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# Fractional order adaptive MRAC controller design for high-accuracy position control of an industrial robot arm


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**Abstract:** Most of conventional feedback controllers become inefficient in hard industrial environments like in steel industry, because of uncertainties in the plant model, or process dynamics variation due to nonlinear actuators, and changes in the character of the disturbances. This paper proposes an adaptive control design based on fractional order model reference adaptive control (FOMRAC) strategy in order to deal with an uncertain horizontal positioning control of an unloading machine in a rotary hearth furnace for hot rolling operation. The proposed FOMRAC scheme uses the MIT rule as an adaptive mechanism with two main modifications comparatively to the conventional MRAC: the reference model is an adequate fractional order system and the parameter adjustment rule contains a fractional order integrator. Stability analysis of the proposed control scheme is performed using the Lyapunov stability theorem. Numerical simulations are presented to show the effectiveness of the proposed fractional adaptive schemes applied to an industrial robot arm loading round steel blocks from inside a rotary hearth furnace. After comparison with the conventional MRAC, it is shown that the performances of FOMRAC are superior to classical control schemes.

**Keywords:** fractional order control; adaptive control; FOMRAC; industrial robot; high-accuracy position control; unloading machine.

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## 1 Introduction

Application of fractional calculus in dynamic systems control is a recent hot research topic (Podlubny, 1999; Raynaud and Zergalnoh, 2000). Fractional adaptive control is one of the most popular novel directions for researches. Initially the model reference approach was developed around 1960 by Whitaker and his colleagues (Osburn et al., 1961). Model reference adaptive control (MRAC) has become since that a standard topic on adaptive control (Landau, 1979; Aström and Wittenmark, 1995). In this approach adaptive algorithms allow the control of systems on which little information is known. The desired performances are set by choosing an adequate reference model.

Recently many automation researchers have focused their efforts on the introduction of fractional order systems and operators in adaptive control schemes, and especially in model reference adaptive control (FOMRAC) (Vinagre et al., 2002; Ladaci and Charef, 2006). At the beginning, they noticed that using a fractional order model reference in the control loop is able to improve the controlled system dynamics mainly because of the advantageous properties of fractional order systems (Ladaci et al., 2008).

Nowadays, there is a growing trend for utilisation of FOMRAC in solution of science and engineering problems. Many applications concern various aspects of physical sciences fields, as mechanics, electricity, chemistry, biology, economics, modelling, time and frequency domain, system identification and notably control theory, mechatronics and robotics, with encouraging results and advantageous performance.

The FOMRAC scheme is applied to conical tank level supervision in Balaska et al. (2018), and a fractional-order insulin-glucose dynamic model control in Coman et al. (2017), in medical anaesthesia in Navarro-Guerrero and Tang (2017) and in Suarez and Vinagre (2008) for controlling the lateral position of an autonomous guided vehicle (AGV). In Ma et al. (2009), FOMRAC is dedicated to controlling a hydraulic driven flight motion simulator, while in He and Gong (2010) it is used to control the temperature of a boiler burning system in, and to supervise a multi-source renewable energy system (Djebbri et al., 2020) and the cruise control system for an electric vehicle (Balaska et al., 2019), etc.

There have been also some works introducing FOMRAC approach in industrial robots control systems, particularly on robotic manipulators (Ladaci and Charef, 2002; Yanling, 2015; Bensafia et al., 2018; Maalej et al., 2021). Robot arms are widely used in industrial plants, as they provide more power, precision and rapidity to the manufacturing systems. Their control problem is among the most attractive and challenging research thematic (Arif and Fu, 2022; Uyulan, 2022).

Most of these fractional order MRAC schemes are based on the gradient approach and particularly the MIT rule for updating the unknown control parameters [so-called because it was developed in 1960 by the researchers of Massachusetts Institute of Technology (MIT)]. By using a fractional integration, they have proven the ability of fractional algorithms to guarantee stability with a highest level of performance comparatively to the original integer order algorithms (Bensafia and Ladaci, 2011; Tepljakov et al., 2018).

This paper proposes an adaptive control design based on fractional order MRAC strategy in order to deal with an uncertain horizontal positioning control of an unloading machine in a rotary hearth furnace for hot rolling operation (Chen and Chai, 2010; Liao et al., 2010).

The proposed FOMRAC scheme uses the MIT rule as an adaptive mechanism with two main modifications comparatively to the conventional MRAC: the reference model is an adequate fractional order system and the parameter adjustment rule contains a fractional order integrator.

These innovations are justified by the superior performance of fractional order systems relatively to integer one as demonstrated in a huge number of research works, and the introduction of a supplementary adjustment parameters which are the fractional orders of the integrator plus the reference model, giving more flexibility for the automation engineer.

Numerical simulations on the positioning system of the unloading arm model are presented to show the effectiveness of the proposed fractional adaptive schemes.

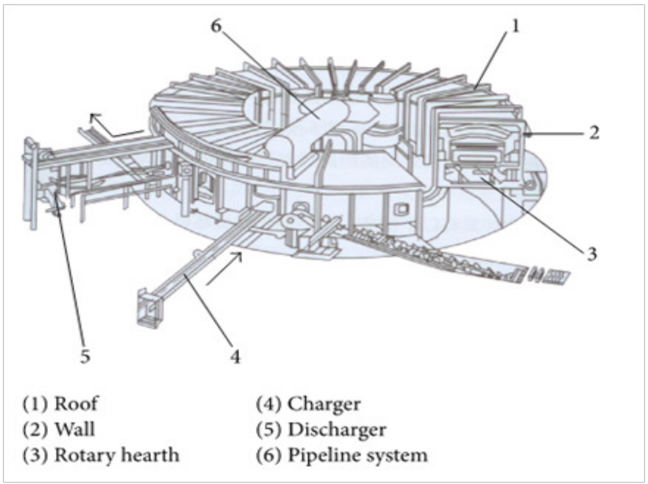
The paper is organised as follows. Section 2 presents the description of a rotary hearth furnace and unloading machine and defines the control problem. Section 3 is an introduction to fractional order systems with short definitions of fractional integration. Section 4 then presents the fractional order model reference adaptive control (FOMRAC) design. Simulation results and discussion are given in Section 5. Finally, some concluding remarks are presented in Section 6.

## 2 Rotary furnace and unloading machine description and problem statement

### *2.1 Machine description*

The rotary hearth furnace is the first element of the technological flow of the hot rolling operation. The role of this furnace is to heat the billet blocks, from room temperature to rolling temperature, about 1,250°C. This oven contains two machines: one used to load the metal blocks and the other to unload them from the furnace. Figure 1 shows the scheme of the machines (Roberts, 1983).

**Figure 1** Rotary hearth furnace equipment




The grip has a fixed jaw and a movable jaw. The grip is designed to pick billets out of the furnace and liberate them on to the output roll table. The arm moves vertically in order to elevate the billets from the oven and place them on to the roll table, and horizontally to control that the grip is in the correct location for the billet grip. Movements are realised using hydraulic cylinders, commanded by Group servo valves, see Figure 2.

