Robust min-norm algorithms for coherent sources DOA estimation based on Toeplitz matrix reconstruction methods

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Abstract: Most of the classical high-resolution algorithms such as ESPRIT, MUSIC or Min-Norm, demonstrate their ability to estimate the directions by which non-coherent signals are arrived on a sensor array. However, the need to enhance this kind of algorithm is becoming increasingly important in order to obtain good estimation also in coherent environments. In this paper, two different algorithms for Direction-of-Arrival (DOA) estimation are devised. These two new algorithms improve the performance of the Min-Norm algorithm by incorporating decorrelation techniques as a tool to overcome coherent source estimation problems. Simulation examples are conducted to validate the robustness and the effectiveness of the new proposed algorithms compared to the conventional Min-Norm high resolution algorithm.

Keywords: array signal processing; DOA estimation; coherent sources; Toeplitz matrix; decorrelation method.

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1 Introduction

Direction-of-Arrival (DOA) estimation is one of the most crucial tasks for numerous classical and modern communication systems. Among the main areas of interest to DOA we mention radio astronomy, radar, sonar, wireless communications, navigation, (Jiang et al., 2013; Krim and Viberg, 1996; Van Trees, 2002; Zheng and Mu, 2020). In the last years, the estimation of DOA has occupied a great place in the array processing area thanks to its solid theoretical basis which is associated with wide practical application diversity. In reality, sophisticated signal processing algorithms play a key role in finding the Angle of Arrival (AOA) of electromagnetic waves which are generated and transmitted from radiating sources, and impinged on one or more antennas (Aounallah et al., 2014; Aounallah, 2018).

In array processing theory, the DOA methods are usually divided into two categories; conventional algorithms which are based on classical beamforming techniques, and subspace-based algorithms which exploit the eigen structure of the input signal matrix (Jeffrey et al., 2008). Additionally, the algorithms of the second class, like Minimum-Norm that will be investigated in this paper, are characterised by their high-resolution capability even in the case that the sources to be estimated are partially correlated. Nevertheless, the performances of the classical subspace decomposition algorithms remain limited for real environment when the signal propagation by multipath are highly correlated and thus causes a coherence limitation (Shan et al., 1985).

To solve the estimation inability problem of subspace-based algorithms in case of coherent signals scenario, numerous techniques have been proposed in literature. The spatial smoothing scheme first suggested by Evans et al. (1981) as a pre-processing method used to decorrelate coherent signals impinging on a uniform linear array. This noteworthy scheme has been then more completely analysed by Shan et al. (1985). Thereafter, it has been shown in the article of Pillai and Kwon (1989) that by reconstituting a smoothed array output covariance matrix which structurally resembles a covariance matrix in some noncoherent condition, eigen-structure algorithms can use this smoothed covariance matrix and become able to precisely estimate directions of arrival irrespective of their correlation. Not long ago, for achieving the purpose of decoherence and restoring the full rank of the received data covariance matrix, a received data matrix has been reconstructed by using a subarray space-time correlation matrix. This idea of space time smoothing algorithm (STSS) has been proposed by Qi and Liu (2021) and Ding et al. (2022) as improved tool to eliminate noise and benefit from the high use of the signal strong correlation in both time and space domains. Consequently, the spatial smoothing decorrelation technique has become more popular over time and has seen various improvements to be able to incorporate into several applications (Changgan and Yumin, 2014; Liu and Vaidyanathan, 2015; Pan et al., 2020; Wen et al., 2022).

On other hand, another kind of decorrelation techniques based on Toeplitz matrices has been alternatively appeared as a sharp rival to the spatial smoothed techniques. In fact, the inexpensive computation of this sort does not cause any loss of resolvability (Pham et al. 2016). The direction finding problem of narrowband source on an equispaced linear array has been addressed by Kung et al. (1986) using Toeplitz's approximation method of stochastic system identification. Low-rank reconstruction of the Toeplitz covariance matrix has been proposed by Liu et al. (2021) to accomplish enhanced DOA estimation performance. The recovered covariance matrix allows the application of subspace-based spectral approaches in coprime array. Bingbing et al. (2021) devised an improved technique to be combined with the ESPRIT algorithm by exploiting all rows of the time-space correlation matrix to recover the Toeplitz matrix. In fact, this decoherence technique exploits strong and weak correlations of signal and noise, respectively, in time and space domains to increase the noise cancellation. As an extension of DOA into two dimensions, Chen et al. (2010) introduced a 2-D ESPRIT-like algorithm that can decorrelate sources by a Toeplitz matrix reconstruction. Likewise, Chen and Zhang (2013) proposed an improved 2D-DOA algorithm based on combination of PM algorithm with Toeplitz Hermitian matrix reconstruction. Also, to fulfil a decoherence purpose and a accurate coherent source DOA estimation of two-dimensional MIMO radar, the paper of Fei et al. (2021) suggested an improved Toeplitz matrix set reconstruction algorithm based on the 2-D creation of Toeplitz class algorithm. Indeed, the whole receiving signal vector is utilised to create two Toeplitz matrix sets including all of the data, as well as their conjugate transposes, resulting in a full-rank matrix.

