Statistical estimation of random failure probability based on stress levels

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Abstract: We develop a new method for estimation of random failure probability; this will inform test design and statistical analysis. First, the random failure mechanism is characterised so that a suitable test can be designed. Second, the experimental results are statistically analysed. This enables the development of a Gompertz model of the failure characteristics, which are then extrapolated, and the failure probability is estimated. The probability of random brake disc failure in the real world was quantitatively estimated using a case study: a reliability test of random failure caused by hot judder designed by a user based on our model.

Keywords: reliability engineering; brake disc; failure mechanism; hot judder; brake torque variation; BTV; random failure; test design; statistical analysis; degradation model; log-normal distribution; failure probability.

Reference to this paper should be made as follows: Sim, H.S., Jung, J.H. and Kim, Y.S. (2022) 'Statistical estimation of random failure probability based on stress levels', *Int. J. Quality Engineering and Technology*, Vol. 8, No. 4, pp.366–378.

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1 Introduction

With the advent of the Fourth Industrial Revolution, manufacturers of items such as automobiles and semiconductors are competing intensely to improve part reliability. The reliability of automobiles and other objects is intimately related to safety, so national standards must be met. It is essential to understand how failure develops when seeking to improve reliability. The various types of product failure include infant mortality, random, and wear-out failure (Kurtz et al., 1989). Failure types are classified by the 'shapes' of the failure rates. Meeker and Escobar (1998) defined infant mortality as early failure due to poor quality. Infant mortality is attributable to manufacturing or design issues (John et al, 2020). Many authors have sought to decrease such mortality using statistical quality control methods (Kwon, 2002) and optimal design theories (Dai and Wang, 2011). Wear-out failures are self-explanatory. Sung (2015) and Nelson (2005a, 2005b) reported that reliability testing, specifically accelerated life testing, has increased lifetime, thus reducing wear-out failures (please see Section 2).

Random failures are caused by external shocks. However, previous studies have assumed that failure rates are constant over time (thus exhibiting exponential distributions); methods used to analyse failure have not considered stress fluctuations in real-world environments. Therefore, we estimate failure rates using exponential distributions while developing new qualitative evaluation methods for random failures. This is essential for modern tests modelling random failure. Statistical methods are required to estimate failure probability.

Here, we develop a new test design and a statistical method for investigation of random failures. Section 2 reviews related works. Section 3 introduces our new test and our statistical analysis; we explore random brake disc failures caused by hot judder. Section 4 presents an empirical case study. Section 5 presents our conclusions and future plans.

2 Related works

The three failure types are infant mortality, random and wear-out failures. Tests for infant mortality and wear-out failures are described below; few works have explored random failures.

2.1 Infant mortality

Vassighi et al. (2008) reported that high-volume manufacturers could eliminate infant mortality by burn-in testing at abnormally high voltages and temperatures. Kim and Lee

(2012) found that the doors of urban railway carriages seemed to be as new after maintenance, but infant mortality then developed. A burn-in case study was presented. Other studies have explored electronics, military supplies, and aerospace components. Choi and Lee (2012) developed environmental stress screening (ESS) to military standards to improve the initial reliability of electronics. Minford (1982) observed some infant mortality during accelerated life-testing (at high voltage and stress) of multilayer ceramic capacitors. Park et al. (2009) subjected light-emitting diodes to six different environmental stresses to detect infant mortality. Lee and Yun (2009) found that burn-in tests improved reliability. Many studies have focused on military and aerospace supplies. Choi (2019) subjected military supplies to quality-assurance testing to reduce infant mortality. Tests included manufacturing preparedness, the use of development control gates, and ESS. Choi (2016) tested each production step of the fire control system (FCS) for the K55A1 howitzer. Burn-in tests reduced infant mortality. Park et al. (2007) developed burn-in tests for the components of domestic space launchers such as the KSLV-I. In summary, infant mortality was reduced by burn-in testing or ESS.

2.2 *Random failure*

Ryu and Chang (2005) suggested that methods detecting overstresses (short of destruction) were essential when evaluating large power systems. A random failure approach was used to analyse and minimise failures. Wang et al. (2005) found that random failures of power systems increased power prices because the failures were unpredicted. Wang et al. (2004) investigated the impact of random failures on the nodal price. Park et al. (2005) reported that products devoid of manufacturing problems accounted for most field failures when exposed to various environments. Thus, environmental tests are essential. However, Anghel et al. (2008) found that experimental studies on random failure are rare. Most have not discussed the tests in detail, focusing rather on the costs of such failures. Although some tests have been suggested, test reliability has not been analysed.

