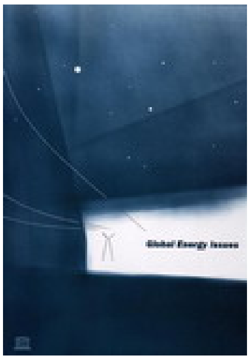

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Crude oil futures tail risk measurement based on extreme value theory

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Abstract: In this paper, we use three common tail risk measurements of Value-at-Risk (VaR), Expected Shortfall (ES) and Spectral Risk Measure (SRM) to calculate the tail risk of crude oil futures based on extreme value theory. Specifically, we propose a method to determine the optimal threshold in the extreme value theory, and further to calculate the values of VaR, ES and SRM based on Peak Over Threshold (POT) model. Empirical results show that the extreme value POT model can be used to characterise the tail risk of the price return under extreme fluctuations in Brent crude oil futures market. Moreover, the risk of VaR, ES and SRM in the Brent crude oil futures market based on extreme value theory is higher than that under the normal distribution assumption, which indicates that the traditional normal distribution assumption underestimates the tail risk. Owing the flexibility and the accuracy, we suggest that investors use ERM to measure the extreme risk of crude oil futures.

Keywords: spectral risk measurement; hyperbolic risk spectral function; extreme value theory; tail risk.

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1 Introduction

As an important part of the energy markets, the energy futures market is commonly used by investors to avoid the risks caused by fluctuations of the energy spot prices. However, while hedging risks, the high leverage feature of the energy futures market makes the energy futures price faces greater risk than the spot price, which affects the realisation and effect of risk hedging. For example, in 2020, influenced by the comprehensive factors such as COVID-19, geopolitics and short-term economic shocks, the international commodity market fluctuated drastically. On 20th April 2020, the official settlement price of CME of WTI crude oil futures contract in May was –37.63 US dollars/barrel, which triggered the 'crude oil treasure' event. On 22nd April, the day after the emergence of 'negative oil price' of Petro China, the Bank of China announced that investors holding multiple open positions of 'crude oil treasure' need to execute at the settlement price of –37.63 US dollars/barrel. It means that investors of crude oil treasure will not only lose the principal, but also double the funds of the bank. Therefore, how to effectively manage the price risk of the energy futures market, and further to prevent the risk of price fluctuations of energy spot has become one of the great concerns to energy investors and energy risk management departments. Since the occurrence of extreme events is likely to lead to investor bankruptcy, and even economic collapse and social unrest, people in today's world pay more and more attention on extreme risk among other types of risks. The premise of effective control of crude oil futures risk is how to accurately measure its extreme risks.

The study of risk measurement can be traced back to Markowitz's paper (1952), which is entitled as 'Asset Selection: Effective Diversification'. The classic mean-variance model opened the beginning of quantifying risk, and the mean-variance model ruled financial risk management for decades till 1990s. However, the use of variance as a risk measure has been criticised in mean-variance model. Firstly, variance cannot be used to describe the asymmetry of risk. Secondly, for a specific distribution (such as credit risk), the second moment may not exist. Since Morgan (1996) first disclosed Risk Metric based on VaR model, VaR then has become the industry standard for risk measurement in the financial industry. Although VaR has a far-reaching influence in the field of risk management, it still has certain limitations. Artzner et al. (1999) proposed the concept of consistent risk measurement and pointed out that VaR is not a consistent risk measurement. However, ES is one of the representatives of adhesion risk measurement. Since then, some new risk measures have been proposed to measure financial risk. For example, Acerbi (2002) proposed a kind of consistent risk called measure-Spectral Risk Measure (SRM). Furthermore, ES and VaR are special cases of SRM. Since SRM takes account both tail risk and investor's subjective risk aversion, and it satisfies internal consistency, so it provides a reasonable choice for effectively measuring financial risk. Since it was put forward, SRM has been widely used in the field of risk management. Sriboonchitta et al. (2010) studied the consistency between the expected utility function reflecting the investor's risk attitude and the spectral risk measure. Brandtner (2018) studied the relationship between risk-free assets and risky assets in SRM-based portfolios. Mozumder et al. (2018) conducted an empirical analysis of major stock index futures using SRM based on the Lévy process.