Basing on the joint diagonalisation structure of a set of Toeplitz matrices, a favourable scheme that does not need to any knowledge of source number has been devised by Qian et al. (2014). This algorithm property allows it to be useful for ractical applications where it is difficult to detect the source number. It is also interesting to note that for coherent scenario, some other approaches like the maximum likelihood algorithms (Choi, 2000) and sparse-representation-based algorithms (Liu et al., 2014) can be usable.

The main contribution that carries our work is summed as follows.

Based on two efficient decorrelation techniques, two different Toeplitz matrices are constructed, by means of which the array signal coherence can be well reduced.

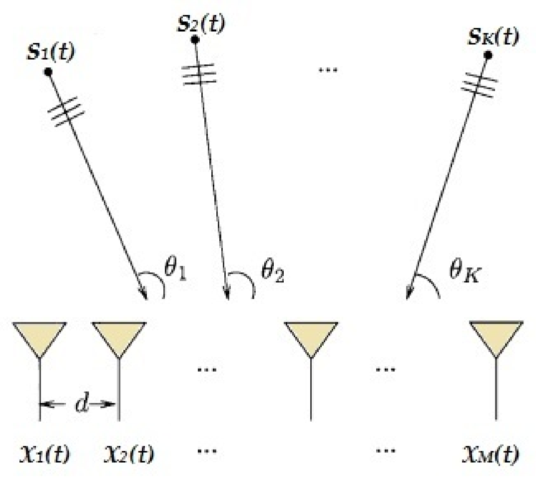
Using the previous reconstituted Toeplitz matrices, two new eigenstructure techniques are derived, by which the estimation of DOAs of coherent sources can be correctly computed.

The present paper is structured as follows. Section 2 develops the signal model which corresponds to a system using linear uniform array. Section 3 is reserved to present DOA estimation algorithms including the standard Min-Norm and the two proposed algorithms. Section 4 is devoted to simulation examples and their results, and finally, Section 5 summarises the main conclusions.

2 Signal model

In this paper section, we introduce the general signal model that is considered for DOA estimation problem. For clarity sake, we consider a Uniform Linear Array (ULA) consists of M isotropic sensors; each two adjacent sensors are equally separated by a distance d. The array receives K narrowband plane waves $s_k(t)$, $1 \le k \le K$, that come at distinct angles θ_k as illustrated in Figure 1.

Figure 1 System model for DOA estimation using a ULA of *M* sensors receiving *K* plane waves



The received signal $x(t)$ at any time t is a $M \times 1$ complex vector often written as:

$$x(t) = \sum_{k=1}^{K} a(\theta_k) s_k(t) + n(t) = A(\theta) S(t) + n(t) \tag{1}$$

where $A(\theta) = [a(\theta_1), a(\theta_2), ..., a(\theta_k)]$ is a $M \times K$ matrix containing the $M \times 1$ array steering vectors $a(\theta_k)$,

$S(t)=\left[s(t_1),s(t_2),...,s(t_k)\right]^T$ is the incident signals vector, and $n(t)$ is the $M\times 1$ complex noise vector modelled as a zero mean Gaussian with a variance σ^2.

For an array of uniform linear geometry, the *k*-th antenna steering vector $a(\theta_k)$ corresponding to the angle of arrivals θ_k is given as:

$$a(\theta_k)=\left[1,e^{2j\pi(d/\lambda)sin\theta_k},...,e^{2j\pi(M-1)(d/\lambda)sin\theta_k}\right]^T \tag{2}$$

where λ is the carrier wavelength and $[\cdot]^T$ is the transpose operator.

