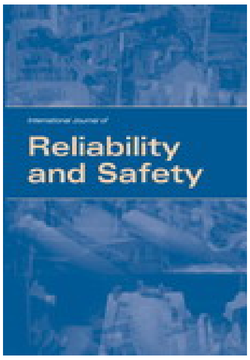


International Journal of Reliability and Safety

ISSN online: 1479-3903 - ISSN print: 1479-389X
https://www.inderscience.com/ijrs

Reliability analysis by Markov model and stochastic estimator of stochastic Petri nets

Hamid El-Moumen, Nabil El Akchioui, Mohammed Hassani Zerrouk

DOI: 10.1504/IJRS.2022.10053783

Article History:

Received:	04 March 2022
Last revised:	30 June 2022
Accepted:	20 September 2022
Published online:	30 January 2023

Reliability analysis by Markov model and stochastic estimator of stochastic Petri nets

Hamid El-Moumen*, Nabil El Akchioui and Mohammed Hassani Zerrouk

Faculty of Sciences and Technology Al Hoceima,
University Abdelmalek Essaadi,
Tétouan and Tangier, Morocco
Email: elmoumen.hmd@gmail.com
Email: n.elakchioui@uae.ac.ma
Email: m.hassani@uae.ac.ma
*Corresponding author

Abstract: In our study, we were interested in the reliability of large discrete systems. These studies can be based on Markov models or on Stochastic Petri Nets (SPNs) that are generally used for the analysis and synthesis of the models used in the different phases of a system's life. Markov models or SPNs are perfect for many cases, still, they suffer from the combinatorial explosion when analytically their state numbers increase as the complexity of the dynamic systems grows accordingly with their components. Such issue reflects itself in the slowness of these models to accomplish convergence. These different modelling tools make it possible to deduce the average behaviour and to obtain the performance indicators of the system studied, either by calculation or by estimation. We will present the Markov analysis of a system whose state space is finite as well as its estimator obtained using SPNs.

Keywords: Petri net; stochastic Petri net; stochastic estimator; Markov model; reliability analysis.

Reference to this paper should be made as follows: El-Moumen, H., El Akchioui, N. and Zerrouk, M.H. (2022) 'Reliability analysis by Markov model and stochastic estimator of stochastic Petri nets', *Int. J. Reliability and Safety*, Vol. 16, Nos. 1/2, pp.110–123.

Biographical notes: Hamid El-Moumen received a master's degree in 2019, specialty, Embedded Systems and Robotics from the Faculty of Science and Technology, AL Hoceima, University Abdelmalek Esaadi of Tetouan, Morocco. He is currently a PhD student at the same faculty. His research focuses on the study of reliability, diagnosis and monitoring of industrial facilities by continuous approaches such as Petri nets.

Nabil El Akchioui received a PhD in Automatic and Computer Science from the University of Sciences and Technologies of Le Havre in 2012, France and an HDR from the University Abdelmalek Esaadi Tetouan, Morocco in 2020. Since 2013, he is a professor at the Faculty of Science and Technology, Al Hoceima University, Morocco. He is part of the research group in Electrical and Automatic Engineering. His research interests include Petri nets, learning processes, adaptive control, fault detection and diagnosis and its applications to electrical.

Mohammed Hassani Zerrouk holds a Master's degree in Natural Resources and Environment (in 2004) & a PhD in Water treatment (in 2007) from Cadiz University, Spain. He was a member in the research group 'Environmental Technologies' of Cadiz University (Spain) from 2001 to 2009. Currently he is a member of the research team: Engineering Environment, Food and Valuation of Biomolecules of the Faculty of Sciences and Technologies Al Hoceima, University Abdelmalek Esaadi Tetouan, Morocco, where he is also a professor from 2013. His research area includes the environment and environmental technologies especially the water sector, advanced water treatment technologies.

1 Introduction

Current industrial systems, in particular petrochemicals, rail transport, telecommunications and nuclear power, must implement an assessment policy including studies of the reliability of their facilities in order to comply with the standards and regulations in force. This assessment also helps to improve the security of these systems by complying with Operational Safety (OS) requirements. However, risk and reliability assessment quickly become complex for large systems. For this reason, it is important to adapt existing methods and tools according to the specificities of the systems. Studies of the reliability of complex dynamical systems generally make use of Stochastic Discrete Event Models such as Stochastic Petri Nets (SPNs) (Marsan and Chiola, 1987; Zerhouni and Alla, 1990; Recalde and Silva, 2004).

The continuous-time homogeneous Markov chain is a method used to analytically determine the steady-state probabilities, in order to compute the usual reliability indicators when the system is of reduced dimension. In large systems this method becomes unpractical because of the increase in the number of states related to the marking graph, this problem is called combinatorial state explosion.