During rotation of the furnace, the billets change their position at the outlet door, the billet is slightly moved to the right, or to the left, from the roll table axis. The robotic arm should be positioned on a billet so that the handle grips the block in the middle. In terms of positioning the grip inside the oven, this necessitates a left-to-right rotational movement, around a static point on the arm. The displacement distance resulting from the rotation of the arm is quite small and is approximated by a linear movement. Because the position of the billet in the oven changes, it is necessary to use a control loop, and because the precision of the positioning of the arm is raised, the arm must attain the desired position without a large overshoot. Exceeding the system can destroy the furnace, hitting the masonry of the unloading door with the arm. Due to the importance of precision in billet handling, the horizontal positioning system has an automatic control structure with controller and hydraulic positioning system.

**Figure 2** Unloading arm in action (see online version for colours)

The reference of the automatic positioning system represents the displacement of the billet relative to the rail axis, while the output of the system is represented by the position of the unloading arm. The set point of the system is given by a billet detection system which is based on a video camera and determines the displacement of the billets relative to the roller way axis in mm.

### *2.2 Unloading arm movement modelling*

By considering the constituent elements of the positioning system for the unloading arm, the plant is considered to be an electro-hydraulic axis and for consideration of time constants we take into account only the dynamics of the hydraulic part. Thus, the leading equations are given below (Inoan et al., 2011).

Linear equation of servo valves:

$$\Delta Q = K_Q \, \Delta x - K_C \, \Delta P_m \tag{1}$$

Equation of flow conservation:

$$\Delta Q = S\frac{d(\Delta y)}{dt} + \alpha \Delta P_m + \frac{V_T}{4E}\frac{d(\Delta P_m)}{dt} \tag{2}$$

Mechanical equation of motion:

$$S\Delta P_m = m\frac{d^2(\Delta y)}{dt^2} + f\frac{d(\Delta y)}{dt} + F_R \tag{3}$$

where the variables and coefficients are presented in Table 1.

**Table 1** Variables and parameters in the arm model

| *Variable* | *Meaning* |
|---|---|
| $K_Q$ | Flow gain |
| $K_C$ | Valve flow-pressure coefficient |
| $\Delta Q$ | Flow differential |
| $\Delta x$ | Billet displacement |
| $\Delta P_m$ | Pressure differential |
| $S$ | Piston area |
| $\alpha$ | Overall rate of oil loss |
| $V_T$ | Total oil volume |
| $\Delta y$ | Output size (arm displacement) |
| $E$ | Coefficient of oil elasticity |
| $m$ | Mass of the piston and the load |
| $f$ | Viscous damping coefficient |
| $F_R$ | Static force strength |

By applying the Laplace transform to equations (1)–(3), and by neglecting the parameters $\alpha$, $K_C$ and $F_R$, the following mathematical model is obtained (Inoan et al., 2011):

$$\Delta y = \frac{k\omega^2}{s^2 + 2\xi\omega s + \omega^2}\Delta x \tag{4}$$

where the mathematical parameters are defined as follows:

$$\begin{cases} k = \frac{K_Q}{S} \\ \omega = \sqrt{\frac{4ES^2}{V_T m}} \\ \xi = \frac{V_T f}{8ES^2}\sqrt{\frac{4ES^2}{V_T m}} \end{cases} \tag{5}$$

By replacing the parameters with the values from the data sheets and considering the approximated value of the mass, the following values are obtained for the transfer function (4) (see Inoan et al., 2011):

$$\begin{cases} k = 13.281 \\ \omega = 797.1495 \\ \xi = 3.0355\ 10^{-7} \end{cases} \tag{6}$$

### 2.3 Problem statement

The usual control system for the billet horizontal positioning uses a PI controller for the movement of the unloading arm. Its performance is 11% overshoot, 2 second settling time, and zero steady state error (Inoan, 2010).

The goal of this paper is to improve the unloading rate of the furnace, which means decreasing the current settling time, by designing a FOMRAC controller.

## 3 Basics of fractional calculus

Fractional calculus including integration and differentiation of arbitrary orders dates back to Cauchy, Riemann Liouville and Leitnikov in the 19th century. It has been used in mechanics since at least the 1930s and in electrochemistry since the 1960s. In control field, pioneering works have been achieved in the Soviet Union (Brin, 1962), and later, several researchers have studied fractional differential operators and their application in systems theory (Srivastava and Saxena, 2001).

Different definitions have been proposed in mathematical literature for the fractional order (non-integer) integral and derivative operators. The most popular are those of Riemann-Liouville, Grünwald-Letnikov and Caputo (see for instance Podlubny, 1999).

### 3.1 Definition of fractional integration

The fractional-order integral in the sense of Riemann-Liouville, is defined as

$$D_a^{-\nu} f(t) = \frac{1}{\Gamma(\nu)}\int_a^t (t-\zeta)^{\nu-1} f(\zeta)d(\zeta) \tag{7}$$

with the Gamma function given by

$$\Gamma(x) = \int_0^\infty y^{x-1}e^{-y}dy \tag{8}$$

such that $(a, t) \in \Re^2$ with $a < t$ and $(0 < \nu < 1,\ \nu \in \Re)$.

Usually, industrial controlled processes are discretised, so we have to use a numerical approximation of equation (7). The literature is very prolific in numerical solutions for fractional order differential equations and discretisation of fractional operators (Ladaci and Charef, 2006; Naifar and Ben Makhlouf, 2022).

### 3.2 Numerical approximation of the fractional order integral

A very simple numerical approximation of Riemann-Liouville integral definition is used here based on the rectangular method (Ladaci and Charef, 2006).

By putting, $t = k\Delta$ where $t$ is the actual time, $k$ an integer and $\Delta$ the sampling period, we obtain:

$$I^\nu\left(f(k\Delta)\right) = \frac{\Delta^\nu}{\Gamma(\nu)}\sum_{\tau=0}^{k-1}(k-\tau)^{\nu-1} f(\tau\Delta) \tag{9}$$

Another widely used numerical approximation is based on the Grünwald-Leitnikov definition (Podlubny, 1999).

For a causal function $f(t)$, with the time operator $t = kh$, the fractional order derivative is computed by Balaska et al. (2019):

$$D_t^\alpha f(t) = h^{-\alpha}\sum_{r=0}^{\frac{t}{h}}(-1)^r \binom{\alpha}{r} f(kh - rh) \tag{10}$$

The fractional order integral is given by:

$$\begin{aligned} I_t^\alpha f(t) &= D_t^{-\alpha} f(kh) \\ &\approx h^\alpha \sum_{r=0}^{\frac{t}{h}}(-1)^r \binom{-\alpha}{r} f(kh - rh) \end{aligned}$$

where $\binom{-\alpha}{r}$ is calculated from equation:

$$\binom{\alpha}{0} = 1,\quad \binom{\alpha}{r} = \frac{\alpha(\alpha-1)\dots(\alpha-r+1)}{r!} \tag{11}$$

with $r \in N$ and $h$ denotes the sampling period.