2.3 *Wear-out failure*

Many studies have addressed wear-out failure; here, we review the literature from 2016 through 2020. Flicker et al. (2017) explored the long-term reliabilities of modular power electronics devices from five different manufacturers via accelerated testing under both powered and unpowered conditions. The thermal cycling test was that of IEC 61215, the damp heat test that of IEC 6215, and static temperature tests were performed at 100 and 125°C. Demir et al. (2016) emphasised that power electronic circuits must be reliable, and showed how to measure the inrush current and calculate the melting point. Wang et al. (2019) found that drive write rates slowed over time; wear-out rendered solid-state drives unable to accept the rated data volumes. A solid-state drive lifetime endurance test was developed and implemented. Sleik et al. (2017) found that most test systems analysed the wear-out of only one specific component (among many relevant components) under defined stress conditions. Choi (2018) developed useful power-cycling test circuits, wear-out failure indicators, and measurement strategies for different test circuits. Lin et al. (2017) presented a new reliability assessment model that subjected heterogeneous components to different accelerated stresses. Lim et al. (2018) developed an accelerated lifetime test method using temperature and humidity to stress switchedmode power supplies. Chang et al. (2019) analysed breakage wear-out of wound hoist wire rope via finite element simulation. In summary, many more studies have explored wear-out than random failure.

3 Estimation of random failure probability

3.1 Introduction

Most tests of product lifetimes apply stresses; the data are used to estimate the shape parameter of the Weibull distribution, which features decreasing, constant, increasing, or bathtub-shaped failure rates (Hjorth, 1980) for infant mortality and random and wear-out failures. In most cases, reliability tests have been used to estimate wear-out failures. Figure 1 presents our suggested tests for wear-out and random failures.

Figure 1 A comparison of wear-out and random failures (see online version for colours)

	Wear-out Failure Test	**Random Failure Test**
Applied Type of Test Item	Increase failure rate on **time(cycle)**	Increase failure rate on **stress**
Independent Variables	Stress, Time(cycle)	Stress
Estimation Target	Failure event at a time(cycle)	Failure event at a stress

3.2 Phase 1: understanding failure mechanisms, and test design

Figure 2 describes Phase 1 of the two-phase process that we use to estimate the probability of random failure. Phase 1 is intended to clarify the failure mechanism and to establish a useful test thereof.

Phase 1 features several steps. First, only items that exhibit random failure are selected. Temperature, voltage, and force all contribute to such failure. Then, the stress factors causing failures are identified. Failures occur at different stress levels. Failure characteristics should be monotonous and the highest level of stress must be the most severe. Failure characteristics and thresholds are then determined using appropriate tests. Each threshold is a specific combination of the characteristics that trigger failure, and can be used to determine design specifications or standards. Figure 3 shows how to measure the independent and dependent variables.

Figure 4 presents the differences between random and wear-out failures. The latter do not occur at low stresses; random failures may. The wear-out failure rate increases by the cycle number. The random failure rate increases as stress increases. In other words, the duration of usage does not affect random failure when stress levels are low. The thresholds are chosen to ensure that products outlive their warranties.

As shown in Figure 5, our random failure test is based on stress levels, failure stress, failure characteristics, and threshold. The test is analogous to a degradation test. The horizontal axis shows the stress levels and the vertical axis shows the failure characteristics. At least three stress measurements should be conducted.

