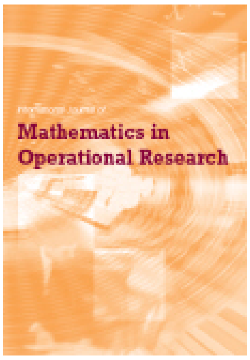

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An imperfect production-inventory model for reworked items with advertisement, time and price dependent demand for non-instantaneous deteriorating item using genetic algorithm

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An imperfect production-inventory model for reworked items with advertisement, time and price dependent demand for non-instantaneous deteriorating item using genetic algorithm

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Abstract: This article deals with an imperfect production inventory system with advertisement, price, and time-dependent demand for a non-instantaneous deteriorating item. Every manufacturing company aims to produce only perfect quality items, but this is not possible in reality. Firstly, the manufacturing system starts producing perfect items, but after some time, it starts producing imperfect items also, because of the manufacturing machine's long run. Perfect products are ready to sell, and some percentages of imperfect products are reworked to become perfect, and the remaining defective products are sold at a cheaper price. An efficiency cost is included to maintain the system efficiency. Considering these conditions, we developed a profit function. This profit function is highly nonlinear. Therefore, we employed a real coded genetic algorithm to obtain the optimum values of production rate, selling price of the perfect quality item, selling price of imperfect item, and inverse efficiency factor to maximise the profit. The model is demonstrated by using numerical example followed by sensitivity analysis.

Keywords: economic production quantity; EPQ; imperfect production; rework; non-instantaneous deterioration; genetic algorithm.

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1 Introduction

Many researchers developed mathematical models on the production-inventory system. The product usually depends on many factors such as raw materials, firm size, various costs, production facilities, etc. Considering all those factors, there is much research work in the field of production inventory systems. Economic production quantity (EPQ) model includes these types of production inventory systems. In the manufacturing sector, the EPQ model is widely used to find the optimal production quantity that maximises the total profit of the manufacturer. Many researchers developed a production model where only perfect items are produced, but this is not possible in reality. A manufacturing process is not always completely perfect; there is always the possibility of damaged items. In the EPQ model, the demand rate is essential to find the optimal production quantity. Some factors affect the demand rate like an advertisement, selling price, inventory stock, time, etc.

Roy et al. (2011) deal with an EOQ model in which defective products have a certain percentage with partial backlogging. Sarkar et al. (2011) considered an EPQ model with imperfect production and stochastic demand under the inflation effect. Sarkar and Moon (2011) deal with an EMQ model with imperfect production and time-varying demand. Mukhopadhyay and Goswami (2014) investigate an EPQ model with imperfect quality items with varying setup costs. Patra and Mondal (2015) developed an EPQ model with variable production, fuzzy demand and selling price depending on production time. Chakrabarty et al. (2017) developed a production inventory model with imperfect production of a single item where money's inflation and time value are considered. Khara et al. (2017) considered an EPQ model with an imperfect production system where demand is a function of selling price and reliability of the product. Manna et al. (2017) considered an EPQ model that produces imperfect and perfect items, and demand depends on advertisement. Patra and Maity (2017) considered a single product model with irregular production rate assuming defective products also have some demand. Patra (2018) developed a production inventory model with defective items where demand depends on advertisement and selling price. Shah et al. (2018) proposed an EPQ model where defective products are either rejected immediately or reworked and sold as good ones with the price and stock-dependent demand. Saha and Sen (2019) formulated an inventory model for deteriorating items where demand is influenced by price and time. Rathore (2019) explored a production reliability model where demand is dependent on selling price and frequency of advertisement. Ruidas et al. (2020) developed an EPQ model with variable production rates, and demand depends on selling price and stock. Udayakumar and Geetha (2019) discussed a production inventory model with a single buyer and vendor under an imperfect production process. Singh (2019) developed a production inventory model for deteriorating products where demand for the product depends on stock, selling price, and time. Khara et al. (2021) formulated an integrated imperfect production system with advertisement dependent demand consisting a manufacturer and retailer.

In the inventory system, deterioration plays an essential part. The damage or decay of the item is known as deterioration. Most researchers deal with instantaneous deterioration, but some items undergo deterioration over time, known as non-instantaneous deterioration items. Teng and Chang (2005) established an EPQ model for deteriorating products where demand is dependent on stock and price. Wu et al. (2006) proposed an optimal replenishment policy where demand is dependent on stock for non-instantaneous deteriorating products with partial backlogging. Sicilia et al. (2014) proposed a deterministic inventory system where the deterioration rate is constant and demand depends on time. Geetha and Udayakumar (2016) formulated an EOQ model for non-instantaneous deteriorating items with price and advertisement-dependent demand. Jaggi (2014) and Maihami and Kamalabadi (2012) proposed an EOQ model for a non-instantaneous deteriorating product with price-dependent demand. Palanivel and Uthayakumar (2015) deal with an EOQ model for a non-instantaneous deteriorating product where demand is a function of advertisement and price. Panda et al. (2019) proposed an EOQ model with price, stock, and advertisement-dependent demand for deteriorating items in a two-warehouse inventory model. Sahoo et al. (2019) deal with an inventory model with a linear deterioration rate where demand is a function of the selling price. Shaikh (2017) developed an inventory model under the mixed type trade credit for deteriorating products with selling price and frequency of advertisement-dependent demand. Dari and Sani (2020) formulated an EPQ model for delayed deteriorating products with time-dependent quadratic demand. Barman et al. (2020) developed a two-layer EPQ model with price-dependent demand for non-instantaneous deteriorating items.

This article expands the EPQ model by considering the imperfect production inventory system, which produces perfect and imperfect products due to various types of factors like labour, raw materials, machinery, etc. Due to these problems, we include the inverse efficiency factor in this paper. Some percentages of defective items are reworked at a cost per unit item to become perfect, and the remaining defective products are sold at a lower cost. In today’s world, advertisement, selling price, and time affect the demand for the product. In this paper, we considered that the demand for the perfect quality items depends on the selling price, advertisement, and time. In the real world, most items have a period in which these items do not deteriorate. After this period, these items start deteriorating, called non-instantaneous deteriorating items. Therefore, we considered non-deteriorating items in our work. To determine the system’s maximum profit, a mathematical model is created. To explain the model numerically, some examples are provided.

The rest of the article is organised as follows: notations and assumption, mathematical formulation for the proposed production inventory model, genetic algorithm and solution methodology, numerical example, sensitivity analysis, conclusions.

Table 1 Brief literature review

Author(s)	*EOQ/ EPQ*	*Deterioration type*	*Demand rate*	*Production rate*	*Defective rate*
Roy et al. (2011)	EOQ	----	Constant	----	Constant
Sarkar et al. (2011)	EPQ	----	Stochastic and uniform	Constant	Weibull distribution
Sarkar and Moon (2011)	EPQ	----	Time	Constant	Time-dependent
Mukhopadhyay and Goswami (2014)	EPQ	----	Constant	Constant	Uniformly distributed random variable
Patra and Mondal (2015)	EPQ	----	Fuzzy	Variable	-----
Chakrabarty et al. (2017)	EPQ	----	Selling price	Constant	Production dependent and random
Khara et al. (2017)	EPQ	-----	Selling price and reliability	Constant	Production dependent
Manna et al. (2017)	EPQ	-----	Advertisement and time	Variable	Production dependent
Patra and Maity (2017)	EPQ	----	Constant	Variable	Constant
Patra (2018)	EPQ	----	Advertisement and selling price	Variable	Production dependent
Shah et al. (2018)	EPQ	----	Stock and price	Constant	Production dependent and random
Saha and Sen (2019)	EOQ	Instantaneous	Time and selling price	----	----
Rathore (2019)	EOQ	Non-instantaneous	Advertisement, time, and selling price	----	----
Ruidas et al. (2020)	EPQ	---	Stock and selling price	Variable	Constant
Udayakumar and Geetha (2019)	EPQ	----	Constant	Constant	Probability density function
Singh (2019)	EPQ	Instantaneous	Stock, selling price and time	Constant	----
Khara et al. (2021)	EPQ	----	Advertisement	Constant	Reliability

