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# Optimising green vehicle routing problem - a real case study 

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#### Abstract

The optimisation of distribution activities in the logistics scheme of various companies, long time based on economic objectives, is widening today to integrate environmental concerns. This paper addresses the fuel consumption minimisation problem for one variant of the green VRP which is the VRP with fuel consumption rate (FCVRP) and considers load and distance as two main factors affecting fuel consumption. The problem is classified as NP-hard, hence, we propose to solve it by an iterated local search meta-heuristic (ILSFC-SP) starting with a heuristic approach that is based on mathematical programming and generates solutions by CPLEX. In order to test its performance, ILSFC-SP was first applied on benchmark instances to minimise fuel consumption as well as travelled distance and compared with the literature where it proved its efficacy, then, it was applied to a real-world application in Tunisia where it suggested operational solutions reducing considerably the fuel costs. [Submitted: 28 June 2019; Accepted: 17 April 2022]


Keywords: fuel consumption; green vehicle routing problem; iterated local search; logistics; set-partitioning problem.

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#### Abstract

This paper is a revised and expanded version of a paper entitled 'Minimizing fuel consumption in capacitated vehicle routing problem' presented at International Conference of the African Federation of Operational Research Societies AFROS 2018, Tunis, Tunisia, July 2018.


## 1 Introduction

The classical vehicle routing problem (VRP) aims to design optimal delivery routes of a fleet of vehicles-based at a depot and serving a set of customers with specified demands. The importance of this problem is two-fold: theoretical since it is an NP-hard combinatorial optimisation problem and considerably difficult to solve, and practical given its various applications in the real world.

By considering all the intricacies of real case distribution, certain challenging constraints were added defining thus different extensions of the VRP. For example, the capacitated VRP fortifies the fact that the vehicle has a limited capacity and should not be exceeded; the VRP with time windows (VRPTWs) imposes that customers must be served within a given time window; the VRP with multiple depots (MDVRP) defines several depots in the supply chain so a vehicle route can start and end at any depot; the VRP with pick-up and delivery (VRPPD) combines the delivery and the collection activities.

For many decades, the VRP objective has focused only on economic issues by minimising the total distance travelled or the total travel costs. Recently, researchers became aware of the dire effects of pollution, therefore, the VRP objective has been extended to deal with environmental sustainability. Consequently, the green vehicle routing problem (green VRP) has emerged as a key to tackle environmental threats from the transportation sector. Thus, research interest in Green logistics is growing.

In this paper, we tackle one variant of the green VRP which is the VRP with fuel consumption rate (FCVRP). This variant is an extension of the well studied capacitated VRP in which the objective is to minimise total fuel consumption rather than travel distance. This problem is NP-Hard as the classical VRP is NP-Hard, thus exact methods can not provide optimal solutions for large scale instances, unlike meta-heuristics which proved their efficiency in solving such problems.

In this context, this work makes two main contributions which can be summarised as follows:

- The first one is to propose an efficient algorithm in order to solve the problem. For this purpose, we develop a new heuristic based on mathematical programming that ends up with a solution generated by CPLEX. Then, an iterated local search method called ILSFC-SP is involved in order to improve the quality of heuristic solutions where different neighbourhood operators such as swap, shift, and $r$-opt were applied and a perturbation based on destroy and repair was handled to diversify the search. The performance of the algorithm is tested on different benchmark instances and compared with state-of-the-art.
- The second contribution is to bring help to the decision maker in a Tunisian company in order to better use its resources and improve its distribution system
based on operational research tools. This company is engaged in the production and distribution of several types of filtres and has more than 2000 customers and a homogeneous fleet of four vehicles.

The remainder of this article is organised as follows. In Section 2, we provide the state-of-the-art. In Section 3, we propose a FCVRP formulation based on set-partitioning model. Then, in Section 4 we introduce a new effective heuristic based on this last formulation. In Section 5, we present our approach. Experimental analysis with the literature is presented and discussed in Section 6. We introduce the case study in Section 7. Finally, we conclude our paper in Section 8.

## 2 State-of-the-art

In the literature, Lin et al. (2014) surveyed the state-of-the-art of the VRP with environmental dimension; they classified research that considers fuel consumption and $\mathrm{CO}_{2}$ emissions minimisation as green VRP. Both issues depend on different factors that can be divided into two categories; the first one consists of vehicle load, speed, distance and road gradient while the second category includes taxes, driver wages, maintenance, etc. (Demir et al., 2014). Although fuel consumption and $\mathrm{CO}_{2}$ emissions are directly related to each other, they have been studied in many works separately.

Concerning fuel consumption problem, studies are developed for different extensions of the VRP and are based on several models with different formulas for estimating fuel consumption based on specific factors affecting the fuel consumption as indicated above. Kara et al. (2007) defined an energy minimisation function for the VRP; this function represents a product of vehicle load and distance. Kuo (2010) proposed a fuel consumption model for the time-dependent vehicle routing problem (TDVRP); the author developed a simulated annealing algorithm to minimise the total fuel consumed where speed and travel time were considered. Kuo and Wang (2011) proposed a tabu search method for the CVRP where distance, speed and load factors have been considered in the fuel consumption model. Xiao et al. (2012) studied the fuel consumption minimisation for the FCVRP; they considered load and distance factors and they determined a fuel consumption rate (FCR) based on statistical data; the authors proposed a mixed integer programming model and developed a simulated annealing algorithm to solve the FCVRP. Li (2012) developed a fuel consumption minimisation model for the VRPTW; the author proposed a tabu search with variable neighbourhood descent procedure to solve it. Later, Ene et al. (2016) defined a mixed integer programming model for the green VRP in the case of heterogeneous fleet with fuel consumption objective where load and distance factors were taken into consideration; the authors tested their approach on different instances generated from benchmark problems of Solomon (1987). Teng and Zhang (2016) developed a simulated annealing algorithm for the green VRP to mitigate both total fuel cost and travelling distance and tested their algorithm on small scale instances. An efficient bi-objective hybrid local search method was developed by Rao et al. (2016); based on the existing FCR model of Barth et al. (2005), the authors presented a mixed integer programming model where the amount of fuel consumed is a road-gradient-based function. Recently Kazemian and Aref (2017) developed a simulated annealing where they proposed a time-dependent green VRP model for the capacitated time-dependent VRPTWs based
on load, distance and vehicle speed; the model accounts for both fuel consumption minimisation and emission reduction; the authors tested their approach on randomly generated problems. Furthermore, Feng et al. (2017) used the fuel consumption model of Barth and Boriboonsomsin (2009) with consideration of load, distance and speed factors; the authors proposed an improved simulated annealing in which different operators were applied to explore the search space; they tested their approach on different problems based on real distances collected from UK cities where they proved its performance to minimise fuel consumption in comparison with the simulated annealing in terms of computational time. On contrary to Feng et al. (2017) and Kuo and Wang (2011), Xiao et al. (2012) has not considered the vehicle speed and provided rich computational experiments on well-known benchmark instances.

We notice that although the number of papers in this field is increasing, the proposed models and algorithms are scattered due to the diversity of the real world applications. Unlike research for the traditional VRP, there is a lack of comparative studies based on common benchmarks between these approaches. In this paper, we remedy this anomaly by providing such a work.

Regarding the minimisation of $\mathrm{CO}_{2}$ emissions in the air, Figliozzi (2010) proposed a greedy heuristic for emissions in the VRP (EVRP) geared to minimise emissions and fuel consumption. Bektas and Laporte (2011) studied the pollution routing problem in which a cost-minimising objective function including cost of emissions, drivers and fuel is considered for the VRPTW. Pitera et al. (2011) formulated an emission minimisation problem and developed a local search algorithm for a case study of the VRP with a heterogeneous fleet. Jemai et al. (2012) proposed a bi-objective formulation considering the minimisation of both total distance travelled and $\mathrm{CO}_{2}$ emissions; they developed an NSGA-II algorithm where they proved its performance for solving green VRP benchmarks. Later, an exact dynamic programming approach was proposed by Xiao and Konak (2015) to minimise weighted tardiness and $\mathrm{CO}_{2}$ emissions for the time-dependent vehicle routing and scheduling problem with a heterogeneous fleet; in order to improve the quality of the solutions, the authors combined their approach with a genetic algorithm. Last but not least, Mirmohammadi et al. (2017) studied the minimisation of $\mathrm{CO}_{2}$ emissions as well as the lateness and earliness penalties costs for the VRP with time-dependent and time windows; the authors formulated the problem as an integer linear mathematical model where they used CPLEX to solve some randomly generated test problems. Recently, Rezaei et al. (2019) proposed a mixed-integer linear programming model to minimise $\mathrm{CO}_{2}$ emissions in the VRPTW with a heterogeneous fleet; the authors developed a genetic algorithm and a population-based simulated annealing algorithm and tested them on different generated instances based on a benchmark database.

As green VRP has triggered a line in logistics research, another strand which concerns alternative fuel vehicles has been proposed. These vehicles consume electricity, natural gas, propane, ethanol, etc. instead of fuel (Yavuz, 2017). Erdogan and Miller-Hooks (2012) examined the possibility of refueling vehicles by environment-friendly fuel stations; they formulated the problem as a mixed integer linear program and in order to solve it they developed a heuristic based on the modification of the Clarke and Wright (1964) savings algorithm called MCWS and density-based clustering algorithm DBCA. Schneider et al. (2014) studied the electric recharging stations for the VRPTW; their resolution method consists of applying a tabu search with a variable neighbourhood algorithm. Later, Lin et al. (2016) surveyed the battery
consumption of an electric VRP; they referred to a case study where the objective was to minimise energy cost as well as travel time. An exact algorithm has been developed by Andelmin and Bartolini (2017) who formulated the green VRP with alternative fuel vehicles as a set-partitioning problem and solved optimally different benchmark instances with up to 110 customers.

We notice that studies about routing such vehicles are scarce in comparison with routing traditional vehicles. This is due to many reasons, such that the limited infrastructure of alternative fuel vehicles throughout the world and the lack of environmental regulations and energy policies supporting ecofriendly paractices.

We have provided in this section the main literature on the green VRP, for further details we refer the reader to the survey of Lin et al. (2014) and the one of Bektas et al. (2016).

## 3 Problem description and mathematical model

Let $G=(V, E)$ be a graph where $V=\{0,1, \ldots, n\}$ is the set of vertices and $E=$ $\{(i, j) \mid i, j \in V, i<j\}$ is the set of arcs. The vertex 0 represents the depot while the remaining vertices represent the $n$ customers. Each vertex $V /\{0\}$ is associated with a non-negative demand $q_{i}\left(q_{i} \leq Q, \forall i \in V\right)$ and each arc $(i, j)$ is associated with a distance $d_{i j}$. A fleet of homogeneous vehicles is available at the depot. Each vehicle has a loading capacity $Q$ that cannot be exceeded. A vehicle route must start and end at the depot after serving a set of customers. Each customer must be served only once by a single vehicle. The objective is to minimise the total fuel consumed while serving all the customers.

