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Threshold of motion of unconventional sediment under unidirectional flow

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Abstract: The threshold of motion of commonly found sediments in streams and rivers, sand and gravel which are mainly siliceous in nature, has been studied thoroughly since the development of the field of sediment transport in fluvial hydraulics. There are other types of sediment, of more irregular shape, such as bioclastic, biogenic, and organic detritus which need a special focus vis-a-vis their transport. This study considers the incipient motion of such sediments and finding a better representation of their threshold of motion. This study considers the empirical curves of some previous researchers and the scatter of data from unconventional sediment to assess the threshold behaviour of such sediment under unidirectional flow. The consideration of the settling velocity in the sediment threshold studies is important in general and for the sediment studied here in particular. The Movability Number is found to be a better representation of the threshold than the Shields parameter or square of Movability Number as data show less scatter about empirical curves.

Keywords: sediment transport; fluvial hydraulics; incipient motion; unconventional sediment; Movability Number; Shields parameter.

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1 Introduction

Shields (1936) made a large impression in the study of the threshold of the sediment motion under streamflow. Studies that followed (Andrews, 1983; Wiberg and Smith, 1987; Kirchner et al., 1990; Houwing and Van Rijn, 1998; Luckner and Zanke, 2007; Dwivedi et al., 2012; Kitsikoudis et al., 2016; Zhou et al., 2021) linked their findings with that seminal work. Most threshold experiments considered traditional sediments like sand and gravel, commonly found in rivers and streams. There are, however, other types of sediment present along with such sediment. Various types can be identified from some of those studies, like detritus, biogenic material, and bioclastic sediment, found in streams, estuaries, and coastal areas. This study reintroduces this unconventional sediment to study its threshold behaviour in some different and broader aspects.

In sediment transportation, there are two conditions which are followed by sediment particles before actual transport takes place. The threshold condition is when the driving forces of the streamflow just equal the resisting forces and the incipient motion condition when the particle starts to move. The most used parameter of the sediment threshold is Shields parameter (Fenton and Abbott, 1977; Buffington and Montgomery, 1997; Dey, 1999; Paphitis, 2001; Paphitis et al., 2002; Zanke, 2003; Vollmer and Kleinhans, 2007; Safari et al., 2017) which is the dimensionless shear stress

$$\theta = \frac{\tau}{(\rho_s - \rho)gd} = \frac{u_*^2}{(\rho_s/\rho - 1)gd} \quad (1)$$

where $\tau = \rho u_*^2$ is the bed shear stress, in which u_* is the shear velocity of the flowing water, ρ_s is the density of sediment, ρ is the density of the water, g is the acceleration due to gravity, and d is the representative diameter of the sediment. The incipient motion condition is the critical condition where critical bed shear stress is represented as $\tau = \rho u_{*c}^2$, where u_{*c} is the critical shear velocity. Many researchers have used another parameter for the threshold of the sediment motion, the Movability Number A , which is the ratio of the shear velocity of the flowing stream to the settling velocity w_s of the sediment under observation (Collins and Rigler, 1982; Komar and Clemens, 1986; Paphitis, 2001; Beheshti and Ataie-Ashtiani, 2008; Simoes, 2014; Rieux et al., 2019). The incipient motion condition is represented by the critical Movability Number given by

$$A_c = u_{*c} / w_s \quad (2)$$

The Movability Number is actually a dimensionless velocity, while as Shields parameter, a dimensionless stress. To make the two quantities comparable, the Movability Number is converted to the dimensionless stress by squaring it (A^2) and the critical condition is represented as shown below.

$$A_c^2 = \frac{u_{*c}^2}{w_s^2} = \frac{\tau_c / \rho}{w_s^2} \quad (3)$$

In the present study, A_c^2 as the sediment incipient motion parameter is also analysed. Even though A^2 considers the effect of all important settling velocity, but still, it is not as better parameter as A . As per Simoes (2014), A reduces the scatter of the experimental data around the empirical curves as it is proportional to the square root of θ , hence τ as per the following equation.

$$A = \sqrt{\frac{3}{4} C_D \theta} \quad (4)$$

where C_D is the drag coefficient.