Despite their shortcomings, Markov chain models have proven over time their reliability to solve complex issues and increase system efficiency. Thus, to be a foundational element to solve optimisation problems for small-and-large-scale industries. Hence, many works have been proposed to improve the sensitivity analysis of Markov processes in reliability studies for steady-state to take full advantage of Markov Chain (MC) models (Do Van et al., 2012).

Stochastic Petri nets are a very powerful construct for specifying systems with concurrent and asynchronous activities, i.e., the ability to model, synthesise and describe discrete event systems (Balbo, 2000). In the case where the Markov process is homogeneous to that of the stochastic Petri net, the Petri net can be seen as an estimator of the Markov model. The benefit of this estimation lies in the inessential requirement to specify a marking graph. However, it suffers from a long time of simulation, which is considered as a weakness for this estimation as this leads to slow convergence of the state probabilities.

In this work, we will focus mainly on running some of the SPNs simulations to obtain estimates of the asymptotic mean flows and markings. Then, we will investigate the limitations and capabilities of our proposed Markovian approach and the stochastic estimator based on the results of the usual reliability indicators from the application example on a manufacturing system, using the Petri net model. Furthermore, we were interested in the reliability of large discrete systems. These studies can be based on Markovian models or on stochastic Petri nets, which are generally used for the analysis and synthesis of the models, used during the different life phases of a system. These various modelling tools make it possible to deduce the average behaviour and to obtain the performance indicators of the system studied, either by calculation or by estimation.

2 Basic concept of Markov model and Petri nets for reliability

2.1 Markov model

A Markov chain is a mathematical structure named after Andrey Markov that transitions from one state to another between a finite or countable number of possible states (Das and Bhuyan, 1985). It is a memoryless random process in which the next state is determined solely by the current state and not by the sequence of events that preceded it. The Markov property describes this kind of 'memorylessness'. As mathematical models of real-world systems, Markov chains have a wide range of applications (Vazquez et al., 2008).

The study of the reliability of systems can be carried out using Markovian analysis to analytically evaluate and quantify the usual performance indicators of repairable systems. Markov processes are often used to quantitatively evaluate the functioning of systems, especially when the transition rates are constant, that is, the instants of failure and repair of components are distributed according to exponential laws (Vazquez et al., 2008).

2.2 Petri Nets (PNs)

A Petri Net (PN) is a mathematical model used to represent various systems operating on discrete variables, is a bipartite directed graph, provided with two types of vertices, places and transitions (Vazquez et al., 2009). A place is represented by a circle and a transition by a line. $P = \{P_1, P_2, ..., P_i\}$ is the finite set of *n* places and $T = \{T_1, T_2, ..., T_q\}$ is the finite set of *q* transitions whose occurrences cause a change of state of the system. Places and transitions are connected by directed arcs that connect either a place to a transition or a transition to a place (David and Alla, 1992) according to the backward and forward incidence applications. We denote the forward incidence application $W_{PR} = \left(w^{PR}{}_{ij}\right) \in IN^{n\times q}$ where $w^{PR}{}_{ij}$ is the weight of the arc directed from P_i to T_j, and the backward incidence application $W_{PO} = \left(w^{PO}{}_{ij}\right) \in IN^{n\times q}$ (Julvez et al., 2005) where $w^{PO}{}_{ij}$ is the weight of the arc directed from T_j to P_i. The incidence matrix *W* of the network is defined by $W = W_{PO} - W_{PR} \in Z^{n\times q}$. Each transition T_j is activated according to its activation degree $n_j\left(M\left(t\right)\right)$ defined for the marking *M(t)* by equation (1) (Lefebvre et al., 2010):

$$n_j\left(M\right) = \min\left(m_i / w^{PR}{}_{ij}\right) \text{ for all } P_i \in {}^\circ T_j \tag{1}$$

where $°T_j$ represents the set of upstream places of T_j. The place P_i such that $i = \text{argmin}\left(m_k\left(t\right)/w^{PR}{}_{kj}\right)$ for all $P_k \in °T_j$ is the critical place for the transition T_j at time *t* (Lefebvre et al., 2010).