### 3.1 Fuel consumption rate

Based on statistical data, Xiao et al. (2012) formulated a FCR as a load-dependent function. Let $Q_{i j}$ be the carried load from customer $i$ to customer $j, c_{0}$ the unit fuel cost, $\rho^{*}$ and $\rho_{0}$ are respectively the rate of fuel consumption when the vehicle is fully loaded and the rate when there is no load carried.

The FCR along the route from $i$ to $j$ is: $\rho_{i j}=\rho_{0}+\alpha \times Q_{i j}$ where $\alpha$ is a constant and equal to $\alpha=\frac{\left(\rho^{*}-\rho_{0}\right)}{Q}$. Thereby, fuel cost $C_{i j}^{f}$ for travelling from customer $i$ to customer $j$ is expressed as: $c_{i j}^{f}=c_{0} \rho_{i j} d_{i j}$.

In this work, we adopted this FCR function as previously expressed.
From a first view, it seems that minimising distance involves fuel consumption reduction which is not always true. In fact, although the distance is an undeniable factor; the carried load of the vehicle is not less important and the rate function depends strongly on the variation of both these factors without exception. The example below helps to better understand that a good solution in terms of distance travelled is not always efficient from point of view fuel consumption if we did not take into consideration the carried load during the vehicle trip.

### 3.1.1 Example

Let the number of customers be equal to 5 , the vehicle capacity equals 100 , the unit fuel cost equals 1 , the fuel rate when the vehicle is totally charged and when there is
no load carried are respectively equal 2 and 1 . The coordinates of each customer and their requests are given in Table 1.

Table 1 Data for the example in Subsection 3.1

|  | Identifier | Coordinate | Demand |
| :--- | :---: | :---: | :---: |
| Depot $_{\text {Customer }_{1}}$ | 0 | $(1,1)$ | 0 |
| Customer $_{2}$ | 1 | $(4,2)$ | 60 |
| Customer $_{3}$ | 2 | $(3,5)$ | 5 |
| Customer $_{4}$ | 3 | $(2,5)$ | 5 |
| Customer $_{5}$ | 4 | $(4,3)$ | 10 |

Figure 1 Routes with the shortest distance and the lowest fuel cost of the example in Subsection 3.1 (see online version for colours)


On the basis of data of customers, two shortest delivery routes are determined by calculating Euclidean distance $: 0 \rightarrow 3 \rightarrow 2 \rightarrow 4 \rightarrow 1 \rightarrow 5 \rightarrow 0$ and $0 \rightarrow 5 \rightarrow 1$ $\rightarrow 4 \rightarrow 2 \rightarrow 3 \rightarrow 0$ [Figures 1(a) and 1(b)]. Although these two routes have identical distance travelled (13.7734), they have two different fuel costs (21.9418 and 19.3784 calculated by considering FCR). In addition, in terms of fuel consumption, none of these routes have the lowest fuel cost, where another route with a minimum fuel cost equals 18.6205 but with long distance (14.1717) can be found: $0 \rightarrow 1 \rightarrow 5 \rightarrow 4 \rightarrow 2 \rightarrow$ $3 \rightarrow 0$ [Figure 1(c)]. The distribution strategy on this last route starts by serving the customers with the heaviest demand without being oblivious of the travelled distance so that the amount of fuel consumed in the route can be lowered later after that the weighty charges have been unloaded.

### 3.2 Mathematical model for FCVRP

Many formulations were proposed in literature to model the classical VRP. For instance, Fisher and Jaikumar (1981) modelled the problem as a generalised assignment problem; Laporte et al. (1985) proposed a vehicle flow formulation; Balinski and Quandt (1964) modelled the problem as a set-partitioning problem. In this study, we propose to extend this last formulation to FCVRP. In this formulation, customers are first assigned to the vehicles which lead to a number of sets of customers, then, for each vehicle, a route
is built in order to visit customers assigned to it. A route is feasible if it satisfies the capacity constraints.

Let $\Omega$ be the set of all the feasible vehicle routes. $X_{k}$ is a decision variable corresponding to the route $k$; its value is 1 if route $k$ is selected in the solution, otherwise it is equal to 0 . The coefficient $a_{i k}$ is binary and equals to 1 if customer $i$ is included in the route, otherwise $a_{i k}=0 . C_{k}^{f}$ is the fuel cost of route $k$. This cost should be calculated as the sum of the fuel cost of all arcs making up the route according to the formula presented in Subsection 3.1.

The mathematical formulation of the FCVRP as a set-partitioning problem is as follows:

$$
\begin{equation*}
\operatorname{Min} \sum_{k \in \Omega} C_{k}^{f} X_{k} \tag{1}
\end{equation*}
$$

Subject to

$$
\begin{align*}
& \sum_{k \in \Omega} a_{i k} X_{k}=1, \quad \forall i \in\{1, \ldots, n\}  \tag{2}\\
& X_{k} \in\{0,1\}, \quad \forall k \in \Omega \tag{3}
\end{align*}
$$

- The objective function (1) aims to minimise the total fuel costs of all the routes.
- Constraint (2) imposes that each customer must be served only one time.
- Constraint (3) is relative to the integrality of the decision variables.


## 4 An effective set-partitioning-based heuristic (HSP) for FCVRP

The proposed heuristic is based on mathematical programming. In this kind of approach, the initial program is simplified for the sake of being solved (Ball and Magazine, 1981).

In our case, and in order to easily solve the set-partitioning model, we build a reduced subset $\Omega^{\prime} \subset \Omega$ of feasible and promising routes, then we solve the restricted model that is limited to $\Omega^{\prime}$ (Tayachi and Jendoubi, 2018).

The key element of our heuristic HSP is how to build the subset $\Omega^{\prime}$ such that routes of good quality are obtained. The basic idea consists in collecting the customers that are close to each other on the same route, then we insert the depot and we solve a travelling salesman problem (TSP) on this route with the depot.

The construction of $\Omega^{\prime}$ is made iteratively as follows.
For each customer $i$, we determine the nearest neighbour $i_{1}$, then the second nearest neighbour $i_{2}$ from $i_{1}$ and we iterate the procedure until reaching the capacity of the vehicle. Thus we obtain a set $S_{i}=\left\{i_{1}, i_{2}, \ldots, i_{k}\right\}$. After that, we form a new set $C_{i}$ by adding the depot and customer $i$ to the set $S_{i}, C_{i}=\{0, i\} \cup S_{i}=\left\{i_{1}, i_{2}, \ldots, i_{k}\right\}=$ $\left\{0, i, i_{1}, i_{2}, \ldots, i_{k}\right\}$. Then, from the set $C_{i}$, we form the initial route $T_{i}=0--i-i_{1}-$ $i_{2}-\ldots-i_{k}-0$. Based on $T_{i}$, we apply a descent method that we call Descent $t_{1}$ as a local search to obtain the route $R_{i}$. This method takes the route $T_{i}$ as input; using $r$-opt, swap and shift moves, it dives in its neighbourhood to search for a route $R_{i}$ of good quality (the algorithm of Descent $t_{1}$ is given in Algorithm 1). Based on the returned route $R_{i}$, we generate more routes by eliminating at each iteration the farthest customer from $i$ until obtaining the route including only customer $i$. This process is repeated for $i=1$ to $i=n$ leading to the set $\Omega^{\prime}$.

The algorithm of generating $\Omega^{\prime}$ is given in Algorithm 2.

```
Algorithm 1 Descent \({ }_{1}\)
    for (each initial route \(T_{i}\) ) do
        repeat
            Search \(R_{i}\) in the neighbourhood of \(T_{i}\)
            if ( Cost \(\left.R_{i}<\operatorname{Cost} T_{i}\right)\) then
                \(T_{i} \leftarrow R_{i}\)
            end if
        until (Cost \(\left.R_{i} \geq \operatorname{Cost} T_{i}\right)\)
        \(R_{i} \leftarrow T_{i}\)
        Return the route \(R_{i}\)
    end for
```

Algorithm 2 Construction of $\Omega^{\prime}$
for (each customer $i$ ) do
Determine the set $S_{i}$ of its nearest neighbours where the sum of requests does not exceed
the capacity of the vehicle
Add the depot and the customer $i$ to form the new set $C_{i}$
Form the initial route $T_{i}$ from the set $C_{i}$
Apply a local search procedure ( Descent $_{1}$ ) to the route $T_{i}$ to obtain the route of good
quality $R_{i}$
Add the route $R_{i}$ to $\Omega^{\prime}$
while (the number of customers in the route $R_{i}$ is greater than 1) do
Remove from the route $R_{i}$ the farthest customer from $i$
Add the resulting route to the set $\Omega^{\prime}$
Update $R_{i}$
end while
end for

Once the feasible set $\Omega^{\prime}$ is built, it will be given as input to the set-partitionning formulation to obtain a solution to the problem. To do so, we determine the binary matrix $A=\left(a_{i j}\right)$ with $n$ rows and $\left|\Omega^{\prime}\right|$ columns. Each row represents a customer and each column represents a route. This matrix indicates to which route each customer is assigned where a solution imposes that each customer belongs to only one route. At each column, a fuel cost is assigned. The objective is to select a set of columns in such a way that the sum of the total fuel costs is the lowest and the number of 1 figuring in each row of the selected columns is equal to 1 . In order to get the solution, the model was solved by CPLEX 12.6 software.

## 5 An iterated local search algorithm ILSFC-SP for FCVRP

The iterated local search is a meta-heuristic proposed by Stützle (1998) which consists of two phases: the local search phase and the perturbation phase. Local search is applied to an initial solution $S_{0}$ until a local optimum is reached. As soon as a local optimum is reached, the perturbation turns it into a new solution that represents a new beginning for the local search. This procedure is repeated until a stopping criterion is satisfied.

In our algorithm, we apply the iterated local search for the initial solution $S_{0}$ obtained by the heuristic HSP. For the local search phase, we propose a descent
with shift, swap and $r$-opt moves which we call Descent $_{2}$. This method performs in inter-route and intra-route modes where intra-route mode modifies a single route while the inter-route mode modifies different routes. Although Descent ${ }_{2}$ seems to be similar to Descent $_{1}$, in fact, they are different; the main difference between them is that Descent $_{1}$ operates in only intra-route mode while Descent $_{2}$ operates in both modes.

For the perturbation phase in our algorithm, we propose the destroy and repair operators. These operators have been popularised by Ropke and Pisinger (2006). They consist of destroying a part of the solution and then recreating it which lends the way to explore widely the search space.

