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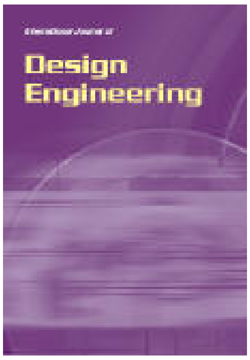

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Flow shop scheduling – especially structured models under fuzzy environment with optimal waiting time of jobs

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Abstract: This paper presents a two-stage flow shop fuzzy scheduling approach under uncertain situations. In the past, flow shop scheduling in fuzzy environment has received little attention. To cope with data in a fuzzy environment and reduce total waiting time of jobs, particularly for structured problems, the research described in this paper is of utmost importance. The processing times are demonstrated by triangular membership function. An exact algorithm is proposed to achieve a schedule that minimises the total waiting time of jobs in especially structured model where the AHR of processing times is not arbitrary but must satisfy a definite condition. Most of the literature in scheduling focuses on minimising the makespan. Significance of the desired objective and effectiveness of proposed algorithm is exhibited in comparison to existing makespan approaches. The results obtained shows the best out of the three approaches wherein the objective of minimising waiting times is concerned.

Keywords: flow shop scheduling; triangular fuzzy numbers; heuristic; total waiting time; job sequencing.

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Biographical notes: Bharat Goyal is presently working as an Assistant Professor in Department of Mathematics, Khalsa College Patiala, Punjab, India. He has ten years of experience in teaching mathematics and has eight years of research experience. His area of expertise is optimisation techniques, scheduling. He has more than ten research papers of international repute and six books of national/international repute to his credit. He has also completed a minor research project in scheduling.

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1 Introduction

Scheduling is a methodical investigation of decision-making issues. Flow shop scheduling is a selection philosophy employed in today's engineering and industrial production services. The job-shop scheduling model includes a variety of jobs as well as some procedures that must be completed on various equipment. Railway lines, ophthalmologists, car production machines, and a variety of other machines are examples of machines. Arrival and departure of trains, patient diagnosis, and sequential assembly of vehicle parts are all examples of jobs. Every job was run via a machine for a specific amount of time. The processing periods of a job's various procedures do not cross each other. Only one work can be completed at a time by a single machine. One of the most common issues is flow shop scheduling. The flow shop scheduling problem is one of the most common scheduling issues. Each job's m-operations must be completed in the same order on m distinct machines. Scheduling is an important aspect of permutable theories and heuristic approaches since it gives many techniques for achieving the goal. Scheduling seeks to achieve one or more goals by distributing tasks across available machines.

The majority of the literature focuses on deterministic processing times, yet there are many challenges in the real world that involve uncertain scenarios. Approaches that deal with exact processing times fail to address concerns that are based on uncertainty. To tackle indeterminist challenges, scheduling systems take advantage of the fuzzy environment, which gives answers for problems involving uncertainty. To display this hazy information, triangular fuzzy membership functions can be employed. Almost every researcher in scheduling theory has been motivated by the goal of finding an optimal or near-optimal solution to minimise makespan. In this research, we offer an exact approach for determining the best sequence for minimising overall job waiting time. To defuzzify fuzzy integers with triangular membership, McCahon and Lee (1990) suggested an approach using generalised mean values (GMVs). Sanja and Xueyan (2006) later enhanced their results by using the α-cut strategy to minimise the makespan in two machine flow shop scheduling problems. In their study of several defuzzification strategies, Van Leekwijck and Kerre (1999) discovered that the maxima methods provide excellent results when referring to the fundamental defuzzification methods. In this study, Yager's (1981) ranking approach is utilised to achieve the best results.