Table 1 Brief literature review (continued)

Author(s)	*EOQ/ EPQ*	*Deterioration type*	*Demand rate*	*Production rate*	*Defective rate*
Teng and Chang (2005)	EPQ	Instantaneous	Stock and price	Constant	----
Wu et al. (2006)	EOQ	Non-instantaneous	Inventory	----	----
Sicilia et al. (2014)	EOQ	Instantaneous	Time	----	----
Geetha and Udayakumar (2016)	EOQ	Non-instantaneous	Selling price and advertisement cost	----	----
Jaggi (2014)	EOQ	Non-instantaneous	Selling price	----	----
Maihami and Kamalabadi (2012)	EOQ	Non-instantaneous	Selling price and time	----	----
Palanivel and Uthayakumar (2015)	EOQ	Non-instantaneous	Selling price and advertisement cost	---	----
Panda et al. (2019)	EOQ	Instantaneous	Advertisement, Inventory and selling price	----	----
Sahoo et al. (2019)	EOQ	Instantaneous	Selling price	----	----
shaikh (2017)	EOQ	Instantaneous	Selling price and advertisement	----	----
Dari and Sani (2020)	EPQ	Non-instantaneous	Time	Constant	----
Barman et al. (2020)	EPQ	Non-instantaneous	Selling price	Constant	----
This paper	EPQ	Non-instantaneous	Advertisement, time, and selling price	Variable	Production dependent

The above researchers have not yet paid attention to the EPQ model with variable production rates and imperfect production inventory system with non-instantaneous deteriorating items and rework of defective items.

To address the above issues and complete the literature gap, this paper is presented.

2 Notations and assumptions

The proposed model has been developed by using the following assumptions and notations.

2.1 Notations

Notations	*Description*
P	Production rate per unit time
C_p	Cost of production per unit item
C_0	Setup cost
C_h	Holding cost per unit item per unit time
C_E	Rate of efficiency cost
C_d	Deterioration cost per unit item
C_r	Reworking cost on per unit defective item to become perfect
C_a	Advertisement cost
A	Frequency of advertisement
α	The shape parameter of advertisement
λ	Inverse efficiency
I_1	Inventory level of perfect items
I_2	Inventory level of defective items
s	The selling price of perfect items
s_1	The selling price of defective items
D_1	Demand rate of perfect items
D_2	Demand rate of defective items
θ	Deterioration rate
β	Percentage of defective items reworked to become perfect
ψ	Defective rate
t_5	Total business period

2.2 Assumptions

1 A single product is considered in the production system.

2 Shortages are not allowed.

3 It is assumed that the product has no deterioration during a specific time period. After this period, the product starts to deteriorate at a constant rate θ.

4 The time horizon is finite.

5 The starting and terminal stock level are zero.

6 The rate of production has been considered as a variable. At the start of the cycle, the production rate is constant as the system's factors are in good condition. After some time, the system will be insufficient, so the production decreases gradually, and the system starts producing defective items. So, it is necessary to increase the efficiencies of all the factors to meet the customer's demand. To increase the efficiencies, a new variable λ is included, known as Inverse efficiency. Considering this fact, production rate P is taken as

$$P = \begin{cases} P & 0 \le t \le t_2 \\ Pe^{-\lambda(t-t_2)} & t_2 \le t \le t_3 \end{cases}$$

where $\lambda = \dfrac{1}{E}$.

7 The defective rate of produced items is proportional to the rate of production. Therefore, the defective rate (ψ) is taken as $\psi = \psi_1 Pe^{-\lambda(t_3-t_2)}$ where ψ_1 is positive constant.

8 The demand rate of the perfect quality item is considered a function of advertisement frequency, selling price, and time. It has been seen that the advertisement of the product helps to increase the sale of the product. Also, the selling price affects the product's demand. If the selling price of the item increases, then the demand for the item decreases. Sometimes, demand is also time-dependent. Considering these facts, the demand rate of the perfect quality item has been taken as

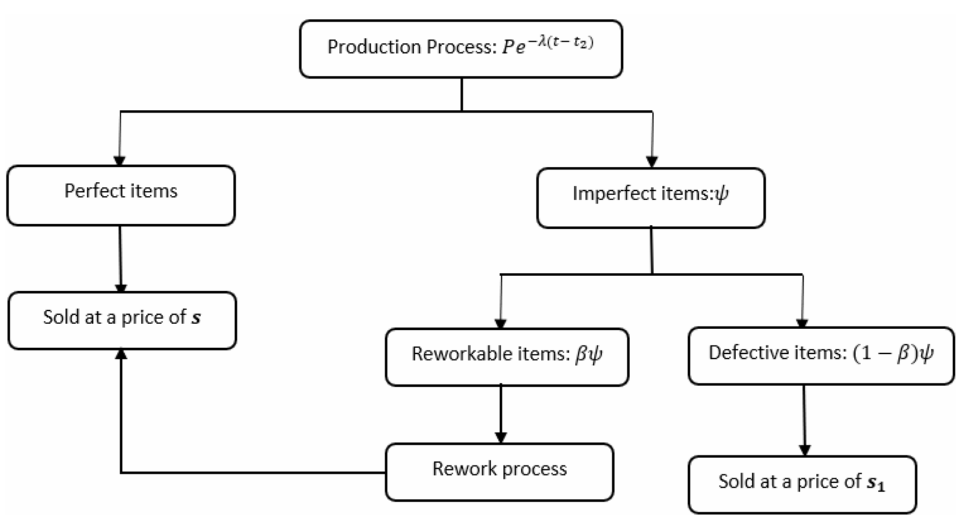
$$D_1(A, t, s) = \frac{A^{\alpha}\left(D_0 + \eta t\right)}{s^m}$$

where D_0 is the initial demand, η is the rate at which demand changes with time, and m is the price elasticity index.

9 Defective items also have some demand $D_2 = \dfrac{D}{s_1^n}$ where n is the price index.

10 In this paper, we neglect the second and higher powers of θ as the value of θ is small.

Figure 1 A schematic diagram of our proposed model



3 Mathematical formulation for the proposed production inventory model

At time $t = 0$, the production inventory system starts producing a single item. Initially, the production is constant P and continued up to time t_2. There was no deterioration during the time period $[0, t_1]$, so the stock level decreased due to demand only. After that period, the product starts to deteriorate at a constant rate. Further, during the period $[t_1, t_2]$, the stock level reduces because of the combined effect of demand and deterioration. As per our assumption, the production starts decreasing after time t_2, and at time t_3 the production stops. During the time interval $[t_2, t_3]$, the system starts producing some defective items due to unavoidable reasons. After regular production, β percent of defective items are repaired in the time interval $[t_3, t_4]$ to become perfect. During the time period $[t_4, t_5]$, the stock level gradually reduces due to customers' demand and deterioration, and at the end of the time period $t = t_5$, the stock level drops to zero.

