India, money lenders use some persons to chase their default customers. Mimicking this system, we have suggested the introduction of a system that reminds the default customer to pay and the retailer incurs some extra cost for this purpose. This additional expenditure is termed as a reminder cost. The system of reminding the customers will also have some adverse effects. Some customers willing to enjoy credit periods may not like this type of reminder system. They become psychologically afraid of chasing by the retailer, so they try to avoid the retailer, and as a result, demand decreases. Thus demand decreases with the increase in reminder level. None attempted to solve this type of problem for the retailer.

Again several Wu et al. (2014) and Mahata et al. (2020) made a mistake evaluating earned interest and interest paid, not taking default risk and item's deterioration effect, respectively. In this investigation, we have corrected these mistakes.

In the conventional approach of trade-credited inventory model formulations (Shah and Cárdenas-Barrón, 2015), some assumptions like "at the end of the credit period, out of the sales revenue, only the purchased price of the sold items is paid to the wholesaler, the rest amount is kept in hand for other expenses", etc. invite several questions such as – what is the amount of other expenses?, is the amount kept in hand sufficient?, etc. (Majumder et al., 2016) tried to remove these questions partly. Here, we suggest a new approach for the trade-credited inventory models, considering the present banking interest system.

In this study, we formulate a mathematical model using the reminder level concept for the first time in two different (conventional and new) approaches. In both formulations, we consider three different scenarios based on trade credit. The conventional approach is the one (Wu et al., 2014) where the retailer pays the dues and interest charges on the unsold items at the end of the business period. In the new approach, the retailer pays the whole due amount to the wholesaler at the end of the credit period by borrowing a loan from a bank because the bank interest rate is normally less than the wholesaler's interest. The retailer pays back the loan amount to the bank in fixed installments between the credit period and the end of the business cycle. In both approaches, default risk and reminder cost are introduced. We evaluated the maximum profit for the retailer by formulating the model as mixed-integer nonlinear programming problems and solved using the generalised reduced gradient (GRG) method through LINGO software. In both approaches, the introduction of reminder cost fetches more profit, and the new approach is better than the conventional one. Trade-offs between 'profits and retailer's trade credit', 'profits and time period', 'profits and reminder level' and 'profits and greening level' are demonstrated. Some managerial decisions are presented.

This investigation introduces two major novel ideas for two-level trade credited inventory models.

1 We present the concept of reminder cost to reduce the default risk, i.e., the number of default customers and its negative effect on demand. Incorporating this, the models are solved through two approaches. As an example, the two-level trade credited inventory model of Wu et al. (2014) is solved by correcting the mistakes.

2 A new payment policy (new approach) for the retailer is proposed taking the present bank loan system into account. Here, at the end of the credit period, the retailer clears the wholesaler's dues and ordering cost (with interest) by taking a loan from a bank. The retailer clears the bank loan at some fixed installments from the sales revenue and earned interest on it at the earliest possible time for maximum profit.

2 Literature review

In the 21st century, the biggest issue for the environment and human life is pollution. Due to government and NGO campaigns, people are conscious and prefer to purchase green products. There are several articles available on the green supply chain inventory model. Recently, Paul et al. (2022) formulated an inventory model with retail investments in green operations in which the demand rate linearly depends on the green level. Hakim et al. (2022) investigated an inventory model for degrading commodities with nonlinear green level-dependent demand.

Almost in every type of business, competition is increasing day by day. So for a growing market, the entities in the supply chain (supplier, manufacturer, wholesaler, retailer, etc.) adopt some promotional activities to increase sales and total profit. Trade credit is an attractive promotional activity that increases sales and profit. For deteriorating items, Chang et al. (2003) developed an EOQ model, in which the supplier permits the customers an acceptable delay if the order quantity is more than or equal to a predetermined amount. In that study, demand is time-dependent, which may be constant or linear, while deterioration is also time-dependent. Chang et al. (2010) proposed the optimal replenishment policies for manufacturers in the supply chain for deteriorating items using downstream and upstream trade credits. They have considered both demand and deterioration are constant and proposed the arithmetic-geometric inequality method to find the optimal solution when the deterioration rate is sufficiently small. Stock dependent demand inventory model under two-level trade credit was developed by Teng et al. (2011), where the deterioration rate is constant and the demand rate is a function of inventory level. Later, Teng et al. (2012) extended the constant demand to a time-dependent non-decreasing linear demand function where the supplier offered a fixed credit period to the retailer and used the bisection method for the solution. Taleizadeh et al. (2013) considered an EOQ problem in the context of partial delayed payment in which the retailer must pay a proportion of the purchase price at the beginning of the credit period, and later the remaining amount is paid. There are many authors (Mahata and Goswami, 2007; De and Goswami, 2009; Mohanty et al., 2018; Pakhira et al., 2018; Pramanik and Maiti, 2019; Zhang et al., 2021) have formulated trade-credit models under different scenarios with different demand and deterioration functions.

Table 1 Features of some trade credited inventory models

References	*Two-level trade credit*	*Default risk*	*Deterioration*	*Reminder cost*	*Formulation approach*
Liao (2008)	✓	–	ct	–	Cov
Teng (2009)	✓	–	–	–	Cov
Chang et al. (2010)	✓	–	ct	–	Cov
Teng et al. (2011)	–	–	ct	–	Cov
Teng et al. (2012)	✓	✓	–	–	Cov
Lou and Wang (2013)	–	✓	–	–	Cov
Teng et al. (2013)	✓	–	–	–	Cov
Wang et al. (2014)	✓	✓	td	–	Cov
Wu et al. (2014)	✓	✓	td	–	Cov
Shah and Cárdenas-Barrón (2015)	✓	✓	ct	–	Cov
Pakhira et al. (2018)	✓	–	–	–	Cov
Zhang et al. (2018)	✓	✓	–	–	Cov
Mahata et al. (2018)	✓	✓	–	–	Cov
Tsao (2018)	–	✓	–	–	Cov
Pramanik and Maiti (2019)	✓	–	td	–	Cov
Molamohamadi et al. (2020)	✓	–	ct	–	Cov
This paper	✓	✓	td	✓	Cov and new

Notes: ct: constant, td: time-dependent, and Cov: conventional.

Retailers' trade credit has many advantages; it attracts new customers and increases profit, but it has some disadvantages also. Because of the credit period, there is a risk in payment by the customers. Some customers do not pay at all, and the retailer's profit is decreased. The default risk or default customer increases with the credit period. Shi and Zhang (2010) investigated the best trade credit period decision using an expanded EOQ model with a default risk component with constant demand. Lee and Rhee (2011) tried to shed light on trade credit from a seller's viewpoint and represented it as a technique for supply chain management under default risk, where demand is uniformly distributed and default risk is constant. Lou and Wang (2013) proposed a two-level trade credit inventory model under default risk. In that study, demand is a positive exponential function of the credit period; default risk is an increasing function of the credit period. Wang et al. (2014) developed an EOQ model for a seller based on the assumption that the credit period increases both demand and default risk. The authors considered that the deteriorating products deteriorate continuously and have a maximum lifetime where demand and default risk are the same as Lou and Wang (2013) with time-dependent deterioration. Wu et al. (2017) developed a retailer-supplier non-cooperative replenishment model wherein demand and default risk are the same as Lou and Wang (2013), Wang et al. (2014) to determine the best trade credit period in a supplier-Stackelberg game. Wang et al. (2018) described three main strategies used by suppliers to resolve credit default issues, screening, checking, and insurance. Tsao (2018) addressed the optimal credit period and replenish options where the credit period increases both demand and default risk.

None of the above studies of two-level trade credited models thought of how to control the default risk. Moreover, the above formulation of models is biased by some unrealistic assumptions. For example, why should we only clear the wholesaler's dues at the end of the business period? If a sufficient amount is available with the retailer, he should clear the dues as soon as the wholesaler's interest is normally higher than the earned interest. Moreover, interest on bank loans nowadays is substantially lower than the other charged interests. This financial scenario may be considered in formulating two-level trade credit models. In this study, an attempt has been made to answer the above questions and formulate the two-level trade credited models with the present banking system. However, Majumder et al. (2016) presented payment policies, different from the conventional one.

In this paper, some two-level trade credited inventory models with credit-dependent demand, default risk, and reminder level are formulated following conventional and new approaches. Here a wholesaler sells a perishable item to a retailer on a trade credit basis, and in return, the retailer also offers trade credit to the customers. Due to retailers' trade credit, some customers become defaulters. Customer's demand increases with trade credit, and default risk also increases as demand increases. To reduce default risk, the concept of reminder level is introduced. The model is formulated by both conventional and new approaches and solved by the generalised reduced gradient (GRG) method through LINGO 19.0 software. The model is numerically illustrated. The introduction of reminder cost fetches more profit in both approaches. Again, the new approach gives better results than the conventional ones. Several trade-offs between the profit and decision variables are presented numerically. Some managerial decisions are made.

3 Notations

The following notations are used in the mathematical formulations:

- C_o: Retailer's per-order set-up costs in \$.
- C_o': Retailer's per-order set-up costs in \$ with interest.
- C_p: Retailer's per-unit purchasing cost in \$.
- C_s: Retailer's per-unit selling price in \$.
- h: Retailer's annual holding cost (HC) per unit in \$ (excluding interest charge).
- I_e: Retailer's annual earned interest rate in \$ year in \$; with $I_c > I_e$.
- I_b: Bank interest rate per year in \$.
- r_c: Reminder level.