2.3 Stochastic Petri Nets (SPNs)

SPNs are timed PNs with randomly distributed transition firing times based on an exponential probability distribution with a parameter that varies round(n_j(M)).μ_j (Lefebvre et al., 2010). Molloy (1982) was the first to introduce this model and several other expansions have been developed for the analysis of the reliability of repairable systems. Fondamentally, a $\text{SPN} =< PN, \mu >$, with $\mu = \left(\mu_j\right) \in \left(R^+\right)^q$ is a vector of crossing rate. The firing rate μ_j characterises every transition T_j such that $\left(\mu_j.dt\right)$ is the estimated probability of triggering the transition T_j in period *t* and $t + dt$ when the transition T_j was triggered, with an activation degree equal to 1 at time *t*. The characteristics of an SPN, such as incidence matrices, firing rates, initial marking, and policy compliance (firing, servers and execution), are all used to describe the process of marking of an SPN (Lefebvre et al., 2010; Molloy, 1982). The vector of the average flow and average marking of an SPN at time *t* will be named $X_s\left(t\right)$ and $M_s\left(t\right)$ (Bobbio et al., 1998). The SPNs in this work have satisfied the hypotheses (H_1) to (H_5) (Lefebvre et al., 2010):

(H_1) the marked SPNs are bounded.

(H_2) the marked SPNs are reinitialisable.

(H_3) the firing policy is a race policy: the transition whose is assumed to be the one that will fire next.

(H_4) the server policy is of type infinite server: influence of the degree of crossing.

(H_5) the execution policy is resampling memory: influence of the transition crossed on the next crossings

3 Application of SPNs to reliability studies

Stochastic Petri nets are tools for analysing the structure and behaviour of dynamic stochastic systems with discrete events (Vazquez et al., 2009).

- The direct analysis of the markings graph makes it possible to characterise the general and specific properties of the model studied (bounded aspect, living, etc.) (Vazquez et al., 2008).
- The exploitation of the stochastic process associated with the Markov model makes it possible to evaluate the behaviour in permanent and transient regimes of the model (average frequency of crossing transitions, average residence time in persistent states, etc.).

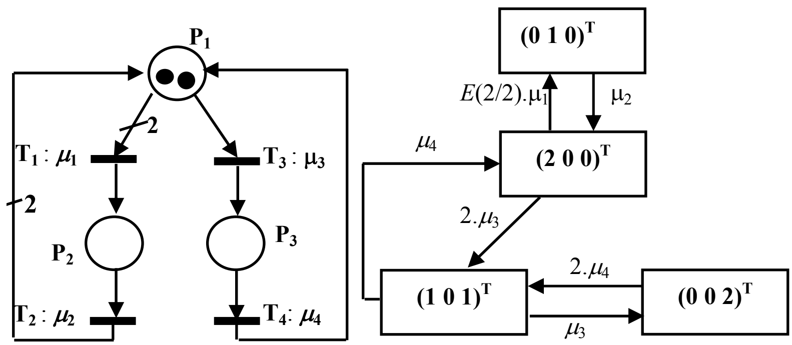
- The simulation of the SPNs makes it possible to obtain approximations of the average flows and markings as well as the usual indicators of reliability.

In this part, we show how to use the Markov analysis of SPNs for reliability studies.

3.1 Markovian analysis of a SPNs

The Markov analysis of an SPNs consists of constructing the graph of reachable markings of the SPNs and labelling each arc by a crossing rate which depends on the rate of the transition taken and the degree of awareness of this transition (Vazquez et al., 2008, 2009). The process of labelling the SPNs is then identical to that of the homogeneous Markov process thus determined. Consider the SPN of Figure 1, of parameters m_1, m_2, m_3, m_4 and the associated Markov process:

Figure 1 SPNs and Markov Chain



The Markov process makes it possible to evaluate the behaviour in steady and transient regimes of the model such as the average marking of each place, the average flow of crossing each transition or the average residence time in each state (Do Van et al., 2012; Vazquez et al., 2009). If we suppose that the Markov model is ergodic then it has a generator which admits a unique stationary solution (Molloy, 1982). It is therefore possible to determine the values of the fluxes and of the mean markings.

3.2 Associated Markov process generator

In the case where the SPNs is bounded, the marking graph is finite and the Markov process has a finite number of states. The generator of the Markov process A is obtained from the markings graph. The states are linked in pairs by arcs with which the probabilities of passing from one state to another are associated (Bobbio et al., 1998; Lefebvre, 2011).

The generator of the Markov process associated with the SPNs is therefore a square matrix $A \in (R)^{N \times N}$, where N is the number of states. Process a depends on the reachability graph, the vector of the crossing rates of the transitions as well as the degree of sensitisation of the transitions (linked to the weights of the arcs of the front incidence matrix) (Kara et al., 2008; Mahulea et al., 2006). This matrix A is constructed as follows:

- The off-diagonal element $a_{ij}\left(i \neq j\right)$ is equal to the crossing rate allowing to pass from the state S_i to the state S_j by crossing the transition $T_k : a_{ij} = n_k\left(M_i\right).m_k$, where M_j represents the marking associated with the state S_i.
- The diagonal element $aii = -\sum_{i \neq j}^{N} a_{ij}$ represents the complement to zero of the sum of the other elements of line *i* (sum of the exit rates from the state S_i).