Fisher et al. (1983) studied the incipient motion of mostly organic detritus matter on sand and gravel beds. The presence of such matter along with inorganic sediment, hence acting also as nutrients for organisms present in the water bodies stimulated those authors to carry out the study. They analysed the scatter of the data around the Shields curve and then formulated a new sediment entrainment function, taking into account the non-uniformity of the mostly organic test sediment and the underlying inorganic bed material, which reduced the scatter and hence largely eliminated the effect of the sediment non-uniformity.

Paphitis et al. (2002), while arguing that biogenic shell fragments are also present along with the natural sedimentary sediment and as they have non-homogenous shapes, stressed the incipient motion behaviour of such sediment by observing the scatter around some already existing sediment threshold curves. After observing the results, they preferred settling-size over the physical sieve size and Movability Number over the Shields parameter.

Rieux et al. (2019) considered bioclastic sediment which may be present along with the inorganic sediment like sand in significant proportions producing high carbonate content in temperate and cool-water bodies. They focused on the various species of such sediment, their varying shapes, and density, and thus showing varying hydrodynamic behaviour. They found that the sieve diameter is a better representative of the sediment size than the equivalent settling diameter for the type of sediment used. On observing the scatter of the data along already existing threshold curves of the siliciclastic sediment, they found that the values of the critical Movability Number fell mostly over the curves.

In this study the scatter of data about some existing empirical curves of the sediment threshold will be examined, the threshold behaviour assessed and a better parameter, representative of the threshold of such sediment, will be found.

The Paphitis (2001) single (mean) threshold curve in terms of the critical Movability Number and particle Reynolds number is given by

$$A_c = \frac{0.75}{R_*} + 14e^{-2R_*} + 0.01 \ln R_* + 0.115, \quad 0.1 < R_* < 10^5 \quad (5)$$

where $R_* = u_*d/\nu$, is the particle Reynolds number and ν is the kinematic viscosity of flowing fluid. Squaring the above equation, A^2 can be expressed as below:

$$A_c^2 = \left[\frac{0.75}{R_*} + 14e^{-2R_*} + 0.01 \ln R_* + 0.115 \right]^2, \quad 0.1 < R_* < 10^5 \quad (6)$$

Simoes (2014) presented the threshold curve in terms of the dimensionless grain diameter as an independent variable, given by (Paphitis, 2001)

$$d_* = d \left[\frac{(s-1)g}{\nu^2} \right]^{1/3} = \left[\frac{R_*^2}{\theta} \right]^{1/3} \quad (7)$$

The Simoes (2014) equation is given by

$$A_c = 0.215 + \frac{6.79}{d_*^{1.70}} - 0.075e^{-2.62 \times 10^{-3} d_*} \quad (8)$$

Squaring the above equation, A^2 can be expressed as below:

$$A_c^2 = \left[0.215 + \frac{6.79}{d_*^{1.70}} - 0.075e^{-2.62 \times 10^{-3} d_*} \right]^2 \quad (9)$$

The Paphitis (2001) mean threshold curve in terms of the critical Shields parameter and the particle Reynolds number is given by

$$\theta_c = \frac{0.188}{1 + R_*} + 0.0475(1 - 0.699e^{-0.015R_*}), \quad 0.01 < R_* < 10^5 \quad (10)$$

The Paphitis (2001) mean threshold curve in terms of the critical Shields parameter and the dimensionless grain diameter is given by

$$\theta_c = \frac{0.273}{1 + 1.2d_*} + 0.0460(1 - 0.576e^{-0.02d_*}), \quad 0.1 < R_* < 10^4 \quad (11)$$

2 Experimental data

The detritus data in this study was acquired from Fisher et al. (1983), of which most are organic. The sediment data, for which the effect of the non-uniformity with respect to the underlying sand and gravel beds has been nullified to large extent, was taken into consideration. They calculated the settling velocity in a settling tube with dimensions 1.4 m long and diameter 0.1 m, considering settling velocity as the average of 10 experiments. The critical shear velocity in the unidirectional flow was measured using the Chézy equation modified by Henderson (1966) using the cross-sectional dimensions (0.3 m width, 0.6 m depth, 6 m length), flow rates and bottom roughness (assumed).