- θ: Greening level.
- β: Rate of default risk.
- S: Wholesaler's trade credit duration for the retailer in years.
- R: Retailer's trade credit duration for the customer in years.
- T: Optimal cycle length in years.
- $\lambda(t)$: Time dependent deterioration rate.
- $I(t)$: Inventory level at unit time t.
- Q: Retailer's order quantity.
- $D(R, r_c, \theta)$: Annual demand of the market.
- r: Annual compound interest per \$.
- n: Number of instalment in which retailer's paying loan amount.

4 Assumptions

The mathematical model is formulated under the following assumption:

1. This investigation is limited to a single-supplier and single-retailer scenario for a single deteriorating product. As deterioration is time-dependent, this investigation is only for a single period.
2. The wholesaler gives the fixed credit period S to the retailer to settle the account, and subsequently, the retailer gives credit period R to the customer.
3. Let θ and r_c be the greening and reminder levels, respectively. Then demand is a function of customers credit period R, greening level (θ) and greenness level (r_c)

$$D(R, r_c, \theta) = Ke^{aR - a_1 r_c} + K_0\big(1 - e^{-k\theta}\big) \tag{1}$$

   where K, a, a_1, K_0, and k all are positive constant. If $\theta = 0$ and $r_c = 0$, then demand function is reduced to Teng and Lou (2012), Chern et al. (2013), Wu et al. (2014) and Mahata et al. (2020). From equation (1) demand is a increasing function of customer credit period R and greening level θ but decreasing w.r.t. r_c, reason is discussed letter on.
4. Expiration dates can be found on all decaying objects. As a result, as time approaches the expiry date m, the deterioration rate must approach to 1. We suppose the same deterioration rate as Sarkar (2012), Wang et al. (2014), Wu and Chan (2014), Wu et al. (2014), Chen and Teng (2014), Mahata (2015), Sarkar et al. (2015):

$$\lambda(t) = \frac{1}{m - t + 1}, \quad 0 \leq t \leq T \leq m. \tag{2}$$

5 If longer credit period is offered to customers, some customer are defaulters. To reduced default customer, the retailer adopts a strategy like sending a people and reminding all customers. Therefore, expression of the default risk is

$$F(R, r_c) = 1 - e^{a_2 r_c - bR}, \quad a_2 r_c - bR \leq 0 \tag{3}$$

where a_2 and b are positive constants. It is clear from equation (3), $F(R,\, r_c)$ is increasing function in R and decreasing function with r_c. If $r_c = 0$, then expression reduced to Teng and Lou (2012), Chern et al. (2013), Wu et al. (2014), Shah and Cárdenas-Barrón (2015) and Mahata et al. (2020).

6 If the retailer adopts strategy as mentioned in Assumption 5 to reduced default customers, and expending some extra cost on it which is termed as 'reminder cost' and given as

$$RC = \rho r_c^2. \tag{4}$$

where ρ is a positive constant and r_c is reminder level.

7 To maintain greenness, the retailer incurs some extra cost (greening cost, say), which is function of greenness level θ and is given as (Ghosh and Shah, 2012; Sinayi and Rasti-Barzoki, 2018).

$$GC = \eta \theta^2. \tag{5}$$

where η is a positive constant and θ is greening level.

8 Following Wu et al. (2014) and Mahata et al. (2020), net revenue of the retailer's after opportunity cost and default risk is

$$\begin{aligned} &C_s D(R, r_c, \theta)(1 - F(R, r_c))e^{-rR} \\ &= C_s\Big(Ke^{aR - a_1 r_c} + K_0\big(1 - e^{-k\theta}\big)\Big)e^{a_2 r_c - (b+r)R} \end{aligned} \tag{6}$$

9 We might presume that in today's time-based economy, shortages are not permitted.

10 In the payment policies [conventional approach (Shah and Cárdenas-Barrón, 2015)], the retailer will have to pay the dues to the wholesaler at S and if he does not have the sufficient amount to clear the dues then the retailer pays interest at the rate I_c on the unsold item.

11 In new payment policies (Majumder et al., 2016), the retailer pays the sales revenue and earned interest on it from 0 to $S - R$ period and clears the due amount with interest at a time (within the business cycle) when retailers have the sufficient sold revenue and interest earned on it. We assumed that the retailer clears the dues of the wholesaler at the end of the credit period S taking a loan from a bank. In addition to the sales revenue for the period 0 to $S - R$, he takes the required amount from the bank for this purpose. The wholesaler interest charge is more than the bank interest on the borrowed loan. The retailer pays the loan amount to the bank at a fixed number of installments between the offered credit period by the wholesaler and the end of the business cycle when retailers will have sufficient sold revenue and interest earned on it.

12 The replenishment time is infinite and lead-time is negligible.

5 Mathematical formulation

The inventory level is depleted by demand and deterioration during the replenishment cycle [0, T], and is thus governed by the differential equation:

$$\frac{dI(t)}{dt} = -D(R, r_c, \theta) - \lambda(t)I(t), \quad 0 \le t \le T, \tag{7}$$

with $I(T)$ = 0 as the boundary condition, we solve differential equation (7), we get

$$I(t) = e^{\zeta(t)} \int_t^T e^{\zeta(x)} D(R, r_c, \theta) dx, \quad 0 \le t \le T, \tag{8}$$

where

$$\begin{aligned} \zeta(t) &= \int_0^t \lambda(x)dx = \int_0^t \frac{1}{1-x+m} dx \\ &= \big[-\ln(1-x+m)\big]_0^t = \ln\Big(\frac{m+1}{1-t+m}\Big) \end{aligned} \tag{9}$$

Using equation (9) in equation (8), at time t, we have the inventory level as

$$\begin{aligned} I(t) &= D(R, r_c, \theta)\Big(\frac{1-t+m}{m+1}\Big) \int_t^T \frac{m+1}{m+1-x} dx \\ &= D(R, r_c, \theta)(1-t+m) \ln\Big(\frac{1-t+m}{1-T+m}\Big), \quad 0 \le t \le T. \end{aligned} \tag{10}$$

Therefore, the retailers purchased quantity

$$Q = I(0) = D(R, r_c, \theta)(m+1) \ln\Big(\frac{m+1}{1-T+m}\Big). \tag{11}$$

So, purchasing price is

$$PC = C_p\Big(Ke^{aR-a_1 r_c} + K_0\big(1-e^{-k\theta}\big)\Big)(m+1) \ln\Big(\frac{m+1}{1-T+m}\Big) \tag{12}$$

and the retailers HC

$$\begin{aligned} HC &= h\int_0^T I(t)dt = hD(R, r_c, \theta) \int_0^T (1-t+m) \ln\Big(\frac{1-t+m}{1-T+m}\Big) dt \\ &= h\Big(Ke^{aR-a_1 r_c} + K_0\big(1-e^{-k\theta}\big)\Big)\Bigg[\frac{(m+1)^2}{2} \ln\Big(\frac{m+1}{1-T+m}\Big) \\ &\quad - \frac{(m+1)T}{2} + \frac{T^2}{4}\Bigg]. \end{aligned} \tag{13}$$

Now we formulate the mathematical model under three different cases as discussed in Shah and Cárdenas-Barrón (2015) and in Wu et al. (2014) which are similar to $R \le S \le T$, $R \le T \le S$ and $S \le R \le T$.

5.1 Scenario-1: $R \le S \le T$

5.1.1 Conventional approach

In this scenario, the retailer has a credit period S to clear all dues. Here, the retailer gets the selling price at R from the first customer and receives the revenue at $T+R$ from the last customer. Since $R \le S$, the retailer does not have a sufficient amount to pay all purchasing prices to the wholesaler, and therefore, he pays only the received amount on sold items and earned interest on it from 0 to $S-R$. Later, he pays high interest on unsold items to the wholesaler at the end of the business period and the purchasing cost as and when the item is sold. The following equation obtains an interest charge (IC) per unit of time

$$\begin{aligned} IC &= \frac{C_p I_c}{T}\int_S^{T+R} I(t-R)dt \\ &= \frac{C_p I_c D(R,r_c,\theta)}{2T}\Big[(m+R-S+1)^2 \ln\Big(\frac{m+R-S+1}{m-T+1}\Big) \\ &+ \frac{(m-T+1)^2-(m+R-S+1)^2}{2}\Big]. \end{aligned} \tag{14}$$

In this system, at time R, the retailer receives the payment of the item which he sells at time $t=0$. As a result, from R to S, the retailer accumulates revenue in an account that earns interest at the rate of I_e per year. This expression is

$$\begin{aligned} IE &= \frac{C_s I_e}{T}(1-F(R,r_c))\int_R^S D(R,r_c,\theta)(t-R)dt \\ &= \frac{C_s I_e D(R,r_c,\theta)e^{a_2 r_c - bR}}{2T}(S-R)^2. \end{aligned} \tag{15}$$

Therefore, the retailer total annual profit $TP_{c_1}(R,T,r_c,\theta)$ = net annual sales revenue + earned interest – purchasing cost per unit time – annual HC – interest charged – ordering cost per unit time – reminder cost per unit time – greening cost per unit time, i.e.,

$$\begin{aligned} TP_{c_1}(R,T,r_c,\theta) = D(R,r_c,\theta)\Big[&C_s e^{a_2 r_c-(b+r)R} \\ &+ \frac{C_s I_e e^{a_2 r_c - bR}}{2T}(S-R)^2 - \frac{C_p}{T}(m+1)\ln\Big(\frac{m+1}{1-T+m}\Big) \\ &- \frac{h}{T}\Big\{\frac{(m+1)^2}{2}\ln\Big(\frac{m+1}{1-T+m}\Big) - \frac{(m+1)T}{2} + \frac{T^2}{4}\Big\} \\ &- \frac{C_p I_c}{2T}\Big\{(m+R-S+1)^2\ln\Big(\frac{m+R-S+1}{m-T+1}\Big) \\ &+ \frac{(m-T+1)^2-(m+R-S+1)^2}{2}\Big\}\Big] \\ &- \frac{C_o+\rho r_c^2+\eta\theta^2}{T}. \end{aligned} \tag{16}$$

5.1.2 New approach

In this approach, we assumed that the wholesaler's interest charged is more than the bank interest charged on the borrowing loan. Here the retailer takes a loan from a bank and clears all dues, including ordering cost at the end of the offered credit period S. Since $R \leq S$, so retailer have some sales revenue. Therefore, loan amount LA (say) is $PC + OC'$ – sales revenue and interest earned on it from 0 to S. The retailer clears all loan amounts in a fixed number of installments at $t = t'$ (to be determined) between S and the end of the business cycle.