The determination of the Markov process generator associated with the SPNs makes it possible to calculate the probabilities of states of the steady state, the steady state of the SPNs and to deduce from them other usual performance indicators of reliability.

3.3 Permanent regime of an SPN

When the reachability graph of the SPN is isomorphic to the state space of a Markov process, the steady state of the SPN can be obtained using the state probabilities of the Markov model.

Let $X_s = \left(x_{sj}\right)\hat{I}\left(R^+\right)^q$ the vector of asymptotic mean flows, $M_s = \left(m_{si}\right)\hat{I}\left(R^+\right)^n$ the vector of asymptotic mean markings and $\Pi = \left(\pi_k\right)\hat{I}\left[0,\ 1\right]^{1\times N}$ the steady-state state probability vector of the associated Markov model with N states. Let A be the generator of the associated Markov process. The vector of steady-state probabilities is the solution of equation (2).

$$\begin{cases} \Pi.A = 0 \\ \sum_{i=1}^{N} \pi_i = 1 \end{cases} \tag{2}$$

From the vector Π, we deduce the asymptotic mean throughput of the transitions as well as the asymptotic mean markings of the places.

Let $M_k = \left(m_{ki}\right)$ be the marking associated with the state S_k, $n_j\left(M_k\right)$ is the degree of activation of the transition T_K and μ_J is the vector of the crossing rate (Mahulea et al., 2006). The asymptotic mean flows of each transition T_J is given by equation (3):

$$x_{sj} = \mu_j.\left(\sum_{k=1...N} n_j\left(M_k\right).\pi_k\right) \tag{3}$$

The asymptotic mean marking of each place P_i is defined by relation (4):

$$m_{si} = \sum_{k=1...N} m_{ki}.\pi_k \tag{4}$$

This method makes it possible to obtain an analytical solution of the steady state of an SPNs, in the case of an ergodic system and when the state space is of reduced dimension (Molloy, 1982). But one of the crucial problems common to all graph-based studies is the Combinatorial Explosion associated with the increase in the number of states (Sandmann, 2004; El Akchioui, 2017). This problem will be illustrated in the next section and a workaround will be presented later.

3.4 Complexity of the reachability graph

As an example, we will consider the example – prepared by Silva and Recalde (2004) presented in Figure 2. This network models a manufacturing system with five machines (T_1 to T_5), and three tools with limited resources (P_1 to P_3). In this Petri net model, the vector of the parameters of the transitions μ and the initial marking M_J are given by:

$$\mu = (1,1,1,1,1)^T, M_I = K(6,6,4,0,3,0,3,0,0)^T \text{ where } k \in IN.$$

Figure 2 Manufacturing system

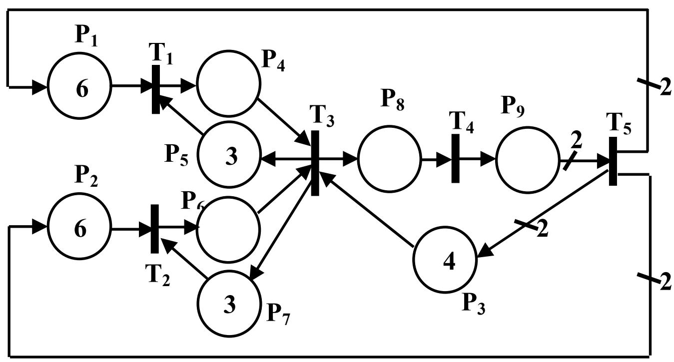


Table 1 illustrates the evolution of the number of states *N* and of the computation time as a function of the parameter *k*. the computational time required to compute the reachability graph increases exponentially and makes Markov analysis difficult if not impossible (Recalde et al., 1999; Molloy, 1982).

Table 1 Number of states and calculation time of the reachability graph in function of k

Coefficient k	*1*	*2*	*3*	*4*	*5*
Number of states (N)	205	1885	7796	22187	50801
Calculation Time (s)	0.113	8.304	164.665	1321.804	6959.009

To work around this problem, we will use the link between SPNs and Markov chain to estimate the state probabilities from the simulation of the SPNs.