Johnson (1954) developed the first optimal two and three step scheduling strategy to optimise the makespan in the 1950s. Palmer (1956) used a heuristic strategy to reduce makespan in an n-job m-machine problem. Nawaz et al. (1983) proposed the Nawaz, Enscore and Ham (NEH) method, which is based on a heuristic approach and aims to reduce total processing time across all machines. By modifying the NEH (Nawaz et al., 1983) algorithm, Chakraborty and Laha (2007) sought to find a good solution in polynomial time. Szwarc (1977) looked at all of the well-known situations of the $m \times n$ flow shop problem and came up with the best solutions for three additional cases. To minimise the makespan, Gupta (1975) considers especially structured models in flow shop scheduling. Gupta and Goyal (2018) found the best total waiting time of jobs, while taking into account the concept of a job block and the time spent travelling in a two-stage flow shop scheduling problem. Again to optimise the waiting time of jobs with deterministic processing times, Gupta and Goyal (2020) used heuristic approaches. Maggu and Das (1985) studied scheduling models with various objectives and parameters. The goal of this study is to offer an especially structured algorithm for minimising total waiting time for tasks in a fuzzy environment, using two machines and an n-jobs flow shop scheduling problem. Goyal et al. (2020) consider especially structured flow shop scheduling in two stage by taking setup times separate from processing times to minimise total waiting time of jobs. Two of jobs grouped together to form a block and made comparison with makespan approaches of Johnson (1954) and Palmer (1956). Goyal and Kaur (2020) proposed a heuristic approach to minimise the waiting time of jobs when the processing times are on the whole arbitrary. Also Goyal and Kaur (2021) proposed a heuristic to minimise total waiting time of jobs under fuzzy environment by considering processing times as trapezoidal fuzzy numbers. The superiority of proposed algorithm was shown by comparing the results of large number of randomly generated problems of various job sizes with the existing approaches for makespan made by Johnson (1954), Palmer (1956), NEH (Nawaz et al., 1983), Nailwal et al. (2016), and waiting time approach by Goyal and Kaur (2020). Liang et al. (2022) developed a computational efficient optimisation approach combining NEH and niche genetic algorithm (NEH-NGA) to minimise makespan.

After reviewing the literature, it was found that most of the literature was concerned with minimising makespan. Only a little amount of work is present that deals with optimisation of total waiting time. Further the fuzzy data is an important issue to deal with. There is lack of research with fuzzy data to minimise total waiting time of jobs. This is the huge gap in the research area. The present paper overcomes this gap by dealing with fuzzy data and considering processing times of jobs as triangular fuzzy numbers and provide an exact algorithm to minimise total waiting time of jobs.

2 Preliminaries

2.1 Fuzzy number

A fuzzy number $\tilde{N}$ is a convex fuzzy set of the real line R along with its membership function $\mu_{\tilde{N}} : R \to [0, 1]$ satisfies the following axioms:

1 $\tilde{N}$ is normal, i.e., there exists exactly one $x \in R$ for which $\mu_{\tilde{N}}(x) = 1$.

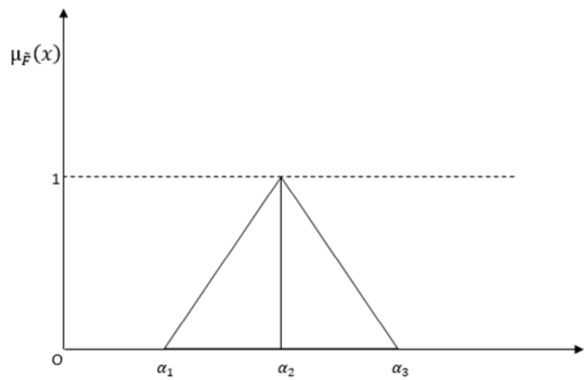
2 $\mu_{\tilde{N}}(x)$ is piecewise continuous.

2.2 Triangular fuzzy number

A fuzzy number $\tilde{F} = (\alpha_1, \alpha_2, \alpha_3)$ is said to be a triangular fuzzy number if its membership function

$$\mu_{\tilde{F}}(x) = \begin{cases} \dfrac{x-\alpha_1}{\alpha_2-\alpha_1}, & \alpha_1 < x < \alpha_2 \\ 1, & x = \alpha_2 \\ \dfrac{\alpha_{3-x}}{\alpha_3-\alpha_2} & \alpha_2 < x < \alpha_3 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

Figure 1 Triangular membership fuzzy number $\tilde{F} = (\alpha_1, \alpha_2, \alpha_3)$



2.3 Yager's ranking method

For a triangular fuzzy number $\tilde{F}$, Yager's (1981) ranking index is given by

$$R(\tilde{F}) = \frac{1}{2}\int_0^1 \left(F_\alpha^l + F_\alpha^u\right) d\alpha \tag{2}$$

where (F_α^l, F_α^u) is the α-level cut for the fuzzy number $\tilde{F}$, $R(\tilde{F})$ is the Yager's ranking index for fuzzy number $\tilde{F}$.