Since $R \leq S$, therefore retailers earns earned interest from R to S, i.e., at S, on the sold amount from 0 to $S - R$, total amount in retailer's hand is TA (say)

$$\begin{aligned} TA &= C_s D(R, r_c, \theta)\big(1 - F(R, r_c)\left[(S-R) + I_e \int_R^S (t-R)dt\right] \\ &= C_s D(R, r_c, \theta) e^{a_2 r_c - bR}(S-R)\left[1 + I_e \frac{S-R}{2}\right]. \end{aligned}$$

The retailer clears all dues at S but have only TA, so remaining amount borrow from bank at compound interest rate I_b. Total loan amount LA, is

$$\begin{aligned} LA = D(R, r_c, \theta)&\left[C_p(m+1)\ln\left(\frac{m+1}{1-T+m}\right)\right. \\ &\left. - C_s e^{a_2 r_c - bR}(S-R)\left(1 + I_e \frac{S-R}{2}\right)\right] + C_o. \end{aligned} \tag{17}$$

Suppose, the retailer pays this loan amount to the bank in fixed number of instalment (n, say) at $t = t'$, $S \leq t' \leq T + R$. Payable amount in each instalment is EPI (say) (cf. http://www.paisabazaar.com/emi-calculator),

$$EPI = LA \frac{I_b'\big(I_b' + 1\big)^n}{\big(I_b' + 1\big)^n - 1}, \text{ where } I_b' = \frac{I_b(t'-S)}{n}. \tag{18}$$

Received amount on sold items from $S - R$ to t'-R including the earned interest on it from S to t' (RA_1, say)

$$\begin{aligned} RA_1 &= C_s D(R, r_c, \theta)\big(1 - F(R, r_c)\big)\left[(t'-S) + nI_e \int_S^{S + \frac{t'-S}{n}} (t-S)dt\right] \\ &= C_s D(R, r_c, \theta)(t'-S) e^{a_2 r_c - bR}\left[1 + I_e \frac{t'-S}{2n}\right] \end{aligned}$$

where t' satisfy the following equation

$$n \times EPI = RA_1 \text{ or } EPI = \frac{RA_1}{n}. \tag{19}$$

Total received amount on sold items from $t' - R$ to T and earned interest on it from t' to $T + R$ (RA_2, say)

$$RA_2 C_s D(R, r_c, \theta)\big(1 - F(R, r_c)\big)\left[(T - t' + R) + I_e \int_{t'}^{T+R} (t - t')dt\right]$$
$$= C_s D(R, r_c, \theta)(T - t' + R)e^{a_2 r_c - bR}\left[1 + I_e \frac{T - t' + R}{2}\right].$$

Therefore, the retailer's total profit amount per unit time,

$$\begin{aligned} TP_{n_1}(R, T, r_c, \theta) &= (RA_2 - HC - RC - GC)/T \\ &= \frac{D(R, r_c, \theta)}{T}\left[C_s e^{a_2 r_c - bR}(T - t' + R)(1 \right. \\ &\quad + I_e \frac{T - t' + R}{2}\bigg) - h\left(\frac{(m+1)^2}{2} \ln\left(\frac{m+1}{1 - T + m}\right) \right. \\ &\quad \left.\left. - \frac{(m+1)T}{2} + \frac{T^2}{4}\right)\right] - \frac{\rho r_c^2 + \eta\theta^2}{T}. \end{aligned} \tag{20}$$

5.2 *Scenario-2: $R \le T \le S$ and $R + T \le S$*

5.2.1 *Conventional approach*

In this scenario, the retailer receives all revenue at S, $S \ge T + R$. Therefore, the interest charge is zero, but the total earned interest at the rate of I_e per year is,

$$\begin{aligned} IE_2 &= \frac{C_s I_e}{T}(1 - F(R, r_c)) \int_0^T D(R, r_c, \theta) t dt \\ &\quad + C_s I_e (1 - F(R, r_c)) D(R, r_c, \theta)(S - T - R) \\ &= \frac{C_s D(R, r_c, \theta) e^{a_2 r_c - bR} I_e}{2}(2S - 2R - T) \end{aligned} \tag{21}$$

Therefore, the retailer's annual total profit function can be expressed as $TP_{c_2}(R, T, r_c, \theta)$ = net annual sells revenue + earned interest – purchasing cost per unit time – annual HC – ordering cost per unit time – reminder cost per unit time – greening cost per unit time, i.e.,

$$\begin{aligned} TP_{c_2}(R, T, r_c, \theta) &= D(R, r_c, \theta)\Big[C_s e^{a_2 r_c - (b+r)R} \\ &\quad + C_s I_e e^{a_2 r_c - bR}\left(S - R - \frac{T}{2}\right) \\ &\quad - \frac{C_p}{T}(m+1) \ln\left(\frac{m+1}{1 - T + m}\right) \\ &\quad - \frac{h}{T}\left(\frac{(m+1)^2}{2} \ln\left(\frac{m+1}{1 - T + m}\right) - \frac{(m+1)T}{2} + \frac{T^2}{4}\right)\Big] \\ &\quad - \frac{C_o + \rho r_c^2 + \eta\theta^2}{T}. \end{aligned} \tag{22}$$

5.2.2 New approach

As the retailer sells all items within the credit period, the question of taking a bank loan to clear the wholesaler's dues does not arise, so, in this case, TP_{n_2} will be the same expression with C_o being replaced by C_o'.

$$\begin{aligned}TP_{n_2}(R,T,r_c,\theta) = D(R,r_c,\theta)\Big[&C_s e^{a_2 r_c-(b+r)R}\\&+ C_s I_e e^{a_2 r_c - bR}\Big(S-R-\frac{T}{2}\Big)\\&- \frac{C_p}{T}(m+1)\ln\Big(\frac{m+1}{1-T+m}\Big)\\&- \frac{h}{T}\Big(\frac{(m+1)^2}{2}\ln\Big(\frac{m+1}{1-T+m}\Big)-\frac{(m+1)T}{2}+\frac{T^2}{4}\Big)\Big]\\&- \frac{C_o'+\rho r_c^2+\eta\theta^2}{T}.\end{aligned} \tag{23}$$

5.3 Scenario-3: $S \le R \le T$ or $S \le T \le R$

5.3.1 Conventional approach

In this case, there is no earned interest as $S \le R$. Moreover, the retailer has to pay an interest charge (IC_1, say) on the full purchasing cost. The interest charge per unit time can be calculated as

$$\begin{aligned}IC_1 &= \frac{C_p I_c}{T}\Big[D(R,r_c,\theta)(m+1)\ln\Big(\frac{m+1}{1-T+m}\Big)T(R-S)+\int_0^T I(t)dt\Big]\\&= C_p I_c D(R,r_c,\theta)\Big[\ln\Big(\frac{m+1}{1-T+m}\Big)\Big\{(m+1)(R-S)+\frac{(m+1)^2}{2T}\Big\}\\&\quad -\frac{(m+1)}{2}+\frac{T}{4}\Big].\end{aligned} \tag{24}$$

Therefore, the retailer's annual total profit per year can be expressed as $TP_{c_2}(R,T,r_c,\theta)$ = net annual sells revenue – purchasing cost per unit time – annual HC – interest charged per year – ordering cost per unit time – reminder cost per unit time – greening cost per unit time, i.e.,

$$\begin{aligned}TP_{c3}(R,T,r_c,\theta) = D(R,r_c,\theta)\Big[&C_s e^{a_2 r_c-(b+r)R}\\&- \frac{C_p}{T}(m+1)\ln\Big(\frac{m+1}{1-T+m}\Big)\\&- C_p I_c (m+1)\ln\Big(\frac{m+1}{1-T+m}\Big)(R-S)\\&- \frac{(h+C_p I_c)}{T}\Big\{\frac{(m+1)^2}{2}\ln\Big(\frac{m+1}{1-T+m}\Big)\\&- \frac{(m+1)T}{2}+\frac{T^2}{4}\Big\}\Big] - \frac{C_o+\rho r_c^2+\eta\theta^2}{T}.\end{aligned} \tag{25}$$

5.3.2 New approach

As $R \geq S$, retailer has no revenue at S. So, He takes loan LA' (say) = PC + OC' from a bank at compound interest rate I_b and pay back at t' (to be determined) in n instalments. Payable amount in each instalment, EPI' (say) (cf. http://www.paisabazaar.com/emi-calculator)

$$EPI' = LA' \frac{I'_b\left(I'_b + 1\right)^n}{\left(I'_b + 1\right)^n - 1}, \text{ where } I'_b = \frac{I_b(t' - S)}{n}. \tag{26}$$

Received amount on sold items from 0 to $t' - R$ and interest earned from R to t' (RA_3, say) is

$$\begin{aligned} RA_3 &= C_s D(R, r_c, \theta)(1 - d(R, r_c)) \left[(t' - R) + n_1 I_e \int_R^{\frac{t'-S}{n}+S} (t - N) dt \right] \\ &= C_s D(R, r_c, \theta) e^{a_2 r_c - bR} \left[(t' - R) + \frac{n I_c}{2} \left(\frac{t' - S}{n} - R + S \right)^2 \right] \end{aligned}$$

