Joint optimisation of production run length and maintenance policy for an imperfect process with multiple correlated quality characteristics

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Abstract: The earliest economic production quantity models assumed that the manufacturing process and the quality of produced items are perfect. While in a real situation, non-conforming products are fabricated and machine failure happens. Hence, the production systems are increasingly engaged in the improvement of machines availability and products quality. In this regard, this paper presents an integrated production and maintenance planning model under monitoring multiple quality characteristics. To adapt to the real production conditions, it is considered that quality characteristics are correlated. Furthermore, to improve the power of process monitoring, a Shewhart control chart is designed by considering both economic and statistical criteria. Due to the complexity of the problem, the particle swarm optimisation algorithm is employed to optimise the expected total cost per time unit, subject to statistical quality constraints. Here, an industrial example is given to show applicability of the presented mathematical programming. Furthermore, to demonstrate the validation and effectiveness of the suggested approach, a comparative study is presented. It confirms that the integration of production planning, maintenance policy, and statistical process monitoring leads to a significant increase in the cost savings.

Keywords: production run length; maintenance policy; statistical process monitoring; multiple-quality characteristics.

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1 Introduction

Production planning, statistical production monitoring (SPM), and maintenance policy are three primary and interdependent tools in any manufacturing system. Production planning and scheduling preventive maintenance (PM) are mutually in a challenge. Because of the interrelated between them often not optimal combination costs, therefore their integration has been shown to be more economical. The PM activity ensured the sustainability of the manufacturing process also decreased quality-related costs, with to maintain equipment in good operating conditions through adequate maintenance programs and to produce conforming items. Indeed, production planning and process monitoring are strongly interrelated to each other. According to the notes mentioned above, it is necessary to develop approaches that study the interdependence among three principal aspects of the manufacturing processes.

In spite of the mentioned facts in the previous paragraph have been existing researches in the literature that three concepts are often considered separately. In this regard, there are many studies dealing with production planning such as Chung and Hou (2003). They proposed a production model to determine an optimum runtime for a deteriorating manufacturing system in which the shortages are allowed. Also, Cheng et al. (2015) and Manna et al. (2017) are the other studies that have been investigated in this context. Lee and Cha (2016) focused only on the maintenance issue and investigated models of different types of periodic preventive maintenance policy that minimise the long-run expected cost. More recent investigations about the maintenance issue can be found in Zhou et al. (2016), Duan et al. (2019). Chen and Yang (2002) investigated the statistical properties of the process and developed a model for economic design control. Moreover, Lee et al. (2012), Nenes et al. (2015) conducted the study on designing the control charts.

In recent years, many integrated models have been presented in the literature to consider difference interactions between two of the three aforementioned basic concepts. For example, Jafari and Makis (2016) suggested an integrated optimisation approach of economic manufacturing quantity and maintenance policy for a deteriorating production system. Shrivastava et al. (2016) designed a CUSUM control chart and determined the optimal value of preventive maintenance interval, simultaneously. Rahim and Ohata (2005), to determine optimum parameters of the inventory and control chart simultaneously, proposed an economic model. Moreover, several other studies such as Yin et al. (2015), Wu et al. (2015), Salmasnia et al. (2016), Cheng et al. (2017), Yang and Lin (2018) and Salmasnia et al. (2018) were conducted in this context.

The joint determination of production run length, maintenance policy, and SPM have attracted the attention of some researchers in the past three decades. In this regard, Ben-daya and Makhdoum (1998) proposed an integrated model of the three suitable functions. They calculated the production costs and EPQ with applied the different maintenance approaches and an X-bar chart. Pan et al. (2012) jointly optimised the production rate and maintenance programming with employed a Shewhart X-bar chart for a deteriorating manufacturing system. Lin et al. (2016) obtained the model for optimised the number of inspection, the inspection intervals, and the economic production quantity for an imperfect production system. Salmasnia et al. (2017a) integrated manufacturing cycle length, maintenance policy, and control chart parameters in a unified model in the production process of a product with a single quality characteristic. Moreover, Bouslah et al. (2016), Gunay and kula (2016), Fakher et al. (2016), and Cheng et al. (2018) are the other researcher's efforts have been devoted to developing this context.

The mentioned papers in the previous paragraph often ignored two important issues in designing the control charts. The first issue is the number of quality characteristics under consideration in the process and the second is the economic statistical design (ESD) of the control charts. This assumption that only a single quality characteristic affects the process efficiency can be very misleading. Actually, nowadays in industry environments, it is necessary that two or more correlated quality characteristics should be monitored simultaneously. This issue is considered by Hotelling (1947), he proposed the T^2 Hotelling chart to monitor multivariate processes. This chart is a development of the univariate Shewhart control chart that it has been one of the simplest statistical process control techniques to monitor multiple characteristics that have led to producing software the T^2 Hotelling chart (Lowry and Montgomery, 1995). Montgomery and Klat (1972) have been investigated the economic design of the T^2 Hotelling control chart. Woodall et al. (2004) and Faraz and Saniga (2011) are other studies that employed the T^2 Hotelling charts for monitoring the manufacturing systems. In the literature introduced another various type of multivariate control charts such as multivariate EWMA chart, multivariate CUSUM chart, multivariate Bayesian chart, and so on, have been developed in the literature. For example, Chen et al. (2015) suggested a multivariate exponentially weighted moving the average chart to monitor Poisson observation.

As mentioned previous paragraph another issue in the most integrated papers is the economic design (ED) of the control charts. The objective of an economic methods design is to minimise the expected cost that ignoring statistical properties. ED control chart at the first time designed by Duncan (1956) for an X-bar control chart. He minimised production cost according to the determination of three parameters, inspection size n, inspection interval h, and control limit k. Chou and Chen (2006) developed a model for the economic design of T^2 Hotelling charts that the expected total cost is minimised. However, Woodall (1986) investigated the ED efficiency that found it has poor statistical performance. To modify the low statistical performance of the economic design control chart, Saniga (1989) expanded the pure economic model by combining additional statistical constraints. Afterward, other authors such as Yin and Makis (2011), and Salmasnia et al. (2017b) applied the economic statistical design method for optimising the control chart parameters. The properties of the proposed scheme in comparative the existing researches in the literature are summarised in Table 1.