Now, if $I_1(t)$ is the stock level of perfect quality items at time t. The Differential equations of the proposed model, according to above-mentioned assumptions, can be written as follows:

$$\frac{dI_1(t)}{dt} = P - D_1, \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_1(t)}{dt} = P - D_1 - \theta I_1(t), \qquad t_1 \le t \le t_2 \tag{2}$$

$$\frac{dI_1(t)}{dt} = Pe^{-\lambda(t-t_2)} - D_1 - \theta I_1(t) - \psi, \qquad t_2 \le t \le t_3 \tag{3}$$

$$\frac{dI_1(t)}{dt} = \beta\psi - D_1 - \theta I_1(t), \qquad t_3 \le t \le t_4 \tag{4}$$

$$\frac{dI_1(t)}{dt} = -D_1 - \theta I_1(t), \qquad t_4 \le t \le t_5 \tag{5}$$

With boundary conditions $I_1(0) = 0$, $I_1(t_5) = 0$.

It is assumed that defective products also have some demand. Since there is the production of defective products during the time period $[t_2, t_3]$ at a rate of ψ. During the time period $[t_3, t_4]$, β percent of defective products are repaired, and the remaining $(1 - \beta)$ percent of defective products are sold at a lower cost. During time period $[t_4, t_5]$, the stock level drops to zero.

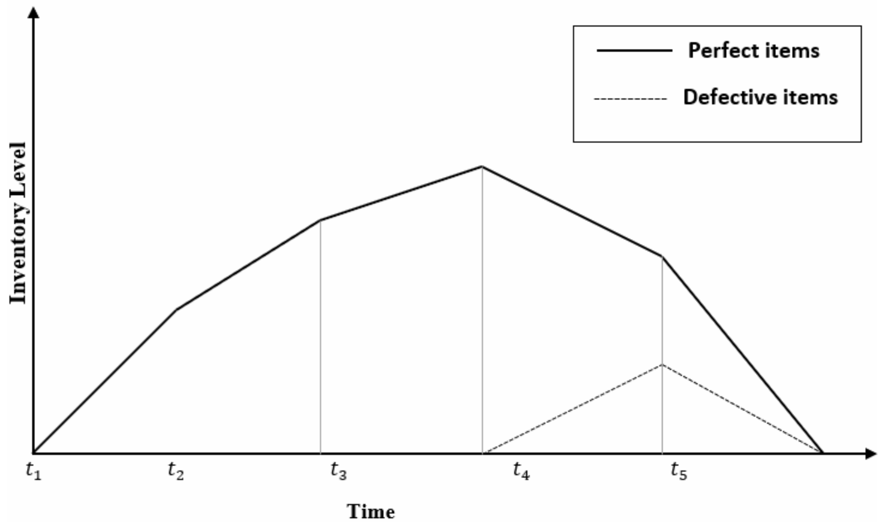
Now, if $I_2(t)$ is the stock level of defective products at time t. According to the assumptions described above, The Differential equations of the proposed model can be written as follows:

$$\frac{dI_2(t)}{dt} = (1 - \beta)\psi - D_2, \qquad t_3 \le t \le t_4 \tag{6}$$

$$\frac{dI_2(t)}{dt} = -D_2, \qquad t_4 \le t \le t_5 \tag{7}$$

With boundary conditions $I_2(t_3) = \psi$, $I_2(t_5) = 0$.

Figure 2 Graphical representation of inventory model



For the interval [0, t_1], integrating the differential equation (1) and using boundary conditions, we have

$$I_1(t) = \left(P - \frac{A^\alpha D_0}{s^m}\right)t - \frac{A^\alpha \eta t^2}{2s^m} \tag{8}$$

For the interval [t_1, t_2], integrating the differential equation (2), we have

$$\begin{aligned} I_1(t) &= \frac{1}{6}A^\alpha s^{-m} t^3 \eta\theta + t\left(P - A^\alpha s^{-m} D_0\right) + \frac{1}{2}t^2\left(-A^\alpha s^{-m}\eta - P\theta + A^\alpha s^{-m}\theta D_0\right) \\ &+ \frac{1}{2}\theta\left(P - A^\alpha s^{-m} D_0\right)t_1^2 - \frac{1}{6}A^\alpha s^{-m}\eta\theta t_1^3 \end{aligned} \tag{9}$$

On solving the differential equation (3) for the interval [t_2, t_3], we get

$$\begin{aligned} I_1(t) &= -\left(\psi + \frac{A^\alpha D_0}{s^m} + \frac{P\theta}{\lambda}\right)t + \left(\frac{\psi\theta}{2} + \frac{A^\alpha \theta D_0}{2s^m} - \frac{A^\alpha \eta}{2s^m}\right)t^2 + \frac{A^\alpha \eta\theta t^3}{6s^m} \\ &+ \frac{Pe^{-\lambda(t-t_2)}}{\lambda}\left(-1 - \frac{\theta}{\lambda}\right) + \frac{P}{\lambda} + \frac{P\theta}{\lambda^2} + \left(P + \frac{P\theta}{\lambda} + \psi\right)t_2 \\ &+ (P+\psi)\frac{\theta t_2^2}{2} + \left(P - \frac{A^\alpha D_0}{s^m}\right)\frac{\theta t_1^2}{2} - \frac{A^\alpha \eta\theta t_1^3}{6s^m} \\ &- (P+\psi)\theta t t_2 \end{aligned} \tag{10}$$

The solution of differential equation (4) for the interval [t_3, t_4] is

$$I_1(t) = \frac{1}{6}A^{\alpha}s^{-m}t^3\eta\theta + \frac{P\theta}{\lambda^2} + \frac{P}{\lambda} + t\left(-\frac{P\theta}{\lambda} + \beta\psi - A^{\alpha}s^{-m}D_0\right)$$
$$+\frac{1}{2}t^2\left(-A^{\alpha}s^{-m}\eta - \beta\theta\psi + A^{\alpha}s^{-m}\theta D_0\right) + \frac{1}{2}\theta\left(P - A^{\alpha}s^{-m}D_0\right)t_1^2$$
$$-\frac{1}{6}A^{\alpha}s^{-m}\eta\theta t_1^3 - t\theta(P+\psi)t_2 + \left(P + \frac{P\theta}{\lambda} + \psi\right)t_2 + \frac{1}{2}\theta(P+\psi)t_2^2 \tag{11}$$
$$-(\psi + \beta\psi)t_3 + t\theta\left(\psi + \beta\psi\right)t_3 - \frac{1}{2}(\theta\psi + \beta\theta\psi)t_3^2$$
$$+\frac{e^{-\lambda(-t_2+t_3)}P\left(-1-\theta\left(-t+\frac{1}{\lambda}+t_3\right)\right)}{\lambda}$$

For the interval $[t_4, t_5]$, integrating the differential equation (5) and using boundary conditions, we get

$$I_1(t) = \frac{1}{6}A^{\alpha}s^{-m}t^3\eta\theta - A^{\alpha}s^{-m}tD_0 + \frac{1}{2}t^2\left(-A^{\alpha}s^{-m}\eta + A^{\alpha}s^{-m}\theta D_0\right) + A^{\alpha}s^{-m}D_0t_5$$
$$-A^{\alpha}s^{-m}t\theta D_0t_5 - \frac{1}{2}A^{\alpha}s^{-m}t\eta\theta t_5^2 + \frac{1}{2}\left(A^{\alpha}s^{-m}\eta + A^{\alpha}s^{-m}\theta D_0\right)t_5^2 \tag{12}$$
$$+\frac{1}{3}A^{\alpha}s^{-m}\eta\theta t_5^3$$

By using boundary conditions and integrating the differential equation (6) for the interval $[t_3, t_4]$, we get

$$I_2(t) = \left(\psi - \beta\psi - \frac{D}{s_1^n}\right)t + \psi - \left(\psi - \beta\psi - D_2\right)t_3 \tag{13}$$

The solution of the differential equation (7) for the interval $[t_4, t_5]$ and by using boundary conditions, we have

$$I_2(t) = \frac{D}{s_1^n}(t_5 - t) \tag{14}$$

The various costs related to the proposed model are the cost of production, setup cost, deterioration cost, efficiency cost, holding cost, reworking cost, and advertisement cost.