The biogenic sediment data was acquired from Paphitis et al. (2002) who carried out experiments on various sieve fractions of the sediment, *Cerastoderma edule*, the

Common Cockle, and *Mytilus edulis*, the Blue Mussel. They calculated the average settling velocity of the shell fractions in a settling tower, 2 m long and with an internal diameter of 0.2 m. The dimensions of the recirculating flume used to calculate the critical shear velocity were 0.3 m wide, 0.3 m deep, and 3.7 m long. For the various unidirectional flow conditions, the velocity profiles were found to be logarithmic in nature. Thus, the critical shear velocity was calculated using the log-of-the-wall formula, considering the fact that the velocity gradient can be used to calculate the shear velocity within the inner layer of the flow, irrespective of the type of the flow regime. The flow regime associated with various flow conditions was either hydrodynamically smooth or transitional.

Table 1 Some features of the sediment acquired from various studies

<i>Fisher et al. (1983)</i>		
<i>Sediment type/name</i>	<i>Representative size (setting diameter), mm</i>	<i>Specific gravity</i>
Walnut shells	0.26, 0.68, 1.56	1.30
Seed	1.50	1.10
Bean	3.71	1.20
Acorn	7.45	1.20
Plastic	2.45	1.20
<i>Paphitis et al. (2002)</i>		
<i>Sediment type/name</i>	<i>Representative size (setting diameter), mm</i>	<i>Specific gravity</i>
<i>Cerastoderma edule</i>	0.230, 0.290, 0.340, 0.390, 0.460, 0.510	2.80
<i>Mytilus edulis</i>	0.210, 0.270, 0.300, 0.340, 0.390, 0.450	2.72
<i>Rieux et al. (2019)</i>		
<i>Sediment type/name</i>	<i>Representative size (sieve diameter), mm</i>	<i>Specific gravity</i>
<i>Crepidula fornicata</i>	0.715, 1.025, 1.625, 2.575, 4.075	2.80 ± 0.0286
<i>Scrobicularia plana</i>	0.715, 1.025, 1.625, 2.575, 4.075	2.781 ± 0.035
<i>Cerastoderma edule</i>	0.715, 1.025, 1.625, 2.575, 4.075	2.771 ± 0.0311
<i>Ruditapes</i>	0.715, 1.025, 1.625, 2.575, 4.075	2.754 ± 0.0308
<i>Mytilus edulis</i>	0.715, 1.025, 1.625, 2.575, 4.075	2.663 ± 0.0374
<i>Anomia ahippium</i>	0.715, 1.025, 1.625, 2.575, 4.075	2.629 ± 0.0688
<i>Magallanagigas</i> (threshold 1)	0.715, 1.025, 1.625, 2.575, 4.075	2.081 ± 0.0298
<i>Magallanagigas</i> (threshold 2)	0.715, 1.025, 1.625, 2.575, 4.075	2.081 ± 0.0298
<i>Ostrea edulis</i> (threshold 1)	0.715, 1.025, 1.625, 2.575, 4.075	2.013 ± 0.0334
<i>Ostrea edulis</i> (threshold 2)	0.715, 1.025, 1.625, 2.575, 4.075	2.013 ± 0.0334

Rieux et al. (2019) obtained some eight species of the bioclastic (mollusc) sediment from the coast of Mont-Saint-Michel Bay, Brittany, France, which showed the area's faunal make-up. The settling velocity of the sediment particles was measured in a settling tube with dimensions 2 m long and 0.2 m wide, following the procedure of Weill et al. (2010). The threshold of motion experiments were carried out in a recirculating flume with dimensions of the test section 2 m long, 0.1 m wide, and 0.25 m deep. The critical bottom shear velocity was calculated using the general form of the law of the wall formula by

considering the measured average-velocity profiles using an acoustic Doppler velocimeter.

Some features of the sediment used in this study are listed in Table 1. For information in greater detail, the original papers of the individual researchers need to be seen. The dimensional grain diameter for the individual runs has been calculated in this study using equation (7). For Fisher et al. (1983) data, critical shear velocity was calculated using equation (1).