3.5 Stochastic estimator by simulation of SPNs

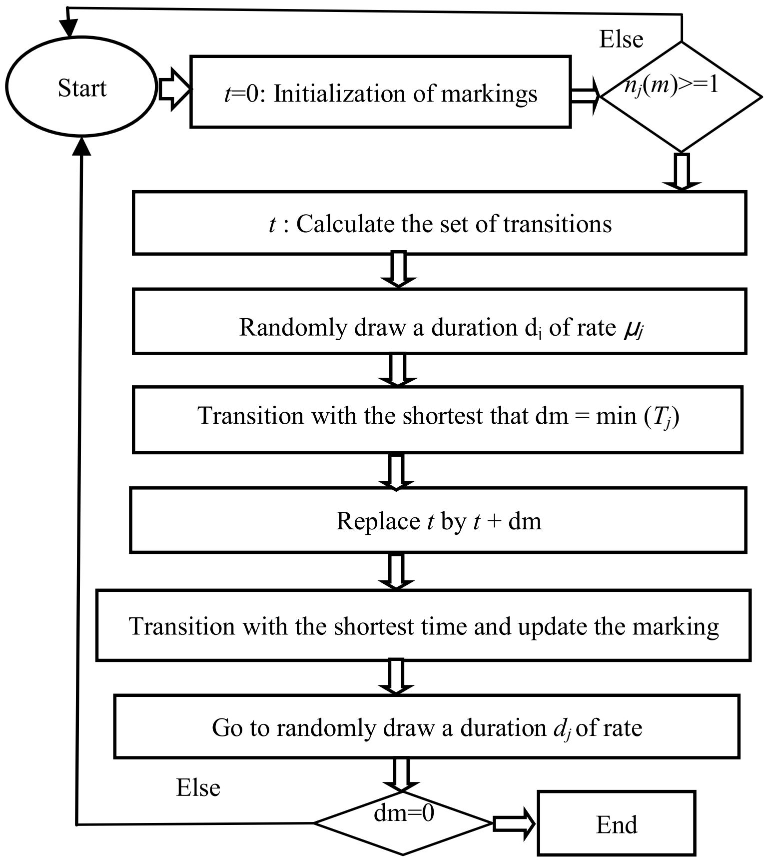
In this part, we are interested in the simulation of stochastic systems in order to determine estimates of indicators related to dependability, such as reliability, Mean Time to Failure (MTTF), Mean Time Between Failures (MTBF) and Mean Up Time (MUT) or availability. We have seen that for large-dimensional systems, the Markov analysis is often unpractical because of the combinatorial explosion due to the passage through the state graph (Recalde and Silva, 2002; Trivedi and Kulkarni, 1993). The simulation of the SPNs does not require the computation of the state graph and the SPNs can be considered as an estimator of the Markov Model (Lefebvre et al., 2009; Mahulea et al., 2008).

We next present the algorithm of this stochastic estimator.

3.5.1 SPNs simulation algorithm

The algorithm of the evolution of a SPNs which makes it possible to determine the steady state is as follows:

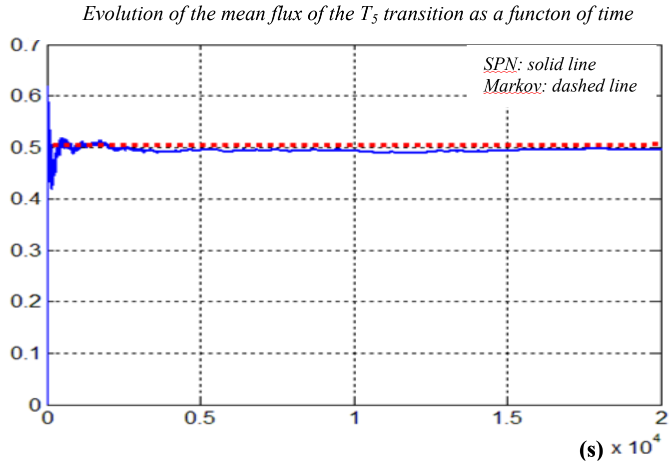
Figure 3 Steady state by SPNs algorithm



3.5.2 Estimation of the OS by simulation

The simulation of the SPNs makes it possible to obtain estimates of the fluxes and of the asymptotic mean markings. Take the example of Figure 2 where the computation time and memory space to find all the states are more expensive.

Figure 4 Stochastic estimator of throughput of the transition T_5 for the system of Figure 2

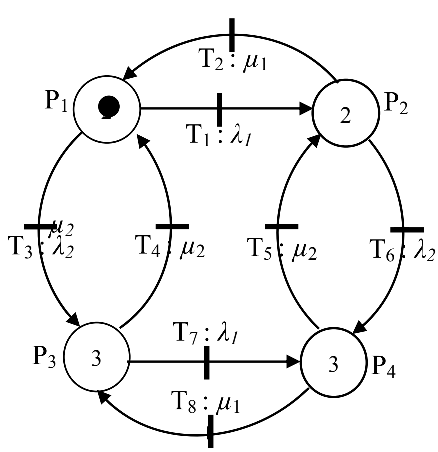


The advantage of this estimator is that the determination of the reachability graph is no longer required, but its major disadvantage is that convergence remains slow especially when the systems have rare events. These events are characterised by very low probabilities of occurrence (Zeng et al., 2019). In these situations, simulation methods are inefficient, since the low probability of the considered event makes its observation improbable, leading to poor precision of the estimate. Usually, the simulation takes a very long time to obtain acceptable results.