The $M\times M$ covariance matrix R of the received signal can be derived as (Krim and Viberg, 1996):

$$\begin{aligned}R&=E\left[x(t)x^H(t)\right]\\&=E\left[\left(A(\theta)S(t)+n(t)\right)\left(A(\theta)S(t)+n(t)\right)^H\right]\\&=A(\theta)E\left[S(t)S^H(t)\right]A^H(\theta)+A(\theta)E\left[S(t)n^H(t)\right]\\&\quad+E\left[n(t)S^H(t)\right]A^H(\theta)+E\left[n(t)n^H(t)\right]\\&=A.R_S.A^H+R_N\end{aligned} \tag{3}$$

where $R_S=E\left[S(t)S^H(t)\right]$ is the signal covariance matrix, and $R_N=E\left[n(t)n^H(t)\right]$ is the noise covariance matrix. $E[\cdot]$ and $(\cdot)^H$ stand for the expectation and the Hermitian transpose, respectively.

Under the uncorrelated signal and noise assumption and the zero-mean noise property, the expectation of the cross-term matrices $\left(R_{SN}=E\left[S(t)n^H(t)\right]\right.$ and $\left.R_{NS}=E\left[n(t)S^H(t)\right]\right)$ between the signal and noise vectors is zero.

Notice that R_S is a diagonal matrix when the signals are uncorrelated, and in this case its rank is equal to the number of sources *K*. The matrix R_S is non-diagonal and non-singular when the signals are partially correlated. The matrix R_S is non-diagonal but singular when some signals are fully correlated (or coherent) (Shan et al., 1985), and in this case R_S has a rank degradation $\left(rank(R_S)<K\right)$.

Under other mathematical matrix writing, the expanded form of the covariance matrix of (3) gives a $M\times M$ coherent data covariance matrix which can be expressed as:

$$R=\begin{bmatrix}r_{xx}(1,1) & r_{xx}(1,2) & \cdots & r_{xx}(1,M)\\ r_{xx}(2,1) & r_{xx}(2,2) & \cdots & r_{xx}(2,M)\\ \vdots & \vdots & \ddots & \vdots\\ r_{xx}(M,1) & r_{xx}(M,2) & \cdots & r_{xx}(M,M)\end{bmatrix} \tag{4}$$

Generally, the true spatial covariance matrix is unknown in practice, and in theory, an estimated sample data covariance matrix for L snapshots should be defined as follows:

$$\hat{R}=\frac{1}{L}\sum_{l=1}^{L}x(l)x^H(l) \tag{5}$$

Once the composite array covariance matrix is defined, the most of narrowband methods can be applied to generate a DOA estimation using this matrix. Accordingly, Eigen-Value Decomposition (EVD) on the said matrix must be necessary for all approaches belonging to the DOA subspace methods category including the conventional Min-Norm. The proposed approaches that will be investigated in this paper will also require an eigenvalue decomposition but on the Toeplitz matrices which will be reconstructed.

The EVD of $\hat{R}$ can be then expressed as:

$$\hat{R}=\sum_{i=1}^{M}\omega_i\cdot e_i\cdot e_i^H=E_S\Omega_S E_S^H+E_N\Omega_N E_N^H \tag{6}$$

where ω_i is the *i*-th eigenvalue associated with the *i*-th eigenvector e_i, and $\Omega=diag(\omega_1,\omega_2,...,\omega_M)$ is a diagonal matrix containing the eigenvalues.

If the M eigenvalues of the matrix $\hat{R}$ are sorted from the largest to the smallest which is equal to the noise variance σ^2, the eigenvectors corresponding to the biggest eigenvalues span the signal subspace E_S which is orthogonal to the noise subspace E_N. This last is spanned by the other eigenvectors corresponding to the smaller eigenvalues.

3 DOA estimation algorithms

The first part of this section reviews briefly the minimum norm DOA method which is based on the eigen decomposition of the covariance matrix into a signal and a noise subspaces, while the second and the third parts explain in detail the transformations that the covariance matrix will be undergone in the context of the proposal of novel algorithms.

3.1 Standard minimum-norm algorithm

The standard Min-Norm algorithm was proposed for the first time by Reddi (Qian et al. (2014) and it saw a development thereafter by Kumaresan and Tufts (Choi, 2000). Min-Norm is referred to as a high resolution algorithm which is based either on the EVD of the auto-covariance matrix or on the Singular-Value Decomposition (SVD) of the signal covariance matrix.