3 Results, comparison, and discussion

Various researchers have replaced the traditional Shields parameter with the Movability Number, citing the inclusion of the shape effects by having settling velocity in the denominator of the dimensionless ratio as the reason (Collins and Rigler, 1982; Paphitis, 2001; Simoes, 2014). The varying shapes and texture of the sediment used in this study (Fisher et al., 1983; Paphitis et al., 2002; Rieux et al., 2019) provide varying hydrodynamic behaviour while settling or moving on the sediment bed. This incorporates the obvious variation in following the threshold curves proposed by various researchers. The Movability Number, having settling velocity in the denominator, provides better results in terms of the degree of the scatter as compared to Shields parameter whether the independent variable is R^* or d^* (Simoes, 2014). As A is dimensionless velocity, to get a parameter which gives the dimensionless stress and also takes the effect of settling velocity into consideration, A^2 is studied in this study. The equations for A_c^2 are obtained by squaring the empirical equations for A_c . Figures 1, 2, 3, and 4 show the results after plotting the experimental data along the various sediment threshold curves. From the visual observation the difference in the scatter can be seen, but to make it clearer for different curves, the mean absolute relative error is calculated.

The magnitude of the dependent variable varies widely. For comparison purpose, the mean errors are calculated relative to the actual values of the dependent variable:

$$\varepsilon = \frac{1}{N} \sum_{n=1}^N \left| \frac{\text{Predicted value} - \text{Actual value}}{\text{Actual value}} \right| \quad (12)$$

where N is the total number of observations.

Table 2 lists the mean absolute error values of various equations representing the curves in Figures 1, 2, 3, and 4, and equations (6) and (9).

Table 2 Error comparison for various threshold equations

<i>Criterion</i>	<i>Equation</i>	ε
Critical Shields parameter, θ_c	Paphitis (2001) equation (10), $\theta_c(R^*)$	0.90
	Paphitis (2001) equation (11), $\theta_c(d^*)$	0.91
Critical Movability Number, A_c	Paphitis (2001) equation (5), $A_c(R^*)$	0.34
	Simoes (2014) equation (8), $A_c(d^*)$	0.30
A_c^2	Equation (6)	0.67
	Equation (9)	0.52

Figure 1 Paphitis (2001) mean threshold curve [equation (10)] and the experimental data (see online version for colours)

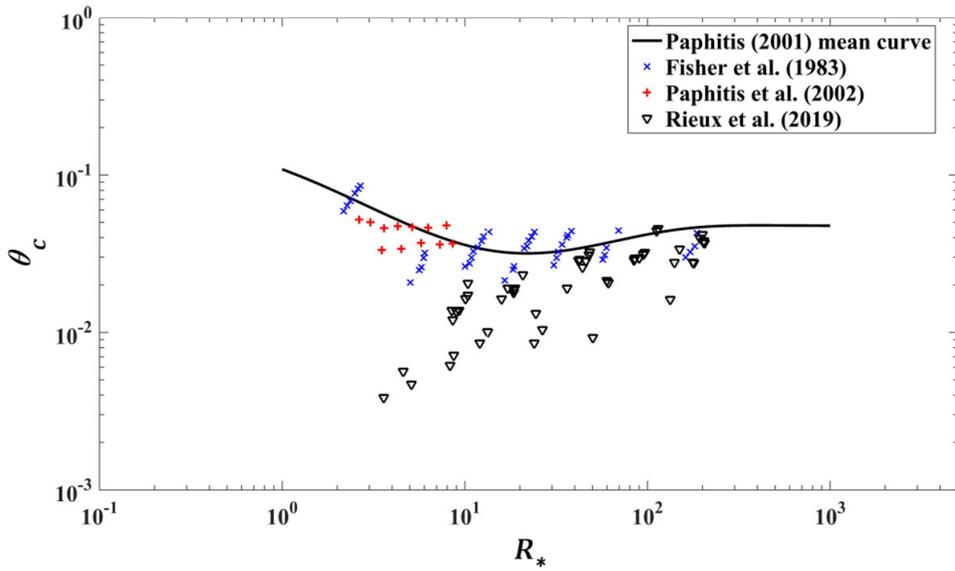


Figure 2 Paphitis (2001) mean threshold curve [equation (11)] and the experimental data (see online version for colours)