### 5.1 Move operators

Three different moves are addressed in this paper:

- $\quad r$-opt consists of removing $r$ arcs of the solution and replacing them with others to reconnect the remaining arcs. In this work, $r$ is equal to 2 .
- Swap consists of exchanging the position of two customers which creates a change in the order of visit.
- Shift involves moving a customer position in a route to another.

Figure 2 2-opt, swap and shift operators


The different moves in inter-route (right side) and intra-route (left side) modes are introduced in Figures 2(a), 2(b) and 2(c) where node 0 indicates the depot. In

Figure 2(a), 2-opt is applied to arcs $\left(a_{0} a_{1}\right)$ and $\left(b_{1} b_{0}\right)$ while in Figure 2(b), node $a_{1}$ is moved from one position to another after applying shift operator and in Figure 2(c), nodes $a_{1}$ and $b_{1}$ are swaped.

### 5.2 Destroy/repair operators

Pisinger and Ropke (2007) defined different types of destroy and repair operators. For the destroy operators, three types of removal were proposed: random removal, related removal, and worst removal. The first one consists of selecting customers randomly. In the second one, customers who are geographically close to each other are removed. The last one consists of removing customers who reduce the cost of the solution as much as possible. For the repair operators, two types of insertion are proposed: greedy and regret insertion. The greedy insertion consists of adding at each iteration the customer that increases the cost of the solution as little as possible while the regret insertion aims to insert in the best position the customer who maximises regret the most. The regret of a customer is the cost difference of the solution when the customer is inserted in the best position and the second-best position in the solution.

Figure 3 Destroy and repair operators


As Figure 3 shows, by applying destroy operators, customers $a_{0}$ and $a_{2}$ are removed which turns the solution to a partial solution. The repair operator aims to reconstruct this partial solution to make it complete by inserting the destroyed customers: $a_{0}$ and $a_{2}$. In our ILSFC-SP approach, we used random removal as a destroy operator and the greedy insertion to repair the solution.

The algorithm of the iterated local search is given in Algorithm 3.
Algorithm 3 ILSFC-SP (see online version for colours)

```
Data \(S_{0}\) : an initial solution generated by HSP (Section 4)
    \(S_{1} \leftarrow \operatorname{Descent}_{2}\left(S_{0}\right)\) (Section 5)
    while Stopping criteria not reached do
        //e.g., a prefixed maximum number of iterations has been performed
        \(S_{1}^{\prime} \leftarrow\) Random removal \(\left(S_{1}\right)\) (Subsection 5.2)
        \(S_{1}^{*} \leftarrow\) Greedy insertion ( \(S_{1}^{\prime}\) ) (Subsection 5.2)
        \(S_{2} \leftarrow\) Descent \(_{2}\left(S_{1}^{*}\right)\) (Section 5)
        if \(\left(C^{f}\left(S_{2}\right)<C^{f}\left(S_{1}\right)\right)\) then
            \(S_{1} \leftarrow S_{2}\)
        end if
    end while
```


## 6 Computational experiments

Our approach was coded by C++ programming language using Microsoft Visual Studio 2010 Compiler and performed on Dell Inspiron N5050 laptop with a 2.4 GHz processor and 4 GB of RAM. The solutions provided by HSP heuristic were generated by CPLEX 12.6 software.

We carried out several experiments on different benchmark instances presented in the literature and on a real-case problem. For the benchmark instances, they consist of two sets of CVRP instances used in Xiao et al. (2012); these instances are classified into two classes. The first class contains seven small/medium-scale CVRP instances from Christofides et al. (1979) with a number of customers ranging from 51 to 200 and the second class consists of 20 large-scale CVRP instances from Golden et al. (1998); these instances range between 240 and 483 customers. For each instance of Christofides et al. (1979) and Golden et al. (1998), ILSFC-SP was run ten times with fuel objective and ten times with distance objective. So the number of runs for the instances of Christofides et al. (1979) is equal to: $2 \times 10 \times 7=140$ and the number of runs for Golden et al. (1998) instances is: $2 \times 10 \times 20=400$. In total, ILSFC-SP was run 540 times.

For the real-world problem, it is a case of a company located in northern Africa (Tunisia) and composed of problems that range between 19 and 64 customers.

### 6.1 Parameters

The fuel parameters are set to the same values fixed by Xiao et al. (2012) in order to make a fair comparison and are as follows: $\rho^{*}=2, \rho_{0}=1$ and $c_{0}=1$.

We limited the execution time of our heuristic HSP to one hour for the instances of Christofides et al. (1979) and three hours for those of Golden et al. (1998) and we fixed the level loop of the iterated local search (the stopping criteria) that is applied to the initial solutions generated by HSP to 1,000 .

Another important parameter in the approach is the percentage of destruction in the destroy/repair procedure. To investigate the effect of this parameter on the quality of the solutions found, we resorted to statistical analysis where we made experimental tests on the seven small/medium-scale instances of Christofides et al. (1979); we have tested destroying a number of customers equal to $10 \%, 20 \%, 30 \%$ and $40 \%$ of the size of each instance. Our algorithm was executed ten times for each instance. Results are presented in the two following subsections.

### 6.1.1 Statistical analysis

In this section, we describe the statistical analysis carried out to better understand the effect of the different destruction percentages on the ILSFC-SP performance.

To evaluate wisely this effect and develop the confidence level, a single factor analysis of variance (ANOVA) was executed on the third instance of Christofides et al. (1979) for both fuel consumption and total distance objectives. The algorithm was executed 30 times under each percentage.

The null and alternative hypotheses are stated as follows: $H_{0}: \mu_{10 \%}=\mu_{20 \%}=$ $\mu_{30 \%}=\mu_{40 \%}$ and $H_{1}$ : not all the destruction percentages generate equal solutions.

The results of the ANOVA tests are shown in Tables 2 and 3.

Table 2 Single-factor ANOVA for instance 3 (101) of Christofides et al. (1979) with fuel objective

| Summary |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :--- |
| Groups | Count | Sum | Average | Variance |  |  |
| $10 \%$ | 30 | $35,467.14$ | 1182.24 | 29.23 |  |  |
| $20 \%$ | 30 | $35,251.80$ | 1175.06 | 4.62 |  |  |
| $30 \%$ | 30 | $35,030.42$ | 1167.68 | 36.45 |  |  |
| $40 \%$ | 30 | $34,804.12$ | 1160.14 | 15.52 |  |  |
| ANOVA |  |  |  |  |  |  |
| Source of variation | SS | $d f$ | MS | $F$ | P-value | F-crit |
| Between groups | $8,144.41$ | 3 | $2,714.80$ | 126.53 | $2.00 \mathrm{E}-36$ | 2.68 |
| Within groups | $2,488.80$ | 116 | 21.46 |  |  |  |
| Total | $10,633.21$ | 119 |  |  |  |  |

Table 3 Single-factor ANOVA for instance 3 (101) of Christofides et al. (1979) with distance objective

| Summary |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Groups | Count | Sum | Average | Variance |  |  |
| $10 \%$ | 30 | $25,109.6$ | 836.98 | 9.7 |  |  |
| $20 \%$ | 30 | $25,043.5$ | 834.78 | 4.39 |  |  |
| $30 \%$ | 30 | $25,034.1$ | 834.46 | 9.45 |  |  |
| $40 \%$ | 30 | $24,985.7$ | 832.86 | 6.52 |  |  |
| ANOVA |  |  |  |  |  |  |
| Source of variation | SS | df | MS | F | P-value | F-crit |
| Between groups | 259.93 | 3 | 86.64 | 11.50 | $1.17 \mathrm{E}-06$ | 2.68 |
| Within groups | 873.61 | 116 | 7.53 |  |  |  |
| Total | 1133.55 | 119 |  |  |  |  |

It can be viewed from Tables 2 and 3 that the null hypothesis is to be rejected for both objectives since the $P$-value is so very close to zero. Let's recall that $P$-value indicates that the probability of $H_{0}$ is true. Thus, there is a noticeable difference between the generated solutions with the different destruction percentages.

### 6.1.2 Testing the percentage of destruction

Since statistical analysis proved that there is a difference between the different destruction percentages and in order to rigorously determine the number of customers to destroy that yields the best performance, we have tested to destroy a number of customers equal to $10 \%, 20 \%, 30 \%$ and $40 \%$ of the size of each instance. Our algorithm was executed ten times for each instance.

The results of the instances of Christofides et al. (1979) are shown in Table 4 for fuel consumption objective and in Table 5 for total distance objective. The first column shows the instance with the number of nodes between parentheses, the second and
the third columns show the mean and the minimum values obtained in ten runs after applying the destroy operator with a determined percentage of destruction.

Table 4 Testing the percentages of destruction with fuel objective on the instances of Christofides et al. (1979)

| Problem | Mean |  |  |  |  |  | Min |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $10 \%$ | $20 \%$ | $30 \%$ | $40 \%$ |  | $10 \%$ | $20 \%$ | $30 \%$ | $40 \%$ |  |  |
| $1(51)$ | 749.52 | 747.35 | 748.074 | 746.388 |  | 746.39 | 746.39 | 746.39 | 746.388 |  |  |
| $2(76)$ | $1,199.212$ | $1,191.12$ | $1,184.81$ | $1,185.4$ |  | $1,184.12$ | $1,184.09$ | $1,177.15$ | $1,177.15$ |  |  |
| $3(101)$ | $1,182.238$ | $1,175.34$ | $1,170.12$ | $1,160.14$ |  | $1,175.33$ | $1,171.66$ | $1,161.43$ | $1,150.48$ |  |  |
| $4(151)$ | $1,464.03$ | $1,461.59$ | $1,462.27$ | $1,459.25$ |  | $1,460.76$ | $1,453.5$ | $1,454.97$ | $1,452.77$ |  |  |
| $5(200)$ | $1,877.412$ | $1,872.15$ | $1,871.79$ | $1,868.39$ |  | $1,865.12$ | $1,863.57$ | $1,864.32$ | $1,859.74$ |  |  |
| $11(121)$ | $1,516.19$ | $1,516.21$ | $1,516.21$ | $1,516.21$ |  | $1,516.19$ | $1,516.21$ | $1,516.21$ | $1,516.21$ |  |  |
| $12(101)$ | $1,176.283$ | $1,174.99$ | $1,174.99$ | $1,174.99$ |  | $1,174.7$ | $1,174.7$ | $1,174.7$ | $1,174.7$ |  |  |