2.4 Waiting time of jobs

The waiting time U_β of a job β in a flow-shop scheduling problem is defined as the time which is consumed on waiting in queue for processing on second machine.

2.5 Total waiting time of jobs

The total waiting time W_t can be stated as the sum of all waiting times, i.e.,

$$W_t = \sum_{i=1}^{n} U_{\beta_i} \tag{3}$$

3 Format of framework

3.1 Notation

Different notations used in the paper in Table 1.

Table 1 Explanation of various notations

Notations	*Explanation*
I	Index for jobs β_i, $i = 1, 2, 3, \ldots, n$
f_i^M	Fuzzy processing time of job i on machine M
p_i^M	AHR value of fuzzy processing time of job i on machine M
C_β^M	Completion time of job β on machine M
U_β	Time consumed on waiting by job β
Y_i^M	starting time of job i on machine M
W_t	Total waiting time of jobs

3.2 Axioms

1 At time $t = 0$, all machines are ready to start working on their tasks (jobs).

2 Any job that has to be processed on the first machine is always available.

3 During the scheduling process, every machine is available without any halts or failures.

4 The time it takes for machines to setup is believed to be included in processing times.

3.3 Description of the problem

In the flow shop process, let n-jobs be carried out on two machines (machines 1 and 2), with the processing time of the i^{th} task on machine M, ($M = 1, 2$) taken as triangular fuzzy numbers and designated as f_i^M. The problem description can be framed mathematically as shown in Table 2.

Table 2 Problem description in matrix form

Job	*Machine 1*	*Machine 2*
I	f_i^1	f_i^2
1	$(\alpha_{11}^1, \alpha_{21}^1, \alpha_{31}^1)$	$(\alpha_{11}^2, \alpha_{21}^2, \alpha_{31}^2)$
2	$(\alpha_{12}^1, \alpha_{22}^1, \alpha_{32}^1)$	$(\alpha_{12}^2, \alpha_{22}^2, \alpha_{32}^2)$
3	$(\alpha_{13}^1, \alpha_{23}^1, \alpha_{33}^1)$	$(\alpha_{13}^2, \alpha_{23}^2, \alpha_{33}^2)$
$\vdots$	$\vdots$	$\vdots$
N	$(\alpha_{1n}^1, \alpha_{2n}^1, \alpha_{3n}^1)$	$(\alpha_{1n}^2, \alpha_{2n}^2, \alpha_{3n}^2)$

The Yager's ranking index of processing times p_i^M (M = 1, 2) are satisfying the condition

$$\max\ p_i^1 \le \min\ p_j^2 \tag{4}$$

The objective is to obtain the best schedule in order to minimise the total waiting time.

3.4 Significance of the objective

In today's competitive market the customer satisfaction is of the utmost importance. The waiting time of the customer then obviously becomes a significant issue to worry in today's modest market. The present paper focuses on to minimise the total waiting time of jobs. It may give rise to other costs like rental cost of machines, machine idle cost, etc. but waiting time is a noteworthy matter from a customer's view point.

4 Theorems and results

Theorem 4.1: Let n-jobs 1, 2, …, n be processed on two machines (machines 1 and 2) in flow shop process without fleeting and satisfying the structural relationship

$$\max\ p_i^1 \le \min\ p_j^2 \tag{4}$$

where p_i^M is the Yager's (1981) ranking index value of the equivalent fuzzy processing time required by job i on machine M, (M = 1, 2): (i, j = 1, 2, 3, …, n), then W_t, the total waiting time of jobs is given by

$$W_t = np_{\beta_1}^1 + \sum_{q=1}^{n-1}(n-q)V_{\beta_q} - \sum_{j=1}^{n} p_{\beta_j}^1 \tag{5}$$

where

$$V_{\beta_q} = \left(p_{\beta_q}^2 - p_{\beta_q}^1\right) \tag{6}$$

Proof: Firstly C_β^M, the completion time of job β on machine M will be evaluated, for the sequence, $S = \beta_1, \beta_2, \beta_3, \ldots, \beta_k, \ldots, \beta_n$.

Claim:

$$C_{\beta_n}^2 = p_{\beta_1}^1 + p_{\beta_1}^2 + p_{\beta_2}^2 + \ldots + p_{\beta_n}^2 \tag{7}$$

Applying mathematical induction on n.