As an initiative to cover the existing research gaps, this paper integrates the concepts of inventory, maintenance policy, and designing a control chart in a unified model. Also, a T^2 Hotelling chart is developed to monitor several correlated quality characteristics in the imperfect manufacturing processes. It aims to minimise the expected total cost of the imperfect production system subject to statistical constraints.

The remainder of this paper is organised as follows: At the end of this section, the state-of-the-art properties are summarised in Table 1. In Section 2, the problem under study will be explained in detail. Section 3 explains the designed mathematical model for the optimisation of production run length, maintenance policy, and Hotelling control chart parameters for multiple characteristics. In Section 4, the solution approach is described. Section 5, is presented a numerical example to demonstrate the applicability of the proposed model then a comparative study is given to show the validation of the presented mathematical programming in comparison with-the-art. Finally, the conclusions and some recommendations for improving function manufacturing systems are laid out in Section 6.

2 Problem description

The traditional EPQ model balances the costs between setup and inventory holding with this assumption that the production system is forever in an in-control state and that all outputs are conforming. In reality, manufacturing processes suffers from deterioration, because of the nature of the process, machine wastage and, etc. This research investigates an imperfect production process that begins from the in-control state and after time may occur an assignable cause that leads to shifts in the out-of-control state.

A T^2 scheme is applied to monitor multiple quality characteristics with an alert signal to inform operators when process produces non-conforming items. A joint model of production run length, statistical process monitoring, and maintenance in a unified model is suggested. Also, to improve the process performance, the simultaneous economic and statistical criteria in the design of the control chart are considered. In the proposed model, the samples are taken with size n, at h time units in each inspection. In production cycle the inspections are taken independently.

According to the occurrence time of the shift is defined three scenarios for the production process. Scenario 1, happens when the process remains in-control during the implementation of the production cycle. In Scenario 2, the manufacturing system starts from the in-control state after time at least one of the quality characteristics exceeds from its target value during manufacturing cycle and control chart detects this deviation before the end of the process. Scenario 3, is similar to Scenario 2 with this difference that shift can't be detected by control chart until the end of the production cycle. Also, at the end of the production cycle in each scenario implement maintenance policy to restore the process to the in control state and to as good as new condition. These three scenarios will explain in the model description section in detail.

In this study, p correlated process variables are considered that follow a p-variate normal distribution with in-control mean vector and variance-covariance matrix . The T^2 control chart is employed for the detection changes quality characteristics that it signals as soon as the statistic. In this paper, for the sake of simplicity, it is assumed that and are known or are estimated from magnitude large enough samples.

Table 1 Summarised literature review

Papers	*Integrated concepts*			*Number of quality characteristics*		*Type of design chart*	
	EPQ	*Maintenance*	*SPM*	*Single*	*Multiple*	*ED*	*ESD*
Chung and Hou (2003)	✓						
Cheng et al. (2015)	✓						
Manna et al. (2017)	✓						
Lee and Cha (2016)		✓					
Zhou et al. (2016)		✓					
Duan et al. (2017)		✓					
Chen and Yang (2002)			✓	✓		✓	
Lee et al. (2012)			✓	✓		✓	
Nenes et al. (2015)			✓		✓	✓	
Rahim and Ohata (2005)	✓		✓	✓		✓	
Salmasnia et al. (2016)	✓		✓	✓		✓	
Yin et al. (2015)		✓	✓	✓			✓
Salmasnia et al. (2018)		✓	✓	✓			✓
Wu et al. (2015)	✓		✓	✓		✓	
Jafari and Makis (2016)	✓	✓					
Cheng et al. (2017)	✓	✓					
Yang and Lin (2018)	✓	✓					
Shrivastava et al. (2016)		✓	✓	✓		✓	
Ben-Daya and Makhdoum (1998)	✓	✓	✓	✓		✓	
Pan et al. (2012)	✓	✓	✓	✓			✓
Lin et al. (2016)	✓	✓	✓	✓			
Salmasnia et al. (2017a)	✓	✓	✓	✓			✓

Table 1 Summarised literature review (continued)

Papers	*Integrated concepts*			*Number of quality characteristics*		*Type of design chart*	
	EPQ	*Maintenance*	*SPM*	*Single*	*Multiple*	*ED*	*ESD*
Bouslah et al. (2016)	✓	✓	✓	✓			
Gunay and Kula (2016)	✓	✓	✓	✓		✓	
Fakher et al. (2016)	✓	✓	✓	✓		✓	
Cheng et al. (2018)	✓	✓	✓	✓			
Hotelling (1947)			✓		✓		
Lowry and Montgomery (1995)			✓		✓	✓	
Faraz et al. (2010)			✓		✓		✓
Chen et al. (2015)			✓		✓		
Duncan (1956)			✓	✓		✓	
Chou and Chen (2006)			✓		✓	✓	
Saniga (1989)			✓	✓			✓
Yin and Makis (2011)		✓	✓		✓		✓
Salmasnia et al. (2017b)			✓		✓		✓
This paper	✓	✓	✓		✓		✓

Table 2 Notations

Notation	*Description*
Indices	
i,j	The indices of inspection intervals
r	The index of Scenarios
Decision variables	
n	The inspected sample size
h	The inspection interval
k	The control limit
K	The number of inspection in a perfect cycle time
Parameters	
A	The set up cost
B	The inventory holding cost per unit per time unit
c_y	The cost of each false alarm
c_f, c_v	The fixed and variable cost of inspection, respectively
c_0	The quality loss cost per unit when the process is in the in-control state
c_1	The quality loss cost per unit when the process is in the out-of-control state
c_{pm}	The cost of preventive maintenance activity
c_{rm}	The cost of reactive maintenance activity
d	The Mahalanobis distance
D_d	The demand rate
$D.$	The annual demand rate
$E(Q)$	The expected quality loss cost
$E(C_Q)$	The expected quality control cost per production cycle
$E(Ins)$	The expected inspection cost

Table 2 Notations (continued)