Note: Italics indicates the best minimum values.
Table 5 Testing the percentages of destruction with distance objective on the instances of Christofides et al. (1979)

| Problem | Mean |  |  |  |  |  |  |  |  |  |  |  | Min |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $10 \%$ | $20 \%$ | $30 \%$ | $40 \%$ |  | $10 \%$ | $20 \%$ | $30 \%$ | $40 \%$ |  |  |  |  |  |  |  |  |
| $1(51)$ | 539.294 | 531.306 | 524.660 | 524.611 |  | 524.611 | 524.611 | 524.611 | 524.611 |  |  |  |  |  |  |  |  |
| $2(76)$ | 849.195 | 851.494 | 850.11 | 852.711 |  | 839.083 | 843.412 | 845.594 | 847.47 |  |  |  |  |  |  |  |  |
| $3(101)$ | 838.83 | 834.7834 | 834.4684 | 832.8577 |  | 832.795 | 830.689 | 828.838 | 827.392 |  |  |  |  |  |  |  |  |
| $4(151)$ | $1,060.84$ | $1,054.123$ | $1,054.6$ | $1,052.9$ |  | $1,049.88$ | $1,049.21$ | $1,050.26$ | $1,049.88$ |  |  |  |  |  |  |  |  |
| $5(200)$ | $1,371.94$ | $1,371.96$ | $1,371.98$ | $1,370.02$ |  | $1,371.94$ | $1,371.96$ | $1,371.98$ | $1,367.23$ |  |  |  |  |  |  |  |  |
| $11(121)$ | $1,047.95$ | $1,049.219$ | $1,049.37$ | $1,049.03$ |  | $1,045.88$ | $1,047.86$ | $1,045.64$ | $1,047.06$ |  |  |  |  |  |  |  |  |
| $12(101)$ | 826.8 | 826.004 | 824.623 | 824.843 |  | 824.181 | 824.777 | 819.558 | 819.558 |  |  |  |  |  |  |  |  |

Note: Italics indicates the best minimum values.
Distributions of solutions of the instance 3 (101) of Christofides et al. (1979) obtained by running ILSFC-SP under the four destruction percentages are shown in Figures 4(a), 4(b), 4(c) and 4(d) for minimising fuel consumption and in Figures 5(a), 5(b), 5(c) and 5(d) for minimising the total distance travelled.

Figure 4 Distributions of the computational results for instance 3 (101) of Christofides et al.
(1979) with fuel cost objective, (a) $10 \%$
(b) $20 \%$
(c) $30 \%$ (d) $40 \%$ (see online version for colours)


Figure 5 Distributions of the computational results for instance 3 (101) of Christofides et al. (1979) with total distance objective, (a) $10 \%$ (b) $20 \%$ (c) $30 \%$ (d) $40 \%$ (see online version for colours)


From Figures 4 and 5, the lowest average value of distance and that of fuel consumption were obviously found with a destruction percentage equal to $40 \%$. In addition to that, from Tables 4 and 5 we can observe that the most minimum values of the different instances were found by $40 \%$; thus, it is wise to choose this destruction percentage.

### 6.2 Comparison of the proposed iterated local search ILSFC-SP with the set-partitioning-based heuristic HSP

In this section, a comparison of ILSFC-SP solutions with the results of HSP for both classes of instances is made in Tables 7 and 8 in terms of fuel and distance. Before that, as HSP was used to generate the initial solutions for ILSFC-SP approach we have compared it with the well known classical savings heuristic of Clarke and Wright (CWS) using the instances of Christofides et al. (1979) for the CVRP. Table 6 displays the results where the first column presents the instance, the second column is reserved for the CWS results, the third column presents the HSP results and the last column is the deviation between both solutions where it is clear that HSP outperforms CWS.

Table 6 Comparison of HSP heuristic with the CWS algorithm on the instances of Christofides et al. (1979) in terms of distance

| Problem | CWS | HSP | Dis-Dev \% |
| :--- | :---: | :---: | :---: |
| $1(51)$ | 625.56 | 586.827 | -6.192 |
| $2(76)$ | $1,005.25$ | 974.373 | -3.072 |
| $3(101)$ | 982.48 | 962.015 | -2.083 |
| $4(151)$ | $1,299.39$ | $1,270.71$ | -2.207 |
| $5(200)$ | 1,708 | $1,539.86$ | -9.844 |
| $11(121)$ | $1,291.33$ | $1,074.91$ | -16.759 |
| $12(101)$ | 939.99 | 827.012 | -12.019 |

Regarding Tables 7 and 8 , the first column defines the instance, the second column presents the vehicle capacity $Q$ and the third column presents the number of the generated tours that compose the reduced set $\left|\Omega^{\prime}\right|$. It is worthwhile to remind that HSP has considerably reduced the number of routes. Indeed, for example for instance 11 (121), HSP operates on a subset of routes $\left|\Omega^{\prime}\right|$ which contains only 2,061 routes (see Table 7) whereas $|\Omega|$ is of order $2^{n}=2^{120}$ if we relax the capacity constraints. The column HSP is the value of the heuristic solution. The column ILSFC-SP presents
the minimum of ten runs obtained by ILSFC-SP. The column Fc-Dev (\%) indicates the deviation between ILSFC-SP and HSP when the objective to minimise is the fuel consumption (fuel oriented) whereas the column Dis-Dev (\%) indicates the deviation between ILSFC-SP and HSP if the objective to minimise is distance (distance oriented).

Table 7 Comparison of ILSFC-SP with HSP on solving Christofides et al. (1979) instances

| Problem | $Q$ | $\left\|\Omega^{\prime}\right\|$ | FCVRP (fuel oriented) |  |  | CVRP (distance oriented) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | HSP | ILSFC-SP | $F \mathrm{c}$-Dev (\%) | HSP | ILSFC-SP | Dis-Dev (\%) |
| 1 (51) | 160 | 487 | 816.015 | 746.388 | -8.53 | 592.452 | 524.611 | -11.45 |
| 2 (76) | 140 | 545 | 1,326.01 | 1,177.15 | -11.23 | 977.803 | 847.47 | -13.33 |
| 3 (101) | 200 | 1,312 | 1,335.93 | 1,150.48 | -13.88 | 986.973 | 827.392 | -16.17 |
| 4 (151) | 200 | 1,928 | 1,744.09 | 1,452.77 | -16.70 | 1,284.14 | 1,049.88 | -18.24 |
| 5 (200) | 200 | 2,377 | 2,144.58 | 1,859.74 | -13.28 | 1,566.21 | 1,367.23 | -12.70 |
| 11 (121) | 200 | 2,061 | 1,555.3 | 1,516.21 | -2.51 | 1,079.53 | 1,047.06 | -3.01 |
| 12 (101) | 200 | 1,150 | 1,176.46 | 1,174.7 | -0.15 | 827.682 | 819.558 | -0.98 |
| Avg |  |  |  |  | -9.47 |  |  | -10.84 |

Table 8 Comparison of ILSFC-SP with HSP on solving Golden et al. (1998) instances

| Problem | $Q$ | $\left\|\Omega^{\prime}\right\|$ | FCVRP (fuel oriented) |  |  | CVRP (distance oriented) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | HSP | ILSFC-SP | Fc-Dev (\%) | HSP | ILSFC-SP | Dis-Dev (\%) |
| 1 (240) | 550 | 6,540 | 8,700.98 | 7,663.69 | -11.92 | 6,607.95 | 5,637.8 | -14.68 |
| 2 (320) | 700 | 11,050 | 13,442.7 | 11,158.3 | -16.99 | 9,095.82 | 8,493.77 | -6.62 |
| 3 (400) | 900 | 17,938 | 19,156 | 14,623.5 | -23.66 | 12,600.2 | 11,199.5 | -11.12 |
| 4 (480) | 1,000 | 23,897 | 25,463.6 | 18,832.1 | -26.04 | 17,077.2 | 13,857.4 | -18.85 |
| 5 (200) | 900 | 8,967 | 10,043.3 | 8,547.1 | -14.90 | 7,340.88 | 6,480.89 | -11.72 |
| 6 (280) | 900 | 12,551 | 13,987.6 | 11,222.5 | -19.77 | 10,530.6 | 8,574.12 | -22.818 |
| 7 (360) | 900 | 16,136 | 16,602.6 | 13,711.3 | -17.41 | 12,199.4 | 10,195.6 | -19.654 |
| 8 (440) | 900 | 19,666 | 19,658.7 | 16,256.4 | -17.31 | 13,533.7 | 11,800.1 | -12.81 |
| 9 (255) | 1,000 | 5,720 | 1,084.16 | 863.767 | -20.33 | 807.703 | 626.542 | -22.43 |
| 10 (323) | 1,000 | 8,190 | 1,377.23 | 1,099.94 | -20.13 | 1,016.2 | 810.337 | -20.26 |
| 11 (399) | 1,000 | 11,326 | 1,533.32 | 1,380.12 | -9.99 | 1,144.45 | 976.811 | -14.65 |
| 12 (483) | 1,000 | 15,244 | 1,811.57 | 1,663.57 | -8.17 | 1,296.49 | 1,194.73 | -7.85 |
| 13 (252) | 1,000 | 2,668 | 1,485.76 | 1,269.65 | -14.55 | 941.608 | 920.487 | -2.24 |
| 14 (320) | 1,000 | 3,860 | 1,781.92 | 1,615.94 | -9.31 | 1,258.13 | 1,168.95 | -7.09 |
| 15 (396) | 1,000 | 5,320 | 2,098.25 | 2,000.19 | -4.67 | 1,695.73 | 1,387.44 | -18.18 |
| 16 (480) | 1,000 | 7,180 | 2,593.92 | 2,408.28 | -7.16 | 1,819.5 | 1,695.98 | -6.79 |
| 17 (240) | 200 | 2,682 | 1,225.9 | 1,044.34 | -14.81 | 887.679 | 734.596 | -17.25 |
| 18 (300) | 200 | 3,342 | 1,720.44 | 1,487.1 | -13.56 | 1,080.65 | 1,052.55 | -2.60 |
| 19 (360) | 200 | 4,002 | 2,315.87 | 2,047.87 | -11.57 | 1,465.94 | 1,461.64 | -0.29 |
| 20 (420) | 200 | 4,662 | 3,126.8 | 2,760.88 | -11.70 | 2,288.61 | 1,944.98 | -15.01 |
| Avg |  |  |  |  | -14.70 |  |  | -12.27 |

For the instances of Christofides et al. (1979) in Table 7, our ILSFC-SP algorithm improved the heuristic solution with $9.47 \%$ on average (Avg) in terms of fuel. This improvement reaches up to $16.70 \%$ for instance 4 (151) while in terms of distance
it improved the heuristic solution with deviation that reaches on maximum $18.24 \%$ [instance 4 (151)] and $10.84 \%$ on average.

For the instances of Golden et al. (1998) in Table 8, ILSFC-SP algorithm improved the heuristic solution with $14.70 \%$ on average in terms of fuel. The fuel consumption decrease reaches more than 20\% [instances 3 (400), 4 (480), 9 (255) and 10 (323)]. In terms of distance, ILSFC-SP improved the HSP solutions up to $22.818 \%$ [instance 6 (280)] on maximum and $12.27 \%$ on average.