The study of extreme value theory was first introduced by Fisher. Fisher and Tippett (1928) gave a detailed proof of extreme value theory, which has laid the foundation for the development of extreme value theory. Longin and Pagliardi (1996) pointed out that the extreme value theory had its favourable property without considering the overall distribution characteristics of the sample sequence, only taking account the tail distribution of the sample. From this point of view, the characterisation of tail distribution is very important. McNeil (1998) combined extreme value theory with GARCH model and found that the results were better than those under normal distribution and *t* distribution through empirical research. Cotter and Dowd (2006) proposed the spectral risk measurement based on extreme value theory, applied the Generalised Pareto Distribution (GPD) to the spectral risk measurement and used the designed measurement method for empirical analysis. Longin and Pagliardi (2016) used the POT model in extreme value theory to study the relationship between the S&P 500 tail rate of return and trading volume. Fuentes et al. (2018) examined extreme comovements between the Australian and Canadian currencies, often known as commodity currencies and gold and oil markets respectively. Ji et al. (2019) focused on investigating financial asset returns' extreme risks, which were defined as the negative log-returns over a certain threshold. Sun et al. (2020) and Ma et al. (2020) measured extreme risk of sustainable financial system. However, at present, there is not much research on combining spectral risk measurement and extreme value theory to study the tail risk of financial markets.

In this paper, the Brent crude oil futures settlement price is used as sample data. After selecting the threshold and parameter estimation, the tail risk at different confidence levels is obtained and the extreme risks under the three measures of VaR, ES and SRM are compared. The structure of the paper is as follows: Section 2 introduces the model building including introducing the risk measure of SRM and the extreme value theory. Section 3 are the empirical results and analysis. Section 4 is the conclusion of this article.

2 Model building

2.1 SRM measurement

Generally, risk hedged investors are risk-averse, and for risk-averse people, the greater the degree of loss, the greater the degree of aversion. In spectral risk measures, one can define the spectrum function precisely to reflect the decision maker's risk preference. SRM is a risk metric constructed with a combination of risk spectrum function and return function in the framework of prospect theory, which is expressed by

$$M_{\phi} = \int_0^1 \phi(p) q_p dp$$

Among them, $\phi(p)$ is the risk spectrum function or weight function, and q_p is the negative value of the $1-p$ quantile of the income distribution, which measures the gain at different confidence levels and indirectly reflects the degree of loss. In addition to inheriting the excellent characteristics of VaR and ES, SRM is a consistent risk measurement, and it can measure the tail risk more accurately. We can find that, for calculating SRM, there are two core problems to be solved: One is to construct an appropriate $\phi(p)$; Another is to obtain an accurate q_p. For the first problem, as a weighting function of SRM, its greatest advantage of $\phi(p)$ is to reflect the investor's risk preference. Then, in order to add this factor to $\phi(p)$, we can refer to the nature of risk preference and related utility functions, investors with different risk preferences correspond to different utility functions, so the effect function is used to represent the investor's risk preference, so that the tail risk is given a weight affected by the preference; for the second problem, it is the accuracy of q_p or VaR estimation. Generally, we can make a hypothesis about the overall distribution of the financial asset's return and calculate VaR based on the assumed distribution. However, the return of the financial asset often has the characteristics of sharp peaks and thick tails. Therefore, an ideal distribution assumption is inconsistent with the true distribution of returns. To solve this problem, the extreme value theory of the tail distribution is introduced as follows. According to extreme value theory, q_p will be calculated more accurately.

2.2 Extreme value theory

The research goal of the extreme value theory is the extreme value distribution of the sample. When applying the extreme value theory, there is no need to make any assumptions about the overall distribution of the sample, only by fitting the distribution of the extreme value of the sample. Based on the tail distribution, q_p in the spectral risk measurement is used to measure the value of an extreme quantile at the tail position. In this way, it can be effectively to measure the tail risk of the sample by using the extreme value theory. There are two main statistical models used in extreme value theory, namely block maximum method and Peaks Over Threshold (POT) distribution model. In this paper, we choose POT model to implement the study.