### 3.3 Approximation of fractional order transfer function

In order to implement our adaptive regulators we need to approximate the fractional order components by rational transfer functions. For this objective, we use the so-called singularity function method proposed by Charef et al. (1992) which provides a simple algorithm for the approximation of transfer function with a single pole or two poles.

In this work we are concerned with a second order plant model, and for the case of fractional second order-like system of the form,

$$G_f(s) = \frac{1}{(\frac{s^2}{\omega_n^2} + 2\xi\frac{s}{\omega_n} + 1)^{\beta}} \tag{12}$$

where $0 < \beta \leq 1$ $\beta$ is constrained in this interval in order to remain in a double fractional-order mode dynamic. $\xi$ is the damping factor and $\omega_n$ the proper pulse.

The fractional order transfer function (12) is approximated by an integer order transfer function by means of the singularity function method. We distinguish two situations:

- *Case* $0 < \beta < 0.5$

  We can express equation (12) as follows:

$$G_e(s) = \frac{(\frac{s}{\omega_n} + 1)(\frac{s}{\omega_n+1})^{\eta}}{(\frac{s^2}{\omega_n^2} + 2\rho\frac{s}{\omega_n} + 1)} \tag{13}$$

  with $\rho = \xi^{\beta}$ and $\eta = 1 - 2\beta$, allowing us to write,

$$G_e(s) \approx \frac{(\frac{s}{\omega_n} + 1)}{(\frac{s^2}{\omega_n^2} + 2\rho\frac{s}{\omega_n} + 1)} \frac{\prod_{i=1}^{N-1}(1 + \frac{s}{z_i})}{\prod_{i=1}^{N}(1 + \frac{s}{p_i})} \tag{14}$$

  The poles $p_i$ and zeros $z_i$ are obtained using the following formulas:

$$p_i = (ab)^{i-1}az_1 \qquad i = 1, 2, 3, ..., \tag{15}$$

$$z_i = (ab)^{i-1}z_1 \qquad i = 2, 3, ..., N-1 \tag{16}$$

  with,

$$z_1 = \omega_n\sqrt{b} \tag{17}$$

$$a = 10^{\frac{\epsilon_p}{10(1-\eta)}}$$

$$b = 10^{\frac{\epsilon_p}{10\eta}}$$

$$\eta = \frac{\log(a)}{\log(ab)}$$

  $\epsilon_p$ is the approximation error in dB (usually, we chose $\epsilon_p < 3dB$ for a good result).

  The approximation order $N$ is calculated by fixing the working frequency bandwidth, specified by $\omega_{\max}$ such that: $p_{N-1} < \omega_{\max} < p_N$, which leads to the following value:

$$N = Integer\ part\ of\ \left[\frac{\log(\frac{\omega_{max}}{p_1})}{\log(ab)} + 1\right] + 1 \tag{18}$$

  $G_e(s)$ becomes a rational function of order $N + 2$ as follows,

$$G_e(s) = \frac{b_{m_0}s^N + b_{m_1}s^{N-1} + ... + b_{m_N}}{s^{N+2} + a_{m_1}s^{N+1} + ... + a_{m_{N+2}}} \tag{19}$$

  The coefficients $a_{m_i}$ and $b_{m_i}$ are computed from the singularities $p_i$, $z_i$.

- *Case* $0.5 < \beta < 1$

  The fractional order transfer becomes:

$$G_e(s) = \frac{(\frac{s}{\omega_n} + 1)}{(\frac{s^2}{\omega_n^2} + 2\rho\frac{s}{\omega_n} + 1)(\frac{s}{\omega_n+1})^{\eta}} \tag{20}$$

  where $\rho = \xi^{\beta}$ and $\eta = 2\beta - 1$. The singularities are computed using the following algorithm:

$$p_i = (ab)^{i-1}p_1 \qquad i = 1, 2, 3, ..., N \tag{21}$$

$$z_i = (ab)^{i-1}ap_1 \qquad i = 2, 3, ..., N-1 \tag{22}$$

$$p_1 = \omega_n\sqrt{b} \tag{23}$$

$$a = 10^{\frac{\epsilon_p}{10(1-\eta)}} \tag{24}$$

$$b = 10^{\frac{\epsilon_p}{10\eta}}$$

$$\eta = \frac{\log(a)}{\log(ab)}$$

  At the end, the transfer function $G_e(s)$ will is expressed as in equation (19).

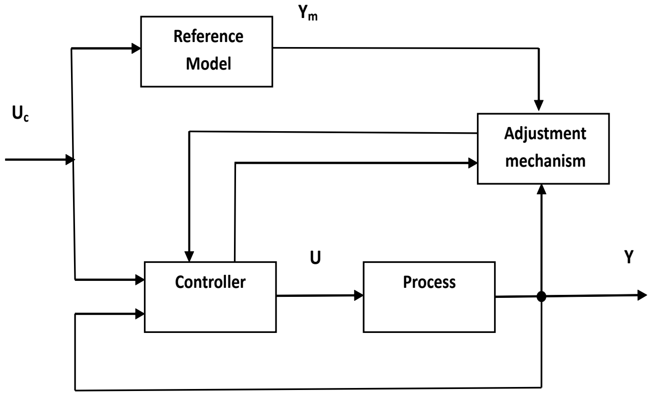
## 4 FOMRAC design

FOMRAC is an extension of MRAC control scheme to the fractional order systems. Since the first propositions in 2002 (Ladaci and Charef, 2002; Vinagre et al., 2002), many authors have proposed FOMRAC schemes based on the Lyapunov stability theorem or the gradient approach. One can cite the works of Abedini et al. (2015) in discrete time, Aguila-Camacho and Gallegos (2020) for a FOMRAC algorithm where the orders are switched among a fractional value in the interval $(0, 1)$ and 1 at certain time instants, Arpacı et al. (2016) for a scheme with an adaptive PI$^{\lambda}$D$^{\mu}$ controller, and many others...

In MRAC approach, the desired performance is expressed in terms of a reference model (a model that describes the desired input-output properties of the closed-loop system) and the parameters of the controller are adjusted based on the error between the reference model output and the system output.

The model reference adaptive control system can be represented by the scheme of Figure 3.

**Figure 3** Direct model reference adaptive control scheme




## *4.1 Direct MRAC with MIT rule*

The plant model is given as:

$$y(t) = G(s)[u](t) = k_p \frac{N(s)}{D(s)}[u](t) \tag{25}$$

where $y$ is the system output and $u$ is the control input.

The expression $G(s)[u](t)$ designates the signal $u(t)$ that is filtered by the filter $G(s)$ and the square brackets are used for avoiding the mixture of the time domain and the frequency domain signals.

Consider a closed loop system where the controller has an adjustable parameter vector $\theta$. We choose a reference model

$$y_m(t) = G_m(s)[u_c](t) = k_m \frac{N_m(s)}{D_m(s)}[u_c](t) \tag{26}$$

which output is $y_m$ specifies the desired closed loop response and $u_c$ the reference signal. Let $e$ be the error between the closed loop system output $y$ and the model one $y_m$, we adjust the parameters such that the cost function:

$$J(\theta) = \frac{1}{2}e^2 = \frac{1}{2}(y - y_m)^2 \tag{27}$$

is minimised. Thus, the parameters are changed in the direction of negative gradient of $J$, so:

$$\frac{d\theta}{dt} = -\gamma \frac{dJ}{d\theta} = -\gamma e \frac{de}{d\theta} \tag{28}$$

where $\theta$ is the parameter vector and $J$ the quadratic error criterion.