### 6.3 Performance evaluation of ILSFC-SP on solving FCVRP and CVRP problems

Table 9 and 10 contain the computational results and the cost deviation of both classes of instances treated as CVRP (distance oriented) and FCVRP (fuel oriented).

Each instance is solved ten times under each objective where the minimum value ( min ) and its related distance/fuel cost are given.

The column BK represents the best-known results in the litterature, ILSFC-SP column indicates the minimum of each objective among ten runs, the columns BK-Dev (\%), Fc-Dev (\%) and Dis-Dev (\%) are respectively the deviation of ILSFC-SP solutions found on solving CVRP from best-known solutions, the mean of ten runs of fuel cost deviation and distance cost deviation. Fc-Dev (\%) and Dis-Dev (\%) are calculated as follows:

$$
\begin{aligned}
& F c-\operatorname{Dev}(\%)= \frac{((\text { fuel oriented })-\text { the related }}{\text { fuel }(\text { distance oriented })) \times 100} \\
& \text { The related fuel }(\text { distance oriented })
\end{aligned}\left(\begin{array}{c}
(\text { The related distance }(\text { fuel oriented })
\end{array}\right.
$$

Table 9 Computational results of ILSFC-SP and deviation comparison between FCVRP and CVRP for Christofides et al. (1979) instances

| Problem | FCVRP (fuel oriented) |  | CVRP (distance oriented) |  |  | BK-Dev Fc-Dev Dis-Dev |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ILSFC-SP Related distance |  | BK | ILSFC- | lated fuel | (\%) | (\%) | (\%) |
| 1 (51) | 746.388 | 555.767 | 524.61 | 524.611 | 778.229 | 0.00 | -4.09 | 5.94 |
| 2 (76) | 1,177.15 | 859.92 | 835.26 | 847.47 | 1,277.37 | 1.46 | -6.85 | 3.42 |
| 3 (101) | 1,150.48 | 850.694 | 826.14 | 827.392 | 1,186.57 | 0.15 | -4.55 | 2.33 |
| 4 (151) | 1,452.77 | 1,077.97 | 1,028.42 | 1,049.88 | 1,548.58 | 2.09 | -6.52 | 2.59 |
| 5 (200) | 1,859.74 | 1,370.43 | 1,291.29 | 1,367.23 | 1,882.28 | 5.88 | -8.32 | 4.01 |
| 11 (121) | 1,516.21 | 1,053.82 | 1,042.11 | 1,047.06 | 1,559.69 | 0.47 | -2.46 | 0.46 |
| 12 (101) | 1,174.7 | 828.411 | 819.56 | 819.558 | 1,194.73 | 0.00 | -1.97 | 0.66 |
| Avg | 1,296.78 | 942.43 | 909.60 | 926.17 | 1,346.78 | 1.82 | -4.97 | 2.77 |

It can be observed from Table 9 that ILSFC-SP succeeds to find best-known solutions in instance 1 (51) and 12 (101) with an average deviation of $1.82 \%$. It can be seen also that the best solutions obtained by solving the problems with the fuel consumption model reduce on average $4.97 \%$ of the total fuel consumed but they require an average
of $2.77 \%$ of additional distance. For example, fuel resolution of instance 2 (76) saves $6.85 \%$ of fuel consumption against a distance increase of $3.42 \%$.

Table 10 Computational results of ILSFC-SP and deviation comparison between FCVRP and CVRP for Golden et al. (1998) instances

| Problem | FCVRP (fuel oriented) |  | CVRP (distance oriented) |  |  | BK-Dev Fc-Dev Dis |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ILSFC-SP | lated distance | BK | ILSFC-S | Related fuel | (\%) | (\%) | (\%) |
| 1 (240) | 7,663.69 | 5,716.89 | 5,627.54 | 5,637.8 | 8,338.89 | 0.182 | -9.01 | 2.04 |
| 2 (320) | 11,158.3 | 8,506.98 | 8,447.92 | 8,493.77 | 12,199.6 | 0.543 | -9.14 | 1.73 |
| 3 (400) | 14,623.5 | 11,351.1 | 11,036.22 | 11,138.6 | 16,517.4 | 0.928 | -11.47 | 1.91 |
| 4 (480) | 18,832.1 | 14,112.1 | 13,624.52 | 13,857.4 | 19,999.5 | 1.709 | -5.84 | 1.84 |
| 5 (200) | 8,547.1 | 6,844.74 | 6,460.98 | 6,480.89 | 8,909.15 | 0.308 | -7.74 | 4.73 |
| 6 (280) | 11,222.5 | 8,554.22 | 8,412.8 | 8,540.19 | 12,517.4 | 1.514 | -10.34 | 0.16 |
| 7 (360) | 13,711.3 | 10,316.3 | 10,181.75 | 10,195.6 | 14,854.7 | 0.136 | -7.70 | 1.18 |
| 8 (440) | 16,256.4 | 12,215.5 | 11,643.9 | 11,800.1 | 17,362.8 | 1.341 | -6.37 | 3.52 |
| 9 (255) | 863.767 | 637.744 | 580.48 | 626.542 | 932.59 | 7.935 | -8.55 | 1.39 |
| 10 (323) | 1,099.94 | 818.996 | 738.73 | 810.337 | 1,194.45 | 9.693 | -8.87 | 0.40 |
| 11 (399) | 1,380.12 | 1,035.69 | 914.75 | 976.811 | 1,459.95 | 6.784 | -5.47 | 6.03 |
| 12 (483) | 1,663.57 | 1,251.8 | 1,106.33 | 1,194.73 | 1,784.54 | 7.990 | -6.78 | 4.78 |
| 13 (252) | 1,269.65 | 943.99 | 857.19 | 920.487 | 1,370.84 | 7.384 | -6.91 | 3.19 |
| 14 (320) | 1,615.94 | 1,209.41 | 1,080.55 | 1,168.95 | 1,751.43 | 8.181 | -6.29 | 3.93 |
| 15 (396) | 2,000.19 | 1,493.03 | 1,340.24 | 1,387.44 | 2,062.46 | 3.522 | -3.02 | 7.61 |
| 16 (480) | 2,408.24 | 1,769.54 | 1,616.33 | 1,695.98 | 2,522.17 | 4.928 | -4.52 | 4.34 |
| 17 (240) | 1,044.34 | 745.711 | 707.76 | 734.596 | 1,090.9 | 3.792 | -5.11 | 1.30 |
| 18 (300) | 1,487.1 | 1,069.27 | 995.39 | 1,052.55 | 1,544.59 | 5.742 | -5.33 | 2.03 |
| 19 (360) | 2,047.87 | 1,488.29 | 1,366.14 | 1,461.64 | 2,195.72 | 6.990 | -5.28 | 2.03 |
| 20 (420) | 2,760.88 | 2,010.18 | 1,819.99 | 1,944.98 | 2,766.08 | 6.868 | -4.22 | 4.22 |
| Avg | 6,082.825 | 4,604.574 | 4,427.97 | 4,505.970 | 6,568.758 | 1.762 | $-6.898$ | 2.918 |

For the large-scale instances of Golden et al. (1998), the experimental results in Table 10 show that the average deviation is of $1.76 \%$. In addition, the fuel cost deviation between the best solutions found by fuel objective and distance objective decreased by $6.89 \%$ on average, on the other hand, distance deviation increased by an average of $2.91 \%$. This decrease of fuel objective reaches almost $10.34 \%$ for instance 6 (280) against a distance increase of $0.16 \%$.

### 6.4 Comparison of the proposed ILSFC-SP with the literature

In order to evaluate the performance of our approach, we have compared it with the string-model-based simulated annealing SMSA algorithm of Xiao et al. (2012). In this last paper, authors presented the formula of FCR that we are adopting in this study as a load dependent function (Subsection 3.1); they considered the minimisation of fuel consumed for the classical CVRP using this FCR function under two factors (load and distance). For the best of our knowledge, their paper is the only one that considered FCR and tested their approach using benchmark instances of literature as previously mentioned for solving the FCVRP whereas the other existing approaches
cited in Section 2 have been developed either for different extensions of the VRP such as the VRPTW or were based on a different calculation formula of fuel consumption integrating other factors such us speed or road gradient. In addition, most of them have been tested on randomly generated instances or real cases. Hence, in order to make a fair comparison, we decided to compare our approach with SMSA algorithm.

The simulated annealing consists of two phases: the heating and the cooling phases. In the first phase, the incumbent solution is stochastically turned into one of its neighbours and becomes the incumbent solution and the initial temperature $T_{0}$ is defined by the maximum deviation objectives between two neighbours while the cooling phase consists of browsing its neighbourhood space in order to accept or reject a neighbour according to the Metropolis probability until reaching the terminating condition. The temperature is lowered according to the cooling schedule which is defined by $T \times \tau$ where $\tau$ is a constant between 0 and 1 . Xiao et al. (2012) tested their method with two values of $\tau: \tau=0.99$ and $\tau=0.999$.

It is important to mention that the comparison of ILSFC-SP and SMSA approaches focuses only on the quality of solution, i.e., fuel and distance.

### 6.4.1 Comparison of the proposed ILSFC-SP algorithm with SMSA in the case of FCVRP

In this subsection, we report the results obtained by our ILSFC-SP approach and those of SMSA algorithm in Tables 11 and 12. Column 1 presents the problem; the columns mean and min respectively indicates the mean and the minimum values obtained among ten runs. Fc1-Dev (\%) is the deviation percentage of the min of ten runs of ILSFC-SP from the min of 100 runs of SMSA when $\tau=0.99$ and Fc2-Dev (\%) is the deviation percentage of the min of ten runs of ILSFC-SP from the min of ten runs of SMSA when $\tau=0.999$.

Table 11 Computational results and comparison with Xiao et al. (2012) on solving Christofides et al. (1979) instances in terms of fuel

| Problem | ILSFC-SP |  | SMSA |  |  |  | Fc1-Dev (\%) Fc2-Dev (\%) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\tau=0.99$ |  | $\tau=0.999$ |  |  |  |
|  | Mean | Min | Mean | Min | Mean | Min |  |  |
| 1 (51) | 746.388 | 746.388 | 756.35 | 751.11 | 751.43 | 751.11 | -0.63 | -0.63 |
| 2 (76) | 1,186 | 1,177.15 | 1,214.62 | 1,181.61 | 1,188.62 | 1,179.53 | -0.38 | -0.20 |
| 3 (101) | 1,158.83 | 1,150.48 | 1,161.07 | 1,147.83 | 1,153.56 | 1,147.83 | 0.23 | 0.23 |
| 4 (151) | 1,458.32 | 1,452.77 | 1,471.71 | 1,449.81 | 1,461.69 | 1,452.88 | 0.20 | -0.01 |
| 5 (200) | 1,865.97 | 1,859.74 | 1,879.66 | 1,842.77 | 1,865.3 | 1,844.87 | 0.92 | 0.81 |
| 11 (121) | 1,516.21 | 1,516.21 | 1,529.93 | 1,514.46 | 1,516.42 | 1,513.48 | 0.12 | 0.18 |
| 12 (101) | 1,174.99 | 1,174.7 | 1,176.1 | 1,174.02 | 1,175.59 | 1,174.02 | 0.06 | 0.06 |
| Avg | 1,300.96 | 1,296.78 | 1,312.78 | 1,294.52 | 1,301.80 | 1,294.82 |  |  |

Notes: Bold indicates the solutions obtained by ILSFC-SP better or equal to SMSA ( $\tau=0.99$ and $\tau=0.999$ ) and italics represent ILSFC-SP values which are better than SMSA in only one case of $\tau(\tau=0.99$ or $\tau=0.999)$.