All of these costs are calculated as follows:

The total cost of production is determined by

$$\begin{aligned} PC &= C_p\left[\int_0^{t_2} Pdt + \int_{t_2}^{t_3} Pe^{-\lambda(t-t_2)}dt\right] \\ &= C_p\left[Pt_2 + \frac{P - Pe^{-\lambda(t_3-t_2)}}{\lambda}\right] \end{aligned} \tag{15}$$

Setup cost is given by

$$SC = C_0 \tag{16}$$

Total deterioration cost is obtained as follows by neglecting higher-order terms of θ

$$
\begin{aligned}
DC &= C_d\left[\int_{t_1}^{t_2}\theta I_1(t)dt+\int_{t_2}^{t_3}\theta I_1(t)dt+\int_{t_3}^{t_4}\theta I_1(t)dt+\int_{t_4}^{t_5}\theta I_1(t)dt\right] \\
&= C_d\left[\frac{1}{6\lambda^2}s^{-m}\theta\left(-6Ps^m+6e^{\lambda(t_2-t_3)}Ps^m-3\lambda^2\left(Ps^m-A^{\alpha}D_0\right)t_1^2\right.\right. \\
&\quad +A^{\alpha}\eta\lambda^2t_1^3-3s^m\lambda^2(P+\psi)t_2^2+6e^{\lambda(t_2-t_3)}Ps^m\lambda t_3 \\
&\quad +3s^m\lambda^2\psi t_3^2+3s^m\beta\lambda^2\psi t_3^2+6Ps^m\lambda t_4 \\
&\quad -6e^{\lambda(t_2-t_3)}Ps^m\lambda t_4-6s^m\lambda^2\psi t_3t_4-6s^m\beta\lambda^2\psi t_3t_4 \\
&\quad +3s^m\beta\lambda^2\psi t_4^2+6s^m\lambda t_2\left(-P+\lambda(P+\psi)t_4\right) \\
&\quad -6A^{\alpha}\lambda^2D_0t_4t_5+3A^{\alpha}\lambda^2D_0t_5^2-3A^{\alpha}\eta\lambda^2t_4t_5^2 \\
&\quad \left.\left.+2A^{\alpha}\eta\lambda^2t_5^3\right)\right]
\end{aligned}
\tag{17}
$$

Total efficiency cost is given by

$$
\begin{aligned}
EFC &= C_E\left[\int_{t_2}^{t_3}Pe^{-\lambda(t-t_2)}dt\right] \\
&= \frac{C_EP\left(1-e^{-\lambda(t_3-t_2)}\right)}{\lambda}
\end{aligned}
\tag{18}
$$

Total holding cost is obtained as

$$
\begin{aligned}
HC &= C_h\left[\int_0^{t_5}I_1(t)dt+\int_{t_3}^{t_5}I_2(t)dt\right] \\
&= C_h\left[\int_0^{t_1}I_1(t)dt+\int_{t_1}^{t_2}I_1(t)dt+\int_{t_2}^{t_3}I_1(t)dt+\int_{t_3}^{t_4}I_1(t)dt\right. \\
&\quad \left.+\int_{t_4}^{t_5}I_1(t)dt+\int_{t_3}^{t_4}I_2(t)dt+\int_{t_4}^{t_5}I_2(t)dt\right] \\
&= C_h\left[\frac{1}{24}\left(-\frac{24P\theta}{\lambda^3}+\frac{24e^{\lambda(t_2-t_3)}P\theta}{\lambda^3}+\frac{24P}{\lambda^2}+\frac{24e^{\lambda(t_2-t_3)}P}{\lambda^2}+3A^{\alpha}s^{-m}\eta\theta t_1^4\right.\right. \\
&\quad -4\theta(P+\psi)t_2^3+\frac{24e^{\lambda(t_2-t_3)}P\theta t_3}{\lambda^2}+\frac{24e^{\lambda(t_2-t_3)}Pt_3}{\lambda}-24\psi t_3 \\
&\quad +\frac{12e^{\lambda(t_2-t_3)}P\theta t_3^2}{\lambda}+24\psi t_3^2-12s_1^{-n}Dt_3^2+4\theta\psi t_3^3+4\beta\theta\psi t_3^3+\frac{24P\theta t_4}{\lambda^2} \\
&\quad -\frac{24e^{\lambda(t_2-t_3)}P\theta t_4}{\lambda^2}+\frac{24Pt_4}{\lambda}-\frac{24e^{\lambda(t_2-t_3)}Pt_4}{\lambda}+24\psi t_4 \\
&\quad +12s^{-m}\theta\left(Ps^m-A^{\alpha}D_0\right)t_1^2t_4-\frac{24e^{\lambda(t_2-t_3)}P\theta t_3t_4}{\lambda}-48\psi t_3t_4+24s_1^{-n}Dt_3t_4 \\
&\quad -12\theta\psi t_3^2t_4-12\beta\theta\psi t_3^2t_4-\frac{12P\theta t_4^2}{\lambda}+\frac{12e^{\lambda(t_2-t_3)}P\theta t_4^2}{\lambda}+12\psi t_4^2+12\theta\psi t_3t_4 \\
&\quad +12\beta\theta\psi t_3t_4^2-4\beta\theta\psi t_4^3-4s^{-m}\theta t_1^3\left(2Ps^m-2A^{\alpha}D_0+A^{\alpha}\eta t_4\right)
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{12t_2\left(2P(\theta+\lambda)-s\lambda(P(\theta+\lambda)+\lambda\psi)t_4+\theta\lambda^2(P+\psi)t_4^2\right)}{\lambda}\\
&-24A^\alpha s^{-m}D_0t_4t_5-24s_1^{-n}Dt_4t_5+12A^\alpha s^{-m}\theta D_0t_4^2t_5+12A^\alpha s^{-m}D_0t_5^2\\
&+12s_1^{-n}Dt_5^2-12A^\alpha s^{-m}\eta t_4t_5^2-12A^\alpha s^{-m}\theta D_0t_4t_5^2+6A^\alpha s^{-m}\eta\theta t_4^2t_5^2\\
&+8A^\alpha s^{-m}\eta t_5^3+4A^\alpha s^{-m}D_0t_5^3-8A^\alpha s^{-m}\eta\theta t_4t_5^3+3A^\alpha s^{-m}\eta\theta t_5^4\Big)\Big]
\end{aligned}
\tag{19}
$$

The total reworking cost is

$$
\begin{aligned}
RWC&=C_r\left[\int_{t_3}^{t_4}\beta\psi dt\right]\\
&=\beta\psi C_r\left(t_4-t_3\right)
\end{aligned}
\tag{20}
$$

Total advertisement cost is given by

$$
AC=AC_a \tag{21}
$$

Now, the total sales revenue generated by selling perfect quality products to the customers at a price of s per item and by selling defective products to the customers at a price of s_1 per item is given by

$$
\begin{aligned}
SR&=s\left[\int_0^{t_2}Pdt+\int_{t_2}^{t_3}\left(Pe^{-\lambda(t-t_2)}-\psi\right)dt+\int_{t_3}^{t_4}\beta\psi dt\right]+s_1\left[\int_{t_3}^{t_4}(1-\beta)\psi dt\right]\\
&=s\left[Pt_2+\frac{\left(1-e^{-\lambda(t_3-t_2)}\right)P}{\lambda}+(t_2-t_3)\psi+\beta\psi(t_4-t_3)\right]\\
&\quad+s_1\left[(1-\beta)\psi(t_4-t_3)\right]\\
&\quad\left(\text{Replace } \psi \text{ by } \psi_1Pe^{-\lambda(t_3-t_2)} \text{ as per our assumption}\right)
\end{aligned}
\tag{22}
$$