The principle of this method is to determine a vector u_1 belonging to the noise subspace whose first element is unity and its other elements are zero. This minimum norm vector is optimum because it can minimise the norm of the antenna array response (Aounallah, 2017).

Then, the DOAs can be estimated by finding the maxima of the following expression:

$$P_{MN}(\theta)=\frac{1}{\left|a^H(\theta)E_N E_N^H u_1\right|^2} \tag{7}$$

The above-mentioned equation is the pseudo-spectrum of the Min-norm DOA estimation method, and the peak angle of the incident signal can be obtained by performing the peak search on the pseudo-spectrum.

In a completely coherent environment there is a rank degradation of the signal covariance matrix which is exhibited by a deficiency in the signal subspace size. Hence, there is no orthogonality between the noise subspace E_N and the array steering vector $a(\theta)$ of coherent sources, and the determination of the true peak angle from the Min-Norm pseudo-spectrum becomes weak and not guaranteed.

In order to tackle such an algorithm weakness in front of coherent sources, it is possible to successfully compensate for a rank deficiency by proposing a pre-processing scheme applying to the received array signals. The purpose of this pre-processing scheme is to ensure a decorrelation (decoherence) between all this signals before eigendecomposition and estimation stages.

3.2 *First proposed algorithm*

Theoretically and under certain constraints, the Min-Norm algorithm can achieve very high resolution in estimating directions of arrival. Furthermore, the weakness of this algorithm becomes apparent when it comes to estimating a signal with low SNR or even strongly correlated or coherent signal sources. To overcome these limitations, this sub-section proposes and explains the first new algorithm which is based on a transformation of estimated Toeplitz spatial covariance matrix.

We develop a first new min-norm DOA scheme employing a technique of decorrelation. This last is an averaging to construct the elements of a Toeplitz matrix. Whereas, the algorithm proposed here is termed the Averaging Toeplitz for Min-Norm (AT-MN).

The decorrelation technique takes the coherent array covariance matrix R to average the oblique diagonal elements of its lower triangular part. The elements $v_a(i)$ of the $M \times 1$ obtained vector can be calculated according to the following expression:

$$v_a(i) = \frac{1}{M-i+1} \sum_{m=i}^{M} r_{xx}(m, m-i+1), \qquad i = 1, 2, \ldots, M \tag{8}$$

Hence, we can define a resulting $M \times M$ Toeplitz matrix T_{va} as:

$$T_{va} = \begin{bmatrix} v_a(1) & v_a^*(2) & \cdots & v_a^*(M) \\ v_a(2) & v_a(1) & \cdots & v_a^*(M-1) \\ \vdots & \vdots & \ddots & \vdots \\ v_a(M) & v_a(M-1) & \cdots & v_a(1) \end{bmatrix} \tag{9}$$

The last new Toeplitz matrix T_{va} is mainly reconstituted from the averages of diagonals of coherent array covariance matrix R. Thus, the superdiagonal entries of the reconstructed matrix T_{va} are a mixture of independent DOA terms and biased terms. This means that every column or row has one entry containing biased terms which leads to resolve the coherence signals problem, but it can result lower accuracy in the DOA estimation.

Now, we use the decorrelated Toeplitz matrix T_{va} instead of the covariance matrix R for the min-norm algorithm to find the DOA. The matrix T_{va} can be described still by its eigenvalues and eigenvectors as:

$$T_{va} = \sum_{i=1}^{M} \omega_{1i} \cdot e_{1i} \cdot e_{1i}^H = E_{1S} \Omega_{1S} E_{1S}^H + E_{1N} \Omega_{1N} E_{1N}^H \tag{10}$$

where E_{1S} and E_{1N} are the new signal and noise subspaces, respectively.