Figure 2 Two-phase process used to estimate the probability of random failure: Phase 1 (see online version for colours)

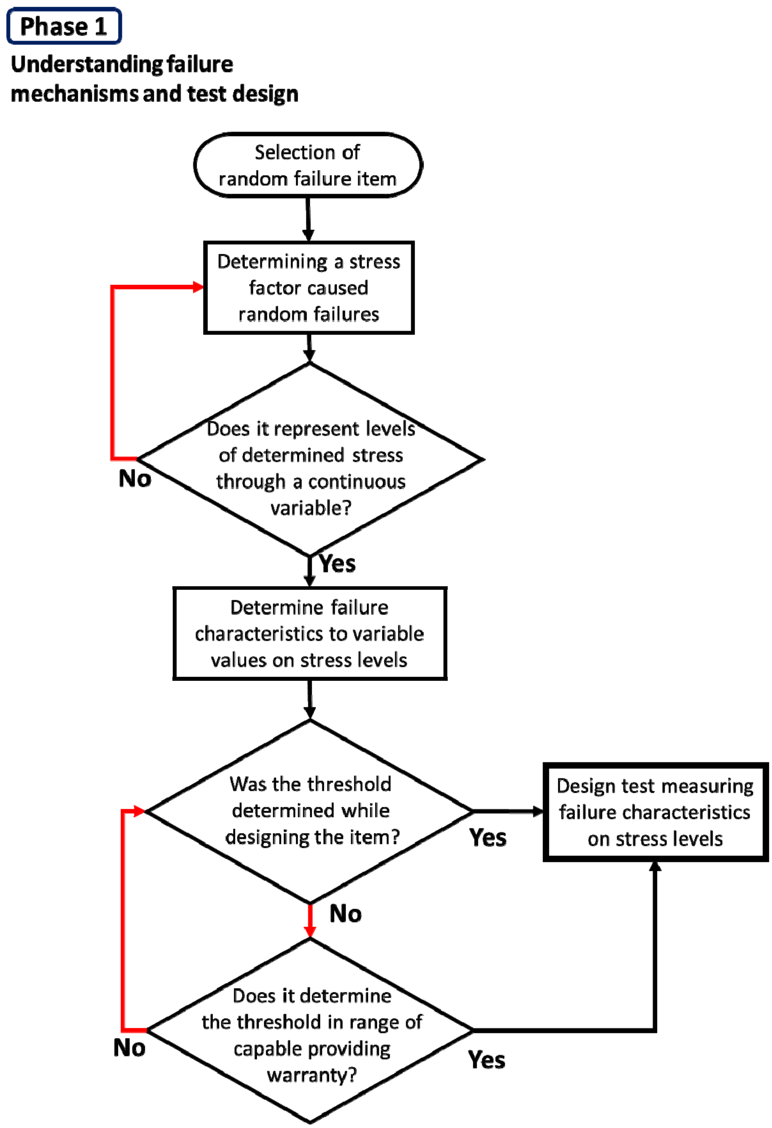


Figure 3 Determination of the independent and dependent variables

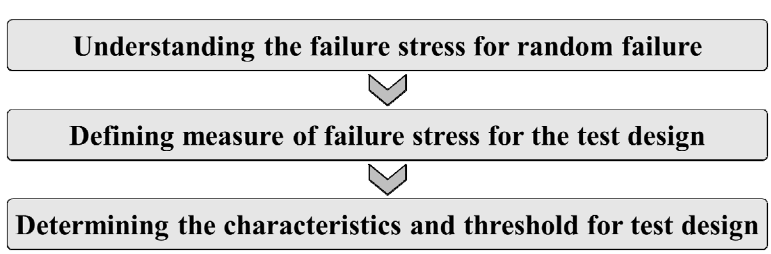


Figure 4 Differences between failure modes (see online version for colours)