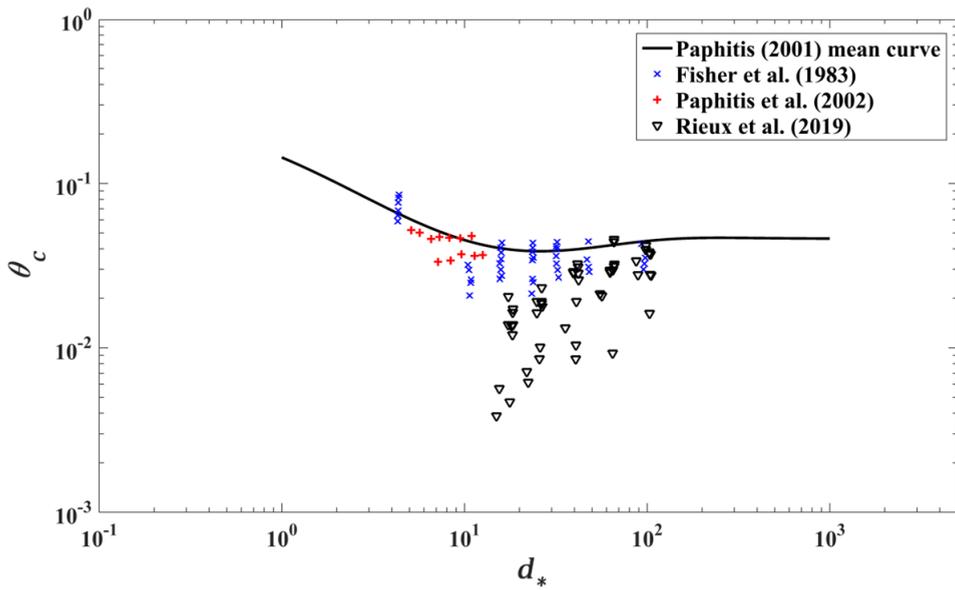


Figure 3 Paphitis (2001) mean threshold curve [equation (5)] and the experimental data (see online version for colours)

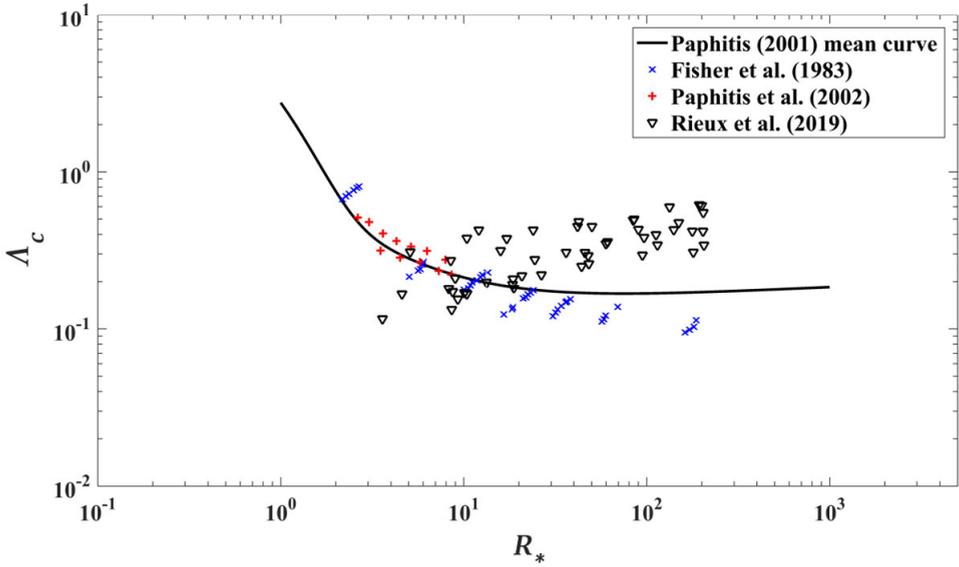
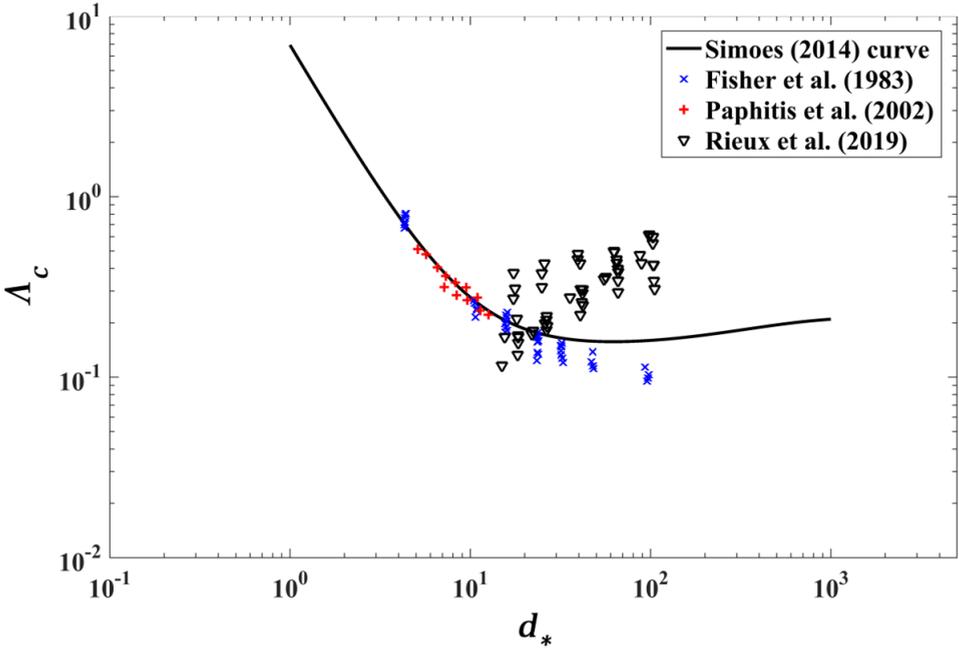