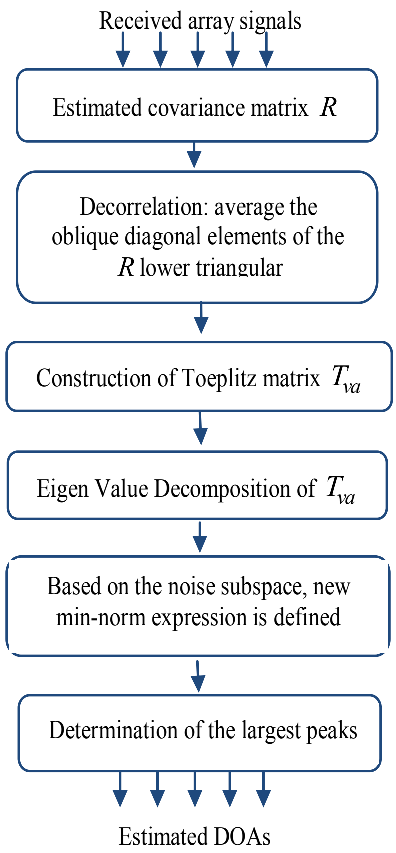
The formulation of the new AT-MN spectrum expression is based on the calculation of a new min-norm vector u_{11} belongs to the noise subspace E_{1N} extracted from the Toeplitz matrix decomposition. Thus, this spectrum is formed as:

$$P_{AT-MN}(\theta) = \frac{1}{\left| a^H(\theta) E_{1N} E_{1N}^H u_{11} \right|^2} \tag{11}$$

The AT-MN algorithm can be classified among the spectral-based approaches. The function expressed in equation (11) can be used to plot the AT-MN spectrum, and the locations of the separated highest peaks are taken as the estimated DOAs.

For simple clarity, the first proposition is summarised in the steps of Algorithm 1, whereas, the flow chart of this proposition for estimating DOA of coherent signals is presented in Figure 2.

Figure 2 The flow chart of our first proposed algorithm



Algorithm 1: AT-MN for estimating DOA of coherent signals	
Step 1:	Estimate the autocorrelation matrix of receiving signals using equation (5)
Step 2:	Compute the average of oblique diagonal elements of the R lower triangular part using equation (8)
Step 3:	Form the Toeplitz matrix T_{va} defined in equation (9)
Step 4:	Apply the EVD on T_{va} to define the signal subspace E_{1S} and the noise subspace E_{1N}
Step 5:	Define new min-norm vector u_{11} from the noise subspace E_{1N}
Step 6:	Estimate the DOAs of coherent signals by using the AT-MN spectrum expression according to equation (11)

3.3 Second proposed algorithm

In this sub-section, a second novel approach is developed for estimating the DOAs of both coherent and non-coherent signals by exploiting another new structure of Toeplitz spatial covariance matrix.

The idea is to make the Min-Norm DOA algorithm works with a reconstituted Toeplitz matrix instead of the spatial covariance matrix. The reconstituted matrix resulting from efficient decorrelation technique which is known as cross-Correlation Vectors Toeplitz (CVT) method. And that's why, the name CVT-Min-Norm is given to our second proposed method.

We remind that the $M \times 1$ received signal vector $x(t)$ can be rewritten as:

$$x(t) = \left[x_1(t), x_2(t), \ldots, x_M(t) \right] \tag{12}$$

Basing on the CVT decorrelation method (Bai et al. 2010), we compute a received cross-correlation vector v_c. This vector comes from the cross-correlation between the first element of $x(t)$, which is the output of the first array sensor, and each element of the received signal vector that represents each sensor output. Therefore, the elements of the received cross-correlation vector v_c can be given as:

$$v_c(i) = x_1(t) x_i^H(t), \quad i = 1, 2, \ldots, M \tag{13}$$

and the received cross-correlation vector is given as:

$$v_c = \left[v_c(1), v_c(2), \ldots, v_c(M) \right] \tag{14}$$

Now, we use the vector expressed in (14) to generate the following constructed Toeplitz matrix:

$$T_{vc} = \begin{bmatrix} v_c(1) & v_c(2) & \cdots & v_c(M) \\ v_c(2) & v_c(1) & \cdots & v_c(M-1) \\ \vdots & \vdots & \ddots & \vdots \\ v_c(M) & v_c(M-1) & \cdots & v_c(1) \end{bmatrix} \tag{15}$$

The reconstruction of a Toeplitz matrix T_{vc} from the first row of R leads to have precise DOA information. In fact, the new matrix row space becomes identical to that of the ideal covariance matrix. The CVT decorrelation method retrieves information on DOA without interaction between sources and thus achieves robust decorrelation for coherent signals. The development of a new Min-Norm algorithm version basing on the decomposition (EVD) of such a decorrelated Toeplitz matrix will lead us to an efficient DOA estimation for coherent sources.