We get (Aström and Wittenmark, 1995),

$$\theta = -\frac{\gamma}{s} y_m e \tag{29}$$

The control law is calculated using this relation,

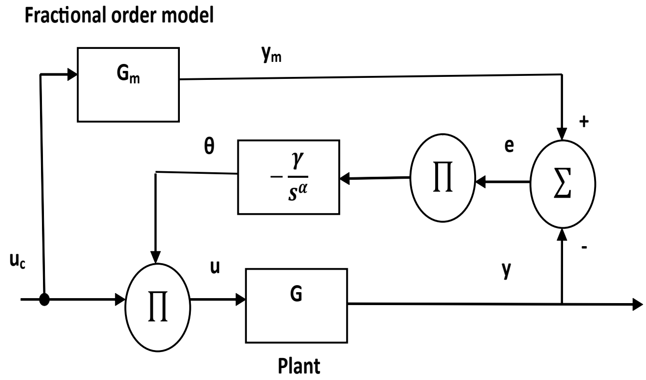
$$u = \phi^T \theta \tag{30}$$

where $\phi$ is the regression vector containing the measured input (control) $u$ and output $y$ signals and the input reference signal $u_c$.

## *4.2 Adaptation law with fractional integration*

In the adjusting algorithm we put a fractional $\alpha$ order integration (Ladaci and Charef, 2003) with $\alpha$ non-zero positive real such that: $0 < \alpha < 2$. We also use a fractional order reference model, which leads to the following block scheme of Figure 4.

**Figure 4** Adaptation algorithm using a fractional order integration




We get,

$$\theta = -\frac{\gamma}{s^\alpha} y_m (y - y_m) = -\frac{\gamma}{s^\alpha} y_m e \tag{31}$$

So,

$$\frac{d^\alpha \theta}{dt^\alpha} = -\gamma\ y_m e \tag{32}$$

and

$$\theta = -\gamma I^\alpha [y_m e] \tag{33}$$

The control law is calculated using the relation (30),

$$u = \phi^T \theta.$$

## *4.3 Stability analysis*

There are various configurations for MRAC adaptive control in literature and many results on stability and robustness analysis of such adaptive control systems (see for instance Rao and Hassan, 2004; Akhtar and Bernstein, 2005; Naeijian and Khosravi, 2020). Even for FOMRAC, interesting results have been obtained recently (Shi et al., 2014; Bourouba et al., 2019), ... and others.

In the case of our study, we have to make certain assumptions on the system and reference model,

*Assumption 1:*

- The sign of $kp$ is known.
- The relative degrees of controlled system and reference model are equal, and the highest order of reference model is not larger than that of the controlled system.
- $D(s)$, $N(s)$, $Dm(s)$, $N_m(s)$ are monic polynomials.

- $G(s)$ is a minimum phase system, thus $N(s)$ is Hurwitz.
- The transfer function $G_m(s)$ is strictly positive real.

In this study, the main purpose is to design a fractional order adaptive control scheme FOMRAC which is able to stabilise a class of linear systems described by equation (25) and force it to behave as the reference model (26). The proposed adaptive control configuration is represented in Figure 4.

The following theorem states its stability properties,

*Theorem 1:* Consider the class of fractional order systems given by equation (25), and let the designed reference model be given by equation (26) under the mentioned assumptions, then the control law defined by equation (30)

$$u = \phi^T \theta.$$

with the gain adaptation law (33)

$$\theta = -\gamma I^{\alpha} [y_m e]$$

guarantees that all closed-loop signals are bounded, the tracking error goes to zero asymptotically.

*Proof of Theorem 1:* See the proof in Balaska et al. (2021) pp.6 for the particular case where the fractional order $\gamma$ is set to be integer ($\gamma = 1$).

## 5 Simulation results and discussion

The unloading machine has for input the displacement of the billet relative to the axis of the rail given by a billet detection system, and for output the position of the unloading arm.

The system is described by the following transfer function (Inoan et al., 2011),

$$G(s) = \frac{8.439\ 10^6}{s^2 + 0.0004839 s + 6.354\ 10^5} \tag{34}$$

we first choose an integer order reference model of the form:

$$G_m(s) = \frac{100}{s^2 + 19s + 100} \tag{35}$$

then we consider a second degree fractional order model

$$G_{mf}(s) = \frac{100}{(s^2 + 19s + 100)^{0.6}} \tag{36}$$

we get the approximate model using the singularity function method (19),

$$G_{mf}(s) \approx \frac{\begin{array}{c} 3.162\ 10^{16}\ s^4 + 5.961\ 10^{20}\ s^3 \\ +6.017\ 10^{23}\ s^2 + 3.758\ 10^{25}\ s \\ +3.162\ 10^{26} \end{array}}{\begin{array}{c} 1\ 10^{11}\ s^6 + 1.884\ 10^{16}\ s^5 \\ +1.894\ 10^{20}\ s^4 + 1.096\ 10^{23}\ s^3 \\ +5.236\ 10^{24}\ s^2 + 7.192\ 10^{25}\ s \\ +3.162\ 10^{26} \end{array}} \tag{37}$$

then, this fractional order model is discretised with a sampling period $\Delta = 0.05\ s$. The sampling period $\Delta$ has been chosen in accordance to the Shannon theorem, so that all the useful information is saved in the discretised signals.

**Figure 5** Open-loop bode diagram for unloading arm

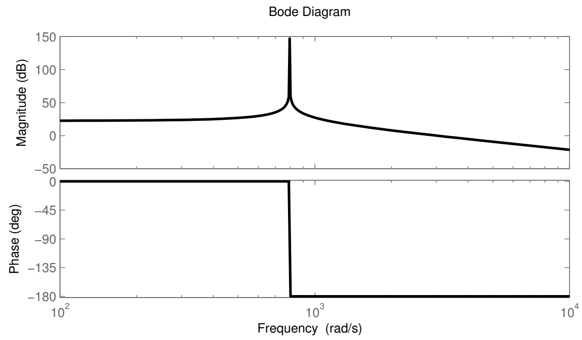
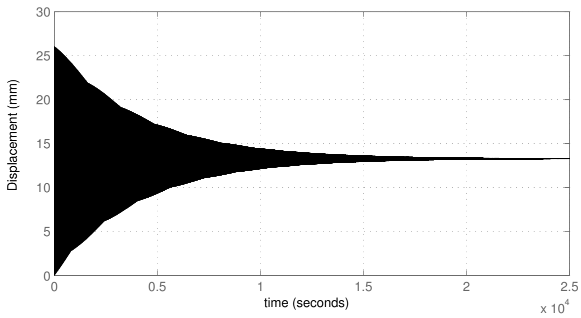



**Figure 6** Open-loop unloading arm displacement




The open loop system frequency response and step response are given in Figures 5 and 6 respectively. They show a badly damped behaviour and a slow convergence rate.