Therefore, the total average profit TP in this proposed system is given by

$$
\begin{aligned}
TP(P,t_3,\lambda)&=SR-PC-SC-HC-DC-EFC-RWC-AC\\
&=\frac{1}{t_5}\left[s\left[Pt_2+\frac{\left(1-e^{-\lambda(t_3-t_2)}\right)P}{\lambda}+\left(\psi_1Pe^{-\lambda(t_{3-t222})}\right)t_2-\left(\psi_1Pe^{-\lambda(t_3-t_2)}\right)t_3\right.\right.\\
&\quad\left.+\beta\left(\psi_1Pe^{-\lambda(t_3-t_2)}\right)(t_4-t_3)\right]+s_1\left[(1-\beta)\left(\psi_1Pe^{-\lambda(t_3-t_2)}\right)(t_4-t_3)\right]\\
&\quad-C_p\left[Pt_2+\frac{P-Pe^{-\lambda(t_3-t_2)}}{\lambda}\right]-C_0-C_h\left[\frac{1}{24}\left(-\frac{24P\theta}{\lambda^3}\right.\right.\\
&\quad+\frac{24e^{\lambda(t_2-t_3)}P\theta}{\lambda^3}-\frac{24P}{\lambda^2}+\frac{24e^{\lambda(t_2-t_3)}P}{\lambda^2}+3A^\alpha s^{-m}\eta\theta t_1^4-4\theta(P+\psi)t_2^3\\
&\quad+\frac{24e^{\lambda(t_2-t_3)}P\theta t_3}{\lambda^2}+\frac{24e^{\lambda(t_2-t_3)}Pt_3}{\lambda}+24\psi t_3+\frac{12e^{\lambda(t_2-t_3)}P\theta t_3^2}{\lambda}\\
&\quad+24\psi t_3^2-12s_1^{-n}Dt_3^2+4\theta\psi t_3^3+4\beta\theta\psi t_3^3+\frac{24P\theta t_4}{\lambda^2}\\
&\quad-\frac{24e^{\lambda(t_2-t_3)}P\theta t_4}{\lambda^2}+\frac{24Pt_4}{\lambda}-\frac{24e^{\lambda(t_2-t_3)}Pt_4}{\lambda}+24\psi t_4
\end{aligned}
$$

$$
\begin{aligned}
&+12s^{-m}\theta\left(Ps^{m}-A^{\alpha}D_0\right)t_1^2t_4-\frac{24e^{\lambda(t_2-t_3)}P\theta t_3t_4}{\lambda}-48\psi t_3t_4\\
&+24s_1^{-n}Dt_3t_4-12\theta\psi t_3^2t_4-12\beta\theta\psi t_3^2t_4-\frac{12P\theta t_4^2}{\lambda}+\frac{12e^{\lambda(t_2-t_3)}P\theta t_4^2}{\lambda}\\
&+12\psi t_4^2+12\theta\psi t_3t_4^2+12\beta\theta\psi t_3t_4^2-4\beta\theta\psi t_4^3\\
&-4s^{-m}\theta t_1^3\left(2Ps^{m}-2A^{\alpha}D_0+A^{\alpha}\eta t_4\right)\\
&+\frac{12t_2^2\left(-P(\theta+\psi)-\lambda\psi+\theta\lambda(P+\psi)t_4\right)}{\lambda}\\
&-\frac{12t_2\left(2P(\theta+\lambda)-2\lambda(P(\theta+\lambda)+\lambda\psi)t_4+\theta\lambda^2(P+\psi)t_4^2\right)}{\lambda^2}\\
&-24A^{\alpha}s^{-m}D_0t_4t_5-24s_1^{-n}Dt_4t_5+12A^{\alpha}s^{-m}\theta D_0t_4^2t_5+12A^{\alpha}s^{-m}D_0t_5^2\\
&+12s_1^{-n}Dt_5^2-12A^{\alpha}s^{-m}\eta t_4t_5^2-12A^{\alpha}s^{-m}\theta D_0t_4t_5^2+6A^{\alpha}s^{-m}\eta\theta t_4^2t_5^2\\
&+8A^{\alpha}s^{-m}\eta t_5^3+4A^{\alpha}s^{-m}\theta D_0t_5^3-8A^{\alpha}s^{-m}\eta\theta t_4t_5^3+3A^{\alpha}s^{-m}\eta\theta t_5^4\Big)\Big]\\
&-C_d\Bigg[\frac{1}{6\lambda^2}s^{-m}\theta\Big(-6Ps^{m}+6e^{\lambda(t_2-t_3)}Ps^{-m}-3\lambda^2\left(Ps^{m}-A^{\alpha}D_0\right)t_1^2\\
&+A^{\alpha}\eta\lambda^2t_1^3-3s^{m}\lambda^2\left(P+\left(\psi_1Pe^{-\lambda(t_2t_3)}\right)\right)t_2^2+6e^{\lambda(t_2-t_3)}Ps^{m}\lambda t_3\\
&+3s^{m}\lambda^2\left(\psi_1Pe^{-\lambda(t_3-t_2)}\right)t_3^2+3s^{m}\beta\lambda^2\left(\psi_1Pe^{-\lambda(t_3-t_2)}\right)t_3^2+6Ps^{m}\lambda t_4\\
&-6e^{\lambda(t_2-t_3)}Ps^{m}\lambda t_4-6s^{m}\lambda^2\left(\psi_1Pe^{-\lambda(t_3-t_2)}\right)t_3t_4\\
&-6s^{m}\beta\lambda^2\left(\psi_1Pe^{-\lambda(t_3-t_2)}\right)t_3t_4+3s^{m}\beta\lambda^2\left(\psi_1Pe^{-\lambda(t_3-t_2)}\right)t_4^2\\
&+6s^{m}\lambda t_2\left(-P+\lambda\left(P+\left(\psi_1Pe^{-\lambda(t_3-t_2)}\right)\right)-t_4\right)-6A^{\alpha}\lambda^2D_0t_4t_5\\
&+3A^{\alpha}\lambda^2D_0t_5^2-3A^{\alpha}\eta\lambda^2t_4t_5^2+2A^{\alpha}\eta\lambda^2t_5^3\Big)\Bigg]-\frac{C_EP\left(1-e^{-\lambda(t_3-t_2)}\right)}{\lambda}\\
&-\beta\left(\psi_1Pe^{-\lambda(t_3-t_2)}\right)C_r\left(t_4-t_3\right)-AC_a\Big]
\end{aligned}
\tag{23}
$$

Equation (23) is the required average profit function. We aim to maximise this highly nonlinear profit function. We solved this problem by using Genetic algorithm. We developed an algorithm to determine the optimal values of P, s, s_1 and λ with the proposed model's net profit by using the real-coded genetic algorithm.

4 Genetic algorithm

It is a stochastic global search method motivated by natural selection and natural genetics principles. The concept of genetic algorithm is based on Darwin's evolutionary theory. It is commonly used to find the solution to optimisation problems by relying on biologically inspired operators such as selection, crossover, and mutation.

This algorithm demonstrates the mechanism of natural selection where the fittest individuals are chosen for reproduction to produce next-generation offspring. This process starts with the selection of the fittest individuals from a population. The fitness value for every individual in the population is calculated in each generation. The fittest individuals are selected from the current population, and each individual is being modified (crossover and mutation) to form a new generation. The selected individuals produce offspring which inherit their parent's characteristics and will be added to the next generation. This process keeps on iterating until the termination condition has been met.

The stepwise procedure is as follows:

4.1 Algorithm

Step 1 Set up the parameters of the real-coded genetic algorithm like the probability of crossover, probability of mutation, maximum generations, population size and variables bound.