So, t' is given by

$$n \times EPI' = RA_3 \text{ or } EPI' = \frac{RA_3}{n}$$

Received amount on sold items from $t' - R$ to T and interest earned from t' to $T + R$ (RA_4, say) is

$$\begin{aligned} RA_4 &= C_s D(R, r_c, \theta)(1 - d(R, r_c)) \left[(T - t' + R) + I_e \int_{t'}^{T+R} (t - t') dt \right] \\ &= C_s D(R, r_c, \theta) e^{a_2 r_c - bR} (T - t' + R) \left(1 + I_e \frac{T - t' + R}{2} \right) \end{aligned}$$

Hence, total profit amount per year is,

$$\begin{aligned} TP_{n_3}(R, T, r_c, \theta) &= RA_4 - HC - RC - GC \\ &= \frac{D(R, r_c, \theta)}{T} \Bigg[C_s e^{a_2 r_c - bR} (T - t' + R)(1 \\ &\quad + I_e \frac{T - t' + R}{2}) - h \Bigg(\frac{(m+1)^2}{2} \ln \left(\frac{m+1}{1 - T + m} \right) \\ &\quad - \frac{(m+1)T}{2} + \frac{T^2}{4} \Bigg) \Bigg] - \frac{\rho r_c^2 + \eta \theta^2}{T}. \end{aligned} \tag{27}$$

6 Optimal solution procedures

We use current theoretical conclusions in concave fractional programming to tackle the problem as in Cambini and Martein (2009). The real value function

$$h(y) = \frac{f_1(y)}{f_2(y)} \tag{28}$$

is pseudo or strictly concave, if $f_1(y)$ differentiable, non-negative, strictly concave or concave and $f_2(y)$ is convex, positive, and differentiable.

6.1 Optimal solution procedure for profit function TP_{c_1}

Theorem 1: For given positive values of R, r_c and θ, $TP_{c_1}(R,T,r_c,\theta)$ is strictly pseudo concave in T and hence exists a unique maximum solution $T = T_1^*$.

Proof: Suppose $TP_1(R,T,r_c,\theta) = \frac{f_1(T)}{f_2(T)}$, where $f_2(T) = T$ and

$$\begin{aligned} f_1(T) = D(R,r_c,\theta)\Bigg[&C_sTe^{a_2r_c-(b+r)R} - C_p(m+1)\ln\Big(\frac{m+1}{1-T+m}\Big) \\ &- h\bigg\{\frac{(m+1)^2}{2}\ln\Big(\frac{m+1}{1-T+m}\Big) - \frac{(m+1)T}{2} + \frac{T^2}{4}\bigg\} \\ &+ \frac{C_sI_ee^{a_2r_c-bR}}{2}(S-R)^2 - \frac{C_pI_c}{2}\bigg\{(m+R-S+1)^2\ln \\ &\Big(\frac{m+R-S+1}{m-T+1}\Big) + \frac{(m-T+1)^2-(m+R-S+1)^2}{2}\bigg\}\Bigg] \\ &- C_o + \rho r_c^2 + \eta\theta^2. \end{aligned}$$

Twice differentiating $f_1(T)$ w.r.t. T, we have

$$\begin{aligned} f_1''(T) = -D(R,r_c,\theta)\Bigg[&\frac{m+1}{(1-T+m)^2} + \frac{h}{2}\bigg[\Big(\frac{m+1}{1-T+m}\Big)^2 + 1\bigg] \\ &+ \frac{C_pI_c}{2}\bigg\{\Big(\frac{m+R-S+1}{m-T+1}\Big)^2 + 1\bigg\}\Bigg]. \end{aligned}$$

Clearly $f_1''(T) \le 0$, hence $TP_1(R,T)$ is strictly pseudo concave in T. □

To find the optimal value T, equating first order partial derivative of TP_{c_1} w.r.t. T to zero, we have

$$\begin{aligned} \frac{\partial TP_{c_1}}{\partial T} = D(R,r_c,\theta)\Bigg[&-\frac{C_sI_ee^{a_2r_c-bR}}{2T^2}(S-R)^2 \\ &+ \frac{C_p}{T^2}(m+1)\ln\Big(\frac{m+1}{1-T+m}\Big) - \frac{C_p}{T}\Big(\frac{m+1}{1-T+m}\Big) \\ &+ \frac{h}{T^2}\bigg\{\frac{(m+1)^2}{2}\ln\Big(\frac{m+1}{1-T+m}\Big) - \frac{(m+1)T}{2} + \frac{T^2}{4}\bigg\} \\ &- \frac{h}{T}\bigg\{\frac{(m+1)^2}{2(1-T+m)} - \frac{(m+1)}{2} + \frac{T}{2}\bigg\} \end{aligned}$$

$$
\begin{aligned}
&+\frac{C_pI_c}{2T^2}\Bigg\{(m+R-S+1)^2\ln\Big(\frac{m+R-S+1}{m-T+1}\Big)\\
&+\frac{(m-T+1)^2-(m+R-S+1)^2}{2}\Bigg\}\\
&-\frac{C_pI_c}{2T}\Bigg\{\frac{(m+R-S+1)^2}{m-T+1}-(m-T+1)\Bigg\}\Bigg]\\
&+\frac{C_o+\rho r_c^2+\eta\theta^2}{T^2}=0.
\end{aligned}\tag{29}
$$

Likewise T, to find the optimal value R^*, equating first order partial derivative of TP_{c_1} w.r.t. R to zero, we have

$$
\begin{aligned}
\frac{\partial TP_{c_1}}{\partial R}&=-D(R,r_c,\theta)\Big[C_s(b+r)e^{a_2r_c-(b+r)R}\\
&+\frac{C_sI_ee^{a_2r_c-bR}}{2T}\{b(S-R)+2\}(S-R)\\
&+\frac{C_pI_c(m+R-S+1)}{T}\ln\Big(\frac{m+R-S+1}{m-T+1}\Big)\Big]\\
&+Kae^{aR-a_1r_c}\Bigg[C_se^{a_2r_c-(b+r)R}+\frac{C_sI_ee^{a_2r_c-bR}}{2T}(S-R)^2\\
&-\frac{C_p}{T}(m+1)\ln\Big(\frac{m+1}{1-T+m}\Big)-\frac{h}{T}\Bigg\{\frac{(m+1)^2}{2}\ln\Big(\frac{m+1}{1-T+m}\Big)\\
&-\frac{(m+1)T}{2}+\frac{T^2}{4}\Bigg\}-\frac{C_pI_c}{2T}\Big\{(m+R-S+1)^2\ln\\
&\quad\Big(\frac{m+R-S+1}{m-T+1}\Big)+\frac{(m-T+1)^2-(m+R-S+1)^2}{2}\Big\}\Bigg]\\
&=0.
\end{aligned}\tag{30}
$$

Now, differentiating twice TP_{c_1} w.r.t. R, we have

$$
\begin{aligned}
\frac{\partial^2 TP_{c_1}}{\partial R^2}&=D(R,r_c,\theta)\Big[C_s(b+r)^2e^{a_2r_c-(b+r)R}\\
&-\frac{C_pI_c}{T}\Big\{\ln\Big(\frac{m+R-S+1}{m-T+1}\Big)+1\Big\}\\
&+\frac{C_sI_ee^{a_2r_c-bR}}{T}\Bigg\{\big(b(S-R)+1\big)^2-\frac{b^2(S-R)^2}{2}\Bigg\}\Bigg]\\
&-2Kae^{aR-a_1r_c}\Big[C_s(b+r)e^{a_2r_c-(b+r)R}\\
&+\frac{C_sI_ee^{a_2r_c-bR}}{2T}\{b(S-R)+2\}(S-R)
\end{aligned}
$$

$$+ \frac{C_pI_c(m+R-S+1)}{T}\ln\left(\frac{m+R-S+1}{m-T+1}\right)\Bigg]$$
$$+ Ka^2e^{aR-a_1r_c}\Bigg[C_se^{a_2r_c-(b+r)R} + \frac{C_sI_ee^{a_2r_c-bR}}{2T}(S-R)^2$$
$$- \frac{C_p}{T}(m+1)\ln\left(\frac{m+1}{1-T+m}\right) - \frac{h}{T}\Bigg\{\frac{(m+1)^2}{2}\ln\left(\frac{m+1}{1-T+m}\right)$$
$$- \frac{(m+1)T}{2} + \frac{T^2}{4}\Bigg\} - \frac{C_pI_c}{2T}\Big\{(m+R-S+1)^2\ln$$
$$\left(\frac{m+R-S+1}{m-T+1}\right) + \frac{(m-T+1)^2-(m+R-S+1)^2}{2}\Big\}\Bigg]$$
$$= X_1, \quad \text{(say)} \tag{31}$$

Let's utilise equation (30) discrimination term to see if R^* is 0 or positive.

$$\Delta_{R_1} = -D(R, r_c, \theta)\Big[C_s(b+r)e^{a_2r_c-(b+r)R}$$
$$+ \frac{C_sI_ee^{a_2r_c-bR}}{2T}\{b(S-R)+2\}(S-R)$$
$$+ \frac{C_pI_c(m+R-S+1)}{T}\ln\left(\frac{m+R-S+1}{m-T+1}\right)\Bigg]$$
$$+ Kae^{aR-a_1r_c}\Bigg[C_se^{a_2r_c-(b+r)R} + \frac{C_sI_ee^{a_2r_c-bR}}{2T}(S-R)^2$$
$$- \frac{C_p}{T}(m+1)\ln\left(\frac{m+1}{1-T+m}\right) - \frac{h}{T}\Bigg\{\frac{(m+1)^2}{2}\ln\left(\frac{m+1}{1-T+m}\right)$$
$$- \frac{(m+1)T}{2} + \frac{T^2}{4}\Bigg\} - \frac{C_pI_c}{2T}\Big\{(m+R-S+1)^2\ln$$
$$\left(\frac{m+R-S+1}{m-T+1}\right) + \frac{(m-T+1)^2-(m+R-S+1)^2}{2}\Big\}\Bigg]. \tag{32}$$

Based on equations (30), (31) and (32), the following theoretical results can be demonstrated:

Theorem 2: For any given positive value of T, r_c and θ, if $X_1 \leq 0$, then we obtain:

a TP_{c_1} is a strictly pseudo-concave function in R, and hence a unique optimum solution R_1^* exist.

b If $\Delta_{R_1} \leq 0$, then TP_{c_1} is maximised at $R_1^* = 0$.

c If $\Delta_{R_1} \geq 0$, then there exist a unique $R_1^* > 0$ such that TP_{c_1} is maximised.