Let $F(x)$ be the overall distribution of a random variable X, and the threshold is η, then the over-limit value is $Y = X - \eta$. Therefore, the GPD of variable Y is

$$G(y;\eta,\theta,\delta) = \begin{cases} 1-\left[1+\delta\left(\dfrac{y}{\theta}\right)\right]^{\frac{1}{\delta}}, & \delta \neq 0 \\ 1-\exp\left(-\dfrac{y}{\theta}\right), & \delta = 0 \end{cases} \tag{1}$$

Here, $\theta > 0$, which is called the scale parameter. When $\delta \geq 0$, then $0 \leq y \leq x - \eta$. Otherwise, the range of y is $0 \leq y \leq \dfrac{\theta}{\delta}$. δ is the shape parameter, which determines the type of GPD. If $\delta > 0$, it is the Pareto type I distribution, and the tail gradually thickens as δ increasing; when , it is the Pareto type II distribution, which is similar to the normal distribution; when , it is the Pareto type III distribution, which is a thin tail distribution.

We can find that variable Y is actually the part where variable X exceeds the threshold η, and the distribution function of Y constitutes a Conditional Excess Distribution Function (CEDF). Let $F\eta(y)$ be the conditional excess distribution function of y, and the conditional probability expression of $F\eta(y)$ is:

$$F\eta(y) = P\{x - \eta \leq y | x > \eta\}$$

Then,

$$F\eta(y) = P\{x \leq \eta + y | x > \eta\} = \frac{F(\eta + y) - F(\eta)}{1 - F(\eta)} = \frac{F(x) - F(\eta)}{1 - F(\eta)}$$

Through deformation, we can obtain

$$F(x) = [1 - F(\eta)]F_\eta(y) + F(\eta)$$

Here, $x = \eta + y$.

Following from Balkema-De Haan-Pickands theorem (Wüthrich, 2004) on the distribution of excesses (over a high threshold), when the threshold η is sufficiently large, the excess distribution converges to GPD. In other words, $G(y)$ can approximately replace $F_\eta(y)$, so

$$F(x) = \left[1 - F(\eta)\right]G(y) + F(\eta) \tag{2}$$

Assuming that *N* is the total number of samples and N_η is the number of samples that exceed the threshold. According to historical simulation methods, $F(\eta)$ can be expressed by the sample frequency that does not exceed the threshold, the expression of $F(\eta)$ is as follows:

$$F(\eta) = \frac{N - N_\eta}{n} \tag{3}$$

And $y = x - \eta$, substituting equations (1) and (3) into equation (2), we can get

$$F(x) = 1\frac{N_\eta}{N}\left[1 + \delta\left(\frac{x - \eta}{\theta}\right)\right]^{-\frac{1}{\delta}}, \quad \delta \neq 0$$

When the tail probability is p, the $(1 - p)$ quantile of the loss distribution is

$$F(q_1 - p) = 1 - p$$

Then, we have

$$q1 - p = \eta - \frac{\theta}{\delta}\left\{1 - \left[\frac{N_\eta}{N_p}\right]\right\} \tag{4}$$

So, the expression of SRM is:

$$M_\phi = \int_0^1 \phi(p)\left[\eta - \frac{\theta}{\delta}\left\{1 - \left[\frac{N_\eta}{N_p}\right]^\delta\right\}\right]dp \tag{5}$$

2.2.1 Determination of the threshold η

The POT model is to model and analyse the part that exceeds the threshold η. The selection of the threshold is an important problem that the POT model needs to solve. According to the PBdH theorem mentioned above, if the set threshold is too small, the CEDF distribution cannot approximate to the GPD. The model established above cannot be applicable; if the set threshold is too large, the number of tail samples behind the threshold is too small to accurately fit the model. Therefore, we propose the method of selecting thresholds to ensure the availability and effectiveness of the POT model.

According to the current research and development of extreme value theory, we conclude that the commonly used threshold selection methods include Mean Excess Function (MEF) method, Hill graph method and kurtosis method. Considering that both the MEF and Hill graph methods use images to subjectively select a certain point as the threshold η, this method has a certain subjective arbitrariness, which makes the obtained threshold value may be not real and accurate; the kurtosis method is from a quantitative perspective to determine the threshold, to a certain extent, it makes up for the shortcomings caused by the observation image judgment threshold. In this paper, the results of the three methods are integrated in the selection of thresholds, and a more comprehensive consideration makes the selected thresholds more reasonable and effective.