3.5.3 Example

Take the example, with an initial marking $M_I = (1,0,0,0)^T$, the stochastic estimator allows us to determine the average operating reliability indicators obtained analytically by Markov analysis.

Figure 5 SPNs equivalent to the Markov model

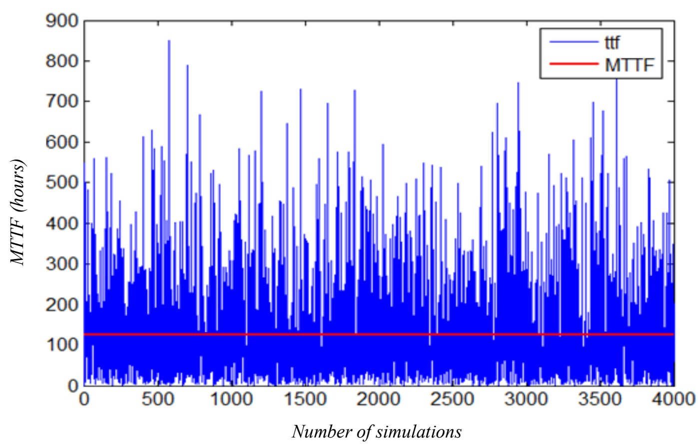


To estimate the MTTF of the system, we performed 4000 simulations by rendering the state four absorbing (stopping the simulation as soon as the state four is reached). The algorithm to determine the MTTF is as follows:

1) Define the network parameters (incidence matrix, initial marking, maximum speed of crossing transitions).
2) Initialise the parameters of the system studied, initial simulation time.
3) Simulate the SPN according to the algorithm of Figure 3.
4) Stop the simulation as soon as the system breaks down (failure states are materialised by the presence of the token in place P_4).
5) Calculate the time achieved until the first failure.
6) Relaunch the program several times (4000 simulations in our example).
7) Calculate the average value obtained during its various simulations (MTTF).

We have obtained the realisations of the times until obtaining a failure, presented in Figure 6 whose average value is MTTF = 122.472 hours.

Figure 6 MTTF estimated by simulation for the model in Figure 5



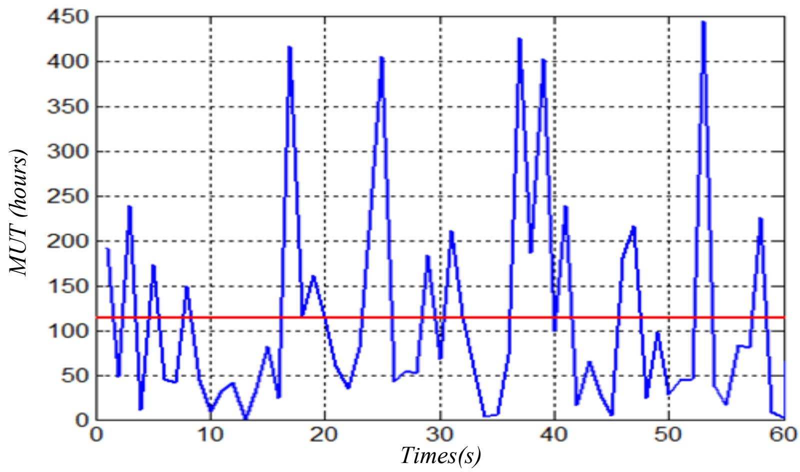
On the other hand, the simulation times are relatively long but they are little influenced by the nature of the probability laws associated with the failures, which allows many technological systems to be taken into account.

The algorithm used to determine the average time between the return of service to the system and its failure state is as follows:

1) Use the results obtained by the simulation of the SPN.
2) Identify the signs of failure states throughout the simulation (materialised by the presence of the token in place P_4).
3) Calculate the average value of all the restart and the time after the system fails. This makes it possible to determine the MUT.

Figure 7 shows the times between a return to service of the system under study from the moment it fails. We estimated MUT = 114.786 hours

Figure 7 MUT estimated by simulation for the model in Figure 5



The algorithm for determining the MTBF is as follows:

1) Use the results obtained by the simulation of the SPN.
2) Identify the indices of failure states throughout the simulation and the indices of good operating states.
3) Calculate the time between two consecutive failures.
4) Calculate the mean of the MTBF times.

Figure 8 represents the times between two consecutive failures obtained by the stochastic estimator. We estimated MTBF = 115.49 hours.