Proof: If $X_1 \leq 0$, then prove of part a is obvious and R_1^* can be obtain from equation (30). For remaining, we have

$$\lim_{R\to\infty} \Delta_{R_1} = \lim_{R\to\infty} \Big\{ -D(R, r_c, \theta)\Big[C_s(b+r)e^{a_2 r_c-(b+r)R} + \frac{C_s I_e e^{a_2 r_c - bR}}{2T}\{b(S-R)+2\}(S-R) + \frac{C_p I_c (m+R-S+1)}{T}\ln\Big(\frac{m+R-S+1}{m-T+1}\Big)\Big] + Kae^{aR-a_1 r_c}\Big[C_s e^{a_2 r_c-(b+r)R} + \frac{C_s I_e e^{a_2 r_c-bR}}{2T}(S-R)^2 - \frac{C_p}{T}(m+1)\ln\Big(\frac{m+1}{1-T+m}\Big) - \frac{h}{T}\Big\{\frac{(m+1)^2}{2}\ln\Big(\frac{m+1}{1-T+m}\Big) - \frac{(m+1)T}{2} + \frac{T^2}{4}\Big\} - \frac{C_p I_c}{2T}\Big\{(m+R-S+1)^2 \ln \Big(\frac{m+R-S+1}{m-T+1}\Big) + \frac{(m-T+1)^2-(m+R-S+1)^2}{2}\Big\}\Big]\Big\} = -\infty.$$

If $\Delta_{R_1} \leq 0$, then $\frac{\partial TP_{c_1}}{\partial R} \leq 0$ for all $R > 0$, and TP_{c_1} is decreasing function in R. Hence the retailer optimal credit period $R_1^* = 0$, which complete the proof of part b.

Now, if $\Delta_{R_1} \geq 0$, and $\lim_{R\to\infty} \Delta_{R_1} = -\infty$. By applying the Mean-value theorem and part a, there exist a unique R_1^* such that Δ_{R_1}=0, at $R = R_1^*$. Consequently, TP_{c_1} is maximised at unique point $R_1^* > 0$. This complete the proof of Theorem 2. □

Now, to find the optimal value r_c^*, equating first order partial derivative of TP_{c_1} w.r.t. r_c to zero,

$$\frac{\partial TP_{c_1}}{\partial r_c} = D(R, r_c, \theta)a_2\Big[C_s e^{a_2 r_c-(b+r)R} + \frac{C_s I_e e^{a_2 r_c-bR}}{2T}(S-R)^2\Big] - Ka_1 e^{aR-a_1 r_c}\Big[C_s e^{a_2 r_c-(b+r)R} + \frac{C_s I_e e^{a_2 r_c-bR}}{2T}(S-R)^2 - \frac{C_p}{T}(m+1)\ln\Big(\frac{m+1}{1-T+m}\Big) - \frac{h}{T}\Big\{\frac{(m+1)^2}{2}\ln\Big(\frac{m+1}{1-T+m}\Big) - \frac{(m+1)T}{2} + \frac{T^2}{4}\Big\} - \frac{C_p I_c}{2T}\Big\{(m+R-S+1)^2 \ln\Big(\frac{m+R-S+1}{m-T+1}\Big) + \frac{(m-T+1)^2-(m+R-S+1)^2}{2}\Big\}\Big] - \frac{2\rho r_c}{T} = 0. \tag{33}$$

Now, differentiating twice TP_{c_1} w.r.t. r_c, we have

$$\frac{\partial^2 TP_{c_1}}{\partial r_c^2} = Ka_1^2 e^{aR-a_1r_c}\Bigg[C_s e^{a_2r_c-(b+r)R} - \frac{C_p}{T}(m+1)\ln\Big(\frac{m+1}{1-T+m}\Big) - \frac{h}{T}\Bigg\{\frac{(m+1)^2}{2}\ln\Big(\frac{m+1}{1-T+m}\Big) - \frac{(m+1)T}{2} + \frac{T^2}{4}\Bigg\} + \frac{C_sI_e e^{a_2r_c-bR}}{2T}(S-R)^2 - \frac{C_pI_c}{2T}\Big\{(m+R-S+1)^2\ln \Big(\frac{m+R-S+1}{m-T+1}\Big) + \frac{(m-T+1)^2-(m+R-S+1)^2}{2}\Bigg\}\Bigg] + \Big(C_s e^{a_2r_c-(b+r)R} + \frac{C_sI_e e^{a_2r_c-bR}}{2T}(S-R)^2\Big)\Big(D(R,r_c,\theta)a_2^2 - 2Ka_1a_2e^{aR-a_1r_c}\Big) - \frac{2\rho}{T} = X_2, \quad \text{(say)}. \tag{34}$$

Let's utilise equation (33) in discrimination form to see if r_c^* is 0 or positive.

$$\Delta_{r_{c1}} = D(R,r_c,\theta)a_2\Bigg[C_s e^{a_2r_c-(b+r)R} + \frac{C_sI_e e^{a_2r_c-bR}}{2T}(S-R)^2\Bigg] - Ka_1e^{aR-a_1r_c}\Bigg[C_s e^{a_2r_c-(b+r)R} + \frac{C_sI_e e^{a_2r_c-bR}}{2T}(S-R)^2 - \frac{C_p}{T}(m+1)\ln\Big(\frac{m+1}{1-T+m}\Big) - \frac{h}{T}\Bigg\{\frac{(m+1)^2}{2}\ln\Big(\frac{m+1}{1-T+m}\Big) - \frac{(m+1)T}{2} + \frac{T^2}{4}\Bigg\} - \frac{C_pI_c}{2T}\Big\{(m+R-S+1)^2\ln \Big(\frac{m+R-S+1}{m-T+1}\Big) + \frac{(m-T+1)^2-(m+R-S+1)^2}{2}\Bigg\}\Bigg] - \frac{2\rho r_c}{T}. \tag{35}$$

Based on equations (33), (34) and (35), the following theoretical results can be demonstrated:

Theorem 3: For any given positive value of R, T and θ, if $X_2 \leq 0$, then we obtain:

a TP_{c_1} is a strictly pseudo-concave function in r_c, and hence exist a unique optimum solution $r_{c_1}^*$.

b If $\Delta_{r_{c1}} \leq 0$, then TP_{c_1} is maximised at $r_{c_1}^* = 0$.

c If $\Delta_{r_{c1}} \geq 0$, then there exist a unique $r_{c_1}^* > 0$ such that TP_{c_1} is maximised.

Proof: If $X_2 \leq 0$, then proof of part a is obvious and $r^*_{c_1}$ can be obtained from equation (33). For remaining parts, we have

$$\lim_{r_c \to \infty} \Delta_{r_{c_1}} = \lim_{r_c \to \infty} \Bigg\{ D(R, r_c, \theta) a_2 \Bigg[C_s e^{a_2 r_c - (b+r)R} + \frac{C_s I_e e^{a_2 r_c - bR}}{2T}(S-R)^2 \Bigg] - K a_1 e^{aR - a_1 r_c} \Bigg[C_s e^{a_2 r_c - (b+r)R} + \frac{C_s I_e e^{a_2 r_c - bR}}{2T}(S-R)^2 - \frac{C_p}{T}(m+1)\ln\Big(\frac{m+1}{1-T+m}\Big) - \frac{h}{T}\Bigg\{ \frac{(m+1)^2}{2}\ln\Big(\frac{m+1}{1-T+m}\Big) - \frac{(m+1)T}{2} + \frac{T^2}{4} \Bigg\} - \frac{C_p I_c}{2T}\Big\{(m+R-S+1)^2 \ln \Big(\frac{m+R-S+1}{m-T+1}\Big) + \frac{(m-T+1)^2 - (m+R-S+1)^2}{2} \Big\}\Bigg] - \frac{2\rho r_c}{T} \Bigg\} = -\infty.$$

If $\Delta_{r_{c_1}} \leq 0$, then $\frac{\partial TP_{c_1}}{\partial r_c} \leq 0$ for all $r_c > 0$, and TP_{c_1} is a decreasing function in r_c. Hence the optimal value of $r^*_{c_1} = 0$, which complete the proof of part b.