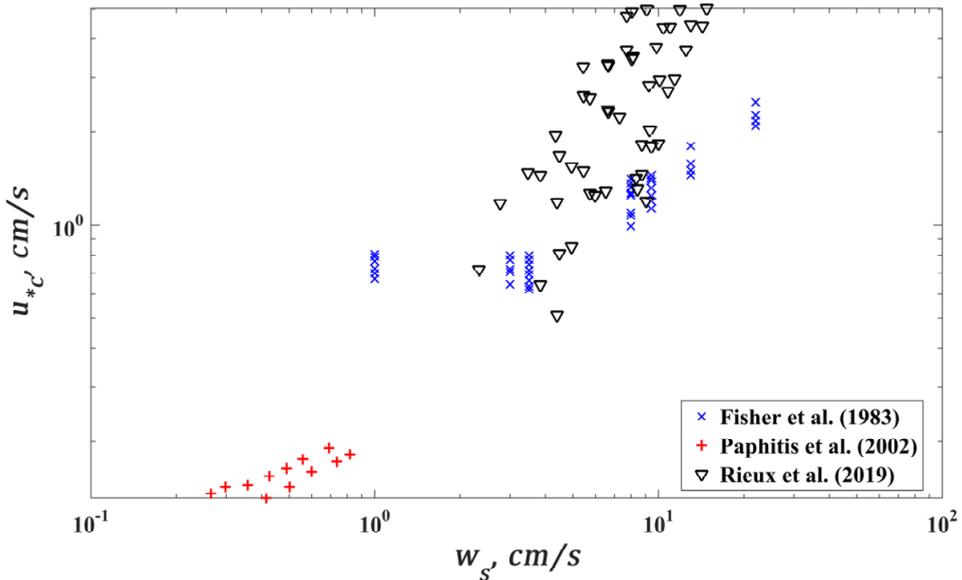
Figure 4 Simoes (2014) curve [equation (8)] and the experimental data (see online version for colours)



On comparing the error values in Table 2, it can be concluded that the Mobility Number A_c is a better criterion than Shields criterion θ_c and the dimensionless grain diameter d^* is slightly better than grain Reynolds number R^* to represent the threshold of the unconventional sediment analysed in this study. From Table 2, it can be inferred that the idea of introducing A^2 as the replacement of the A to get the two identical quantities as both θ and A^2 are dimensionless stress does not hold of much significance as A proves to be a better sediment threshold parameter than A^2 (Simoes, 2014), but A^2 shows less error than θ and it also takes into account the effect of settling velocity. A^2 also shows better behaviour with d^* than R^* as independent variables with the unconventional sediment under consideration.