2.2.2 Parameters estimation of GPD distribution

There are three important parameters denoted by η, δ and θ in the GPD. After determining these three parameters, the specific form of GPD can be determined. The threshold η is determined by the MEF function graph, Hill graph and kurtosis mentioned in the previous section. δ and θ are estimated by applying the maximum likelihood estimation method.

According to the steps of the maximum likelihood estimation method, the GPD function shown in formula (1) is derived, and its probability density function is obtained as follows:

$$g(y) = \frac{1}{\theta}\left[1+\frac{\delta}{\theta}y\right]^{-\left(1+\frac{1}{\delta}\right)}, \quad \delta \neq 0$$

Therefore, the maximum likelihood equation of the GPD according to the probability density function is

$$L(\delta,\theta) = -n\ln(\theta) - \left(1+\frac{1}{\delta}\right)\sum_{i=1}^{n}\ln\left(1+\frac{\delta}{\theta}y_i\right), \quad \delta \neq 0$$

After obtaining the values of threshold η, the parameters of δ and θ are measured by the maximum likelihood estimation method. Then, the estimated values of the three parameters are obtained. We substitute the three parameters into formula (4) to obtain the tail loss function based on the POT model.

3 Empirical research

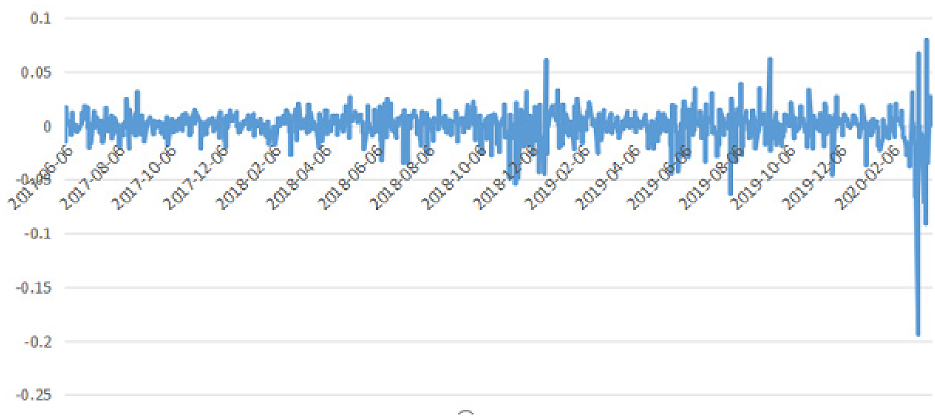
3.1 Sample selection

First, the energy futures market is based on petroleum futures products. Second, market reports provided by Intercontinental Exchange ICE, Reuters and other institutions have revealed that the market's benchmark pricing and main target contracts have gradually changed from the West Texas Crude Oil Futures WTI listed on the US New York Mercantile Exchange to the Intercontinental Exchange listed Brent crude oil futures. Therefore, this paper selects Brent crude oil futures as the international energy futures price to study the extreme risks of energy futures markets. The sample period is from 5th June 2017 to 25th March 2020. There are 724 data in the Brent market. The data comes from the US Energy Information Administration (EIA) and SINA Finance.

3.1.1 Descriptive statistics of data

The log-return of the settlement price of Brent crude oil futures is shown in Figure 1.

Figure 1 Log-return of Brent crude oil futures price



It can be intuitively seen from Figure 1 that the return of Brent crude oil futures has experienced several violent fluctuations, and the frequency and amplitude of negative yield changes are significantly greater than the frequency and amplitude of positive yield changes, reflecting the price of Brent futures implies extreme risks. Therefore, it is feasible and valuable to use spectral risk measurement to conduct tail risk research. Descriptive statistics on the return of Brent futures price are shown in Table 1.