The EVD of T_{vc} can be then expressed as:

$$T_{vc} = \sum_{i=1}^{M} \omega_{2i} \cdot e_{2i} \cdot_{2i}^{H} = E_{2S} \Omega_{2S} E_{2S}^H + E_{2N} \Omega_{2N} E_{2N}^H \tag{16}$$

where E_{2S} and E_{2N} are the signal and noise subspaces obtained thanks to the last matrix eigenvalue decomposition.

The CVT-Min-Norm spatial-spectrum can be formulated as:

$$P_{CVT-MN}(\theta) = \frac{1}{\left| a^H(\theta) E_{2N} E_{2N}^H u_{21} \right|^2} \tag{17}$$

where u_{21} is the minimum norm vector belonging to the noise subspace E_{2N}.

The expression (17) is used to plot the CVT-Min-Norm spectrum. The locations of the peaks on CVT-Min-Norm spectrum indicate the correct DOAs of the incident signals.

Thereupon, we can summarise this second proposition in the steps of Algorithm 2.

Algorithm 2: CVT-MN for estimating DOA of coherent signals	
Step 1:	Estimates the autocorrelation matrix of receiving signals using equation (5)
Step 2:	Computes the received cross-correlation vector v_c via equation (13)
Step 3:	Constructs the Toeplitz matrix T_{vc} defined in equation (15)
Step 4:	Generates the EVD of T_{vc} to determinate the signal subspace E_{2S} and the noise subspace E_{2N}
Step 5:	Define new min-norm vector u_{21} from the noise subspace E_{2N}
Step 6:	Estimate the DOAs of coherent signals by employing the CVT-MN spectrum expression via equation (17)

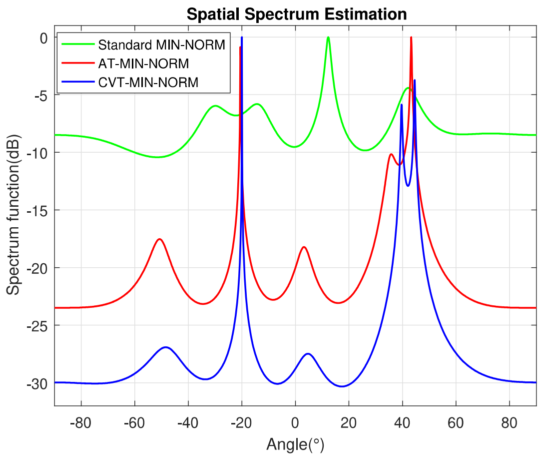
3 Simulation and results

In this section, the results obtained by means of computer simulations are provided to verify the performance of the proposed algorithms by comparing with the original Min-norm algorithm. Simulation examples are carried out for Uniform Linear Array (ULA) of 8 sensors equidistant by half wavelength. We assume that the noise is ideal additive white Gaussian.

Example 1: *Three narrowband coherent signals are considered in the far field with SNR =10 dB. They arrive at the array elements from directions –20°, 40° and 45°. The phases of coherent signals are [* $\pi/5, \pi/4, \pi/4$ *]. We set 300 for the number of snapshots.*

Figure 3 shows the normalised space spectrum versus Direction of Arrival (DOA) which are obtained with the three investigated methods.

Figure 3 Normalised space spectrum versus DOA with three investigated methods



As is clear from Figure 3, both the AT-Min-Norm and the CVT-Min-Norm estimator methods exhibit certain degree of robustness in the presence of the coherent signals, whereas the classical Min-norm estimator yields a poor representation of the spatial spectrum. In other words, due to the coherency of signals, Min-norm spectrum may fail to produce peaks at the DOA locations. The ability of the AT-Min-Norm to resolve spaced signals is good but it can be reduced in the case of closely spaced sources. The results prove also that the CVT-Min-Norm approach performs reasonably well in producing three clear peaks exactly at the corresponding DOA locations. Thus, the CVT-Min-Norm is more robust and has the highest resolution for estimating coherent sources either distant or even closely spaced.

Example 2: *To illustrate the effectiveness of the proposed techniques in terms of Root-Mean-Square Error (RMSE) and probability of resolution, we consider two coherent signals with equal power impinging on the 8-element ULA from sources directions* $\theta_1 = 15°$ *and* $\theta_2 = 10°$. *In this example we change the value of the SNR and we set 1000 for the number of snapshots. At each SNR, 500 Monte Carlo trials are performed to obtain the statistic results.*

The RMSE of the estimated DOAs is defined as:

$$RMSE = \sqrt{\frac{1}{QK}\sum_{q=1}^{Q}\sum_{k=1}^{K}\left(\hat{\theta}_k(q) - \theta_k\right)^2} \quad (18)$$

where $\hat{\theta}_k(q)$ is the estimated angle of θ_k for the q trial, θ_k is the number of Monte Carlo runs, and $K = 2$ is the number of all sources.