Figure 8 MTBF estimated by simulation for the model in Figure 5

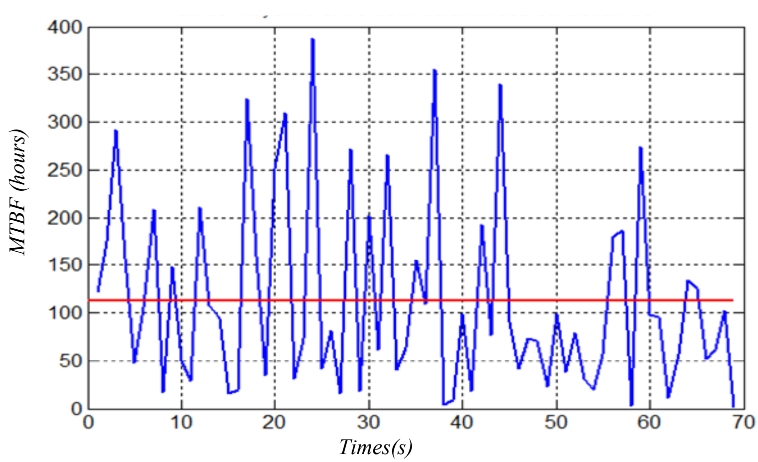


Table 2 summarises the results obtained by the Markovian analysis and by the stochastic estimator.

Table 2 Average reliability indicators obtained by the stochastic estimator and Markovian analysis

Reliability indicators	*MTTF(hours)*	*MUT(hours)*	*MTBF(hours)*
Markovian analysis	122.5	115	115.5
Stochastic estimator	122.472	114.786	115.378
Relative error	0.028	0.214	0.122

The Markov analysis and the stochastic estimator by simulation of the SPNs made it possible to evaluate the usual performance indicators of this system (Giua and Silva, 2018; El Akchioui et al., 2020). If the state space is large (and no approximate model reducing the state space is available), the reachablity graph cannot be generated. Simulation is then still the only possibility.

We can show that for state spaces, analytical-numerical methods are efficient. When the state space becomes larger, there is always a threshold at which the simulation time becomes much longer. Usually, the simulation requires a very long time to obtain good results, note that even if the computation time for the analytical-numerical methods is a little greater than that of the simulation, it is still relevant to use them because they give a precise result instead of a confidence interval.

4 Conclusions

In this paper, we have presented the basic concepts of system dependability, as well as obtaining various reliability indicators from Markov analysis, the limits of which we have highlighted. These performance indicators can be obtained using the state probability vector of the Markov process, but this operation requires the preliminary computation of the state graph, in order to overcome the problem of the combinatorial explosion of the number of states, in the case of complex systems.

The advantage of this estimator is that the determination of the reachability graph is not necessary, but its major disadvantage is its low speed of convergence, especially in the presence of rare events. Regarding reliability studies, as soon as a place is identified as a default place, an estimate of the different indicators can be obtained. The quality of this estimate will be associated to the duration of the simulation. For complex systems, the link between fault states and network places must be established to enable estimates of these indicators to be obtained.

The limitations due to the combinatorial explosion and to convergence problems have given rise to several works based on the fluidification of discrete behaviours. The forthcoming work explores this approach.

References

Balbo, G. (2000) 'Introduction to stochastic Petri Nets', *Conference: Lectures on Formal Methods and Performance Analysis, First EEF/Euro Summer School on Trends in Computer Science*, Berg en Dal, The Netherlands, pp.84–155.

Bobbio, A., Puliafito, A., Telek, M. and Trivedi, K. (1998) 'Recent developments in stochastic Petri Nets', *The Journal of Circuits, Systems and Computers*, Vol. 8, No. 1, pp.119–158.

Das, C.R. and Bhuyan, L.N. (1985) 'Reliability simulation of multiprocessor systems', *Proceedings of the International Conference on Parallel Processing*, pp.591–598.

David, R. and Alla, H. (1992) *Petri Nets and Grafcet – Tools for Modelling Discrete Events Systems*, Prentice Hall, London.

Do Van, P., Barros, A. and Bérenguer, C. (2012) 'Reliability importance analysis of Markovian systems at steady state using perturbation analysis', *Reliability Engineering and System Safety*, Elsevier, Vol. 93, pp.1605–1615.

El Akchioui, N. (2017) 'Fluidification of stochastic Petri Nets by no linear timed continuous Petri nets', *American Journal of Embedded Systems and Applications*, Vol. 5, No. 4, pp.29–34.

El Akchioui, N., Dib, F., Lefebvre, D., Ncir, N., Sebbane, S. and Leclercq, E. (2020) 'About the fluifidication of SPN by CPN for complex dynamical systems: critical regions', *Proceedings of the 1st International Conference on Innovative Research in Applied Science, Engineering and Technology (IRASET)*, IEEE, Morocco.