Table 1 Descriptive statistics of Brent crude oil futures logarithmic returns

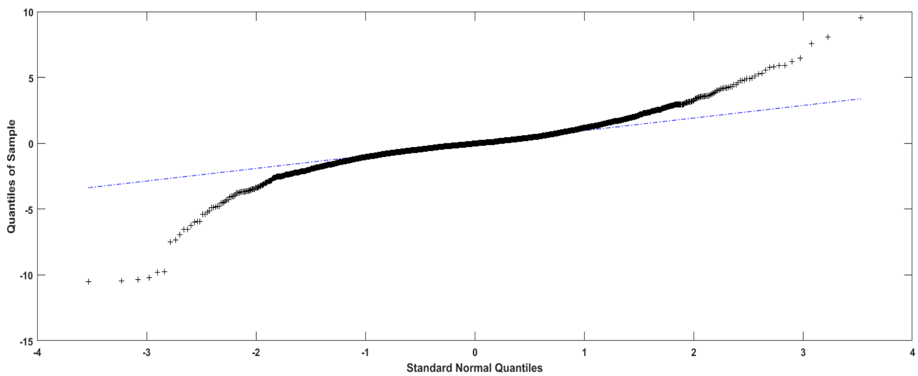
Mean	*Standard deviation*	*Maximum*	*Minimum*	*Skewness*	*Kurtosis*	*J-B statistics*	*P-value*
–0.00047	0.01615	0.079	–0.194	–2.832	32.158	5701.7393	0.0001

According to relevant regulations of futures markets, the daily price settlement limit of crude oil futures is 10%, while the maximum and minimum yields of Brent crude oil futures price are 7.9 and –19.4, respectively, which touches the boundary of price fluctuations. It reflects that the price of Brent crude oil futures has extreme risks and the losses caused by extreme risks are extremely large. If only the measurement of VaR is used to measure the tail risk of Brent crude oil futures price, the tail risk is destined to be far below the actual situation. The skewness of the sample data is –2.832, and the kurtosis is 32.158, indicating that the overall distribution of the Brent crude oil futures yield series is negatively skewed. The results indicate that distribution of the Brent crude oil futures return is non-normal. In order to further examine whether the distribution of the sample rate series is non-normal, the following is to test the normality of the return.

3.1.2 Normal distribution test

The return sequences of financial assets often do not obey the normal distribution. The tail risk calculated under the assumption of normal distribution will be inconsistent with the actual situation. In order to ensure that it is reasonable to use extreme value theory instead of normal distribution to calculate the risk of crude oil futures price, it is necessary to test whether the return of Brent crude oil futures price is normally distributed. The normality of the time series is usually tested by Jarque–Bera test and Q-Q chart.

Figure 2 Q-Q chart of sample distribution and standard normal distribution



In summary, based on the results of JB test and the Q-Q chart, combined with the frequency of occurrence of extreme risks, it can be inferred that the return of Brent crude oil futures is not normal, and according to the characteristics of the Q-Q chart, the sample yield series is especially the data in the tail part is seriously not suitable for normal distribution. Therefore, according to the previous description of extreme value theory, the POT model in extreme value theory is a suitable alternative to normal distribution.

3.1.3 Stationary test

Although the extreme value theory can better describe the tail risk, the premise of applying the extreme value theory POT model is that the related variables in the series are all independent and identically distributed. However, variables in the time series of financial asset returns often have very strong autocorrelation. According to the research of McNeil (1998) and other scholars, as long as the sequence satisfies the condition of stationary, even if there is a certain autocorrelation in the time series, the extreme value theory also holds under this condition.

ADF test is usually used to test the stationarity of the sequence, which is to determine whether the unit root exists in the sequence: if the sequence is stationary, there is no unit root; otherwise, there is a unit root. The original hypothesis of ADF test is that there is a unit root. When the original hypothesis is rejected, the unit root does not exist and the sequence is stable.

According to the results shown in Table 2, the *t*-statistic in the ADF test of the Brent crude oil futures return is –46.8311, which is far less than the critical value at the 1% confidence level. Therefore, the return of Brent oil futures price is stationary.

Table 2 ADF test results of Brent crude oil futures return

		t-statistic	*P-value*
		–46.8311	*0.0001*
Critical value	1% Confidence level	–2.3279	0.000
	5% Confidence level	–1.6455	0.000
	10% Confidence level	–1.2819	0.000

3.2 Parameter estimation of the POT model

(1) *Threshold selection*: The first step in parameter estimation of the POT model is to determine the appropriate threshold. This paper will comprehensively determine the threshold based on the threshold results obtained by Hill graph and kurtosis method.