Step 2 Set generation $T = 0$.

Step 3 Initialise $P(T)$.

Step 4 Evaluate the fitness function of $P(T)$.

Step 5 For mating pool, select N solutions from $P(T)$ by using tournament selection.

Step 6 For crossover depending on Pc, choose solutions from $P(T)$ and make crossover on selected solutions.

Step 7 For mutation depending on Pm, choose solutions from $P(T)$ and make mutation on selected solutions to get $P_1(T)$.

Step 8 Evaluate $P_1(T)$.

Step 9 Choose the best N solutions by comparing $P(T)$ and $P_1(T)$.

Step 10 Set $T = T + 1$

Step 11 If $T >$ maximum generations, then go to step 13; otherwise, go to step 5.

Step 12 Find the best result found from N solutions selected in step 9.

Step 13 End algorithm

4.2 Parameters

The genetic algorithm depends on various parameters like population size, Maximum number of generations, crossover probability, and probability of mutation.

4.3 Chromosome representation

Chromosome representation is an important factor in GA. In nonlinear problems, Binary chromosome representation is not very effective. So, we used a real number representation in our proposed model as our problem is highly nonlinear. Each chromosome U_i is a string of n number of genes G_{ij} where n denotes the number of decision variables ($i = 1, 2, 3, \ldots, popsize$) ($j = 1, 2, 3, \ldots, n$).

4.4 Initial population

Every gene G_{ij} is generated randomly for each chromosome between the lower and upper bound of the variables X_j ($i = 1, 2, 3, \ldots, popsize$) ($j = 1, 2, 3, \ldots, n$).

4.5 Evaluation

Function evaluation (*FEV*) for the chromosome U_i is the same as the objective function value at any point.

These steps are used for function evaluation:

1 Find the $FEC(U_i)$ value such that $FEV(U_i) = f(X_1, X_2, \ldots, X_n)$ where f is the objective function.

2 Find the total fitness of the population $\left(F = \sum_{i=1}^{popsize} FEV(U_i)\right)$.

3 $p_i = \dfrac{FEV(U_i)}{F}$ determines the selection probability (p_i) for each chromosome (U_i).

4 Find the cumulative probability $Z_i = \sum_{j=1}^{n} p_j$ of selection for each chromosome.

4.6 Selection

This phase decides which solutions are chosen in the current population to breed a new generation. There are many selection methods available. In our paper, we used Tournament selection. Tournaments are held between two solutions, with the winner being selected and put in the mating pool. The exact process is repeated for other solutions. Each solution participated in precisely two tournaments. The best solution will win both tournaments, while the worst solution will lose both tournaments.

4.7 Crossover

It is a genetic operator used to produce new offspring by merging the genetic knowledge of two parents. A crossover point is selected randomly from within the genes for each parent's genes to be mated. The crossover operator is applied after selecting the chromosomes. In this paper, simulated binary crossover (SBX) is used. This operator simulates the working principle of single-point crossover. For SBX crossover, we require two parents to generate two offspring.

The offspring have a spread that is proportional to that of parents:

$$O_a - O_b = \beta\left(P_a' - P_b'\right)$$

To create two offspring from two parents, we follow this procedure:

1 Randomly select a pair of parents (say P_a' and P_b') from the mating pool.

2 Generate a number r_i randomly between 0 and 1 for each pair of selected parents.

3 If $r \geq Pc$, then copy the parent solutions as offspring.

4 If $r < Pc$, generate n random numbers (v_j) for each variable.

5 Calculate β_j for each variable using the following equation:

$$\beta_j = \begin{cases} (2v_j)^{\frac{1}{(\eta c+1)}} & \text{if } v_j \le 0.5 \\ \left(\dfrac{1}{2(1-v_j)}\right)^{\frac{1}{(\eta c+1)}} & \text{otherwise} \end{cases}$$

η_c is the distribution index.

6 Generate two offspring (O_a and O_b) by using the following equation:

$$O_a = 0.5\left[(1+\beta_j)P'_a + (1-\beta_j)P'_b\right]$$

$$O_b = 0.5\left[(1-\beta_j)P'_a + (1+\beta_j)P'_b\right]$$

4.8 Mutation

It is a genetic operator used to preserve genetic variation in a genetic algorithm population from one generation to another. One or more gene values changes in a chromosome from its original state during mutation. In the case of mutation, sometimes the solution is entirely different from the previous solution. As a result, GA will come to a better answer by using a mutation. Here we perform Polynomial mutation. The following steps are performed to select the chromosome for mutation:

1 Generate a number $r \in [0, 1]$ randomly for every variable.

2 If $r \ge Pm$, then no change in offspring.

3 If $r < Pm$, generate n random numbers (k_j) corresponding to each variable.

4 Calculate δ_j for each variable using the following equation:

$$\delta_j = \begin{cases} (2k_j)^{\frac{1}{(\eta m+1)}} - 1 & \text{if } k_j \le 0.5 \\ 1 - \left[2(1-k_j)\right]^{\frac{1}{(\eta m+1)}} & \text{if } k_j \ge 0.5 \end{cases}$$

η_m is the distribution index.

5 Modify offspring by using the following equation:

$$O_i = O_i + (X^U - X^L)\delta_j$$

where X^U and X^L are upper bound and lower bound of the variable.

4.9 Termination

The process continues until the maximum number of generations reached.

5 Solution methodology

Genetic algorithm is used to solve the above proposed model with the parameters *popsize* = 50, *Pc* = 0.8, *Pm* = 0.2 $\eta_c = 20$, $\eta_m = 20$ and maximum generations = 1,000. We used a real number presentation. Each chromosome is a string of *n* (here *n* = 4) number of genes where *n* represents decision variables. Here we generated random values for decision variables between their boundaries until it is feasible. In this problem, we used Tournament selection, SBX crossover and polynomial mutation.

Here, we take P, s, s_1 and λ as decision variables.

6 Numerical illustration

A numerical example has been considered here to demonstrate the feasibility of our proposed model. Suppose a company is manufacturing a single product and the related parameters have the following values:

$$D_0 = 1{,}000;\ A = 7;\ \alpha = 0.8;\ \eta = 5,\ m = 1.2;\ n = 1;\ D_2 = 400;\ C_p - 25;\ C_0 = 300;$$
$$C_h = 4;\ C_d = 8;\ C_E = 10;\ C_r = 12;\ C_a = 30;\ \beta = 0.75;\ \theta = 0.08;\ \psi_1 = 0.4;$$
$$t_1 = 0.3;\ t_2 = 0.8;\ t_3 = 1.4;\ t_4 = 1.7;\ t_5 = 2.$$

The optimal values of P, s, s_1, λ and maximum total average profit have been calculated by considering these initial values and results displayed in Table 2.