Notation	*Description*
Parameters	
$E(C_{Ins})$	The expected inspection cost per production cycle
$E(M)$	The expected maintenance cost
$E(C_M)$	The expected maintenance cost per production cycle
$E(I)$	The expected inventory holding and ordering cost per production cycle
ETC	The expected total cost per production cycle
$E(T)$	The expected time of production cycle
$f(t)$	The time to shift probability density function (PDF)
$F(t)$	The time to shift cumulative distribution function (CDF)
p	The number of quality characteristics
p'	The production rate
$Pr(Sc_r)$	The probability of happening r^{th} scenario
$Pr(sig)$	The probability that chart issues a signal when the process shifts to the out-of-control state
Q'	The economic production quantity
Sr	The expected number of inspection points in the in-control state in r^{th} scenario
t	The time to shift from the in-control state to the out-of-control state
Tin	The time that the process is in the in-control state in a given cycle
Tout	The time that the process is in the out-of-control state in a given cycle
α	Probability of type I error
β	Probability of type II error
λ	The parameter of exponential distribution
$\mu0$, $\mu1$	The process mean vector in the in-control and the out-of-control states, respectively

Table 2 Notations (continued)

Notation	*Description*
Parameters	
$\sum$	The constant variance–covariance matrix
τ	The expected in-control time during each inspection interval
ARL0	The average run length when the process is in the in-control state
ARL1	The average run length when the process is in the out-control state
ARLl	The lower bound of
ARLu	The upper bound of
ATS	The average time to signal
AATS	The adjusted average time to signal

2.1 Notations

Before developing the proposed model mathematically, the notations used to formulate the problem are presented in Table 2.

2.2 Assumptions

The underlying assumptions of aforementioned problem are described in the following:

a The process starts in the in-control state with $\mu = \mu_0$ in initial of each cycle.

b Only One type assignable cause occurs that leads to the process mean vector deviation from the target $\mu = \mu_0$ to a known value $\mu = \mu_1$. Hence, the values of the variance-covariance matrix are stable.

c If at the end of the K^{th} inspection interval, the process declared as an in-control state preventive maintenance is implemented on the system at the end of the $(K + 1)^{\text{th}}$ interval. An alert signal in the j^{th} interval $(0 < j < K)$, indicates that the process shifts to the out-of-control state. From this moment, an investigation is carried out to discover the assignable cause and eventually reactive maintenance is implemented to restore the process to the as-good-as-new condition.

d The in-control time of process follows a truncated exponential distribution in which the probability density function (PDF) is given by equation 1.

$$f(t \mid (K+1)h) = \frac{\lambda e^{-\lambda t}}{1 - e^{-\lambda((K+1)h)}} \tag{1}$$

e The production cycle time ends either with a true alarm or after K+1 intervals.

f The following three can be ignored due to those are the very negligible in comparison with production cycle time:

  a the time of inspection

  b time to detect a false alarm

  c the time to implement preventive and reactive maintenance.

3 Model description

In this section, the inspection, quality and the maintenance costs are investigated in addition to the classic EPQ cost in each of the mentioned scenarios. Also, it is calculated the production run length and the occurrence probability of each Scenario.

- Scenario 1: In this scenario, the assignable cause does not occur during K inspection intervals. Nevertheless, the scheduled preventive maintenance will be carried out at the $(K$+1$)^{\text{th}}$ inspection interval. Production run length is equal to the expected in-control time and $E(T_{out} \mid Sc_1)$ is zero. As shown in Figure 1,

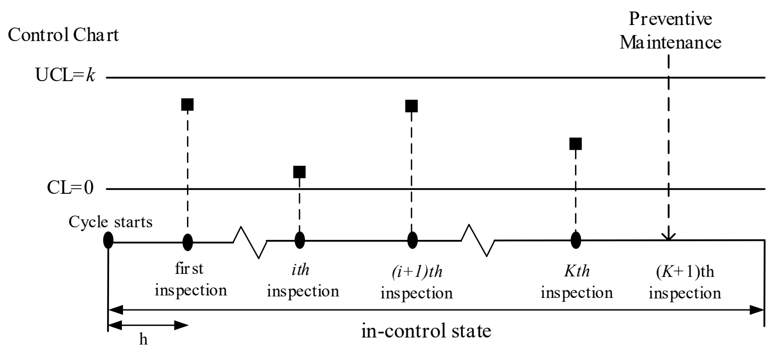
$$E\left(T_{in} \mid Sc_1\right) = (K+1)h \tag{2}$$

$$E\left(T_{out}\left|Sc_1\right.\right)=0 \tag{3}$$

The occurrence probability of scenario1 is equal to the occurrence probability of a shift after at least $(K+1)^{th}$ inspection interval [equation (4)].

$$\Pr\left(Sc_1\right)=1-F((K+1)h) \tag{4}$$

Figure 1 Graphical representation of scenario 1 in the production cycle time



- Scenario 2: According to Figure 2, in this scenario, the manufacturing cycle begins and stays in the in-control state until i^{th} inspection interval. After that, due to the occurrence of an assignable cause at least one of the quality characteristics under consideration at a time between the i^{th} and $(i+1)^{th}$ inspection shifts to the out-of-control state. Therefore, the mean vector of the quality characteristics shifts from μ_0 to μ_1. Due to the weakness of the control chart, it cannot emit the alert signal immediately. The process continues until the $(i+1)^{th}$ inspection that the control chart alerts a signal. In this condition, to discover the assignable cause and restore the process to as good as a new situation, reactive maintenance is implemented. Therefore, the expected in-control time in this scenario is:

$$E\left(T_{in}\left|Sc_2\right.\right)=\int_0^{Kh} t\times f(t\left|(K+1)h\right.)dt \tag{5}$$

and the expected out-of-control time is the average time between the process mean shift and the chart's signal that is called average time to signal (AATS).

$$E\left(T_{out}\left|Sc_2\right.\right)=AATS \tag{6}$$

Here, the AATS is calculated as follows:

$$AATS=ATS-\tau \tag{7}$$

Figure 2 The relationship between AATS and ATS (see online version for colours)