Let the statement $P(n)$:

$$C_{\beta_n}^2 = p_{\beta_1}^1 + p_{\beta_1}^2 + p_{\beta_2}^2 + \ldots + p_{\beta_n}^2 \tag{7}$$

Now, for n = 1,

$$C_{\beta_1}^2 = p_{\beta_1}^1 + p_{\beta_1}^2 \tag{8}$$

Now, let for $n = k$, $P(k)$ be true.

Then for $P(k + 1)$, using equation (4)

$$C^2_{\beta_{k+1}} = \max\left(C^1_{\beta_{k+1}}, C^2_{\beta_k}\right) + p^2_{\beta_{k+1}} \tag{9}$$

Proving,

$$C^2_{\beta_{k+1}} = p^1_{\beta_1} + p^2_{\beta_1} + p^2_{\beta_2} + \ldots + p^2_{\beta_k} + p^2_{\beta_{k+1}} \tag{10}$$

Secondly U_β, the time consumed on waiting by job β will be evaluated.

Claim: For the sequence $S = \beta_1, \beta_2, \beta_3, \ldots, \beta_k, \ldots, \beta_n$ of jobs

$$U_{\beta_k} = p^1_{\beta_1} + \sum_{q=1}^{k-1} V_{\beta_q} - p^1_{\beta_k}, \quad k = 2, 3, \ldots, n \tag{11}$$

Obviously

$$U_{\beta_1} = 0 \tag{12}$$

and

$$U_{\beta_k} = Y^2_{\beta_k} - C^1_{\beta_k}, \quad k = 2, 3, \ldots, n \tag{13}$$

Implies,

$$U_{\beta_k} = \max\left(C^2_{\beta_{k-1}}, C^1_{\beta_k}\right) - C^1_{\beta_k} \tag{14}$$

According to the condition (4) of especially structured model we have

$$U_{\beta_k} = p^1_{\beta_1} + \sum_{q=1}^{k-1} V_{\beta_q} - p^1_{\beta_k}, \quad k = 2, 3, \ldots, n \tag{15}$$

Approaching to the main proof of the theorem

$$W_t = U_{\beta_1} + U_{\beta_2} + U_{\beta_3} + \ldots + U_{\beta_n} \tag{16}$$

$$W_t = np^1_{\beta_1} + \sum_{q=1}^{n-1} (n-q) V_{\beta_q} - \sum_{j=1}^{n} p^1_{\beta_j} \tag{17}$$

Theorem 4.2: For a natural number k and real numbers $y_1, y_2, \ldots, y_k$ such that $y_1 \le y_2 \le \ldots \le y_k$, the value $ky_1 + (k-1)y_2 + (k-2)y_3 + \ldots + 2y_{k-1} + y_k$ is minimum.

Proof: Applying induction hypothesis on k.

The result holds trivially for $k = 1$.

Assume that the result comes true for less than k real numbers.

Now,

$$ky_1 + (k-1)y_2 + (k-2)y_3 + \ldots + 2y_{k-1} + y_k$$
$$= (k-1)y_1 + (k-2)y_2 + (k-3)y_3 + \ldots + y_{k-1} + \sum_{i=1}^{k} y_i$$

As last term $\sum_{i=1}^{k} y_i$ is constant, therefore hypothesis assumption implies $ky_1 + (k-1)y_2 + (k-2)y_3 + \ldots + 2y_{k-1} + y_k$ is minimum.

Remark: Based on the conclusion from Theorem 4.2, the term $\sum_{q=1}^{n-1}(n-q)V_{\beta_q}$ in equation (5) will be minimum for an n-job sequence S: $\beta_1, \beta_2, \ldots, \beta_n$ if n-jobs in sequence S are arranged in non-decreasing order of the values V_{β_q} and $\sum_{j=1}^{n} p_{\beta_j}^1$ is constant for every sequence of jobs. Taking into account these in Section 5, an explicit technique for minimising the total waiting time W_t for two-machine specifically structured flow-shop scheduling problems is provided, discoveries.

Based on the result from Theorem 4.2, we observe that for a n-job sequence S: $\beta_1, \beta_2, \ldots, \beta_n$, the term $\sum_{q=1}^{n-1}(n-q)V_{\beta_q}$ in equation (5) will be minimum if n-jobs in sequence S are arranged in non-decreasing order of the values V_{β_q} and $\sum_{j=1}^{n} p_{\beta_j}^1$ is constant for every sequence of jobs. In keeping mind these observations, an exact method is proposed in Section 5 to minimise the total waiting time W_t for two-machine especially structured flow-shop scheduling problems.