From Figures 1 and 2, it can be observed that the curves of critical Shields stress mostly overestimate the actual threshold values, irrespective of the independent variable. The prime reason for this disparity is that the hydrodynamic behaviour of the sediment particles (Dey and Papanicolaou, 2008; Ali and Dey, 2016; Dey and Ali, 2017; Rieux et al., 2019), and hence their stability in a flow, is better incorporated by their settling velocity, used in the Mobility Number, than the grain diameter and specific gravity of the dimensionless shear stress. To further understand the effect of the settling velocity on the initial motion of the sediment, the variation of the critical shear velocity with respect to the settling velocity is now examined.

Figure 5 Variation of the critical shear velocity with the settling velocity (see online version for colours)



The general trend is that the greater the settling velocity greater the shear velocity required to move the sediment at the threshold. The reasons for this trend could be that the flat-shaped particles settle at lower velocity and when on a bed do not provide as great a resisting surface to the flow, thus are more difficult to move. For particles of a more regular shape, the converse is true, they settle relatively more quickly, and when on the bed are easier to move.

4 Conclusions

The following conclusions are derived from the current study.

- Unconventional sediment accompanying siliceous sediment like sand and gravel in various water bodies needs a special focus concerning their transport, especially the initial stage.
- Settling velocity incorporates the varying shape and textures of the sediment, and the hydrodynamic behaviour of the sediment thus is important for the sediment threshold and its transport.
- Hence, the Movability Number, incorporating settling velocity, is in principle a better parameter of incipient motion than Shields parameter or A^2 , although A^2 denotes the dimensionless stress like Shields parameter.
- This is supported by a comparison between data, three different threshold motion criteria, and previous threshold curves. The critical Movability Number is best expressed as a function of dimensionless grain diameter.

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References

- Ali, S.Z. and Dey, S. (2016) 'Hydrodynamics of sediment threshold', *Physics of Fluids*, Vol. 28, No. 7, p.075103.
- Andrews, E.D. (1983) 'Entrainment of gravel from naturally sorted riverbed material', *Geological Society of America Bulletin*, Vol. 94, No. 10, pp.1225–1231.
- Beheshti, A.A. and Ataie-Ashtiani, B. (2008) 'Analysis of threshold and incipient conditions for sediment movement', *Coastal Engineering*, Vol. 55, No. 5, pp.423–430.
- Buffington, J.M. and Montgomery, D.R. (1997) 'A systematic analysis of eight decades of incipient motion studies, with special reference to gravel-bedded rivers', *Water Resources Research*, Vol. 33, No. 8, pp.1993–2029.
- Collins, M.B. and Rigler, J. (1982) 'The use of settling velocity in defining the initiation of motion of heavy mineral grains, under unidirectional flow', *Sedimentology*, Vol. 29, No. 3, pp.419–426.
- Dey, S. (1999) 'Sediment threshold', *Applied Mathematical Modelling*, Vol. 23, No. 5, pp.399–417.
- Dey, S. and Ali, S.Z. (2017) 'Mechanics of sediment transport: particle scale of entrainment to continuum scale of bedload flux', *Journal of Engineering Mechanics*, Vol. 143, No. 11, p.04017127.
- Dey, S. and Papanicolaou, A. (2008) 'Sediment threshold under stream flow: a state-of-the-art review', *KSCE Journal of Civil Engineering*, Vol. 12, No. 1, pp.45–60.
- Dwivedi, A., Melville, B., Raudkivi, A.J., Shamseldin, A.Y. and Chiew, Y.M. (2012) 'Role of turbulence and particle exposure on entrainment of large spherical particles in flows with low relative submergence', *Journal of Hydraulic Engineering*, Vol. 138, No. 12, pp.1022–1030.