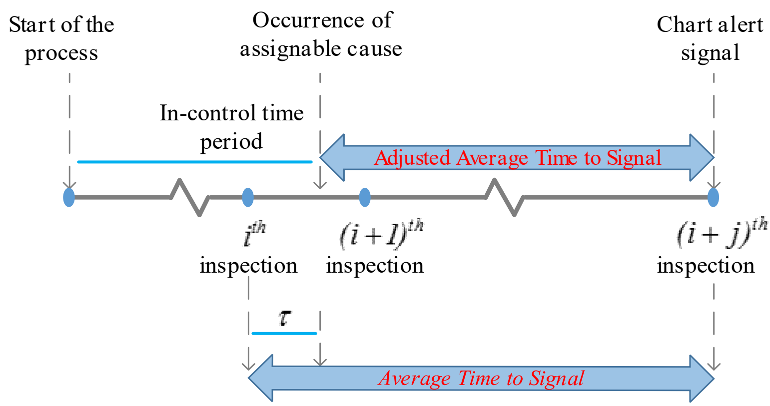
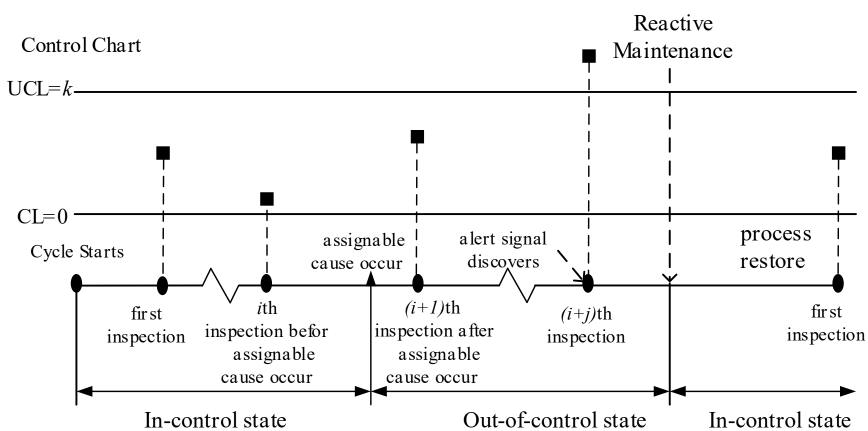


Figure 3 Graphical representation of scenario 2 in the production cycle time



where that ATS is the average time to signal, it is the average time from the last inspection before the occurrence of the shift to signal issued from the chart. It is calculated by product the inspection interval and average run length (ARL) when the process is in the out-control state.

$$ATS = \frac{h}{1-\beta} = h \times ARL_1 \tag{8}$$

$$\beta = \Pr\left(T^2 < k \middle| d \neq 0\right) = \Pr\left(\chi^2(p, nd) < k\right)\Big) \tag{9}$$

Let be the expected time of assignable cause occurrence within an interval.

$$\tau = \frac{\displaystyle\int_{ih}^{(i+1)h} te^{-\lambda t} dt}{\displaystyle\int_{ih}^{(i+1)h} e^{-\lambda t} dt} \tag{10}$$

$\Pr(Sc_2)$ is computed by using the occurrence probability a shift before K^{th} inspection interval, given that the shift is detected before $(K+1)^{\text{th}}$ inspection interval. The probability of releasing an alarm signal when the process shifts to the out-of-control state is called as Pr(*sig*) and is formulated as equation (12).

$$\Pr(Sc_2) = F(Kh) \times \Pr(sig) \tag{11}$$

$$\Pr(sig) = 1 - \beta^{K-1} \tag{12}$$

- Scenario 3: In Scenario3, the process begins in the in-control state and at a time between the i^{th} and $(i+1)^{th}$ inspections, the mean process shift to the out-of-control state. Because of the probability of type II error, the chart cannot discovers the shift, so the process continues until end of the K^{th} inspection interval. Therefore, at the $(K+1)^{th}$ inspection interval, the process is identified in the out-of-control state, which for restoring the process to in-control state, the scheduled PM has to be replaced by RM. Figure 4 demonstrates the manufacturing cycle in Scenario 3. The expected in-control time is formulated by equation 13.

$$E\left(T_{in} \middle| Sc_3\right) = \int_0^{(K+1)h} t \times f\left(t \middle| (K+1)h\right) dt \tag{13}$$

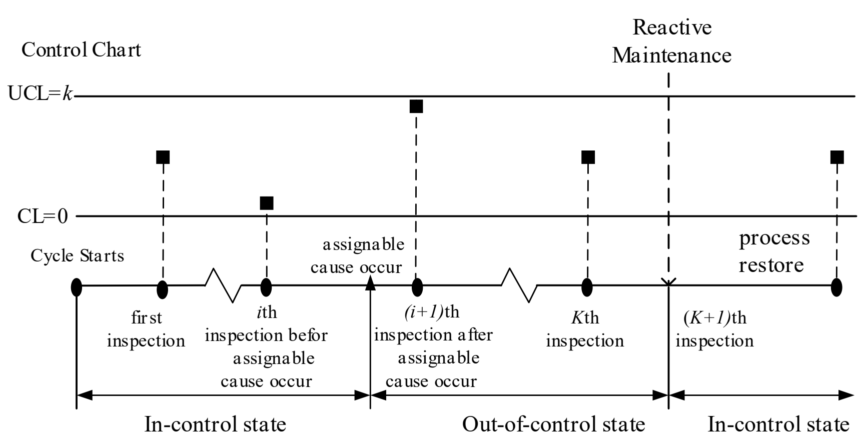
Therefore, to compute the expected out-of-control time in this scenario, the expected in-control time is subtracted from the production run length:

$$E\left(T_{out} \middle| Sc_3\right) = (K+1)h - E\left(T_{in} \middle| Sc_3\right) \tag{14}$$

The occurrence probability of Scenario 3 is equal to the probability of happening an assignable cause before $(K+1)^{th}$ inspection interval while it is not detected until the end of the cycle. Thus, $\Pr(Sc_3)$ can be obtained as:

$$\Pr(Sc_3) = F((K+1)h) - F(Kh)\Pr(sig) \tag{15}$$

Figure 4 Graphical representation of scenario 3 in the production cycle time



As a result, the expected time of the manufacturing cycle is computed by multiplying the duration of the cycle in each scenario and the happening probability of each scenario, the occurrence probability of each scenario and the duration of the production cycle as shown in equation (16) and (17), respectively.