5 Algorithm

The proposed algorithm involves the following procedure:

Step 1 Compute the ranking index value of fuzzy processing time $f_i^M = (\alpha_1, \alpha_2, \alpha_3)$ for all jobs j_i, $i = 1, 2, 3, \ldots, n$ by using the Yager's (1981) ranking index.

Step 2 Check the structural condition, i.e., $\max p_i^1 \le \min p_j^2$.

Step 3 Compute $d_{i_q} = (n-q)V_i$ where $V_i = p_i^2 - p_i^1$ for $m = 1, 2, 3, \ldots, n-1$ and get the computed entries in that tabulated in Table 3.

Step 4 Arrange the jobs in ascending order of V_i to get the sequence $S_1 = \{\beta_1, \beta_2, \beta_3, \ldots, \beta_n\}$.

Step 5 Locate minimum of processing time of machine 1 and call it p_x^1. Further, check the condition $p_x^1 = p_{\beta_1}^1$.

Table 3 Format of the computed entries

Job	*Machine 1*	*Machine 2*	V_i	$d_{i_q} = (n-q)\,V_i$				
i	p_i^1	p_i^2	$p_i^2 - p_i^1$	$q = 1$	$q = 2$	$q = 3$	…	$q = n - 1$
1	p_1^1	p_1^2	V_1	d_{1_1}	d_{1_2}	d_{1_3}	…	$d_{1_{n-1}}$
2	p_2^1	p_2^2	V_2	d_{2_1}	d_{2_2}	d_{2_3}	…	$d_{2_{n-1}}$
3	p_3^1	p_3^2	V_3	d_{3_1}	d_{3_2}	d_{3_3}	…	$d_{3_{n-1}}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	…	⋮
n	p_n^1	p_n^2	V_n	d_{n_1}	d_{n_2}	d_{n_3}	…	$d_{n_{n-1}}$

If this condition is met then the sequence obtained in previous step is optimal otherwise go to next step.

Step 6 Now get sequences S_i, $i = 2, 3, 4, \ldots, n$ by swapping the i^{th} job with first one of the sequence S_{i-1} and keeping the rest of the job sequence unaltered.

Step 7 Using the formula described in equation (5), calculate the total waiting time W_t for all the sequences $S_1, S_2, S_3, \ldots, S_n$.

Step 8 From the list mentioned in the previous step, choose the sequence with minimum total waiting time and this is the desired optimal sequence.

6 Numerical illustration

To evaluate the performance of the solution method of proposed algorithm, a numerical illustration of randomly generated problem with five jobs and two machines is described below:

Let five jobs 1, 2, 3, 4, 5 (say) are carried upon two machines namely 1 and 2.

Table 4 Fuzzy processing times for jobs

Job	*Machine 1*	*Machine 2*
i	f_i^1	f_i^2
1	(5, 7, 9)	(25, 27, 28)
2	(7, 8, 10)	(12, 24, 28)
3	(6, 12, 18)	(16, 17, 28)
4	(10, 16, 18)	(27, 28, 29)
5	(7, 14, 21)	(31, 32, 35)

Yager's (1981) ranking index of above mentioned fuzzy processing times are represented in Table 5.

It can be seen that $\max p_i^1 \leq \min p_j^2$ so the structural condition is met. Table 6 present the detail of all calculated V_i and d_{i_q}.

According to step 4, we get the sequence

$$S_1 = \{\beta_3, \beta_4, \beta_2, \beta_5, \beta_1\}$$

Table 5 Crisp values of fuzzy processing times

Job	*Machine 1*	*Machine 2*
i	p_i^1	p_i^2
1	7.00	26.75
2	8.25	22.00
3	12.00	19.50
4	15.00	28.00
5	14.00	32.50

Table 6 Computed entries

Job	*Machine 1*	*Machine 2*	V_i	$d_{i_q} = (5-q)\,V_i$			
i	p_i^1	p_i^2	$p_i^2 - p_i^1$	$d_{i_q} = 4V_i$	$d_{i_q} = 3V_i$	$d_{i_q} = 2V_i$	$d_{i_q} = V_i$
β_1	7.00	26.75	19.75	79.00	59.25	39.50	19.75
β_2	8.25	22.00	13.75	55.00	41.25	27.50	13.75
β_3	12.00	19.50	7.50	30.00	22.50	15.00	7.50
β_4	15.00	28.00	13.00	52.00	39.00	26.00	13.00
β_5	14.00	32.50	18.50	74.00	55.50	37.00	18.50