Table 2 Optimum values of P, s, s_1, λ

P	s	s_1	λ	*TAP*
298	39.91	26.61	0.96	637.857

Figure 3 Total average profit with respect to production rate (P) and selling price of perfect item (s) (see online version for colours)

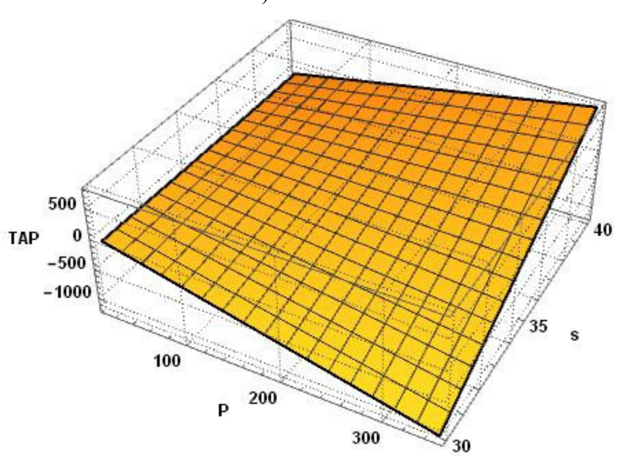


Figure 4 Total average profit with respect to production rate (P) and selling price of imperfect item (s_1) (see online version for colours)

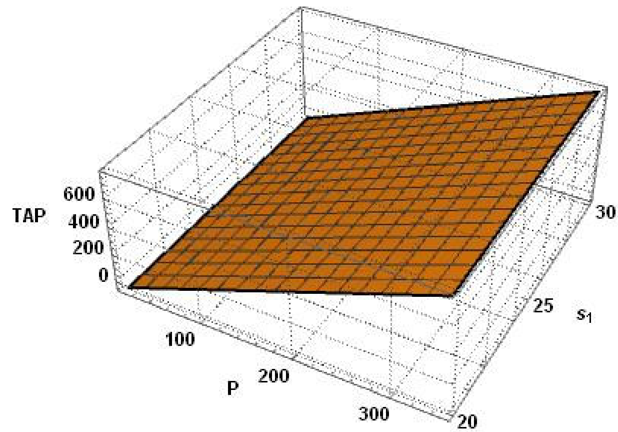
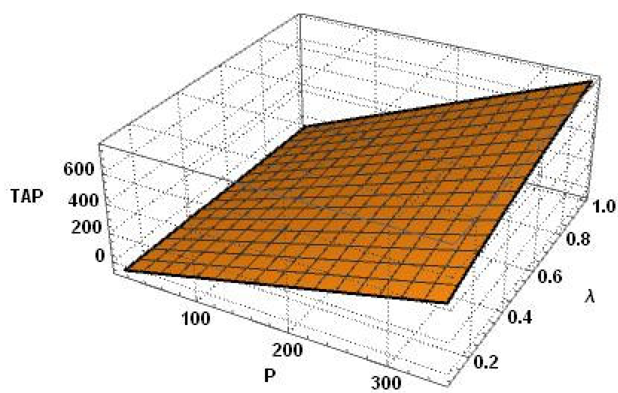


Figure 5 Total average profit with respect to production rate (P) and inverse efficiency (λ) (see online version for colours)



Figures 3, 4 and 5 graphically show the total average profit with respect to production rate (P), selling price of perfect items (s), selling price of defective items (s_1), and inverse efficiency (λ).

7 Sensitivity analysis

We performed a detailed sensitivity analysis to examine the effects of changes in the numerical values taken in the above example. By changing the values of some parameters from –10% to 10%, this analysis has been carried out. However, one parameter is changed at a time, and the values of other parameters remain the same.

Table 3 Total average profit corresponding to different parameters

Parameter	*Percentage change in parameter*	*TAP*	*Percentage change in TAP*
C_p	–10	1,105.76	+73.3555
	–5	871.806	+36.6773
	0	637.857	0.0
	5	403.909	–36.6772
	10	169.96	–73.3545
C_h	–10	698.114	+9.44679
	–5	667.986	+4.72347
	0	637.857	0.0
	5	607.729	–4.72332
	10	577.601	–9.44663
C_d	–10	646.908	+1.41897
	–5	642.383	+0.709563
	0	637.857	0.0
	5	633.332	–0.709407
	10	628.806	–1.41897
C_E	–10	705.817	+10.6544
	–5	671.837	+5.32721
	0	637.857	0.0
	5	603.878	–5.32706
	10	569.898	–10.6543
C_a	–10	648.357	+1.64614
	–5	643.107	+0.823068
	0	637.857	0.0
	5	632.607	–0.823068
	10	627.357	–1.64614
C_r	–10	646.903	+1.41819
	–5	642.38	+0.709093
	0	637.857	0.0
	5	633.334	–0.709093
	10	628.811	–1.41819
A	–10	634.014	–0.602486
	–5	635.974	–0.295207
	0	637.857	0.0
	5	639.67	+0.284233
	10	641.416	+0.557962

Table 3 Total average profit corresponding to different parameters (continued)

Parameter	*Percentage change in parameter*	*TAP*	*Percentage change in TAP*
D_0	–10	620.16	–2.77445
	–5	629.009	–1.38714
	0	637.857	0.0
	5	646.706	+1.3873
	10	655.555	+2.7746
β	–10	636.946	–0.142822
	–5	637.402	–0.0713326
	0	637.857	0.0
	5	638.313	+0.714894
	10	638.769	+0.142979

Figure 6 Graph of total average profit vs. production cost (see online version for colours)

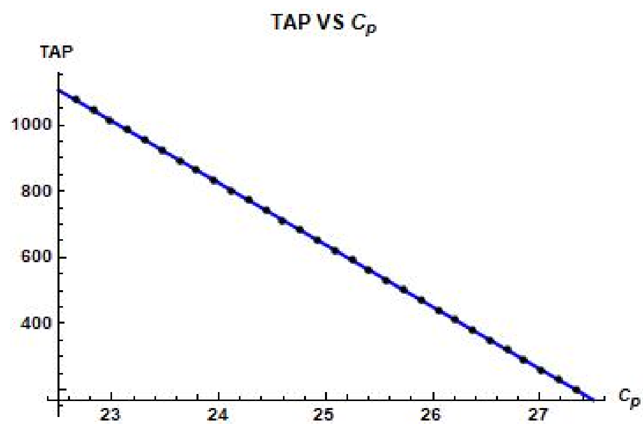


Figure 7 Graph of total average profit vs. holding cost (see online version for colours)

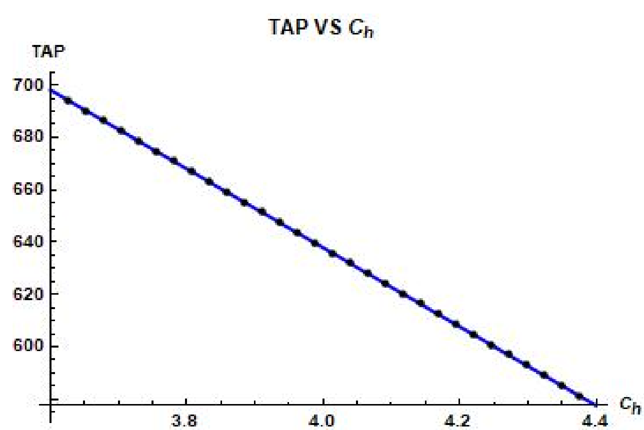


Figure 8 Graph of total average profit vs. deterioration cost (see online version for colours)

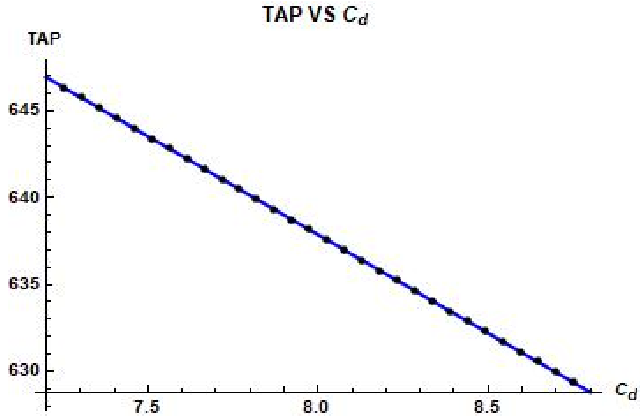


Figure 9 Graph of total average profit vs. efficiency cost (see online version for colours)