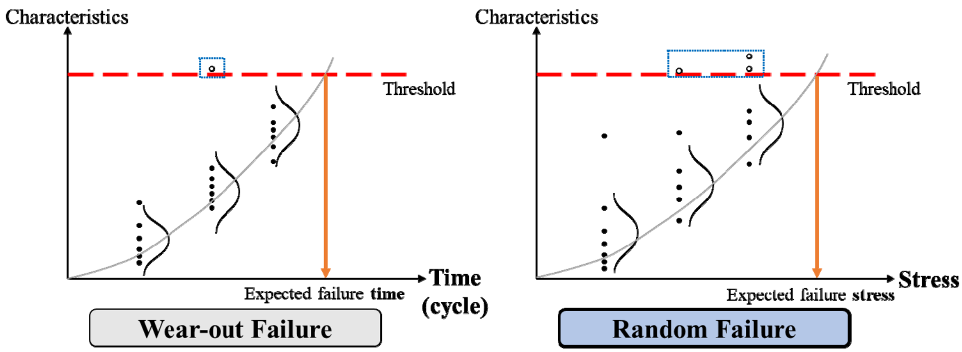
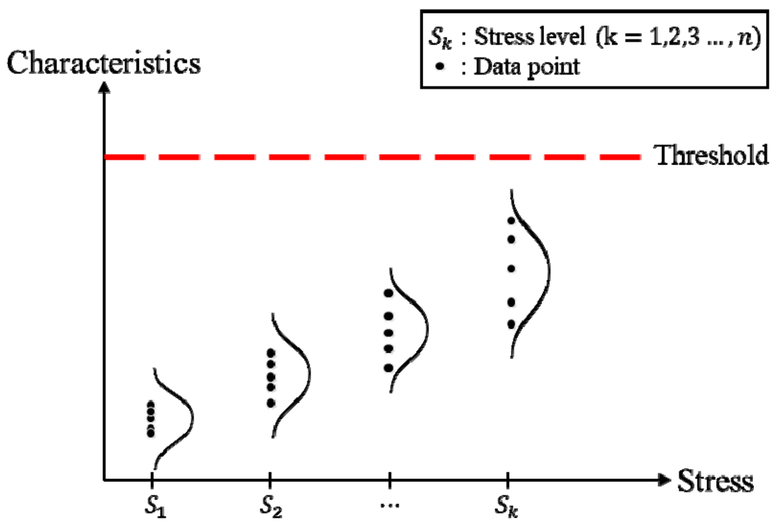


Figure 5 Random failure test (see online version for colours)



3.3 Phase 2: estimating failure probability based on statistical analysis

Figure 6 presents Phase 2 of the two-phase process: the statistical estimation of failure probability. The probability of random failure can be determined using the experimental results of Phase 1.

Herein, we measure both stress levels and failure characteristics. Failure stresses are estimated by extrapolation because long testing times are required before failures occur. As shown in Figure 7, the experimental results were fitted to a regression model and the failure stress was estimated at each threshold value.

The regression models favoured by reliability engineers are listed below (a, b, and c are model parameters, x is the independent variable, and y is the dependent variable):

1 linear model: $y = ax + b$

2 exponential model: $y = b \exp(ax)$

3 power model: $y = bx^a$

4 logarithmic model: $y = a \ln(x) + b$

5 Gompertz model: $y = ab^{c^x}$

6 Lloyd-Lipow model: $y = a - \frac{b}{x}.$

Figure 6 Two-phase process used to estimate random failure probability: Phase 2 (see online version for colours)

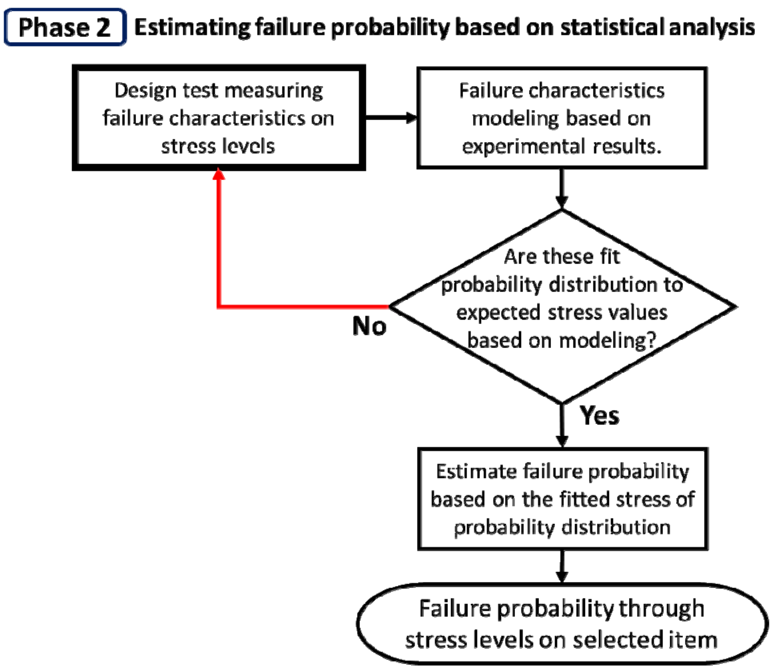


Figure 7 Estimation of failure stress values (see online version for colours)