Now, if $\Delta_{r_{c_1}} \geq 0$, and $\lim_{r_c \to \infty} \Delta_{r_{c_1}} = -\infty$. By applying the mean-value theorem and part a, there exists a unique $r^*_{c_1}$ such that $\Delta_{r_{c_1}}$=0, at $r_c = r^*_{c_1}$. Consequently, TP_{c_1} is maximised at unique point $r^*_{c_1} > 0$. This completes the proof of Theorem 3. □

Again, to find optimum value of θ, equating first order partial derivative of TP_{c_1} w.r.t. θ to zero, we have

$$\frac{\partial TP_{c_1}}{\partial \theta} = K_0 k e^{-k\theta} \Bigg[C_s e^{a_2 r_c - (b+r)R} + \frac{C_s I_e e^{a_2 r_c - bR}}{2T}(S-R)^2 - \frac{C_p}{T}(m+1)\ln\Big(\frac{m+1}{1-T+m}\Big) - \frac{h}{T}\Bigg\{ \frac{(m+1)^2}{2}\ln\Big(\frac{m+1}{1-T+m}\Big) - \frac{(m+1)T}{2} + \frac{T^2}{4} \Bigg\} - \frac{C_p I_c}{2T}\Big\{(m+R-S+1)^2 \ln \Big(\frac{m+R-S+1}{m-T+1}\Big) + \frac{(m-T+1)^2 - (m+R-S+1)^2}{2} \Big\}\Bigg] - \frac{2\eta\theta}{T} = 0. \tag{36}$$

Now, differentiating twice TP_{c_1} w.r.t. θ, we have

$$\frac{\partial^2 TP_{c_1}}{\partial \theta^2} = -K_0 k^2 e^{-k\theta} \Bigg[C_s e^{a_2 r_c - (b+r)R} + \frac{C_s I_e e^{a_2 r_c - bR}}{2T}(S-R)^2$$

$$
\begin{aligned}
&- \frac{C_p}{T}(m+1)\ln\Big(\frac{m+1}{1-T+m}\Big) - \frac{h}{T}\Big\{\frac{(m+1)^2}{2}\ln\Big(\frac{m+1}{1-T+m}\Big) \\
&- \frac{(m+1)T}{2} + \frac{T^2}{4}\Big\} - \frac{C_p I_c}{2T}\Big\{(m+R-S+1)^2 \ln \\
&\Big(\frac{m+R-S+1}{m-T+1}\Big) + \frac{(m-T+1)^2 - (m+R-S+1)^2}{2}\Big\}\Big] \\
&- \frac{2\eta}{T} = X_3, \quad \text{(say)}.
\end{aligned} \tag{37}
$$

Theorem 4: For given positive values of R, T and r_c, TP_{c_1} is strictly pseudo concave in θ and hence exists a unique optimum solution $\theta = \theta_1^*$.

Proof: Clearly from equation (37), $\frac{\partial^2 TP_{c_1}}{\partial \theta^2} = X_3 \leq 0$. Hence TP_{c_1} is strictly pseudo concave in θ and optimum value of θ is obtain from equation (36). □

Similarly, we can find optimal solution for profit function TP_{c_2} and TP_{c_3}.

6.2 Optimal solution procedure for new approach

As the expressions of the profit functions TP_{n_1} and TP_{n_3} are highly nonlinear, so it is difficult to prove analytically, the concavity of the above profit functions can be established numerically and graphically w.r.t. retailer's credit period (R), time period (T), reminder level (r_c) and greening level (θ) (cf. Figures 1 and 2).

7 Numerical example and sensitivity analysis

We maximise the profit functions given by expressions (16), (20), (22), (23), (25) and (27) of different models using the GRG methods through LINGO 19.0 software for different sets of data.

- *Experiment-1:* Let's consider an inventory model under the scenario-1 with the following parameters: K = 1,000, a = 1.2, a_1 = 0.01, K_0 = 625, k = 0.05, a_2 = 0.30, b = 0.20, r = 8.7%, C_o = 250, C'_o = 2,500, C_s = 10, C_p = 16, n = 1, S = 0.16, I_e = 8%, I_c = 13%, I_b = 8.7%, h = 0.2, m = 1, $\rho = 5 \times 10^4$ and η = 315. Maximising profit functions (16) and (20) using LINGO 19.0, optimal solutions for TP_{c_1} and TP_{n_1} are given in Table 2.
- *Experiment-2 and experiment-3:* Changing S = 0.50 and S = 0.08 in the dataset of experiment-1 (for experiment-2 and experiment-3 respectively). We maximise the profit functions (22), (23) and (25), (27) for the profit functions (TP_{c_2}, TP_{n_2}) and (TP_{c3}, TP_{n_3}) under the scenario-2 and scenario-3 respectively. the optimal solution are presented in Table 2.

Table 2 Optimal profits under different scenarios

Scenarios	*Profit*	*For* R^*	*Conventional Approach* T^*	θ^*	r_c^*	TP_c	*For* R^*	*New Approach* T^*	θ^*	r_c^*	t'	TP_n
1	w r_c	0.092	0.248	0.059	0.013	4,395.102	0.160	0.248	0.056	0.014	0.345	4,475.969
	wo r_c	0.086	0.245	0.059	0	4,362.108	0.160	0.243	0.054	0	0.342	4,436.673
2	w r_c	0.208	0.235	0.054	0.014	4,915.896	0.208	0.235	0.054	0.014	—	4,915.896
	wo r_c	0.200	0.232	0.054	0	4,876.307	0.200	0.232	0.054	0	—	4,876.307
3	w r_c	0.363	0.202	0.037	0.013	4,435.852	0.232	0.232	0.048	0.014	0.409	4,456.663
	wo r_c	0.353	0.200	0.037	0	4,391.749	0.227	0.227	0.047	0	0.401	4,416.205

Notes: w: with, wo: without.

For the compound interest, if the number of installments (n) is changed, then profits under scenario-1 and scenario-3 are calculated and presented in Table 3, keeping all other data same.

Table 3 Optimal profits with different no's of instalments

Scenarios ↓	*Profits* ↓	*n* →	2	3	4	5	6	7	8
1	TP_{n_1}	w r_c	4,532.692	4,551.858	4,561.491	4,567.287	4,571.158	4,573.926	4,576.005
		wo r_c	4,492.612	4,511.504	4,520.998	4,526.710	4,530.525	4,533.252	4,533.300
3	TP_{n_3}	w r_c	4,507.453	4,526.872	4,534.535	4,558.293	4,562.935	4,564.372	4,565.241
		wo r_c	4,454.536	4,463.546	4,473.241	4,477.980	4,475.263	44,774.989	4,474.024

Notes: w: with, wo: without.

7.1 Concavity of profit functions

Here, we prove the concavity of the profit functions, TP_{c_1} numerically w.r.t. the decision variables R, T, θ and r_c. For the optimal values of R (= 0.158), T (= 0.163), θ (= 0.011) and r_c (= 0.238), the Hessian matrix H, (say) of TP_{c_1} is

$$H = \begin{bmatrix} \frac{\partial^2 TP_{c_1}}{\partial R^2} & \frac{\partial^2 TP_{c_1}}{\partial R \partial T} & \frac{\partial^2 TP_{c_1}}{\partial R \partial \theta} & \frac{\partial^2 TP_{c_1}}{\partial R \partial r_c} \\ \frac{\partial^2 TP_{c_1}}{\partial T \partial R} & \frac{\partial^2 TP_{c_1}}{\partial T^2} & \frac{\partial^2 TP_{c_1}}{\partial T \partial \theta} & \frac{\partial^2 TP_{c_1}}{\partial T \partial r_c} \\ \frac{\partial^2 TP_{c_1}}{\partial r_c \partial R} & \frac{\partial^2 TP_{c_1}}{\partial r_c \partial T} & \frac{\partial^2 TP_{c_1}}{\partial r_c^2} & \frac{\partial^2 TP_{c_1}}{\partial r_c \partial \theta} \\ \frac{\partial^2 TP_{c_1}}{\partial \theta \partial R} & \frac{\partial^2 TP_{c_1}}{\partial \theta \partial T} & \frac{\partial^2 TP_{c_1}}{\partial \theta \partial r_c} & \frac{\partial^2 TP_{c_1}}{\partial \theta^2} \end{bmatrix}$$

$$= \begin{bmatrix} -428.24 & -369.05 & 00 & -378.59 \\ -369.05 & -36{,}626.19 & 611.50 & 1.79 \\ 00 & 611.50 & -2{,}541.56 & 00 \\ -378.59 & 1.79 & 00 & -402{,}785.40 \end{bmatrix}$$

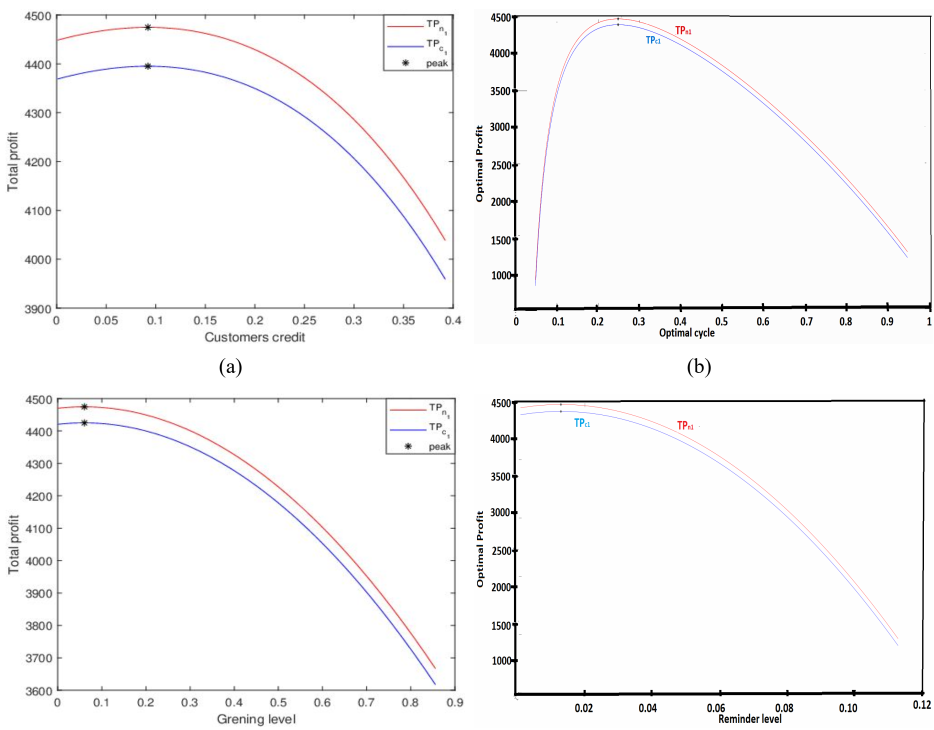
For concavity, the first, second and third principle minor of H should be negative, positive and negative respectively. The value of determinant of H also becomes positive. Clearly, the first minors $\frac{\partial^2 TP_{c_1}}{\partial R^2}$, $\frac{\partial^2 TP_{c_1}}{\partial T^2}$, $\frac{\partial^2 TP_{c_1}}{\partial r_c^2}$ and $\frac{\partial^2 TP_{c_1}}{\partial \theta^2}$ all are negative.