Table 12 Computational results and comparison with Xiao et al. (2012) on solving Golden et al. (1998) instances in terms of fuel

| Problem | ILSFC-SP |  | SMSA |  |  |  | Fcl-Dev (\%) Fc2-Dev (\%) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\tau=0.99$ |  | $\tau=0.999$ |  |  |  |
|  | Mean | Min | Mean | Min | Mean | Min |  |  |
| 1 (240) | 7,686.05 | 7,663.69 | 7,772.92 | 7,694.06 | 7,714.29 | 7,683.95 | -0.39 | -0.26 |
| 2 (320) | 11,239.93 | 11,158.3 | 11,361.27 | 11,174.2 | 11,195.02 | 11,172.7 | -0.14 | -0.13 |
| 3 (400) | 14,910.3 | 14,623.5 | 15,121.38 | 14,647.01 | 14,566.73 | 14,497.64 | -0.16 | 0.87 |
| 4 (480) | 18,832.1 | 18,832.1 | 19,412.56 | 18,825.48 | 18,605.37 | 18,327.03 | 0.035 | 2.756 |
| 5 (200) | 8,592.76 | 8,547.1 | 8,665.07 | 8,561.53 | 8,576.91 | 8,561.53 | -0.17 | -0.17 |
| 6 (280) | 11,222.5 | 11,222.5 | 11,443.51 | 11,105.98 | 11,121.04 | 11,102.22 | 1.049 | 1.083 |
| 7 (360) | 13,711.3 | 13,711.3 | 13,895.87 | 13,437.42 | 13,477.07 | 13,422.16 | 2.038 | 2.154 |
| 8 (440) | 16,256.4 | 16,256.4 | 16,694.53 | 16,300.74 | 16,098.60 | 15,928.26 | -0.272 | 2.060 |
| 9 (255) | 8,66.21 | 863.767 | 874.47 | 854.14 | 858.34 | 850.8 | 1.13 | 1.52 |
| 10 (323) | 1,103.68 | 1,099.94 | 1,113.55 | 1,096.34 | 1,090.85 | 1,083 | 0.33 | 1.56 |
| 11 (399) | 1,380.12 | 1,380.12 | 1,393.83 | 1,365.80 | 1,360.20 | 1,352.32 | 1.048 | 2.056 |
| 12 (483) | 1,663.57 | 1,663.57 | 1,699.33 | 1,660.77 | 1,661.07 | 1,630.81 | 0.169 | 2.009 |
| 13 (252) | 1,274.76 | 1,269.65 | 1,296.67 | 1,269.73 | 1,269.37 | 1,261.93 | -0.01 | 0.61 |
| 14 (320) | 1,622.70 | 1,615.94 | 1,637.93 | 1,613.61 | 1,604.83 | 1,595.48 | 0.14 | 1.28 |
| 15 (396) | 2,000.19 | 2,000.19 | 2,033.93 | 2,005.26 | 1,987.76 | 1,970.43 | -0.253 | 1.510 |
| 16 (480) | 2,408.24 | 2,408.24 | 2,470.80 | 2,419.57 | 2,408.72 | 2,391.12 | -0.468 | 0.716 |
| 17 (240) | 1,048.31 | 1,044.34 | 1,057.42 | 1,038.63 | 1,033.88 | 1,027.21 | 0.55 | 1.67 |
| 18 (300) | 1,492.45 | 1,487.1 | 1,494.16 | 1,476.8 | 1,469.97 | 1,462.31 | 0.70 | 1.70 |
| 19 (360) | 2,053.74 | 2,047.87 | 2,055.6 | 2,030.68 | 2.014.26 | 2,007.62 | 0.85 | 2.00 |
| 20 (420) | 2,767.47 | 2,760.88 | 2,760.98 | 2,720.2 | 2,699.29 | 2,687.85 | 1.50 | 2.71 |
| Avg | 6,106.64 | 6,082.82 | 6,212.79 | 6,064.90 | 6,040.68 | 6,000.82 |  |  |

Notes: Bold indicates the solutions obtained by ILSFC-SP better or equal to SMSA
( $\tau=0.99$ and $\tau=0.999$ ) and italics represent ILSFC-SP values which are better than SMSA in only one case of $\tau(\tau=0.99$ or $\tau=0.999)$.

Regarding the comparison results on the instances of Christofides et al. (1979) with the literature in Table 11, our algorithm is clearly better than SMSA in terms of mean. Indeed, the mean average of our approach is lower than the mean average of SMSA in both cases of $\tau$. In addition, ILSFC-SP succeeds in finding strict improvements of the best-known results for the instance 1 (51) and the instance 2 (76) and three solutions better than SMSA for $\tau=0.999$ in 1 (51), 2 (76) and 4 (151) instances.

Table 12 shows that for the instances of Golden et al. (1998) the average of means of ILSFC-SP is less than that obtained by SMSA when $\tau=0.99$. Furthermore, strict improvements were found for the three benchmark instances 1 (240), 2 (320) and 5 (200). Moreover, our ILSFC-SP finds eight better solutions than the SMSA when $\tau=$ 0.99 in 1 (240), 2 (320), 3 (400), 5 (200), 8 (440), 13 (252), 15 (396) and 16 (480) instances.

### 6.4.2 Comparison of the proposed ILSFC-SP with SMSA in the case of CVRP

Tables 13 and 14 report the results obtained by our ILSFC-SP approach and those of SMSA algorithm in terms of distance. Column 1 presents the problem; the column Min
is the minimum distance value in ten runs. Dis1-Dev (\%) shows the distance deviation between the min of ten runs of ILSFC-SP and the min of 100 runs of SMSA when $\tau=$ 0.99 whereas Dis2-Dev (\%) shows the distance deviation between the min of ten runs of ILSFC-SP and the min of ten runs of SMSA when $\tau=0.999$.

Table 13 Computational results and comparison with Xiao et al. (2012) on solving Christofides et al. (1979) instances in terms of distance

| Problem | ILSFC-SP |  | SMSA |  |  |  | Fcl-Dev (\%) Fc2-Dev (\%) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\tau=0.99$ |  | $\tau=0.999$ |  |  |  |
|  | Mean | Min | Mean | Min | Mean | Min |  |  |
| 1 (51) | 524.61 | 524.611 | 526.78 | 524.61 | 524.61 | 524.61 | 0.00 | 0.00 |
| 2 (76) | 852.71 | 847.47 | 847.81 | 835.45 | 840.9 | 835.45 | 1.44 | 1.44 |
| 3 (101) | 832.99 | 827.392 | 829.97 | 826.14 | 827.49 | 826.14 | 0.15 | 0.15 |
| 4 (151) | 1,052.92 | 1,049.88 | 1,543.71 | 1,044.15 | 1,038.43 | 1,034.37 | 0.55 | 1.50 |
| 5 (200) | 1,371.94 | 1,367.23 | 1,326.57 | 1,311.23 | 1,316.42 | 1,308.08 | 4.27 | 4.52 |
| 11 (121) | 1,049.03 | 1,047.06 | 1,047.66 | 1,042.12 | 1,043.28 | 1,042.12 | 0.47 | 0.47 |
| 12 (101) | 824.84 | 819.56 | 820.74 | 819.56 | 819.56 | 819.56 | 0.00 | 0.00 |
| Avg | 929.86 | 926.17 | 991.89 | 914.75 | 915.81 | 912.90 | 0.98 | 1.15 |
| Comp. time | 3.9 min | per run | 6 s p | er run | 1.2 min | per run |  |  |

Notes: Bold indicates the solutions obtained by ILSFC-SP better or equal to SMSA
( $\tau=0.99$ and $\tau=0.999$ ) and italics represent ILSFC-SP values which
are better than SMSA in only one case of $\tau(\tau=0.99$ or $\tau=0.999)$.
Due to the difference between computers, data structures, compiler options, etc. we present the computational time in Table 13 only and that is just for indicative purpose.

Table 13 shows that for the instances of Christofides et al. (1979) and in terms of distance travelled, the mean average of our approach is clearly lower than that of SMSA when $\tau=0.99$. Furthermore, our algorithm finds two minimum solutions [1 (51) and 12 (101) instances] equal to those found by SMSA under both values of $\tau$. The rest of instances are slightly worse than SMSA where the average deviation is of $0.98 \%$ when $\tau=0.99$ and $1.15 \%$ when $\tau=0.999$. Concerning the computational time, ILSFC-SP is much time-consuming.

For the instances of Golden et al. (1998) as it is shown in Table 14, our ILSFC-SP finds one better solution compared with SMSA when $\tau=0.99$ in the instance 7 (360) and one better solution when $\tau=0.999$ in the instance 1 (240) where deviations are respectively of $0.10 \%$ and $0.13 \%$.

As it can be observed, results of this section confirm the difference between the consideration of either fuel consumption or distance as an objective to minimise. It is clear that our algorithm is much better for the objective of minimising fuel consumption which is the main purpose of this paper.

## 7 A real case study

A case of a company located in Tunisia has been explored in this paper. The company produces air, oil and fuel filtres for trucks, commercial and industrial vehicles. This company has more than 2,000 customers and it receives purchase orders weekly; each
one which represent an instance consists of a number of filtres of different types; these purchase orders range between 19 and 64 customers. A homogeneous fleet of 4 vehicles of type Fiat Ducato is available at the depot; each vehicle has two constraints to be considered: the maximum weight allowed which is equal to 1,570 kilograms and the maximum allowable volume which is equal to $8 \mathrm{~m}^{3}$. The driver ensures the distribution activity based on their professional experience, which consists in serving the nearest customer at each time, thus the distribution system has no prior study, and it lacks to take it wisely. Therefore for reasons of competitiveness and performance, the company should improve its distribution system.