The Root-Mean-Square Error (RMSE) versus SNR is depicted in Figure 4. From this Monte Carlo simulation results, we can see that the RMSE of the Min-Norm is of important value comparing with those of the other methods, and this means that this classical algorithm cannot work for a scenario of coherent sources. However, the two proposed algorithms have a reduced RMSE which indicates their accurate estimation performance. In fact, the CVT-Min-Norm algorithm significantly outperforms the AT-Min-Norm with its better decorrelation ability for the coherent sources, and this indicates its better estimation precision over the entire SNR region.

Figure 4 RMSE of DOA estimates versus SNR for two coherent sources, where *L*=1000 snapshots and *Q*=500 Monte-Carlo trials

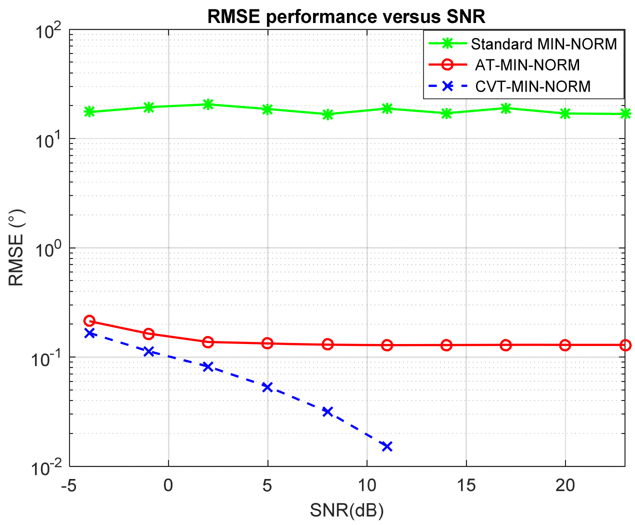


Figure 5 illustrates that the CVT-Min-Norm method has higher resolution DOA estimation than the AT-Min-Norm and the conventional methods, and it obviously outperforms them across a wide range of SNR. Additionally, we can say that in the presence of coherency between sources, serious difficulties which manifest as a loss of resolution can face the functional performance of the conventional Min-norm algorithm.

Example 3: *To assess the performance of the three studied techniques in terms of RMSE and probability of resolution versus the number of snapshots, we consider a scenario with the same parameters as those used for example 2, except that the SNR is fixed at 10 dB and we compute the results for different number of snapshots.*

It can be clearly seen from results of Figure 6 that with the increase of snapshots number the AT-Min-Norm scheme achieves a much better RMSE performance than the conventional method. Meanwhile, the CVT-Min-Norm scheme can give better RMSE than the AT-Min-Norm scheme especially with the increase in the number of snapshots.

Figure 5 Probability of resolution of DOA estimates versus SNR for two coherent sources, where L=1000 snapshots and Q=500 Monte-Carlo trials