- Fenton, J.D. and Abbott, J.E. (1977) 'Initial movement of grains on a stream bed: the effect of relative protrusion', *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, Vol. 352, No. 1671, pp.523–537.
- Fisher, J.S., Sill, B.L. and Clark, D.F. (1983) 'Organic detritus particles: initiation of motion criteria on sand and gravel beds', *Water Resources Research*, Vol. 19, No. 6, pp.1627–1631.
- Henderson, F.M. (1966) *Open Channel Flow*, Macmillan, New York.
- Houwing, E.J. and Van Rijn, L.C. (1998) 'In Situ Erosion Flume (ISEF): determination of bed-shear stress and erosion of a kaolinite bed', *Journal of Sea Research*, Vol. 39, Nos. 3–4, pp.243–253.
- Kirchner, J.W., Dietrich, W.E., Iseya, F. and Ikeda, H. (1990) 'The variability of critical shear stress, friction angle, and grain protrusion in water-worked sediments', *Sedimentology*, Vol. 37, No. 4, pp.647–672.
- Kitsikoudis, V., Spiliotis, M. and Hrissanthou, V. (2016) 'Fuzzy regression analysis for sediment incipient motion under turbulent flow conditions', *Environmental Processes*, Vol. 3, No. 3, pp.663–679.
- Komar, P.D. and Clemens, K.E. (1986) 'The relationship between a grain's settling velocity and threshold of motion under unidirectional currents', *Journal of Sedimentary Research*, Vol. 56, No. 2, pp.258–266.
- Luckner, T. and Zanke, U. (2007) 'An analytical solution for calculating the initiation of sediment motion', *International Journal of Sediment Research*, Vol. 22, No. 2, pp.87–102.
- Paphitis, D. (2001) 'Sediment movement under unidirectional flows: an assessment of empirical threshold curves', *Coastal Engineering*, Vol. 43, Nos. 3–4, pp.227–245.
- Paphitis, D., Collins, M.B., Nash, L.A. and Wallbridge, S. (2002) 'Settling velocities and entrainment thresholds of biogenic sands (shell fragments) under unidirectional flow', *Sedimentology*, Vol. 49, No. 1, pp.211–225.
- Rieux, A., Weill, P., Mouaze, D., Poirier, C., Nechenache, F., Perez, L. and Tessier, B. (2019) 'Threshold of motion and settling velocities of mollusc shell debris: influence of faunal composition', *Sedimentology*, Vol. 66, No. 3, pp.895–916.
- Safari, M.J.S., Aksoy, H., Unal, N.E. and Mohammadi, M. (2017) 'Experimental analysis of sediment incipient motion in rigid boundary open channels', *Environmental Fluid Mechanics*, Vol. 17, No. 6, pp.1281–1298.
- Shields, A. (1936) 'Application of similarity principles and turbulence research to bed-load movement', *Mitteilungen der Preussischen Versuchsanstalt für Wasserbau und Schiffbau*, Vol. 26, pp.5–24, Berlin.
- Simoes, F.J. (2014) 'Shear velocity criterion for incipient motion of sediment', *Water Science and Engineering*, Vol. 7, No. 2, pp.183–193.
- Vollmer, S. and Kleinhans, M.G. (2007) 'Predicting incipient motion, including the effect of turbulent pressure fluctuations in the bed', *Water Resources Research*, Vol. 43, No. 5, W05410, doi:10.1029/2006WR004919.
- Weill, P., Mouazé, D., Tessier, B. and Brun-Cottan, J.C. (2010) 'Hydrodynamic behaviour of coarse bioclastic sand from shelly cheniers', *Earth Surface Processes and Landforms*, Vol. 35, No. 14, pp.1642–1654.
- Wiberg, P.L. and Smith, J.D. (1987) 'Calculations of the critical shear stress for motion of uniform and heterogeneous sediments', *Water Resources Research*, Vol. 23, No. 8, pp.1471–1480.
- Zanke, U.C.E. (2003) 'On the influence of turbulence on the initiation of sediment motion', *International Journal of Sediment Research*, Vol. 18, No. 1, pp.17–31.
- Zhou, Y., Yao, X., Gu, Y., Qian, D., Tang, Y., Zhang, Y., Zhu, D.Z. and Zhang, T. (2021) 'Biological effects on incipient motion behavior of sediments with different organic matter content', *Journal of Soils and Sediments*, Vol. 21, No. 1, pp.626–640, DOI: <https://doi.org/10.1007/s11368-020-02807-9>.