$$E\left(T\left|Sc_r\right.\right)=\begin{cases}(K+1)h & r=1,3\\ \int_0^{Kh} t\times f(t\left|(K+1)h\right.)dt+AATS & r=2\end{cases} \tag{16}$$

$$E(T)=\sum_{r=1}^{3}E\left(T\left|Sc_r\right.\right)\Pr\left(Sc_r\right) \tag{17}$$

3.1 Quality loss cost

The quality loss cost is imposed on manufacturer in both the in-control and the out-of-control states because of producing the non-conforming items. Therefore, this cost includes of the quality loss cost when the process is in the in-control state, and the quality loss cost when the process is in the out-of-control state. Let $c_0\left(c_1\right)$ the quality loss cost per unit when the process is in the in-control (out-of-control) state thus, according to the mentioned explanation, it can be accounted so equations (18) for each of the three scenarios. Finally, the expected quality loss cost can be computed according to equation (19).

$$E\left(C_Q\left|Sc_r\right.\right)=\begin{cases}c_0\times E\left(T_{in}\left|Sc_r\right.\right) & r=1\\ c_0\times E\left(T_{in}\left|sc_r\right.\right)+c_1\times E\left(T_{out}\left|Sc_r\right.\right) & r=2,3\end{cases} \tag{18}$$

$$E\left(C_Q\right)=\sum_{r=1}^{3}E\left(C_Q\left|Sc_r\right.\right)\Pr\left(Sc_r\right) \tag{19}$$

3.2 Inspection cost

The inspection cost in each scenario is calculated by product the average number of inspections during of the production cycle in the summation of the fixed $\left(c_f\right)$ and variable $\left(c_v\right)$ costs of each inspection. The average number of inspection in Scenarios 1 and 3 is equal to *K*, while it in Scenario 2 is obtained by summation of the expected number of inspection in the in-control and out-of-control states. Therefore, for all three scenarios, $E\left(C_{Ins}\left|C_r\right.\right)$ is formulated in equation (20). Eventually, the total expected inspection cost per cycle time is calculated as equation (21).

$$E(C_{Ins}\left|Sc_r\right.)=\begin{cases}\left(c_f+c_v n\right)K & r=1,3\\ \left(c_f+c_v n\right)\left(\dfrac{E\left(T_{in}\left|Sc_r\right.\right)+E\left(T_{out}\left|Sc_r\right.\right)}{h}\right) & r=2\end{cases} \tag{20}$$

$$E\left(C_{Ins}\right)=\sum_{r=1}^{3}E\left(C_{Ins}\left|Sc_r\right.\right)\times\Pr\left(Sc_r\right) \tag{21}$$

3.3 Maintenance cost

Maintenance cost includes of two parts:

1 the false alarm cost

2 planned preventive (PM) and reactive maintenance (RM) costs and depends on the scenario that occurs.

To compute the false alarm cost the number of inspection in the in-control state must be multiplied by the average run length when process is in the in-control state (ARL_0) and the cost of each false alarm (c_y). Since in the Scenario 1 the average number of inspection points in the in-control state is K and the PM cost is implemented to manufacturer at the end of cycle. In the Scenarios 2 and 3 due to the production process shifts to the out-of-control state it is less than K and can be calculated by equations (22) and (23), respectively and the RM is conducted to the manufacturer at the end of cycle. Based on the aforementioned descriptions, the expected maintenance cost in each scenario is formulated by equations (25).

$$s_2 = \sum_{l=1}^{K-1} e^{-\lambda lh} - (K-1)e^{-\lambda Kh} \tag{22}$$

$$s_3 = \sum_{l=1}^{K} e^{-\lambda lh} - Ke^{-\lambda(K+1)h} \tag{23}$$

$$ARL_0 = \frac{1}{\alpha},\ \alpha = \Pr\left(T^2 > k \middle| d = 0\right) \tag{24}$$

$$E\left(C_M \middle| Sc_r\right) = \begin{cases} \dfrac{K}{ARL_0}.c_y + c_{pm} & r = 1 \\ \dfrac{s_2}{ARL_0}.c_y + c_{rm} & r = 2 \\ \dfrac{s_3}{ARL_0}.c_y + c_{rm} & r = 3 \end{cases} \tag{25}$$

Finally, the expected total maintenance cost per manufacturing cycle is obtained according to equation (26).

$$E(C_M) = \sum_{r=1}^{3} E\left(C_M \middle| Sc_r\right) \times \Pr(Sc_r) \tag{26}$$

3.4 Inventory holding and set up costs

According to the classical *EPQ* model, the expected inventory holding cost and the set up cost are given by:

$$E(C_I) = \frac{D \times A}{p' \times E(T)} + \frac{B\left(p' - D_d\right) \times E(T)}{2} \tag{27}$$

where that p' is the production rate, D_d demand rates, B the inventory holding cost per unit per time unit, and the set up cost.

Finally, the expected total cost (ETC) and the economic production quantity (EPQ) can be attained as equations (28) and (29).

$$ETC = E(C_I) + E(C_Q) + E(C_{Ins}) + E(C_M) \tag{28}$$

$$Q' = p' \times E(T) \tag{29}$$

3.5 Mathematical modelling

An integrated model of statistical process monitoring, economic production quantity, and maintenance policy can be described as that the economic cost function is minimised subject to the constrained minimum value ARL_1 as well as the maximum value of ARL_0 in the production run length. The objective function with regard to the costs that were explained in the previous paragraphs can be formulated as the following:

$$Min\ ETC \tag{30}$$

$$Subject\ to:$$

$$ARL_0 > ARL_1 \tag{30.1}$$

$$ARL_1 > ARL_u \tag{30.2}$$

$$1 \le n \le n_{\max},\ \ h_{\min} \le h \le h_{\max}, K \ge K_L \tag{30.3}$$

$$n \in Z^+, K \in Z^+ \tag{30.4}$$

As aforementioned in equation (30), the objective function is equal to minimise ETC that displays the total expected cost per production cycle time. Also, the necessary constraints (30.1)–(30.4) should be incurred in the mathematical programming to make the model more adapted to real industry situation. These constraints are as follows:

1. To decrease the number of false alarms per time unit without influencing the performance of control chart, ARL_0 must be bigger than a pre-determined value of ARL_l, as illustrated in equation (30.1).
2. In order to increase the power of control chart to detect shift in the process, ARL_1 must be lower than pre-determined value of ARL_u as shown in equation (30.2).
3. Typically, in the practical conditions because of economic reasons, the inspected sample size and the inspection interval must be limited between two certain upper and lower limits as shown in equation (30.3). Also, this constraint guarantees that the process continued, according to the number of inspection intervals in a perfect cycle must be greater than K_L.
4. The equation (30.4) ensures that the inspected sample size and the number of inspection in a perfect cycle time to be a positive integer value.