### 5.1 MRAC control with integer order model

In the first simulation experiment, we apply the MRAC control law (29) and (30) to the robot system (34) using the integer order reference model (35).

Figure 7 shows the responses obtained for the case of integer order model in the ideal case whereas Figure 8 illustrates the responses in presence of additive output random noise of 5% reference signal amplitude.

Although the response time remains unchanged, we have managed to stabilise the Closed Loop system, while ensuring a good precision of the output.

### 5.2 MRAC control with fractional order model

In this simulation experiment, we apply the MRAC control law (29) and (30) to the robot system (34) using the fractional order reference model (36).

Figure 9 shows the responses obtained in the ideal case whereas Figure 10 illustrates the responses in presence of additive output random noise of 5% reference signal amplitude.

**Figure 7** Output of the process with integer order model without perturbations, $\theta$ parameters, control signal, error signal

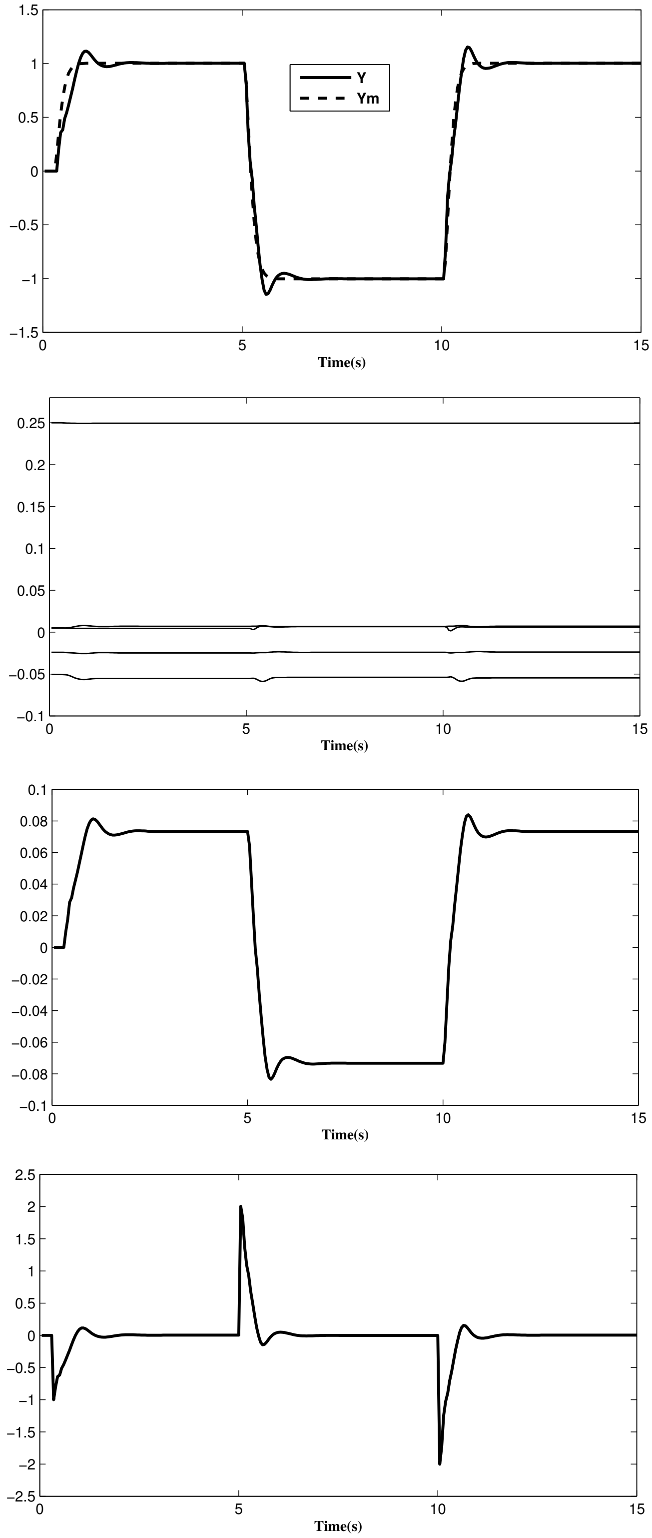



**Figure 8** Output of the process with integer order model with random noise of 5% reference signal amplitude, $\theta$ parameters, control signal, error signal

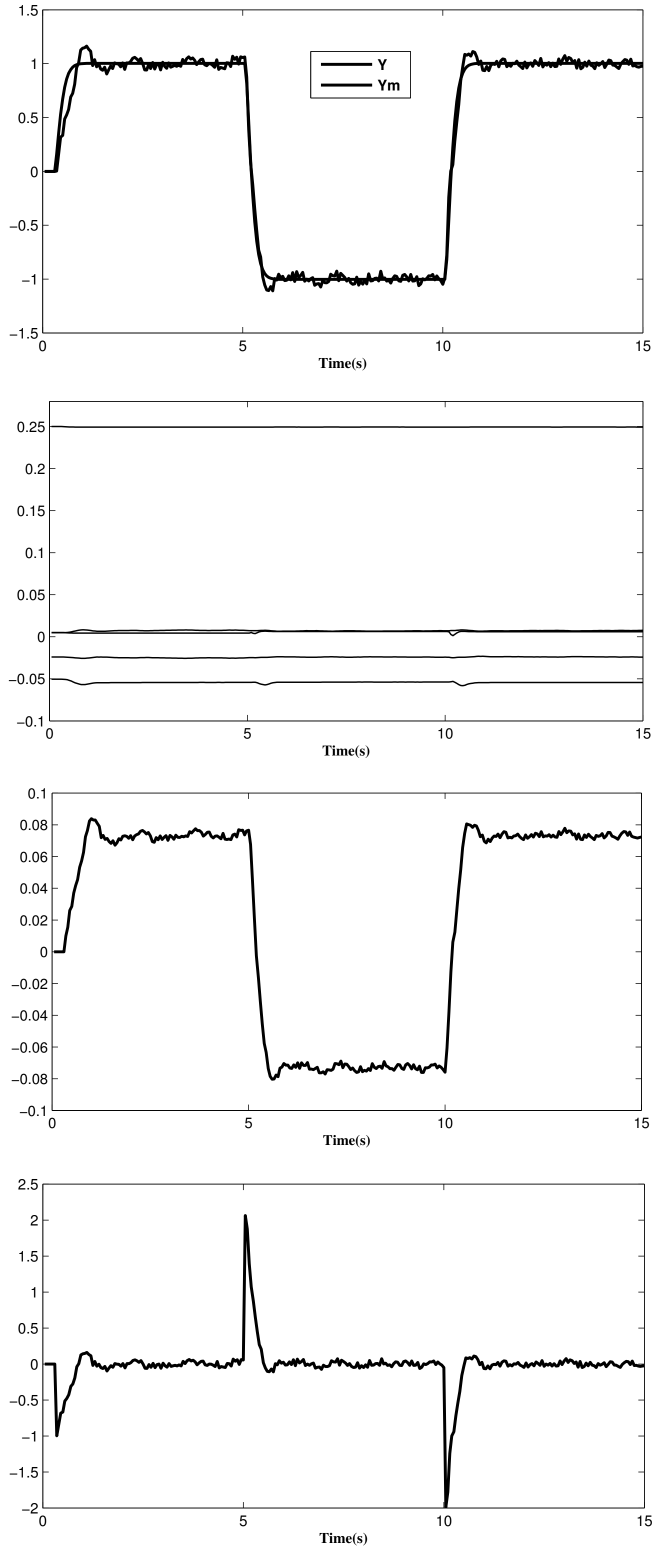


**Figure 9** Output of the process with fractional order model without perturbations, $\theta$ parameters, control signal, error signal