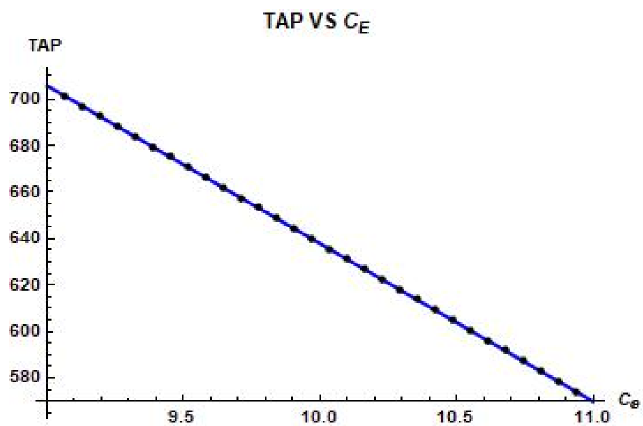


Figure 10 Graph of total average profit vs. advertisement cost (see online version for colours)

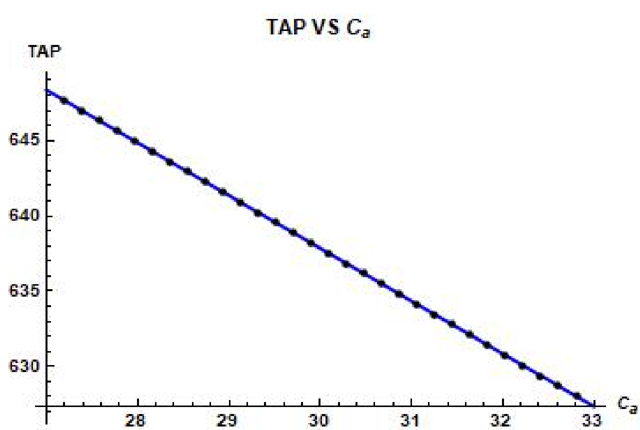


Figure 11 Graph of total average profit vs. reworking cost (see online version for colours)

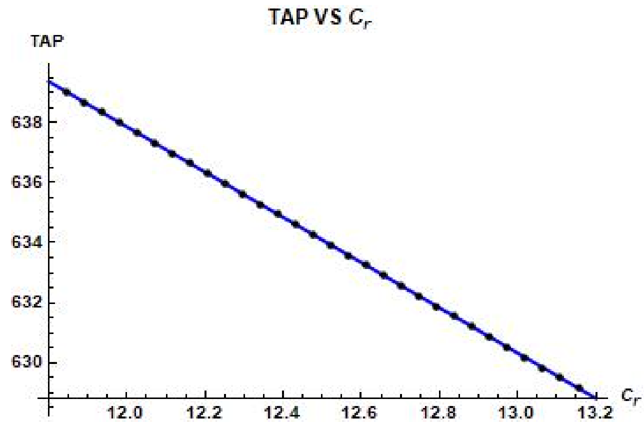


Figure 12 Graph of total average profit vs. advertisement frequency (see online version for colours)

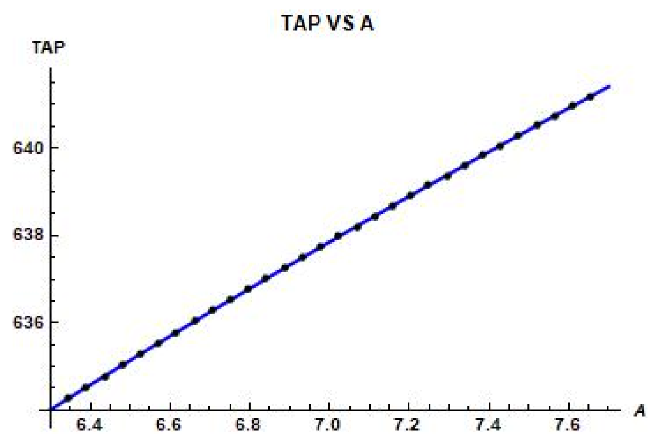


Figure 13 Graph of total average profit vs. demand (see online version for colours)

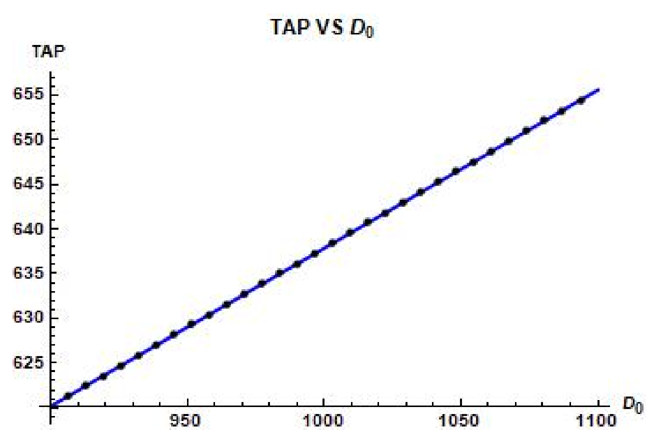
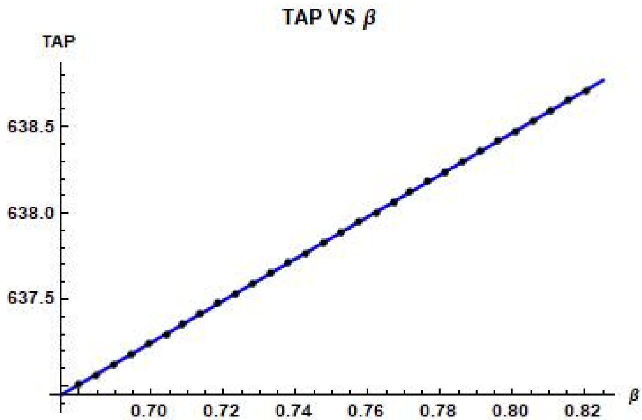


Figure 14 Graph of total average profit vs percentage of reworking items (see online version for colours)



8 Observations

The following changes are observed from Table 3:

1 In Figures 6, 7, 8, 9, 10 and 11, we conclude that the total average profit increases when the value of any cost related to our proposed model decreases. When the value of any cost related to our proposed model increases, the total average profit decreases, a real-life phenomenon.

2 In the case of advertisement frequency (Figure 12), when we increase A's value, the total average profit increases. When we decrease A's value, then total average profit decreases, which satisfies the phenomenon that more advertisement of the product will increase the product sales.

3 In the case of demand (Figure 13), when we decrease the value of demand for perfect items, the profit decreases and vice-versa. This demand analysis also follows the real-life phenomenon.

4 In Figure 14, when we decrease the value of reworked defective items, the total average profit decreases, and when we increase the value of β, the total average profit increases. This point also satisfies the phenomenon that the more reworked done of defective items it will increase the profit.

9 Conclusions

Every manufacturing company aims to manufacture only perfect quality items, but this is not possible. In general, a production process cannot be perfect. Each company wants to increase its profit; to increase profit, the company needs to run the machine for a longer duration. However, when the machine runs for a long duration, it starts producing imperfect and perfect items because of so many factors present in the system. In real life, advertising of the product and the selling price of the product affect the product's demand. By considering all these factors, we developed an imperfect production inventory model with advertisement, time, and price-dependent demand for a non-instantaneous deteriorating item. Here, we considered that imperfect items also have some demand. The perfect quality products are sold directly, and some imperfect products are reworked at a cost to become perfect. The remaining imperfect items are sold at a lower price. Also, to maintain the efficiency of the system, an efficiency cost has been included. The demand for perfect quality items depends on the advertisement, selling price, and time. On the other hand, the demand for imperfect items is selling price-dependent. A profit function has been made by considering all those factors. This profit function is highly nonlinear and has been maximised by using genetic algorithm. A numerical example and graph illustration have been given to study the model. The proposed work helps the manufacturer determine the optimal production quantity and selling price by considering factors that affect production.

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