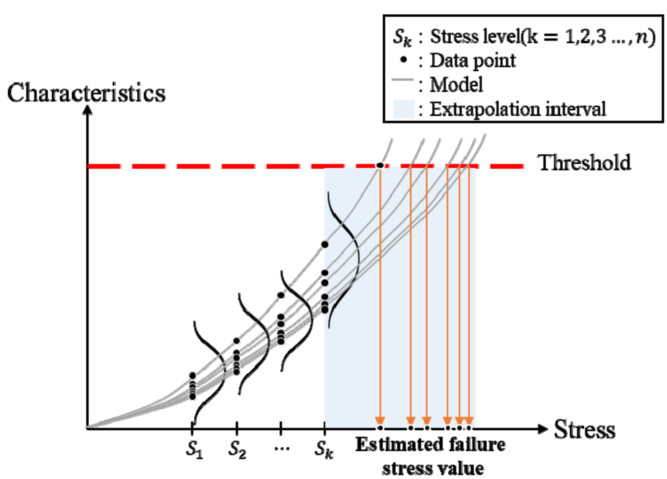
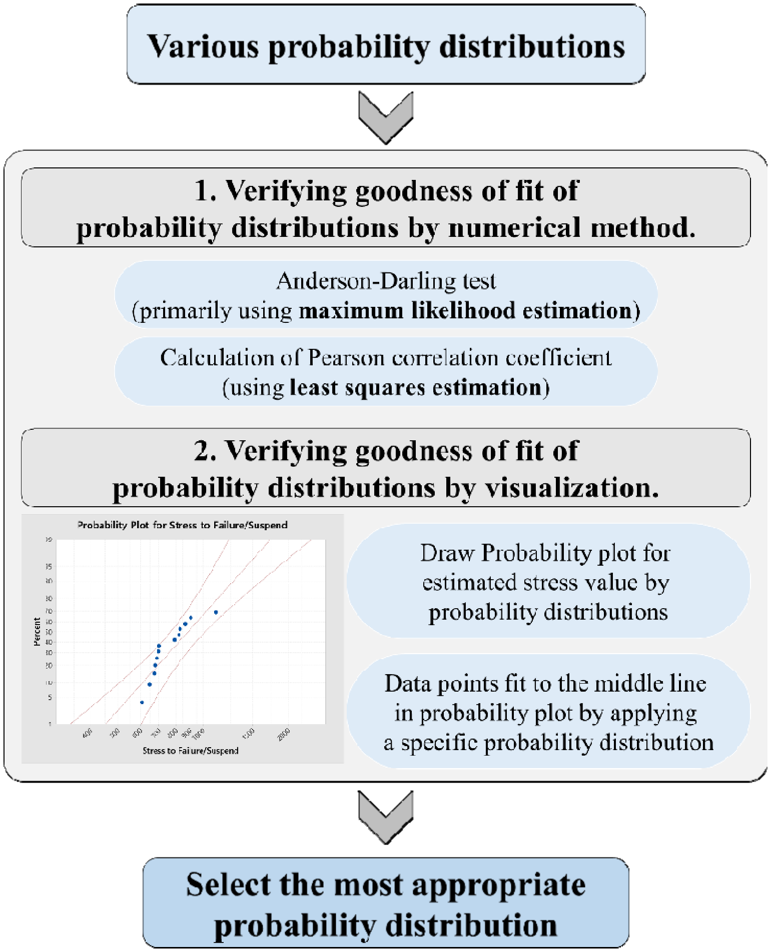


Figure 8 Selection of a probability distribution (see online version for colours)



The predictive power of each model is optimised by minimising the sums of the squared errors (SSEs). The failure characteristics at each stress level are then extrapolated. Random failure probability was calculated using the probability distributions obtained, as shown in Figure 8. The distributions favoured by reliability engineers include the Weibull, log-normal, exponential, and normal distributions. All distributions are explored numerically and visualised by plotting. The goodness-of-fit is verified numerically; two such methods are available. The Pearson correlation coefficient is derived via least-squares estimation (LSE) and evaluates the linearity of the relationship between the midline and the data points of the probability plot. Such a correlation coefficient cannot be calculated if the exponential distribution follows a fixed gradient. The Anderson-Darling (A-D) test features maximum-likelihood estimation (MLE). The distributional tails are heavily weighted and the summed, weighted squared distances between the fitted line and the plotted points are calculated. Thus, the distribution with the smallest A-D statistic is optimal. In general, the maximum-likelihood method is more precise than the least-squares method. Next, the probability distributions are visualised on plots drawn using the estimated stress values yielded by the distributions. If the data points align well with the midline of a probability plot, the chosen probability measure is appropriate.