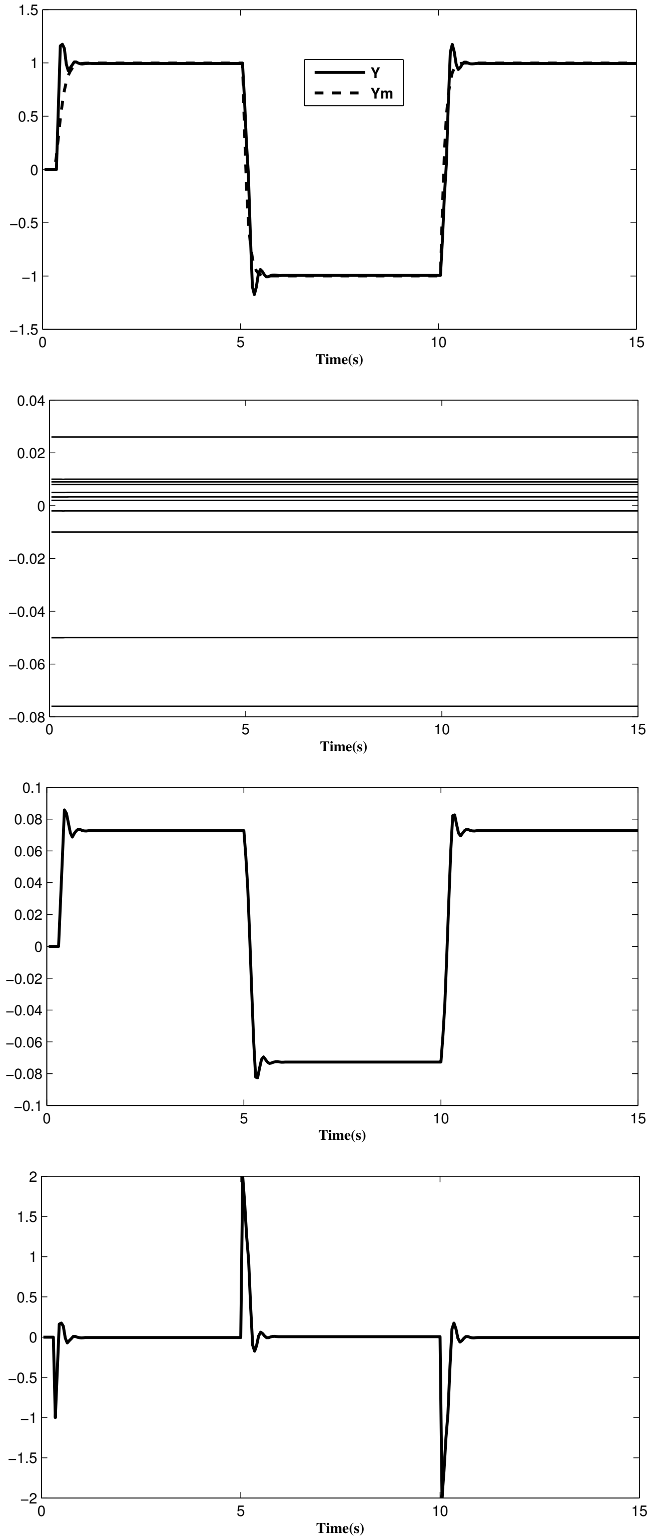
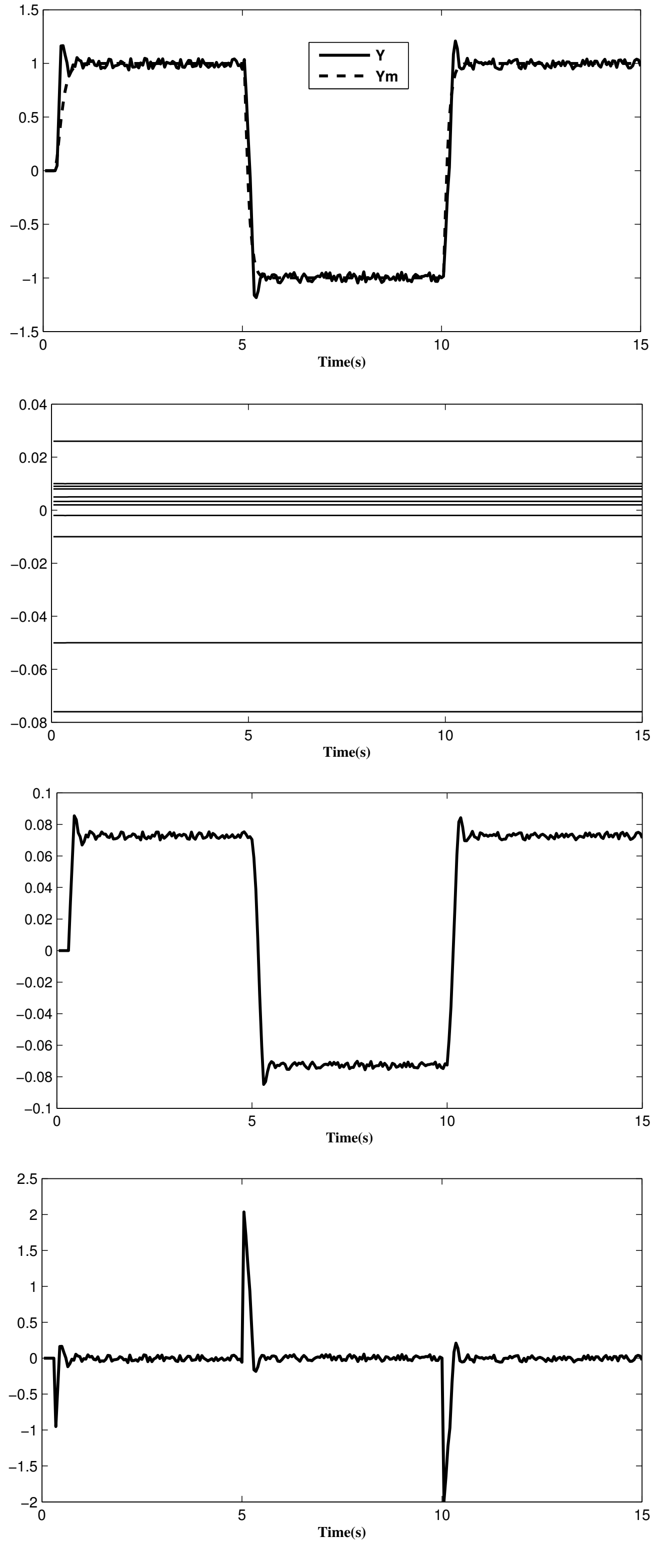



**Figure 10** Output of the process with fractional order model with random noise of 5% reference signal amplitude, $\theta$ parameters, control signal, error signal



For the case of fractional order model, results show a marked improvement in the dynamics of the closed loop system, with a small adaptation gain (a ratio of the order of $1/10$ with that of the classic case), and a response time equal to 1 s.