Table 14 Computational results and comparison with Xiao et al. (2012) on solving Golden et al. (1998) instances in terms of distance

| Problem | ILSFC-SP |  | SMSA |  |  |  | $\begin{gathered} \text { Fcl-Dev } \\ \text { (\%) } \end{gathered}$ | Fc2-Dev <br> (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\tau=0.99$ |  | $\tau=0.999$ |  |  |  |
|  | Mean | Min | Mean | Min | Mean | Min |  |  |
| 1 (240) | 5,669.91 | 5,637.8 | 5,662.23 | 5,628.58 | 5,652.23 | 5,645.17 | 0.16 | -0.13 |
| 2 (320) | 8,516.42 | 8,493.77 | 8,488.92 | 8,453.91 | 8,466.03 | 8,452.72 | 0.47 | 0.49 |
| 3 (400) | 11,199.5 | 11,138.6 | 11,163.08 | 11,045.81 | 11,097 | 11,045.81 | 0.84 | 0.84 |
| 4 (480) | 13,857.4 | 13,857.4 | 13,888.81 | 13,746.61 | 13,725.79 | 13,630.52 | 0.81 | 1.66 |
| 5 (200) | 6,544.13 | 6,480.89 | 6,532.11 | 6,460.98 | 6,460.98 | 6,460.98 | 0.31 | 0.31 |
| 6 (280) | 8,574.12 | 8,540.19 | 8,515.26 | 8,413.82 | 8,451.49 | 8,413.82 | 1.50 | 1.50 |
| 7 (360) | 10,195.59 | 10,195.59 | 10,309.63 | 10,206.05 | 10,230.43 | 10,195.59 | -0.10 | 0.00 |
| 8 (440) | 11,800.1 | 11,800.1 | 11,859.32 | 11,713.59 | 11,703.72 | 11,689.08 | 0.74 | 0.95 |
| 9 (255) | 633.45 | 626.542 | 602.73 | 591.53 | 595.09 | 590.6 | 5.92 | 6.09 |
| 10 (323) | 814.70 | 810.337 | 769.1 | 756.42 | 754.8 | 750.18 | 7.13 | 8.02 |
| 11 (399) | 976.811 | 976.811 | 960.47 | 943.13 | 937 | 931.21 | 3.57 | 4.90 |
| 12 (483) | 1,194.73 | 1,194.73 | 1,173.17 | 1,148.26 | 1,139.52 | 1,127.18 | 4.05 | 5.99 |
| 13 (252) | 924.08 | 920.487 | 886.61 | 872.17 | 872.16 | 869.07 | 5.54 | 5.92 |
| 14 (320) | 1,173.42 | 1,168.95 | 1,125.33 | 1,107.99 | 1,108.14 | 1,101.51 | 5.50 | 6.12 |
| 15 (396) | 1,404.47 | 1,387.44 | 1,399.56 | 1,380.65 | 1,371.65 | 1,363.42 | 0.49 | 1.761 |
| 16 (480) | 1,695.98 | 1,695.98 | 1,697.94 | 1,669.68 | 1,658.03 | 1,646.14 | 1.58 | 3.03 |
| 17 (240) | 738.72 | 734.596 | 719.79 | 713.06 | 712.68 | 710.19 | 3.02 | 3.44 |
| 18 (300) | 1,057.51 | 1,052.55 | 1,028.98 | 1,018.86 | 1,015.08 | 1,006.69 | 3.31 | 4.56 |
| 19 (360) | 1,464.51 | 1,461.64 | 1,412.26 | 1,397.66 | 1,385.38 | 1,377.58 | 4.58 | 6.10 |
| 20 (420) | 1,944.98 | 1,944.98 | 1,898.96 | 1,873.81 | 1,857.85 | 1,849.6 | 3.80 | 5.16 |
| Avg | 4,519.027 | 4,505.970 | 4,504.713 | 4,457.129 | 4,459.753 | 4,442.853 |  |  |

Notes: Bold indicates the solutions obtained by ILSFC-SP better or equal to SMSA ( $\tau=0.99$ and $\tau=0.999$ ) and italics represent ILSFC-SP values which are better than SMSA in only one case of $\tau(\tau=0.99$ or $\tau=0.999)$.

We have built from scratch the VRP necessary database such that the purchase orders of customers and the distances. The company provides for the different products only some details such as weight, length and width and for the customers, it provides only their addresses and the quantity of the purchase orders of each of the required products. Then, based on the given addresses of customers, we determined approximately the matrix of distances between each pair of customers using Google Maps and based on the required
quantity, the length and width of products, we determined the purchase orders, their weights as well as their volumes.

After reviewing the built purchase orders, we noticed that customers and their demands are different only in ten weeks; hereafter they seem to be repeated. Thus, we considered the ten purchase orders as ten real-world CVRP instances.

As for the fuel parameters, the provided vehicles consume diesel as fuel. The unit diesel cost $\left(c_{0}\right)$ is of 1.75 TND ; the diesel consumption rate when the vehicle is totally charged $\left(\rho^{*}\right)$ is 0.15 litre per kilometre and diesel consumption rate when there is no load carried $\left(\rho_{0}\right)$ is 0.11 litre per kilometre.

### 7.1 Solving the case study by ILSFC-SP with FCVRP and CVRP objectives

We have applied our approach to solve a real-world problem by considering both fuel and distance objectives. For each real instance, we have run our algorithm ten times with fuel objective and ten times with distance objective, thus we have $10 \times 2$ solutions for each instance. The results are shown in Table 15. The first column indicates the identifier of the purchase order, the second column is for FCVRP (fuel oriented objective) while the third column is for CVRP (distance oriented) where for each objective the mean, the min cost (the best in ten runs) and the related min cost of ten runs are calculated. The last two columns are fuel cost and distance cost deviations calculated in the same way as in Subsection 6.3.

Table 15 Computational results of ILSFC-SP on solving real-case problem and deviation comparison between FCVRP and CVRP resolution

| Problem | ILSFC-SP |  |  |  |  |  | $\begin{aligned} & \text { Fc-Dev Dis-Dev } \\ & (\%) \quad(\%) \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | FCVRP (fuel oriented) |  |  | CVRP (distance oriented) |  |  |  |  |
|  | Mean | Min | Related distance | Mean | Min | Related fuel |  |  |
| 1 (64) | 158,980.8 | 158,561 | 787.846 | 787.98 | 784.106 | 169,598 | $-6.51$ | 0.48 |
| 2 (49) | 108,715.5 | 108,477 | 541.344 | 542.9 | 541.224 | 109,128 | $-0.60$ | 0.02 |
| 3 (19) | 77,642.6 | 77,642.6 | 382.35 | 382.35 | 382.35 | 77,642.6 | 0.00 | 0.00 |
| 4 (43) | 96,036.9 | 95,903.4 | 482.994 | 482.59 | 482.59 | 98,271.6 | -2.41 | 0.08 |
| 5 (31) | 84,007.6 | 83,978.1 | 423.62 | 423.62 | 423.62 | 83,978.2 | 0.00 | 0.00 |
| 6 (54) | 158,124 | 158,124 | 793.02 | 774.24 | 770.12 | 161,382 | -2.02 | 2.97 |
| 7 (44) | 150,240.5 | 150,179 | 760.69 | 758.9 | 754.69 | 150,407 | -0.15 | 0.80 |
| 8 (60) | 106,671.6 | 106,484 | 512.36 | 511.37 | 510.39 | 109,449 | -2.71 | 0.39 |
| 9 (40) | 146,536.2 | 145,984 | 719.29 | 716.85 | 716.39 | 153,545 | -4.92 | 0.40 |
| 10 (20) | 92,541.3 | 92,541.3 | 472.25 | 472.25 | 472.25 | 92,541.3 | 0.00 | 0.00 |
| Avg | 117,949.70 | 117,787.44 | 587.58 | 585.31 | 583.77 | 120,594.27 |  |  |

It is noticeable that the solutions generated by solving a fuel consumption model have different costs than those generated by solving a distance minimisation model. It is corroborative that a good solution for a fuel consumption problem is not always of good quality in terms of the capacitated problem. This is due to the fact that fuel consumption is affected not only by distance but also by other factors such as the load.

Comparing deviation cost between fuel and distance objectives, we notice that solving an FCVRP model generates solutions that can save fuel consumption up to $6.51 \%$ with a slightly longer distance that gets up to $0.48 \%$ [instance 1 (64)].

### 7.2 Comparison between ILSFC-SP and real-life provided solutions

In this section, a comparison between the solutions obtained by ILSFC-SP and those provided by the company is shown in Tables 16 and 17 in terms of fuel and distance respectively. The first column defines the real-world problem (instance) where the number of customers is between parentheses. The second column presents the cost of the best solutions found by the ILSFC-SP algorithm in ten runs. The third column presents the costs of company solutions. Columns Fc-Dev (\%) and Dis-Dev (\%) indicate respectively the deviations between ILSFC-SP and company solution costs under fuel and distance objectives.

Table 16 Comparison between ILSFC-SP and company solutions in terms of fuel

| Problem | ILSFC-SP | Company solutions | Fc-Dev (\%) |
| :--- | :---: | :---: | :---: |
| $1(64)$ | 158,561 | 201,453 | -21.29 |
| $2(49)$ | 108,477 | 250,554 | -56.71 |
| $3(19)$ | $77,642.6$ | 114,914 | -32.43 |
| $4(43)$ | $95,903.4$ | 187,388 | -48.82 |
| $5(31)$ | $83,978.1$ | 140,214 | -40.11 |
| $6(54)$ | 158,124 | 246,306 | -35.80 |
| $7(44)$ | 150,179 | 193,104 | -22.23 |
| $8(60)$ | 106,484 | 234,679 | -54.63 |
| $9(40)$ | 145,984 | 216,170 | -32.47 |
| $10(20)$ | $92,541.3$ | 123,416 | -25.02 |
| Avg | $117,787.44$ | $190,819.8$ | -36.95 |

Table 17 Comparison between ILSFC-SP and company solutions in terms of distance

| Problem | ILSFC-SP | Company solutions | Dis-Dev (\%) |
| :--- | :---: | :---: | :---: |
| $1(64)$ | 784.106 | 991.516 | -20.92 |
| $2(49)$ | 541.224 | $1,278.09$ | -57.65 |
| $3(19)$ | 382.35 | 564.65 | -32.29 |
| $4(43)$ | 482.59 | 942.394 | -48.79 |
| $5(31)$ | 423.62 | 707.52 | -40.13 |
| $6(54)$ | 770.12 | $1,193.39$ | -35.47 |
| $7(44)$ | 754.69 | $9,83.94$ | -23.30 |
| $8(60)$ | 510.396 | $1,127.78$ | -54.74 |
| $9(40)$ | 716.39 | $1,099.44$ | -34.84 |
| $10(20)$ | 472.25 | 632.85 | -25.38 |
| Avg | 583.77 | 952.16 | -37.35 |

From Tables 16 and 17, it is shown that ILSFC-SP approach explored wisely the search space where it succeeds in finding significantly better solutions for all of the

10 real-world instances compared with company solutions. For the FCVRP objective, deviation ranges between $21.29 \%$ and $56.71 \%$ while for the CVRP objective it ranges between $20.92 \%$ and $57.65 \%$.