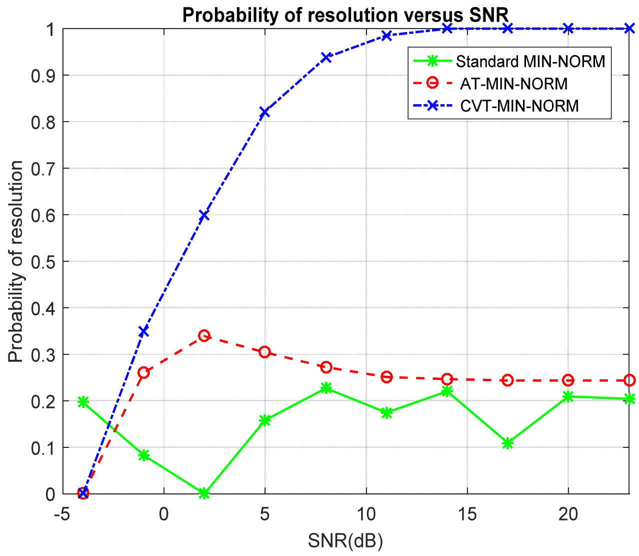
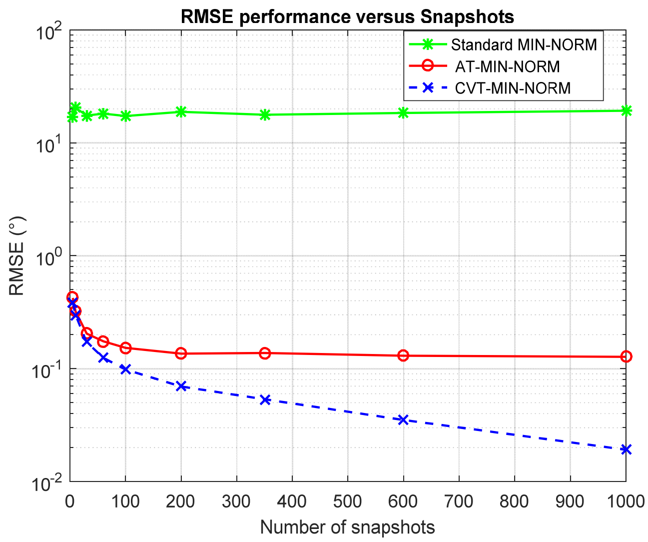
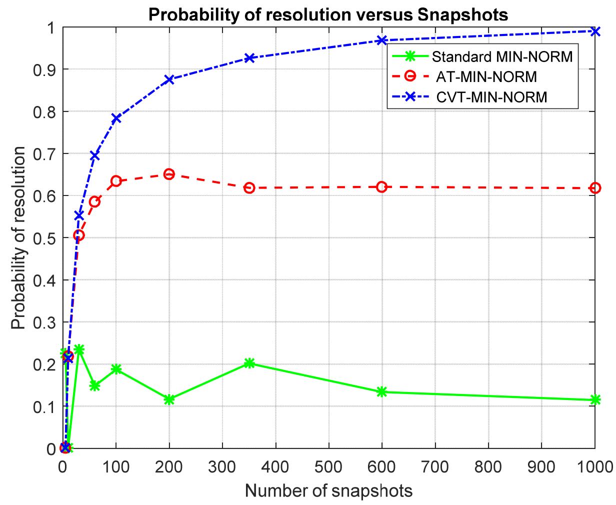


Figure 6 RMSE of DOA estimates versus number of snapshots for two coherent sources, where SNR=10 dB and Q= 500 Monte-Carlo trials



As shown in Figure 7, the CVT-Min-Norm method has higher resolution DOA estimation than the AT-Min-Norm and the conventional methods whatever the number of snapshots. The resolution probability of the AT-Min-Norm increases with the increase of snapshot number and it remains between 60% and 70% when the number of snapshots exceeds 100. The resolution probability of the CVT-Min-Norm increases proportionally with the snapshot number, it reaches over 90% for a number of snapshots that exceeds about 250. The resolution probability of the Min-Norm is weak and not stable.

Figure 7 Probability of resolution of DOA estimates versus number of snapshots for two coherent sources, where SNR =10 dB and Q = 500 Monte-Carlo trials



4 Conclusions

The correctness of the DOA estimation by means of eigenvalue decomposition algorithms like the Min-Norm depends essentially on the non-singularity propriety of the signal covariance matrix. In the case of non-coherent signals, the rank of this matrix is identical to the number of sources and thus non-singularity is ensured. Otherwise, in the case of coherent signals, the rank of this matrix degrades and becomes less than the number of sources, and therefore the non-singularity is not guaranteed.

In this paper, two attractive solutions are proposed for the particular difficulties encountered by the Min-Norm algorithm in DOA estimation of coherent signals. These solutions are a pre-processing scheme applying to the received array signals, the purpose of which is to ensure a decorrelation between all this signals before the estimation step. The performance of the two devised algorithms is evaluated through some simulation examples in terms of spatial spectrum, RMSE and probability of resolution. In many simulations cases, we have found that the named CVT-Min-Norm algorithm is more accurate, robust and it offers better resolution than the called AT-Min-Norm algorithm.

In the future, it is planned to apply the scheme proposed in this paper for other antenna array geometries. Also, it is considered to look at more advanced de-coherence approaches basing on the exploitation of deep learning technology for DOA estimation. In other words, many deep learning models can be integrated with robust DOA estimation schemes, and different evaluation criteria can be derived according to various parameters which affect the estimation to analyse the DOA topic.

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