### *5.3 MRAC control with fractional order model and fractional order adaptation rule*

Now we apply the MRAC control law with fractional order integrator (33) and (30) to the robot system (34) using the fractional order reference model (36).

In order to evaluate the performance of the different control schemes results, we consider the following index functions on the simulation window:

- The quadratic error criterion $J_e$:

$$J_e = \sum e^2 \tag{38}$$

- The quadratic control input (energy) criterion $J_u$:

$$J_u = \sum u^2 \tag{39}$$

- The combined quadratic error and energy criterion $J_{eu}$:

$$J_{eu} = \sum \left(\lambda\, e^2 + \mu\, u^2\right) \tag{40}$$

  where $\lambda$ and $\mu$ are two positive real numbers to be chosen.

In Table 2, we present the comparative results obtained for the criteria $J_e$, $J_u$ and $J_{eu}$ with $\lambda = 0.6$ and $\mu = 0.4$ when applying different values of $\alpha$ from 0.1 to 1.9 with adequate values of the adaptation gain.

It can be noted from the values presented in Table 2 that the best value resulted for $\alpha$ is 1.5, for which the quadratic error criterion and the combined quadratic criterion are equal to $J_e = 4.4451$, $J_{eu} = 3.5307$ respectively. Figure 13 illustrates the evolution of the quadratic error criterion versus the fractional integration order.

Figure 11 shows the responses obtained for the case of fractional order model with $\alpha = 1.5$ and a very small adaptation gain (a ratio of the order of $1/1{,}000$ with that of the classic case). From the numerical simulations, we can notice that the performance level is better with the FOMRAC and adaptation law with fractional order integral in terms of response time and peak overshoot. Besides, the time response is doubled when the integration is of integer order, which is a major advantage of introducing a fractional order integrator in the adaptation law.

Figure 12 illustrates also the robustness of this adaptive control scheme against additive output noises.

Table 3 clearly shows the improvement obtained with the proposed fractional order adaptive control scheme.

In Table 3, the step response of the proposed fractional order adaptive command gives an overshoot of order $13.49\%$, billets can be shifted by up to $24,57$ mm to the left, or to the right, of the rail way axis.

**Table 2** Adaptive control comparative performance evaluation vs. integration order

| $\alpha$ | $J_e$ | $J_u$ | $J_{eu}$ | *Peak overshoot* |
|---|---|---|---|---|
| 0.1 | 20.1117 | 1.5207 | 12.6750 | 1.1743 |
| 0.2 | 20.1084 | 1.5210 | 12.6736 | 1.1746 |
| 0.3 | 20.1050 | 1.5213 | 12.6714 | 1.1746 |
| 0.4 | 20.1015 | 1.5216 | $12,6693$ | 1.1746 |
| 0.5 | 20.0977 | 1.5219 | $12,6671$ | 1.1746 |
| 0.6 | 20.0977 | 1.5221 | $12,6671$ | 1.1746 |
| 0.7 | 20.0280 | 1.5183 | $12,6238$ | 1.1689 |
| 0.8 | 19.7929 | 1.5346 | $12,4898$ | 1.1530 |
| 0.9 | 19.9156 | 1.5092 | $12,5528$ | 1.1187 |
| *1.0* | *26.9910* | *1.4900* | *16.7281* | *1.1143* |
| 1.1 | 19.9069 | 1.5110 | 12.5485 | 1.1187 |
| 1.2 | 19.8925 | 1.5112 | $12,5400$ | 1.1187 |
| 1.3 | 19.8829 | 1.5121 | $12,5344$ | 1.1187 |
| 1.4 | 19.8723 | 1.5131 | $12,5287$ | 1.1187 |
| *1.5* | *19.7590* | *1.5254* | $12,4658$ | *1.1349* |
| 1.6 | 19.8477 | 1.5155 | $12,5146$ | 1.1187 |
| 1.7 | 19.8337 | 1.5170 | $12,5068$ | 1.1187 |
| 1.8 | 19.8743 | 1.5208 | $12,5330$ | 1.1187 |
| 1.9 | 19.5631 | 1.5260 | 12.3481 | 1.2003 |

**Table 3** Comparative performance evaluation of the different adaptive control schemes

| *Control scheme* | $J_e$ | $J_u$ | $J_{eu}$ | *Peak overshoot* |
|---|---|---|---|---|
| MRAC ideal | 26.991 | 1.4903 | 16.7903 | 1.0970 |
| MRAC + noises | 27.3291 | 1.4862 | 16.9917 | 1.1230 |
| FOMRAC ideal | 20.095 | 1.5222 | 12.6657 | 1.1755 |
| FOMRAC + noises | 20.871 | 1.5228 | 13.1319 | 1.1977 |
| FOMRAC + frac int i deal | 19.759 | 1.5254 | 12.4658 | 1.1349 |
| FOMRAC + frac int + noises | 20.094 | 1.5310 | 12.6686 | 1.1582 |

The distance between two consecutive billets on the hearth (distance considered on the medium circle at the half length between the internal and the external wall) is constant and equal to the billet's diameter. There are only two possible diameters of the billets that can be used, 150 mm, respectively 180 mm.

The horizontal positioning distance of 364 mm achieved by the unloading arm is limited because of the outlet door. If the arm would perform a broader positioning movement it is likely to hit the outlet door's walls.

We can notice that the controller is very sensitive to the change of reference input.

**Figure 11** Output of the process with fractional order model and fractional integral $\beta = 1.5$ without perturbations, $\theta$ parameters, control signal, error signal