Giua, A. and Silva, M. (2018) 'Petri Nets and automatic control: a historical perspective', *Annual Reviews in Control*, Vol. 45, pp.223–239.

Julvez, J., Recalde, L. and Silva, M. (2005) 'Steady-state performance evaluation of continuous mono-T-semiflow Petri Nets', *Automatica*, Vol. 41, No. 4, pp.605–616.

Kara, R., Loiseau, J.J. and Djennoune, S. (2008) 'Quantitative analysis of continuous weighted marked graphs', *Nonlinear Analysis, Hybrid Systems*, Vol. 2, pp.1010–1020.

Lefebvre, D. (2011) 'About the stochastic and continuous Petri Nets equivalence in long run, non-linear analysis, hybrid systems', *Non-Linear Analysis, Hybrid Systems*, Vol. 5, pp.394–406.

Lefebvre, D., Leclercq, E., El Akchioui, N., Khalij, L. and Souza de Cursi, E. (2010) 'A geometric approach for the homothetic approximation of stochastic Petri Nets', *Proceedings of the IFAC, Discrete-Event Systems*, Berlin, Germany, Vol. 43, No. 12, pp.235–240.

Lefebvre, D., Leclercq, E., Khalij, L., Souza de Cursi, E., El Akchioui, N. (2009) 'Approximation of MTS stochastic Petri Nets steady state by means of continuous Petri Nets: a numerical approach', *Proceedings of the IFAC, Analysis and Design of Hybrid Systems*, Zaragoza, Spain, pp.62–67.

Mahulea, C., Giua, A., Recalde, L., Seatzu, C. and Silva, M. (2006) 'On sampling continuous timed Petri Nets: reachability 'equivalence' under infinite servers semantics', *Proceeding of the IFAC, Analysis and Design of Hybrid Systems*, Alghero, Italy, Vol. 37–43.

Mahulea, C., Ramirez-Trevino, A., Recalde, L. and Silva, M., (2008) 'Steady state control reference and token conservation laws in continuous Petri Nets', *Transactions on IEEE, Transactions on Automation Science and Engineering*, Vol. 5, No. 2, pp.307–320.

Marsan, M.A. and Chiola, G. (1987) 'On Petri Nets with deterministic and exponentially distributed firing times', in Rozenberg, G. (Ed.): *Advances in Petri Nets*, Springer Verlag, pp.132–145.

Molloy M.K. (1982) 'Performance analysis using stochastic Petri Nets', *IEEE Transactions on Computers C*, Vol. 31, pp.913–917.

Molloy, M.K. (1982) 'Performance analysis using stochastic Petri Nets', *IEEE Transaction on Computers*, pp.913–917.

Recalde, L. and Silva, M. (2002) 'Petri nets and integrality relaxations: a view of continuous Petri nets', *Transactions on IEEE, Man, and Cybernetics, Part C*, Vol. 32, No. 4, pp.314–326.

Recalde, L. and Silva, M. (2004) 'On fluidification of Petri Nets: from discrete to hybrid and continuous models', *Reviews in Control*, Vol. 28, No. 2, pp.253–266.

Recalde, L., Teruel, E. and Silva, M. (1999) 'Autonomous continuous P/T systems', *International Conference on Application and Theory of Petri Nets*, Vol. 39, No. 16, pp.107–126.

Sandmann, W. (2004) 'Relative error and asymptotic optimality in estimating rare event probabilities by importance sampling', *Proceedings of the OR Society Simulation Workshop (SW04) Held in Cooperation with the ACM SIGSIM, The Operational Research Society*, Birmingham, UK, pp.49–57.

Trivedi, K. and Kulkarni, V. (1993) 'Fluid stochastic Petri Nets', in Marsan, M.A. (Ed.): *Lecture Notes in Computer Science*, Vol. 691.

Vazquez, C.R., Recalde, L. and Silva, M. (2008) 'Stochastic continuous-state approximation of Markovian Petri Net systems', *Proceedings of the IEEE Conference on Decision and Control*, Cancun, Mexico, No. 8, pp.901–906.

Vazquez, C.R., Recalde, L. and Silva, M. (2009) 'Hybrid approximations of markovian Petri nets', *Proceedings of the IFAC, Conference on Analysis and Design of hybrid Systems*, Zaragoza, Spain, pp.901–906.

Zeng, Y., Duan, R., Feng, T. and Huang, S. (2019) 'Reliability analysis for complex systems based on generalized stochastic petri net and EDA approach considering common cause failure', *Engineering Computations*, Vol. 37, No. 5, pp.1513–1530.

Zerhouni, N. and Alla, H. (1990) 'Dynamic analysis of manufacturing systems using continuous Petri Nets', *Proceedings of the IEEE, International Conference on Robotics and Automation*, Cincinnati, Ohio, USA, pp.1070–1075.