We can observe the change trend of Hill statistics through the Hill graph and determine the size of the threshold. According to Reiss and Thomas (2001), when the tail data of the sample is small, the Hill chart shows strong fluctuations due to the extreme risk. As the number of tail samples increases, the amplitude of the fluctuation will gradually decrease and stabilise. Therefore, by capturing the starting point when the curve in the Hill graph begins to stabilise, an appropriate threshold can be accurately obtained, thereby reducing the influence of tail extreme data on tail risk measurement.

According to the Hill chart patterns of the left tail distribution and the right tail distribution in Figures 3 and 4, the Hill tail statistics of the left tail distribution begins to show a stable trend between the 120th variable and the 140th variable. The threshold value is roughly between 2.13 and 2.25; the right-tailed Hill statistics begin to show a stable state between the 80th and 110th variables, and the threshold value is roughly between 2.55 and 2.93.

Figure 3 Hill graph of left tail distribution

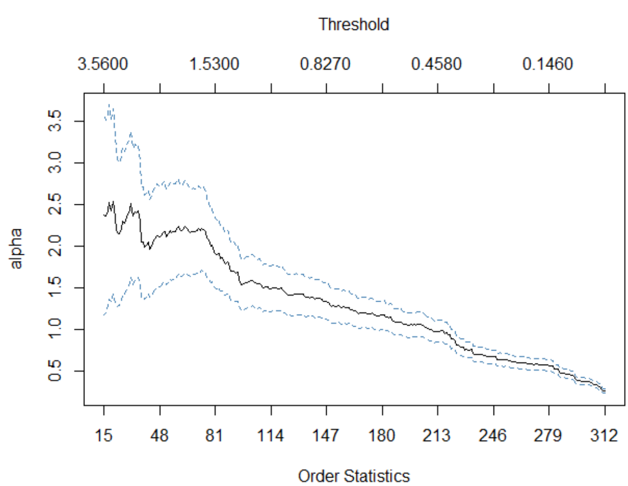
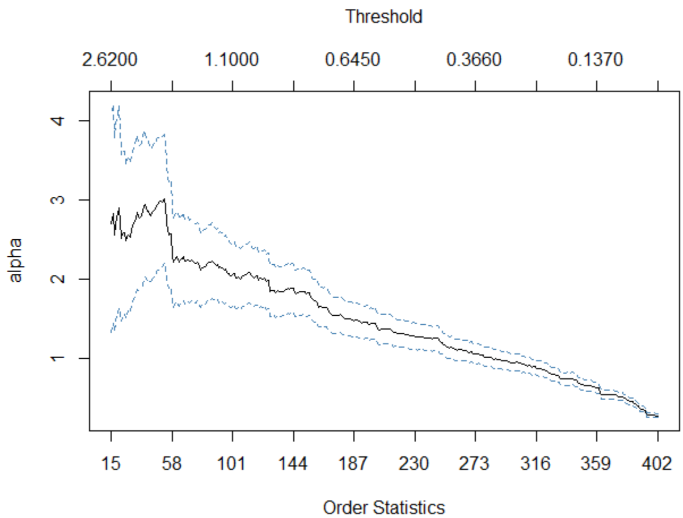


Figure 4 Hill plot of right tail distribution



Based on the principle of the kurtosis method, we remove the value that deviates the most from the mean value in the time series, and continue to shrink the kurtosis until the kurtosis of the remaining samples is equal to when the normal distribution is 3, from the quantitative point of view, we find the intersection of the peak distribution of the Brent crude oil futures distribution and the normal distribution. According to the principle of kurtosis method, the variables in the left tail distribution and the right tail distribution are screened, and finally the threshold values of the left and right tail portions are 2.0182 and 2.9341, respectively.

In summary, it can be found that the threshold value obtained by the kurtosis method is significantly higher than the threshold values obtained by the other two methods. The results obtained by the kurtosis method are used as the basis for selecting the threshold value. In this paper, the threshold value of the tail distribution is $\eta_{\text{left}} = 1.5736$, and the threshold value of the right tail distribution is $\eta_{\text{right}} = 1.7047$.