Again, the second, third principle minor and value of determinant of H are 1.55×10^7(>0), -3.93×10^{10}(<0) and 1.58×10^{16}(>0) respectively. So, TP_{c_1} is concave w.r.t. R, T, θ and r_c. Moreover, as $\frac{\partial^2 TP_{c_1}}{\partial R^2}$, $\frac{\partial^2 TP_{c_1}}{\partial T^2}$, $\frac{\partial^2 TP_{c_1}}{\partial r_c^2}$ and $\frac{\partial^2 TP_{c_1}}{\partial \theta^2}$ all are negative for optimal values, we conclude that TP_{n_1} is concave separately in R, T, θ and r_c for the fixed values of other three variable. Similarly, the concavity of all other profit functions can be established numerically.

7.2 Graphical representations of profit functions

Here we present the graphical representations of profit function TP_{c_1} and TP_{n_1} for scenario-1 w.r.t. decision variable R, T, θ and r_c separately in Figure 1. The profit functions are concave separately w.r.t. R [Figure 1(a)], T [Figure 1(b)], θ [Figure (c)] and r_c [Figure 1(d)] keeping corresponding other three variable at their optimum values. The graphical representations support the deductions of the previous Theorems 1, 2, 3 and 4. Similarly, for the other scenarios, we can represent the profit function graphically, which are again concave w.r.t. decision variables.

Figure 1 Concavity of TP_{c_1} and TP_{n_1} in single decision variable keeping other decision variable constant, (a) concavity of TP_{n_1} and TP_{c_1} in R (b) concavity of TP_{n_1} and TP_{c_1} in T (c) concavity of TP_{n_1} and TP_{c_1} in θ (d) concavity of TP_{n_1} and TP_{c_1} in r_c (see online version for colours)



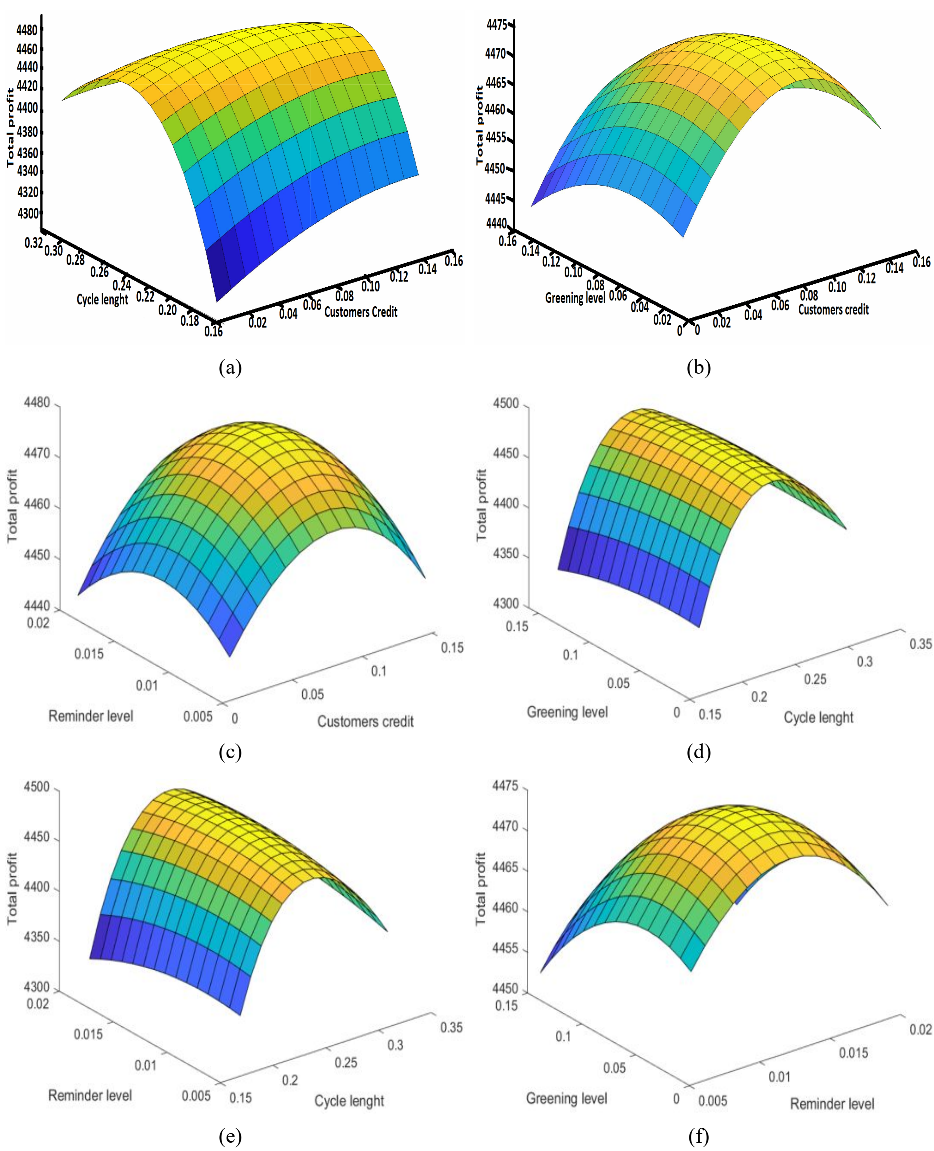
(a) (b) (c) (d)

As four and five dimensional graphs are not possible to draw, we present six three dimensional graphs to represent concavity of profit function TP_{n_1} w.r.t. two decision variable keeping other decision variables constant. These are shown in Figure 2, i.e., TP_{n_1} w.r.t. R and T (for fixed values of θ, r_c), TP_{n_1} w.r.t. R and θ (for fixed values of T, r_c), TP_{n_1} w.r.t. R and r_c (for fixed values of T, θ), TP_{n_1} w.r.t. T and θ (for fixed values of R, r_c), TP_{n_1} w.r.t. T and r_c (for fixed values of N, θ) and TP_{n_1} w.r.t. θ and r_c (for fixed values of T, R) in Figures 2(a), 2(b), 2(c), 2(d), 2(e) and 2(f) respectively.

7.3 Dissection of components of profit functions

Here we analysed the behaviours of components of the profit function, TP_{c_1} for the scenario-1 w.r.t. decision variables R, T, θ and r_c (cf. Table 4).

Figure 2 Concavity of TP_{n_1} in two decision variable keeping other decision variable constant, (a) concavity of TP_{n_1} in R, T (b) concavity of TP_{n_1} in R, θ (c) concavity of TP_{n_1} in R, r_c (d) concavity of TP_{n_1} in T, θ (e) concavity of TP_{n_1} in T, r_c (f) concavity of TP_{n_1} in θ, r_c (see online version for colours)



8 Discussion

- *Profit with and without r_c:* Optimum results for all profit functions are given under different scenarios with and without r_c in Tables 2 and 3. Profits with and without r_c in the new approach are more compared to the conventional one in all scenarios. In each scenario, profits with r_c are more than those without r_c for both approaches. This justifies considering reminder costs in a trade-credited system for more profit. But in scenario-2, the retailer can pay all the costs, including the purchasing cost, ordering cost, and HC within S as $R + T \leq S$. He does not take any loans in the new approach. For this reason, profits in both approaches are the same under the scenario $R \leq T \leq S$ and $R + T \leq S$. Within a fixed time, the increase in installments for payment in a bank always reduces the total payable amount, so profit increases. This is reflected in Table 3 (scenario-1 and scenario-3).
- *Trade off between profit and retailer's trade credit:* The expression (1) implies that demand increases with the retailer's credit period R. As a result, the total sales revenue/profit increases along with R. But from equation (3), R increases the default risk, which has a negative effect on the sales revenue/profit. Thus, there is a trade-off between R and profit. For the small values of R, demand has more effect on sales revenue than default risk, and so profit increases. After a certain level of R, the default risk dominates, and sales revenue goes down along with the profit. Therefore, profit is concave in nature w.r.t. R as shown in Figure 1(a) and demonstrated in Table 4.
- *Trade off between profit and reminder level:* The reminder level (r_c) affects the profit components. It has a negative effect on demand, and hence the number of total customers decreases as r_c increases. But it positively affects default risk, i.e., it decreases the number of total defaulters. These two behaviours of r_c have the opposite effects on sales revenue, i.e., profit. For this reason, for small values of r_c, the effect of r_c on default risk is more, and hence profit increases. After a certain value of r_c, its effect on demand dominates, so profit goes down. It is reflected in Table 4. Changes in demand will directly affect purchasing and HCs (these depend on procured quantities), which are reduced with increases of r_c. Hence, the profit is concave w.r.t. r_c [cf. Figure 1(d)].
- *Trade off between time period and profit:* Due to changes in T (time period), though demand remains constant, economic order quantity increases with it, and so do the purchasing cost, HC, and interest charged. On the other hand, as T increases, ordering cost and reminder cost per unit time decrease. It is interesting to note that the behaviour of total expenditure changes with T, i.e., it decreases initially with it, and after a certain value of T, it increases [cf. Table 4, Figure 1(b)]. As a result, initially, profit increases with T, and after a certain value of T, profit starts to decrease.
- *Trade off between greening level and profit:* From equation (1) demand is an increasing function of greening level but to maintain greenness, the retailer spends some amount on it. So the greening level has both positive (increased demand) and negative (increasing expenditure) effects on total sales revenue. Thus,

initially, profit increases with greening up to a certain level of θ. After that profit goes down, which is reflected in Figure 1(c) and Table 4.