In order to illustrate the difference between our algorithm and the company solution, we have represented the problem 3 (19) on a map in Figures 6(a) and 6(b).

From Figures 6(a) and 6(b) although the number of tours that compose ILSFC-SP and the company solutions are equal, the difference in the distribution strategy is clear.

Figure 6 Difference between vehicle trips of the ILSFC-SP solution and the company solution for the third real case problem (see online version for colours)


## 8 Conclusions

With the world growth and the increase of human needs, access to a healthy and balanced environment has become a strategic issue. The green VRP aims to minimise environmental harms that come from the transportation sector. Indeed, it addresses the VRP with objectives that are not only based on economic considerations but it also seeks to minimise the risks of pollution by reducing its origins. One of its contributors is the over-exploitation of fuel consumption in the distribution activities.

In this paper, we have proposed an approach of two-fold; first, we developed a new heuristic based on mathematical programming where we have proposed a new model that serves to reduce the amount of fuel consumed under distance and load factors. Second, a local search meta-heuristic handling destroy and repair operators was applied to the heuristic solutions.

We have compared the performance of our approach with an efficient simulated annealing method of Xiao et al. (2012) on the two well known sets of CVRP benchmark instances such as Christofides et al. (1979) instances and Golden et al. (1998) instances. The algorithm was considered as a fuel consumption minimisation as well as a total distance minimisation. Computational experiments prove that our method compares well for fuel consumption objective since it succeeds to find new best results for many benchmark instances, but slightly lower in terms of minimising distance.

Our intervention was not limited to literature where in addition, we have responded to worldwide directives by providing good solutions in a practical case study of the VRP. The case is a CVRP where vehicles constraints are dilated to include volume restrictions. The proposed approach succeeds to reduce both the amount of fuel consumed and total distance travelled compared with the solutions currently applied by the company.

In terms of fuel, our algorithm saved up to $36 \%$ on average. The deviation reaches in some cases more than $56 \%$. While in terms of distance, it decreased the travelled distance with more than $37 \%$ on average.

As future research, we plan to study different extensions of the VRP such as the case of a heterogeneous fleet. In addition, in order to make the model more realsitic we will integrate additional factors, other than load and distance, that affect fuel consumption such as speed of the vehicles. We plan also to consider a multi-objective model considering fuel consumption, distance and time together.

## References

Andelmin, J. and Bartolini, E. (2017) 'An exact algorithm for the green vehicle routing problem', Transportation Science, Vol. 51, No. 4, pp.1288-1303.
Balinski, M.L. and Quandt, R.E. (1964) 'On an integer program for a delivery problem', Operations Research, Vol. 12, No. 2, pp.300-304.
Ball, M. and Magazine, M. (1981) 'The design and analysis of heuristics', Networks, Vol. 11, No. 2, pp.215-219.
Barth, M. and Boriboonsomsin, K. (2009) 'Energy and emissions impacts of a freeway-based dynamic eco-driving system', Transportation Research Part D: Transport and Environment, Vol. 14, No. 6, pp.400-410.
Barth, M., Younglove, T. and Scora, G. (2005) 'Development of a heavy duty diesel modal emissions and fuel consumption model', Tech. Rep., California Partners for Advanced Transit and Highways, San Francisco, Calif, USA.
Bektas, T., Demir, E. and Laporte, G. (2016) 'Green vehicle routing', International Series in Operations Research \& Management Science, Vol. 226, pp.243-265.
Bektas, T. and Laporte, G. (2011) 'The pollution-routing problem', Transportation Research Part B: Methodological, Vol. 45, No. 8, pp.1232-1250.
Christofides, N., Mingozzi, A. and Toth, P. (1979) 'The vehicle routing problem', in Christofides, N., Mingozzi, A., Toth, P. and Sandi, C. (Eds.): Combinatorial Optimization, pp.315-338, Wiley, Chichester, UK.
Clarke, G. and Wright, J.W. (1964) 'Scheduling of vehicles from a central depot to a number of delivery points', Operations Research, Vol. 12, No. 4, pp.568-581.
Demir, E., Bektas, T. and Laporte, G. (2014) 'A review of recent research on greenroad freight transportation', European Journal of Operational Research, Vol. 237, No. 3, pp.775-793.
Ene, S., Kucukoglu, I., Aksoy, A. and Ozturk, N. (2016) 'A hybrid metaheuristic algorithm for the green vehicle routing problem with a heterogeneous fleet', International Journal of Vehicle Design, Vol. 71, Nos. 1-4, pp.75-102.
Erdogan, S. and Miller-Hooks, E. (2012) 'A green vehicle routing problem', Transportation Research Part E: Logistics and Transportation Review, Vol. 48, No. 1, pp.100-114.
Feng, Y., Zhang, R.Q. and Jia, G. (2017) 'Vehicle routing problems with fuel consumptionand stochastic travel speeds', Mathematical Problems in Engineering, Article ID 6329203 [online] https://doi.org/10.1155/2017/6329203.
Figliozzi, M. (2010) 'Vehicle routing problem for emissions minimization', Transportation Research Record: Journal of the Transportation Research Board, Vol. 2197, No. 1, pp.1-7.
Fisher, M.L. and Jaikumar, R. (1981) 'A generalized assignment heuristic for vehicle routing', Networks, Vol. 11, No. 2, pp.109-124.

Golden, B.L., Wasil, E.A., Kelly, J.P. and Chao, I.M. (1998) 'The impact of metaheuristics on solving the vehicle routing problem: algorithms, problem sets and computational results', Fleet Management and Logistics, pp.33-56, Springer, Boston, MA.
Jemai, J., Zekri, M. and Mellouli, K. (2012) 'An NSGA-II algorithm for the green vehicle routing problem', in Hao, J.K. and Middendorf, M. (Eds.): EvoCOP 2012, Lecture Notes in Computer Science, Springer, Berlin, Heidelberg, p. 7245.
Kara, I., Kara, B.Y. and Yetis, M.K. (2007) 'Energy minimizing vehicle routing problem', in Dress, A., Xu, Y.F. and Zhu, B. (Eds.): COCOA 2007, Lecture Notes in Computer Science, Springer, Berlin, Heidelberg, p. 4616.
Kazemian, I. and Aref, S. (2017) 'A green perspective on capacitated time-dependent vehicle routing problem with time windows', International Journal of Supply Chain and Inventory Management, Vol. 2, No. 1, pp.20-38.
Kuo, Y. (2010) 'Using simulated annealing to minimize fuel consumption for the time dependent vehicle routing problem', Computers \& Industrial Engineering, Vol. 59, No. 1, pp.157-165.
Kuo, Y. and Wang, C.C. (2011) 'Optimizing the VRP by minimizing fuel consumption', Management of Environmental Quality: An International Journal, Vol. 22, No. 4, pp.440-450.
Laporte, G., Nobert, Y. and Desrochers, M. (1985) 'Optimal routing under capacity and distance restrictions', Operations Research, Vol. 33, No. 5, pp.1050-1073.
Li, J. (2012) 'Vehicle routing problem with time windows for reducing fuel consumption', $J C P$, Vol. 7, No. 12, pp.3020-3027.
Lin, C., Choy, K. L., Ho, G.T., Chung, S.H. and Lam, H. (2014) 'Survey of green vehicle routing problem: past and future trends', Expert Systems with Applications, Vol. 41, No. 4, pp.1118-1138.
Lin, J., Zhou, W. and Wolfson, O. (2016) 'Electric vehicle routing problem', Transportation Research Procedia, Vol. 12, pp.508-521.
Mirmohammadi, S., BabaeeTirkolaee, E., Goli, A. and Dehnavi-Arani, S. (2017) 'The periodic green vehicle routing problem with considering of time-dependent urban traffic and time windows', Iran University of Science \& Technology, Vol. 7, No. 1, pp.143-156.
Pisinger, D. and Ropke, S. (2007) 'A general heuristic for vehicle routing problems', Computers \& Operations Research, Vol. 34, No. 8, pp.2403-2435.
Pitera, K., Sandoval, F. and Goodchild, A. (2011) 'Evaluation of emissions reduction in urban pickup systems: heterogeneous fleet case study', Transportation Research Record: Journal of the Transportation Research Board, Vol. 2224, No. 1, pp.8-16.
Rao, W., Liu, F. and Wang, S. (2016) 'An efficient two-objective hybrid local search algorithm for solving the fuel consumption vehicle routing problem', Applied Computational Intelligence and Soft Computing, Article ID 3713918 [online] https://doi.org/10.1155/2016/3713918.
Rezaei, N., Ebrahimnejad, S., Moosavi, A. and Nikfarjam, A. (2019) 'A green vehicle routing problem with time windows considering the heterogeneous fleet of vehicles: two metaheuristic algorithms', European Journal of Industrial Engineering, Vol. 13, No. 4, pp.507-535.
Ropke, S. and Pisinger, D. (2006) 'A unified heuristic for a large class of vehicle routing problems with backhauls', European Journal of Operational Research, Vol. 171, No. 3, pp.750-775.
Schneider, M., Stenger, A. and Goeke, D. (2014) 'The electric vehicle-routing problem with time windows and recharging stations', Transportation Science, Vol. 48, No. 4, pp.500-520.
Solomon, M.M. (1987) 'Algorithms for the vehicle routing and scheduling problems with time window constraints', Operations Research, Vol. 35, No. 2, pp.254-265.
Stützle, T. (1998) Applying Iterated Local Search to the Permutation Flow Shop Problem, Technical Report AIDA-98-04, FG Intellektik, TU Darmstadt, Darmstadt, Germany.
Tayachi, D. and Jendoubi, C. (2018) 'Minimizing fuel consumption in capacitated vehicle routing problem', in Proceedings of the International Conference of the African Federation of Operational Research Socities AFROS 2018, Tunis, Tunisia, July.

Teng, L. and Zhang, Z. (2016) 'Green vehicle routing problem with load factor', Advances in Transportation Studies, Vol. 3, Special Issue, pp.75-82.
Xiao, Y. and Konak, A. (2015) 'A simulating annealing algorithm to solve the green vehicle routing \& scheduling problem with hierarchical objectives and weighted tardiness', Applied Soft Computing, Vol. 34, No. C, pp.372-388.
Xiao, Y., Zhao, Q., Kaku, I. and Xu, Y. (2012) 'Development of a fuel consumption optimization model for the capacitated vehicle routing problem', Computers \& Operations Research, Vol. 39, No. 7, pp.1419-1431.
Yavuz, M. (2017) 'An iterated beam search algorithm for the green vehicle routing problem', Networks, Vol. 69, No. 3, pp.317-328.