The failure probability at each stress level was estimated by fitting the failure rate to the probability distribution. Equation (1) is used to estimate the random failure probability: $F(\cdot)$is the distribution function of the selected probability; S is the failure probability at a certain stress level; s_p is a specific stress level; and p is the failure probability at s_p.

$$F\left(s_p\right) = \Pr\left\{S \le s_p\right\} = p \tag{1}$$

4 Case study: brake discs

Brakes are critical in terms of automobile safety, and brake discs are key components of brakes. Brake disc failure can be classified by failure mechanism. Kao and Richmond (2000) found that one such mechanism is fracture at high rotor speed, causing cold judder; and the other fracture type is high-temperature fracture, triggering hot judder. The latter failures predominate (Abdelhamid, 1997). Various hot judder tests have been devised. Park et al. (2011) used finite element analysis to this end. Sim et al. (2013) simulated the relationship between hot judder and heat transfer. Hot judder is a random failure (Barber, 1969). Cold judder is a wear-out failure and can be tested via either accelerated lifetime or accelerated degradation tests.

4.1 Random failure testing and temperature stress results

The probability of random failure caused by hot judder is not affected by time. The test was repeated three times using six brake discs at six temperatures (150, 200, 250, 300, 400, and 500 degrees Celsius); the brake torque variation (BTV) differed. Table 1 lists the experimental results.

Table 1 Experimental results

No.	1	1	1	1	1	1	2	2	⋯	18	18	18	18	18	18
Temperature (°C)	150	200	250	300	400	500	150	200	⋯	150	200	250	300	400	500
BTV (kgfm)	1.1	0.9	0.9	1.2	3.8	4.1	1.1	1	⋯	1.1	1.7	2	2	4.1	5.8

4.2 Statistical analysis

The threshold is a BTV of 10 and the experimental results are interpreted accordingly. Of the various models, the Gompertz model afforded the lowest SSE; the specified BTV was attained at higher stress levels (Figure 9).

Of 18 fitted Gompertz models, 5 do not attain the thresholds and are thus censored. The estimated failure temperatures are fitted to the probability distribution via MLE (Figure 10). A log-normal distribution is fitted based on the probability plot and the A-D statistic.

The random failure probability is thus estimated at each stress level based on the log-normal distribution. Figure 10 and Table 2 were used to derive the data.

Figure 9 BTV by stress level (see online version for colours)