4 Solution approach

Since the mathematical model is non-linear and includes both continuous and discrete decision variables, it falls under the category of NP-hard problems, which is difficult to solve using exact algorithms.

Evolutionary algorithms (EAs) are very suitable to optimally solve problems where there exists no known exact algorithm that can produce the outcome in polynomial time. EAs are meta-heuristic approaches to solving optimisation problems by imitating the biological nature of evolution such as genetic algorithm (GA), particle swarm optimisation (PSO), and, etc. Because the integrated models of the production process, maintenance policy, and control chart design contains complicated optimisation models, various heuristic and meta-heuristic algorithms have been suggested in the literature to obtain near-optimal solutions. For example, Safaei et al. (2012) designed an X-bar control chart using Taguchi loss function with an economic-statistical approach using multi-objective GA. Saghaei et al. (2014) applied GA for designed economic EWMA control chart according to evaluation error. Niaki et al. (2012) used a PSO for optimising the models of both the economic and the economic-statistical design problems of MEWMA control charts. Also, Zhang et al. (2015), Liu et al. (2017), Salmasnia et al. (2018), and Chalabi et al. (2016) applied of the PSO algorithm in their studies.

Particle swarm optimisation (PSO) is used to optimise the suggested model due to features of PSO algorithm that can be summarised as follows:

1 To use the performance index for search in the problem space lead to that it get suitable dealing with non-differentiable objective functions.

2 Because of it uses probabilistic transition rules, this algorithm has high flexibility and good capability in search a compacted and uncertain area.

3 One of the unique features of PSO is the balance between the global and local exploration ability of the search space that leads to dominating the premature convergence and increase the search capability.

4 The solution quality of PSO algorithm does not depend on the initial population. When it starts anywhere in the search space, the algorithm still ensures the convergence to the optimal solution.

5 It has good performance in optimising non-linear mathematical programming models.

Also, PSO has been extensively used in various optimisation problems due to unique features that mentioned in the previous paragraph. Barzinpour et al. (2013) displayed that the particle swarm can lead the simplex-based Nelder–Mead algorithm to better results.

4.1 Particle swarm optimisation

The PSO algorithm has been based on the metaphor of the social behaviour of birds and fish procession that firstly proposed by Kennedy and Eberhart (1995). This algorithm employs collaboration among a population called particles, to find optimum decision variable values particle moving within a multidimensional space. Unlike other heuristic methods, PSO has increasingly engaged a flexible and well-balanced mechanism which to enhance global and local exploration abilities. Generation of initial PSO is generated with a population of random particle with random positions and velocities inside the problem space.

This algorithm subsequently searches for an optimal solution by updating consecutive particles based on the force of inertia and the two ''best'' values. The first value is the best value experienced by the i th particle which is called personal best (*pbest*). Another ''best'' value is best solution observed so far which is called global best (*gbest*). It means that, in this iterative process, the behaviour of a particle is a compromise among three possible alternatives:

1 following its current velocity

2 going towards its personal best

3 going towards the global best.

In each iteration, after finding the two best values, the particle will update its velocity and position.

4.1.1 PSO Notations:

In this section, as can be observed in Table 3, the notation used to explain PSO algorithm is shown.

Table 3 PSO notations

PSO notations	*Description*
N	The number of particle in the swarm
$x_i^t = [n,h,k,K]$	Position decision variables of the i th particle in iteration t
$pbest_i^{t-1}$	Personal best of the i th particle in iteration t
$gbest^t$	The best solution founded until iteration t
$b_{lo}\,(b_{up})$	The lower (upper) boundary for decision variables
v_i^t	Velocity of the i *th* particle in iteration t
c_1, c_2	Cognition and social learning factors
w	Inertia weight
w_{damp}	Fixed factor less than 1

4.1.2 Velocity update

According to pbest and gbest, the i *th* particle velocity with respect to the t *th* iteration is updated by the following equation:

$$v_i^t = w.v_i^{t-1} + c_1 r_p \left(pbest_i^{t-1} - x_i^{t-1} \right) + c_1 r_g \left(gbest_i^{t-1} - x_i^{t-1} \right) \tag{31}$$

where r_p and r_g are uniformly random numbers selected from the interval [0,1]. The inertia weight's value (w) controls the impact velocity and changes in each iteration. Its value in the first iteration is a predetermined value between 0 and 1, then in each iteration, it is decreased to $w * w_{damp}$. Also, the summation of c_1 and c_2 values with regard to Kennedy et al. (2001) is generally investigated equal to 4.

4.1.3 Position update

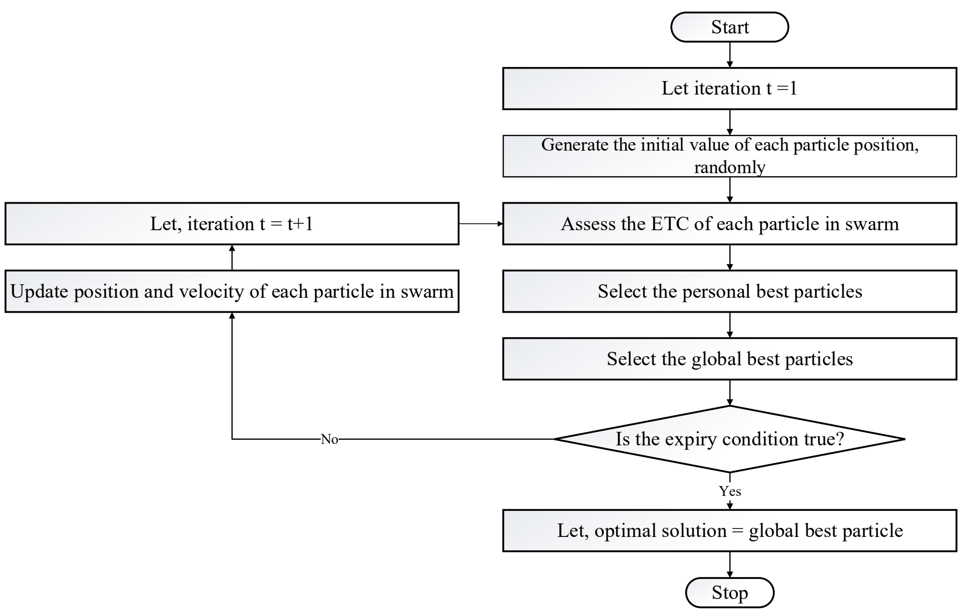
Actually, the initial value of continuous variables are generated randomly from a uniform distribution: $x_i \sim U(b_{lo}, b_{up})$. The particle position updating is formulated based on the value of velocity updating and particle position in the previous iteration. It can be obtained as following:

$$x_i^t = x_i^{t-1} + v_i^t \tag{32}$$

4.1.4 Termination condition

The searching is a repeat process until conditions to terminate is met. Generally, there are two stop criteria for PSO algorithm that one of the conditions is the maximum iteration number reached and the other is a solution with the smallest fitness function value found. Figure 5 is illustrated a Summary of the computational method in the PSO algorithm.

Figure 5 Complete computational procedure of the PSO algorithm



Source: Dos Santos Coelho (2009)

4.2 Application of PSO in the proposed model

The solution representation for the presented model consists four-dimensional particles that each dimension refers to a certain decision variable. In the presented model, the inspected sample size and the number of inspection in a perfect cycle time (*n,K*) are an integer, while the other decision variables (*h,k*) are real numbers. As mentioned earlier, to generate the initial value of each continuous decision variable, a uniformly distributed random value is produced between the lower and upper limits of the considered decision variable. Therefore, to generate an initial value for discrete decision variables, a random value from a uniform distribution in the interval [0, 1] is generated. The values of discrete, i.e., the sample size (n) and the number of inspection in a perfect cycle time (K), decision variables in the proposed model are computed based on equations (33)–(34)

$$n = \min\left(n_{\min} + \left\lfloor (n_{\max} - n_{\min} + 1) \times R_1 \right\rfloor, n_{\max}\right) \tag{33}$$

$$K = \min\left(K_{\min} + \left\lfloor (K_{\max} - K_{\min} + 1)) \times R_2 \right\rfloor, K_{\max}\right) \tag{34}$$

where $(n_{\min}, K_{\min})$ and $(n_{\max}, K_{\min})$ are the lower and upper limits of (n,K), respectively. Furthermore, R_1 and R_2 are two random number that follow the uniform distribution with R_1R_2~$U(0,1)$.

Table 4 The values of the parameters in the numerical example

Parameter	p'	d'	d	D	A	B	λ
Value	100	80	1	10,000	80	10	0.01
Parameter	c_0	c_1	c_y	c_f	c_v	c_{pm}	c_{rm}
Value	115	950	200	1	0.2	2,400	5,000
Parameter	p	ARL_l	ARL_u	h_{min}	h_{max}	n_{max}	K_l
Value	3	100	10	0.01	0.6	20	40

Table 5 The optimal results of the case study

Decision variables				*EPQ*	*Production run length*	*Objective function*
n*	h*	k*	K*	Q*	ET*	ETC*
11	0.15	4.50	25	390	3.84	5785.16

5 Experimental results

To illustrate the applicability and credibility of the presented optimisation model an industrial example from Pan et al. (2012) is used. Firstly to solve and to optimise this case study the PSO algorithm is run. Then, a comparative study is conducted between the integrated model and an mathematical programming in which the decision variables related to the three concepts of production planning, maintenance policy and statistical process monitoring are optimised, separately. The results of two models for 27 instances that are generated by a Taguchi experimental design are compared.

Table 6 The generated instances with the Taguchi L27 design

Instance	*The process parameters*									
	p	λ	d	c_0	c_1	c_y	c_f	c_v	c_{pm}	c_{rm}
1	3	0.01	0.5	50	100	50	0.5	0.1	1,000	2,500
2	3	0.01	0.5	50	900	300	2	0.5	2,500	5,000
3	3	0.01	0.5	50	1,500	500	4	1	4,000	7,500
4	3	0.03	1	400	100	50	0.5	0.5	2,500	5,000
5	3	0.03	1	400	900	300	2	1	4,000	7,500
6	3	0.03	1	400	1,500	500	4	0.1	1,000	2,500
7	3	0.05	2	700	100	50	0.5	1	4,000	7,500
8	3	0.05	2	700	900	300	2	0.1	1,000	2,500
9	3	0.05	2	700	1,500	500	4	2	2,500	5,000
10	4	0.01	1	700	100	300	4	0.5	2,500	7,500
11	4	0.01	1	700	900	500	0.5	0.5	4,000	2,500
12	4	0.01	1	700	1,500	50	2	1	1,000	5,000
13	4	0.03	2	50	100	300	4	0.5	4,000	2,500
14	4	0.03	2	50	900	500	0.5	1	1,000	5,000
15	4	0.03	2	50	1,500	50	2	0.1	2,500	7,500
16	4	0.05	0.5	400	100	300	4	1	1,000	5,000
17	4	0.05	0.5	400	900	500	0.5	0.1	2,500	7,500
18	4	0.05	0.5	400	1,500	50	2	0.5	4,000	2,500
19	5	0.01	2	400	100	500	2	0.1	4,000	5,000
20	5	0.01	2	400	900	50	4	0.5	1,000	7,500
21	5	0.01	2	400	1,500	300	0.5	1	2,500	2,500
22	5	0.03	0.5	700	100	500	2	0.5	1,000	7,500
23	5	0.03	0.5	700	900	50	4	1	2,500	2,500
24	5	0.03	0.5	700	1,500	300	0.5	0.1	4,000	5,000
25	5	0.05	1	50	100	500	2	1	2,500	2,500
26	5	0.05	1	50	900	50	4	0.1	4,000	5,000
27	5	0.05	1	50	1,500	300	0.5	0.5	1,000	7,500

Table 7 The ETC for the proposed model (A) and the optimisation model of three concepts separately (B)