Since $p_x^1 \neq p_\mu^1$, so all the possible produced sequences according to step 6 are

$$S_1 = \{\beta_3, \beta_4, \beta_2, \beta_5, \beta_1\};\ S_2 = \{\beta_4, \beta_3, \beta_2, \beta_5, \beta_1\};\ S_3 = \{\beta_2, \beta_3, \beta_4, \beta_5, \beta_1\};$$
$$S_4 = \{\beta_5, \beta_3, \beta_4, \beta_2, \beta_1\};\ S_5 = \{\beta_1, \beta_3, \beta_4, \beta_2, \beta_5\}$$

Table 7 present the total waiting time for above produced sequences.

Table 7 Optimal schedule table

Sequence	*Total waiting time (W_t)*
S_1	118.75
S_2	139.25
S_3	*107.00*
S_4	150.00
S_5	120.00

Thus $\min\{W_t\}$ = 107 units of time corresponding to the sequence S_3.

Hence, $S_3 = \{\beta_2, \beta_3, \beta_4, \beta_5, \beta_1\}$ is the desired optimal schedule of jobs satisfying the goal of the proposed numerical.

7 Computational analysis

To look over the suitability of the proposed heuristic, numerous examples of various groups are randomly generated in which each group varies upon different number of jobs. Here ten groups are generated with job sizes 5, 10, 15, 20, 30, 40, 50, 55, 60, 80 and each group is studied over ten different randomly generated problems with processing times as triangular fuzzy numbers. For all groups, mean of the total waiting time of each problem for proposed algorithm is compared with the mean of already existed makespan approaches of Johnson (1954), Palmer (1956) and NEH (Nawaz et al., 1983) and are plotted in graph as shown in Figure 2, which demonstrate that the curve of proposed heuristic is below than the all other curves whereas Palmer (1956) algorithm curve is high among all. Furthermore, the curve of NEH (Nawaz et al., 1983) is closer than others to the proposed algorithm's curve.

Figure 2 Comparison of proposed heuristic with existing algorithms (see online version for colours)

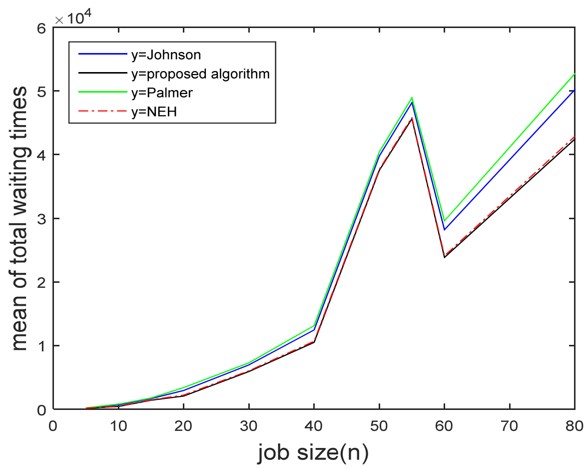


Table 8 Computational results of proposed algorithm compared with existed one's

n	*Mean of the total waiting times in Palmer (1956) algorithm*	*Mean of the total waiting times in Johnson (1954) algorithm*	*Mean of the total waiting times in NEH (Nawaz et al., 1983) algorithm*	*Mean of the total waiting times in proposed algorithm*
5	186.03	163.25	144.07	137.05
10	787.25	681.33	468.60	413.10
15	1,749.58	1,654.75	1,461.08	1,406.70
20	3,395.70	2,936.43	2,205.07	2,063.78
30	7,281.45	6,955.68	6,025.65	5,907.35
40	13,125.03	12,442.48	10,668.60	10,487.05
50	40,428.72	39,762.75	37,665.97	37,525.50
55	48,921.53	48,165.22	45,814.22	45,632.82
60	29,585.50	28,191.22	24,106.20	23,841.10
80	52,703.82	45,086.00	42,850.75	42,430.38

In addition, the percentage of error for each of the problem is also calculated by using the formula

$$e_{rr} = \left[(W_\delta - W_\theta) / W_\theta \right] * 100 \tag{18}$$

where W_δ is the total waiting time of existed algorithms and W_θ is the total waiting time of the same job computed by using proposed algorithm. For the sake of measuring the wellness of the proposed algorithm, mean of percentage error is calculated for all job groups and then figured out in the graph shown in Figure 3.