3.3 Tail risk measurement

The parameters δ, θ and the threshold η calculated based on the maximum likelihood estimation has been obtained under the premise of a given confidence level $(1-p)$, and the corresponding VaR along with ES and SRM based on the POT model are also obtained. Tables 3 and 4 present the tail risks under different distribution assumptions. Table 3 is the values of VaR, ES and SRM obtained under the POT model. Among them, the confidence levels are 95%, 97.5% and 99%, respectively. The risk aversion factor γ of the spectral function is respectively 0.01, 0.1 and 0.5. Table 4 is the values of VaR, ES, SRM obtained under the assumption of normal distribution. The confidence level selected and risk aversion factor in Table 4 are the same as in Table 3.

Tables 3 and 4 reflect the tail risks measured by different risk measurements at the same confidence level. We find that the risk measured by SRM is larger than those by ES and VaR. The reason for this sort is that, VaR only measures the value of a certain position in the return, and ES measures the average value of the part that exceeds VaR. Then, ES is larger than VaR. Since SRM gives greater weights to the greater risk of the tail, it makes sense that SRM is larger than ES. We can also find that as the confidence level increases, the risks measured by the three risk measurement tools are increasing, intuitively reflecting that the closer to the end of the tail, the greater the severity of the risk. Under different risk aversion factors, the risk measurement results of SRM are also changing, and as the risk aversion factor γ increases, the SRM results also increase. Since γ reflects the degree of risk aversion of investors, when γ increases, even if the risk level remains the same, the investor's aversion to the same risk level or the confidence level, will increase, which also makes SRM increase. So, the risk measurement tool of SRM is a flexibility tail risk measurement by adjusting the risk aversion factor γ.

Table 3 VaR, ES, SRM based on POT model

		VaR	*ES*	*SRM*		
				$\gamma = 0.02$	$\gamma = 0.1$	$\gamma = 0.5$
Left tail	$p = 0.05$	3.5358	6.2971	7.3756	7.4919	8.8505
	$p = 0.025$	4.8247	8.5185	9.9611	10.1168	11.9342
	$p = 0.01$	7.2195	12.6465	14.7652	14.9938	17.6637
Right tail	$p = 0.05$	2.3972	3.6856	4.1341	4.1809	4.7083
	$p = 0.025$	3.1054	4.6719	5.2169	6.5347	6.7672
	$p = 0.01$	4.2813	6.3097	7.0146	7.0883	7.9181

Table 4 VaR, ES, SRM under the assumption of normal distribution

		VaR	*ES*	*SRM*		
				$\gamma = 0.02$	$\gamma = 0.1$	$\gamma = 0.5$
Left tail	$p = 0.05$	3.8634	4.2996	8.8284	8.8557	9.1379
	$p = 0.025$	4.3731	4.5389	9.5244	9.5495	9.8097
	$p = 0.01$	4.9656	5.6517	10.3651	10.4371	10.6177
Right tail	$p = 0.05$	2.3396	3.6793	3.4010	3.4095	3.4981
	$p = 0.025$	2.6251	4.0796	3.6193	3.6272	3.7089
	$p = 0.01$	2.9570	4.4130	3.8805	3.8877	3.9623

4 Conclusions

Based on the HARA family of utility functions, this paper proposes a general hyperbolic risk weight function, establishes a hyperbolic spectral risk measure, and combines extreme value theory POT model to measure the extreme risk of crude oil futures.

Research results show that the tail risks measured by the three measurement tools of VaR, ES and SRM under the normal distribution are all less than those corresponding to the POT model. The reason is that the return series of Brent crude oil futures does not follow the normal distribution, but it shows the characteristic of peak and thick tail, the tail risk brought by this feature is higher than the tail risk in the case of normal distribution. In addition, the VaR, ES and SRM based on the extreme value theory POT model are higher than those based on the normal distribution. Based on the POT model, the results of SRM are affected by both the confidence level and the risk aversion factor.

Although the price of crude oil futures fluctuated violently in the past two years, crude oil futures is still an effective hedging tool for crude oil spot price risk control. When using crude oil futures for risk hedging, investors need to pay close attention to its tail risk. We suggest that investors use ERM risk measure to measure the extreme risk of crude oil futures in order to better capture possible extreme losses. ERM increases with the increase of risk aversion coefficient. Investors can flexibly choose different risk aversion coefficients according to their own risk aversion attitude to achieve the purpose of effective risk management.

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