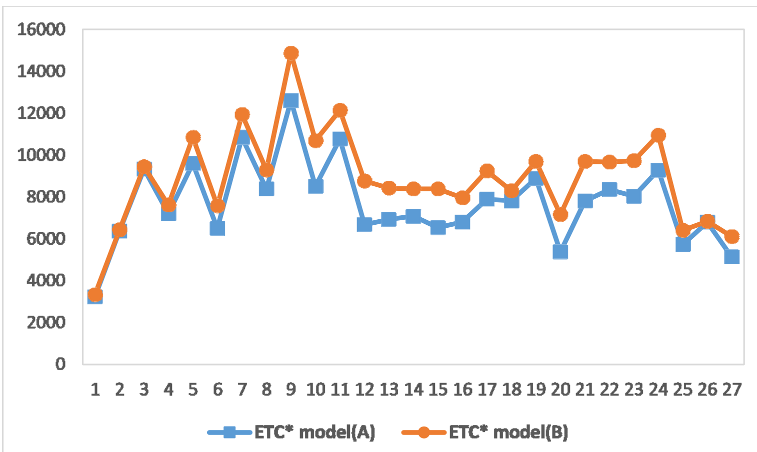
Instance	*Decision variables*				*Objective function*		*Cost saving %*
	*n**	*h**	*k**	*K**	*ETC* model (A)*	*ETC* model (B)*	
1	3	0.18	5	28	3,215	3,331	3
2	6	0.26	5	37	6,365	6,423	1
3	18	0.34	5	28	9,324	9,442	1
4	11	0.06	5	39	7,193	7,609	5
5	5	0.07	5	32	9,596	10,843	12
6	16	0.10	5	27	6,492	7,570	14
7	3	0.19	5	33	10,841	11,935	9
8	13	0.18	5	30	8,372	9,282	10
9	12	0.04	5	29	12,597	14,865	15
10	9	0.09	4.67	32	8,494	10,687	21
11	3	0.18	4.91	31	10,759	12,137	11
12	12	0.08	4.57	28	6,667	8,760	24
13	6	0.28	4.45	30	6,918	8,418	18
14	14	0.12	4.46	37	7,075	8,381	16
15	18	0.11	4.07	38	6,532	8,379	22
16	7	0.11	4.62	35	6,798	7,951	14
17	6	0.12	4.91	30	7,884	9,240	15
18	9	0.12	4.49	29	7,813	8,287	6
19	7	0.09	4.12	31	8,882	9,691	8
20	9	0.11	4.82	30	5,374	7,152	25
21	8	0.12	3.93	33	7,802	9,691	19
22	14	0.12	4.14	34	8,353	9,661	14
23	9	0.10	3.55	31	8,026	9,723	17
24	8	0.11	4.73	33	9,277	10,948	15
25	2	0.25	3.79	30	5,716	6,403	11
26	5	0.16	4.77	31	6,786	6,830	1
27	4	0.11	4.77	29	5,128	6,100	16
Average							13

5.1 Case study

The case study is about a company that sells a determined food product to a wholesaler in packages marked with the specific weight, volume, and length as the quality characteristics. This modified example is taken from Pan et al. (2012) to display the applicability of the integrated model. It is supposed that the shift in three mentioned characteristics affect the product quality. The occurrence of assignable cause leads to change only in the mean of at least one of the quality characteristics. As mentioned before, T^2 Hotelling chart is employed for monitoring the process. Also, the maintenance is implemented at the end of the production cycle according to the condition of the process. The values of the main parameters related to the production system are recorded in Table 4.

With regard to the complexity of the model and the robust performance of the PSO algorithm to find the optimal solution in such models, this algorithm is implemented to solve the model in MATLAB software. The number of iterations and population size are obtained 100 and 50, respectively by the trial-and-error process. The optimal results of the case study consist of decision variables, EPQ, production run length, and ETC are recorded in Table 5.

Figure 6 The graphical representation related to the obtained results of comparing study (see online version for colours)



5.2 Comparative study

In this section, to illustrate the efficiency of the integrated model, it is compared with a model in which decision variables related to the three concepts are separately obtained called hereafter model *B*. For this purpose, firstly the optimal values of the control chart parameters consisting of (n, h, k) are obtained by minimising the summation of quality loss and inspection costs subject to the statistical constraints. The obtained values are equal to $n = 13$, $h = 0.21$, $k = 3.95$. Then, according to these values, the cycle run length is calculated $T = 6.1$. Afterward, the number of sampling in a perfect cycle time is computed $K = 31$. Finally, the result of the integrated model, hereafter called model *A*, and model *B* are compared in terms of the expected cost per time unit.

To enhance the validity of the presented comparison, 27 instances generated using design of experiment method as shown in Table 6. In these instances, the values of the other parameters are considered according to Table 4. Eventually, the ETC values in both above-mentioned models are given in Table 7.

According to the obtained results in Table 7, ETC per time unit in all instances is reduced between 1% to 25% with an average of 13% by the presented model, which is remarkable value in cost-saving. So, to clarify the difference between two above-mentioned models from point-of-view of cost, the values of the expected total cost in all instances are illustrated graphically in Figure 6.

6 Conclusions

This study developed a joint model by integrating three issues of manufacturing cycle length, maintenance policy, and the control chart design. To monitor several correlated quality characteristics was employed the T^2-Hotelling chart. Moreover, the suggested mathematical programming not only investigated economic considerations in the process optimisation but also applied statistical criteria in optimising the control chart parameters. With respect to the model is nonlinear and the solution space is non-convex, the PSO algorithm was employed to compute the optimal values of process variables in a way that the expected total cost per production cycle is minimised. Eventually, to show the importance of integration of statistical process monitoring, maintenance policy and production planning, the presented model, a comparative study between integrated model and a model in which the decision variables related to each of the three concepts obtain separately on twenty-seven different instances was performed. The outcome of two models was compared in terms of the expected total cost per time unit. The results confirmed that the integrated model has better performance in cost savings.

As a suggestion for future research, a similar study can be conducted on more sophisticated production systems such as an inventory system with shortage allowed. Moreover, a process with multiple assignable causes investigates, which leads to the model more adapted to real manufacturing environments.

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