Figure 3 Average percentage of error in the computational experiments (see online version for colours)

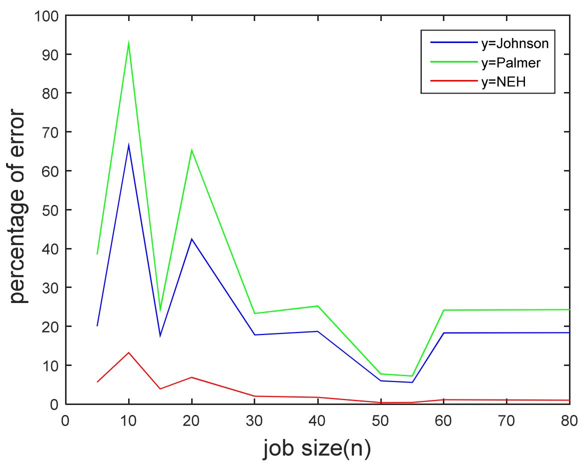


Table 9 Mean of percentage errors

n	*Mean of percentage error of the total waiting times in Palmer (1956) algorithm*	*Mean of percentage error of the total waiting times in Johnson (1954) algorithm*	*Mean of percentage error of the total waiting times in NEH (Nawaz et al., 1983) algorithm*
5	38.47	20.01	5.63
10	92.70	66.50	13.22
15	24.38	17.61	3.88
20	65.25	42.43	6.85
30	23.30	17.79	2.01
40	25.20	18.70	1.73
50	7.74	5.97	0.37
55	7.21	5.55	0.40
60	24.17	18.29	1.11
80	24.28	18.35	0.99

Figure 3 shows that the existed algorithms considered for comparison in this paper yields large total waiting time than the proposed one. Also, the error curve shows that NEH (Nawaz et al., 1983) algorithm with makespan approach returns less total waiting time than the Johnson (1954) and Palmer (1956) algorithm.

From the computational experiments it is noted that the error is independent of job sizes as it can be seen in Table 9, that group with ten jobs has mean of percentage errors as 92.70 units in Palmer (1956) algorithm but when job size is increased to 15 with another data set of problems, it reduces to 24.38 units. Going further, for group with job size 40, it again increases and in like manner it decreases with big difference for group of 50 and 55 jobs. This shows that error is independent of job size but it depends upon the choice of randomly generated fuzzy processing times. A key point is also noted that the mean error increases and decreases in the same manner for both the Palmer (1956) and Johnson (1954) algorithm for different job groups but this is not so in the case of NEH (Nawaz et al., 1983) algorithm.

Table 10 Average of mean percentage errors

Algorithm	*Average of mean percentage errors*
Palmer (1956) algorithm	33.27
Johnson (1954) algorithm	23.12
NEH (Nawaz et al., 1983) algorithm	3.619

Furthermore, it can be seen from Table 10 that NEH (Nawaz et al., 1983) algorithm is very close to the exact solution whereas Palmer (1956) algorithm produces a large amount of error than the Johnson (1954) algorithm. Also, the significant less error in NEH (Nawaz et al., 1983) algorithm clarifies that the algorithm produces a near optimal solution to minimise the idle time of jobs as well.

8 Conclusions

In this paper, the research expands knowledge by offering an exact algorithm to achieve the aim of minimising total waiting time of jobs and has been carried upon fuzzy data in which processing times are triangular fuzzy numbers. But there may be some possibilities that makespan or other costs such as machine idle cost, etc. may increases. From the commercial point of view, it is the primary need in the industries, when industry's manager has promise with the consumer to make their wait as less as possible for completing a project. The computational experiments manifest the propriety of proposed algorithm when compared with the existing approaches for makespan made by Johnson (1954), Palmer (1956) and NEH (Nawaz et al., 1983). Further it can be concluded that NEH (Nawaz et al., 1983) algorithm minimises the makespan by reducing the idle time of jobs consumed in queue for processing on second machine. Paper includes information about the closeness of NEH (Nawaz et al., 1983) algorithm to the exact solution in comparison with Johnson (1954) and Palmer (1956). The present work can be enhanced by taking setup times for machines, or considering the models with random processing times.

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