- *Behaviors of total sales revenue and expenditure:* For each R, θ and r_c, keeping others fixed at their optimal values, both total sales revenue and expenditure increases with them (cf. Table 4). But initially, the increase in sales revenue is higher than that of expenditure. After some value (optimal level), this scenario changes, i.e., increased rate in expenditure dominates over the revenue's increase. For T, keeping R, θ, and r_c at optimal values, the sales revenue decreases very slowly, almost constant, say with an increase in T. Still, expenditure decreases initially, and after a certain value (optimal), it increases. These are because of the changes in the ordering cost, HC, purchasing cost, etc. due to the changes in the ordered amount. However, profit initially increases with T, and after the optimal value, it decreases.

9 Managerial insight

1 From Table 2, it is seen that the length of the retailer's credit period and cycle period are essential for maximum profit. If the wholesaler credit period (S) is highest, i.e., more than the time period, then the profit is maximum. Usually, it does not happen. A retailer can decide his credit period w.r.t. wholesaler's one. For higher retailer's credit ($R > S$), the item will be sold quickly, and profit per unit time will be more. This is important as the item is a deteriorating one. Thus, management (w.r.t. retailer) may decide the length of the credit period depending on the item's condition. This analysis helps the management to determine the inventory of perishable items.

2 With the introduction of reminder cost, the retailer is benefited up to a certain reminder level. Through this analysis, management can decide the optimum investment in reminder cost.

10 Conclusions and future extension

In this study, we have analysed a two-level trade credited EOQ model with default risk and reminder cost under greenness. The model has been mathematically formulated under three different scenarios following conventional and new approaches. Optimum results are obtained using the GRG method through LINGO 19.0 software. For the first time, in a trade credited model, the concept of reminder cost is introduced to reduce the number of default customers and hence for more profit, though it has a negative effect on demand. Moreover, a new alternative approach for trade credited models is proposed and illustrated to avoid some ambiguities in the conventional method. For all scenarios, the new approach gives a better result (more profit) than the conventional one with and without reminder cost. Changing behaviours of the profit w.r.t. R (customer's trade credit), T (time period), r_c (reminder level), and θ, (greening level) are demonstrated both numerically and graphically.

Table 4 Effects of R, T, θ and r_c on profit components

Parameter		$D(N,\theta,r_c)$	$F(R,r_c)$	y_1	y_2	y_3	y_4	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	TP_{c_1}
R	0.032	1,040.97	0.0025	16,566.78	43.87	16,610.65	–	11,113.56	40.16	26.98	1,008.55	33.50	4.50	12,227.27	–	4,383.380
	0.052	1,066.21	0.0065	16,871.35	31.85	16,903.20	292.55	11,383.03	56.19	27.63	1,008.55	33.50	4.52	12,513.43	286.16	4,389.779
	0.072	1,092.06	0.0105	17,181.54	21.57	17,203.11	299.91	11,659.04	75.45	28.30	1,008.55	33.50	4.52	12,809.37	295.94	4,393.742
	0.092	1,118.54	0.0144	17,497.45	13.13	17,510.58	307.47	11,941.76	98.15	28.99	1,008.55	33.50	4.52	13,115.48	306.11	4,395.102
	0.112	1,145.67	0.0184	17,819.18	6.67	17,825.85	315.27	12,231.35	124.56	29.69	1,008.55	33.50	4.52	13,432.17	316.69	4,393.682
	0.132	1,173.45	0.0223	18,146.85	2.31	18,149.16	323.31	12,527.97	154.91	30.41	1,008.55	33.50	4.52	13,759.86	327.69	4,389.297
	0.152	1,201.91	0.0262	18,480.56	0.19	18,480.75	331.59	12,831.79	189.41	31.15	1,008.55	33.50	4.52	14,099.00	339.14	4,381.752
T	0.187	1,118.54	0.0144	17,497.45	17.32	17,514.77	–	11,746.20	56.89	21.72	1,330.64	44.20	5.96	13,205.62	–	4,309.157
	0.207	1,118.54	0.0144	17,497.45	15.66	17,513.11	–1.66	11,810.43	70.25	24.13	1,202.62	39.94	5.39	13,152.76	–52.86	4,360.346
	0.227	1,118.54	0.0144	17,497.45	14.28	17,511.73	–1.38	11,875.60	84.03	26.55	1,097.07	36.44	4.92	13,124.61	–28.15	4,387.121
	0.247	1,118.54	0.0144	17,497.45	13.13	17,510.58	–1.15	11,941.76	98.15	28.99	1,008.55	33.50	4.52	13,115.48	–9.13	4,395.102
	0.267	1,118.54	0.0144	17,497.45	12.15	17,509.60	–0.98	12,008.92	112.57	31.45	933.25	31.00	4.18	13,121.38	5.9	4,388.219
	0.287	1,118.54	0.0144	17,497.45	11.31	17,508.76	–0.84	12,077.11	127.26	33.93	868.42	28.84	3.89	13,139.45	18.07	4,369.301
	0.307	1,118.54	0.0144	17,497.45	10.57	17,508.02	–0.74	12,146.35	142.18	36.44	812.00	26.97	3.64	13,167.59	28.14	4,340.434
θ	0.029	1,117.61	0.0144	17,482.82	13.12	17,495.94	–	11,931.77	98.07	28.97	1,008.55	33.50	1.12	13,101.98	–	4,393.955
	0.039	1,117.92	0.0144	17,487.70	13.12	17,500.82	4.88	11,935.10	98.10	28.98	1,008.55	33.50	2.00	13,106.23	4.25	4,394.592
	0.049	1,118.23	0.0144	17,492.57	13.13	17,505.70	4.88	11,938.43	98.13	28.98	1,008.55	33.50	3.13	13,110.73	4.50	4,349.975
	0.059	1,118.54	0.0144	17,497.45	13.13	17,510.58	8.88	11,941.76	98.15	28.99	1,008.55	33.50	4.52	13,115.48	4.75	4,395.102
	0.069	1,118.85	0.0144	17,502.32	13.14	17,515.46	4.88	11,945.09	98.18	29.00	1,008.55	33.50	6.16	13,120.48	5.00	4,394.975
	0.079	1,119.16	0.0144	17,507.19	13.14	17,520.33	4.87	11,948.41	98.21	29.01	1,008.55	33.50	8.06	13,125.74	5.26	4,394.592
	0.089	1,119.48	0.0144	17,512.06	13.14	17,525.20	4.87	11,951.73	98.24	29.01	1,008.55	33.50	10.21	13,131.25	5.51	4,393.955
r_c	0.003	1,118.64	0.017	17,451.84	13.10	17,464.93	–	11,942.83	98.16	28.99	1,008.55	3.05	4.52	13,086.11	–	4,378.823
	0.006	1,118.61	0.016	17,467.03	13.11	17,480.14	15.10	11,942.48	98.16	28.99	1,008.55	9.57	4.52	13,092.27	6.16	4,387.867
	0.009	1,118.57	0.015	17,482.23	13.12	17,495.35	15.20	11,942.12	98.16	28.99	1,008.55	19.72	4.52	13,102.06	7.79	4,393.293
	0.012	1,118.54	0.0144	17,497.45	13.13	17,510.58	15.23	11,941.76	98.15	28.99	1,008.55	33.50	4.52	13,115.48	13.42	4,395.102
	0.015	1,118.51	0.0136	17,512.68	13.14	17,525.82	15.24	11,941.40	98.15	28.99	1,008.55	50.91	4.52	13,132.53	17.05	4,393.293
	0.018	1,118.47	0.0127	17,527.92	13.15	17,541.08	15.26	11,941.04	98.15	28.99	1,008.55	71.95	4.52	13,153.21	20.68	4,387.867
	0.021	1,118.44	0.0118	17,543.18	13.17	17,556.35	15.27	11,940.69	98.15	28.99	1,008.55	96.62	4.52	13,177.52	24.31	4,378.823

Notes: y_1 = net revenue of the retailer, y_2 = earned interest per unit time, $y_3 = y_1 + y_2$, y_4 = difference in two consecutive values of y_3,
z_1 = purchasing price per unit time, z_2 = interest charge per unit time, z_3 = HC per unit time, z_4 = ordering cost per unit time,
z_5 = reminder cost per unit time, z_6 = greening cost per unit time, $z_7 = z_1 + z_2 + z_3 + z_4 + z_5 + z_6$,
z_8 = difference in two consecutive values of z_7.

The concept of reminder cost can be introduced in all the trade-credited models. Moreover, all trade credited models – EOQ, EPQ, two-warehouse, etc. with full or partial trade credits can be formulated following the new approach presented here as the new approach gives a better return than the conventional one.

The trade credited model for time-dependent deteriorated items can be formulated with a reminder level for a finite time horizon. Depending on the level of deterioration, the retailer's credit period can be determined for his maximum profit.

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