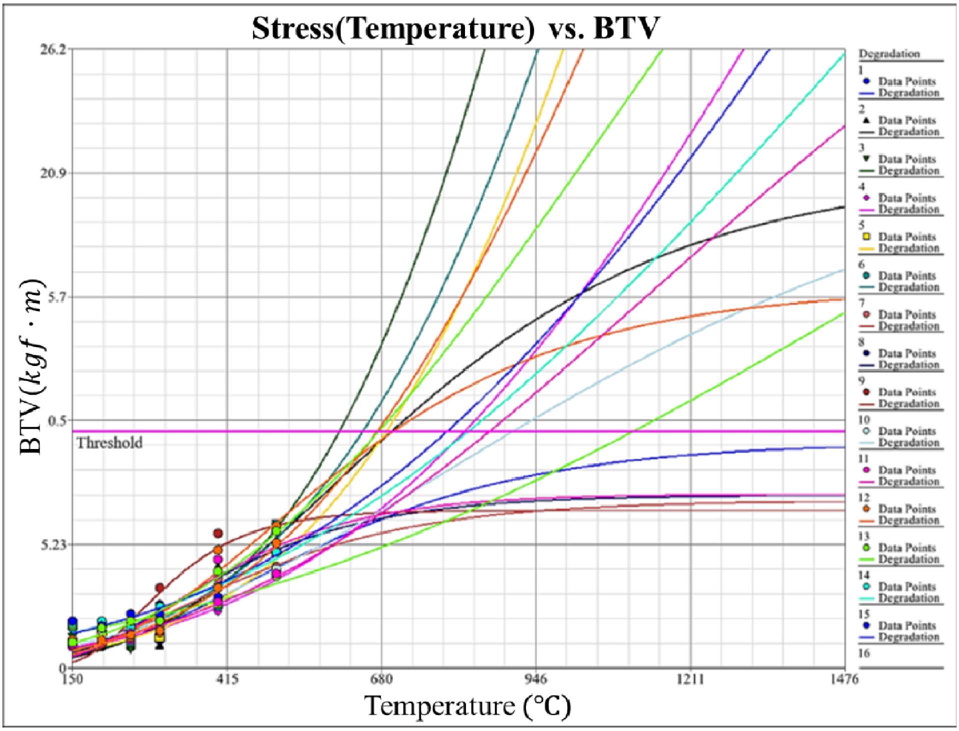


Figure 10 Failure plot and A-D statistics for the estimated stresses (see online version for colours)

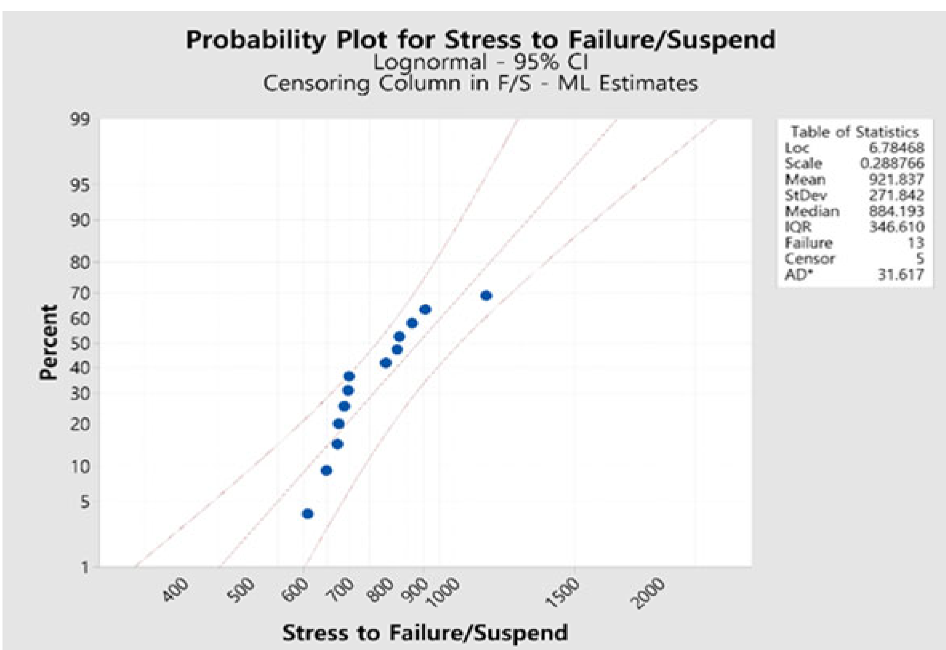


Table 2 Random failure probability by brake disc temperature

Temperature (°C)	300	320	340	360	380	400	440	480	520	560	600
Failure probability (%)	0.01	0.02	0.05	0.09	0.17	0.30	0.78	1.72	3.30	5.69	8.97

5 Conclusions

We use a two-phase process to estimate random failure probabilities. This can inform test design; we present various modes of statistical analysis. Real-world random failure probabilities were estimated at various stress levels. Existing methods assume that the failure rate is constant over time, thus based on an exponential distribution. Prior to statistical analysis, we develop a new reliability test. The horizontal axis was the stress level; random failures occur at various stresses. The vertical axis shows (displays) the failure characteristics. Reliability engineers tend to estimate failure probability over time (or cycles). However, we estimate failure probability by stress level. We statistically analysed the experimental data to estimate failure probabilities at various environmental stresses.

The case study features the random failure probability of a brake disc caused by hot judder. Increases in brake disc temperatures cause random failures in the real world. If the brake disc temperature is known under real operating conditions, the failure probability can be estimated.

Our method is applicable to various scenarios. For example, many electronic failures are caused by voltage deviations. If the failure mechanism can be replicated, no special test facility is required. Hence, our method will find applications in many manufacturing industries.

Acknowledgements

This work was supported by Kyonggi research grant 2019.

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