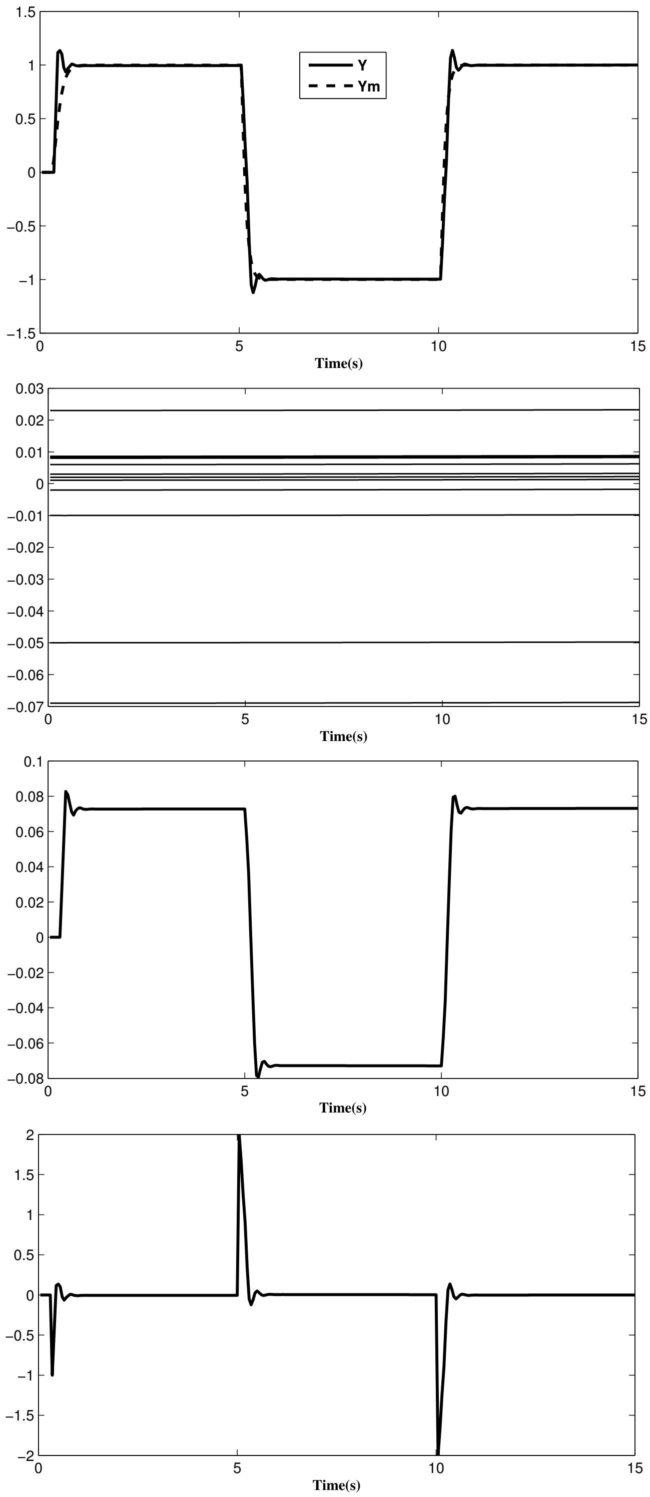



**Figure 12** Output of the process with fractional order model and fractional integral $\beta = 1.5$ with random noise of 5% reference signal amplitude, $\theta$ parameters, control signal, error signal

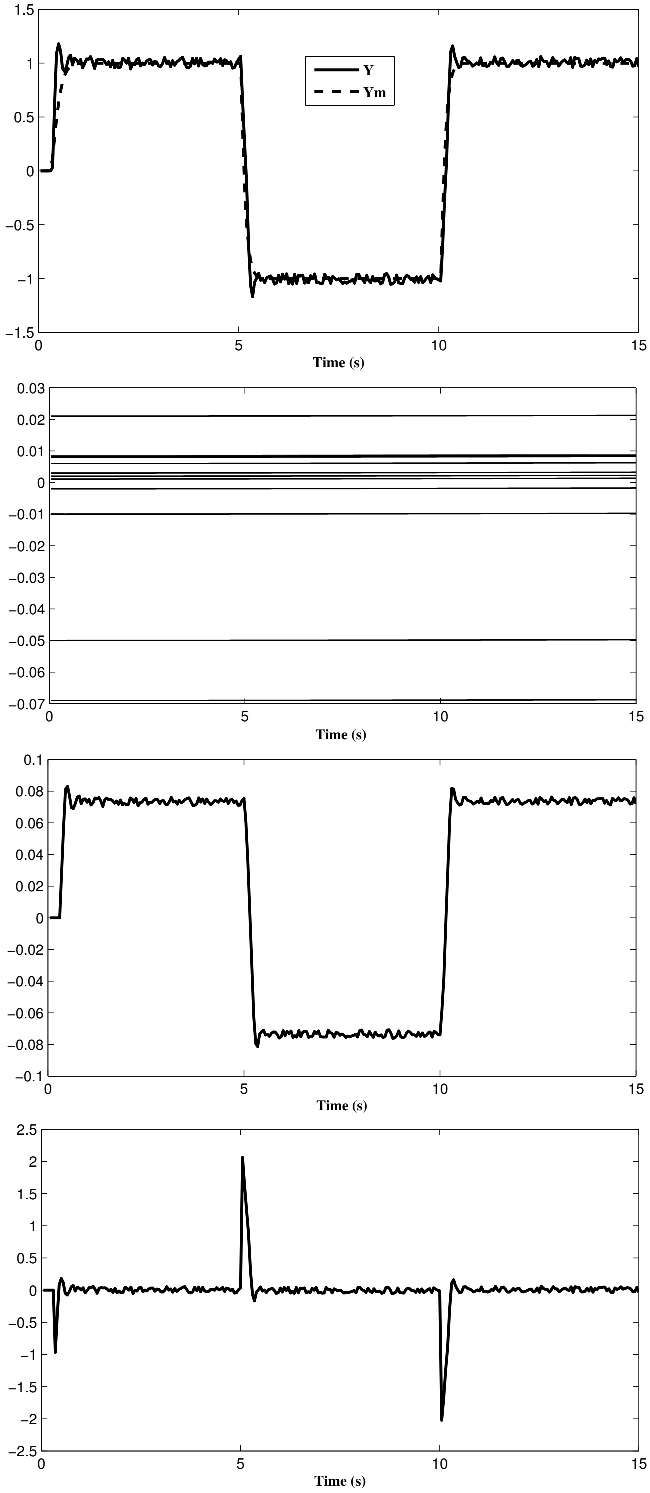
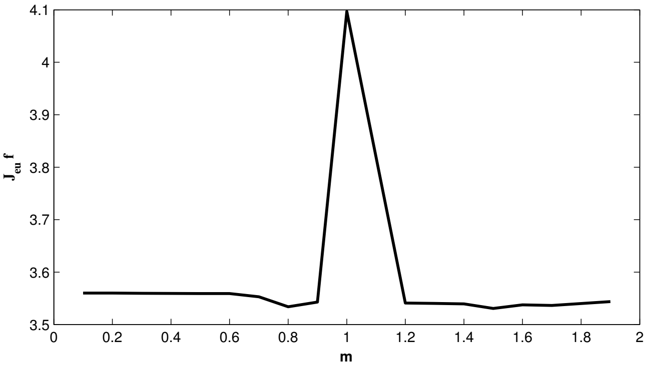


**Figure 13** Quadratic error criteria $J_e$ vs. the fractional integration order




## 6 Conclusions

In this paper, a fractional model reference adaptive control design which includes the use of fractional order integrators, has been proposed for a horizontal positioning system of an unloading machine. The proposed fractional adaptive controller guarantees the closed loop stability with a satisfying level of performances, where time response is priority.

According to the numerical simulation study, the settling time is reduced from 2 s to 1 s and the discharge rate of the billets from the furnace is improved by 50% relatively to the classical integer order control system.

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