Design of EWMA control chart for monitoring transformed Rayleigh distributed data

Olatunde Adebayo Adeoti

Department of Statistics,
Federal University of Technology,
Akure, Nigeria
Email: oaadeoti@futa.edu.ng
Email: tim_deot@yahoo.com

Abstract: Monitoring statistical process for the detection of assignable causes of variation is based on the assumption that the process characteristic follows the normal distribution. But, in practice, this is often not the case as process characteristic seldom follows the non-normal distribution. This paper designs a new control chart to monitor quality characteristic that follow the non-normal distribution. The proposed control chart based on the EWMA statistic is constructed after transforming the Rayleigh distributed data to approximate normal using the power transformation method. The ARL and SDRL values of the proposed control chart are evaluated for different shift sizes. The performance of the proposed chart is compared with the recent CUSUM chart for transformed Rayleigh distributed data. The study shows that the proposed chart outperforms the recent CUSUM control chart for transformed Rayleigh data. Real-life and simulated dataset to illustrate the design and applications of the proposed control chart is given.

Keywords: control chart; transformed Rayleigh data; exponentially weighted moving average; EWMA; average run length; ARL; power transformation.

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Biographical notes: Olatunde Adebayo Adeoti obtained his BSc degree (First Class Honours) in Mathematics from the University of Ilorin, MSc in Statistics from the University of Lagos and PhD in Statistics from the University of Ilorin with specialisation in quality control and management. He is a member of the Nigerian Statistical Association, American Society for Quality and the European Network for Business and Industrial Statistics. He is a recipient of the postdoctoral fellowship of the University of South Africa. His research interests include statistical process control, artificial neural network in statistical process control and statistical quality control and management in healthcare. Currently, he is an Associate Professor at the Department of Statistics, Federal University of Technology, Akure, Nigeria.

1 Introduction

The control chart is the main tool of statistical process control (SPC) that is used to monitor the parameters of a process. The application of control chart is not limited to monitoring production process only, but have been applied in many other applications including healthcare (Woodall, 2006) and engineering (Hwang et al., 2008). Three popular control charts have been developed so far with several modifications and enhancement to them. These are Shewhart (Shewhart, 1931), exponentially weighted moving average (EWMA) (Roberts, 1959) and cumulative sum (CUSUM) (Page, 1954) control charts. The Shewhart chart is user-friendly and efficient for detecting large process shifts, but, inefficient for detecting small-to-moderate shifts in the process parameters (Montgomery, 2013). On the other hand, the EWMA and the CUSUM control charts are efficient for detecting small-to-moderate shifts in the process, but the EWMA chart is preferred over the CUSUM chart by most researchers because its application is straight forward and easy to operate and understand by quality control personnel (Graham et al., 2011; Saghir et al., 2019).

The EWMA control chart is design by assigning more weight to current observation and exponentially smaller weights are assigned to the previous observations. The EWMA control chart has attracted a lot of interest from researchers who have developed several procedures to increase its efficiency of quickly detecting small-to-moderate shifts in process parameters for different situations. Readers are referred to the paper of Jiang et al. (2005), Khan et al. (2016), Aslam et al. (2017) and Abbasi et al. (2018).

Usually control charts are developed based on the assumption that the process quality characteristic follows the normal distribution. However, in practice, some processes generate data that are best represented by non-normal distributions because the quality characteristic deviates from the normal distributional assumption. Hence, control charts for monitoring asymmetric distributed process are required. Many researchers have examined and developed control charts in the SPC literature to handle situations where the quality characteristic is unknown or violates the assumption of normality. Lucas (1985) and Vardeman and Ray (1985) developed the exponential CUSUM chart, Gan (1998) studied the exponential EWMA control chart, Chan et al. (2000) developed the cumulative quality control (CQC) chart, Liu et al. (2007) studied EWMA control chart with transformed exponential data, Derya and Canan (2012) studied control charts for Weibull, gamma and log-normal distributions, Alkahtani (2013) examined the robustness of the EWMA control chart to non-normality, Abbasi et al. (2015) proposed EWMA control chart for monitoring process dispersion under normal and non-normal processes, Akhundjanov and Pascual (2015) proposed the moving range EWMA control charts for Weibull shape parameter, Tyagi and Singh (2016) proposed the transformed Rayleigh CUSUM chart, Aslam (2016) proposed a mixed EWMA-CUSUM control chart for Weibull distributed quality characteristic, Khan et al. (2017) proposed control chart for gamma distributed variable using repetitive sampling, Saghir et al. (2019) proposed the modified EWMA control chart for transformed gamma distribution, and Alevizakos and Koukouvinos (2019) proposed the DEWMA control chart based on the gamma distribution.

For a skewed distributed quality characteristic, the Rayleigh distribution (a particular case of the Weibull distribution) is best-fitted to describe correctly the coaxial defects in mechanics of the manufacturing of cylindrical pieces for automobile where exact characteristics are mandatory (Tyagi and Singh, 2016). The usefulness of the Rayleigh distribution in reliability engineering for modelling lifetime failure of product, manufacturing of electronic devices and physiological sensing systems has been explored by many researchers. Mutlu (2014) used the Rayleigh distribution for fitting signal voltage data from receivers. Regular respiratory signals are modelled using Rayleigh distribution by Li and Li (2015). Raza et al. (2016) designed control chart under type I censoring for Rayleigh distribution. Hossain et al. (2020) designed a Shewhart control chart for monitoring scale parameter of a Rayleigh distributed process. In these papers, the use of Rayleigh random variable as a monitoring statistic have shown the effectiveness of the Rayleigh distribution in reliability engineering for non-normal processes. However, the transformation method can help to solve other problems as pointed out by Chen et al. (2005) and Castagliola and Tsung (2005). Moreover, monitoring process using EWMA chart is beneficial for transformation scheme because of its robustness to non-normality (Borror et al., 1999; Maravelakis et al., 2005). In fact, Montgomery (2007) highlighted the importance of monitoring non-normal data by transformation method and stated that "in many cases the CUSUM and EWMA control charts would be better alternatives because these charts are more effective in detecting smaller shifts in process mean".

Though, a lot of research has been done to investigate the performance of EWMA control chart for monitoring non-normal distribution processes. The study on the performance of EWMA control chart for monitoring process parameter using the transformed Rayleigh distribution has not been investigated in the SPC literature to the best of author's knowledge.

In this paper, the power transformation method is used to transform the Rayleigh distribution with known scale parameter into normal distributed data, and then the EWMA control chart is applied to monitor shifts in process parameters after transformation.

The rest of this article is organised as follows: the Rayleigh distribution and design of EWMA control chart for monitoring the Rayleigh distributed data is given in Section 2. The performance of the proposed control chart is evaluated based on the average run length (ARL) and standard deviation of the run length (SDRL) computed by Monte Carlo simulation technique, and compared with the recent CUSUM control chart based on the transformed Rayleigh distributed data and the non-transformed Rayleigh data in Section 3. Illustrative examples are presented in Section 4 to demonstrate the application of the proposed control chart. Section 5 presents the concluding remarks.

2 Description of the Rayleigh distribution and transformation method

In this section, the Rayleigh distribution is presented as well as the power transformation method. Also, the design of the EWMA control chart based on the Rayleigh distribution is given.

2.1 The Rayleigh distribution

The Rayleigh distribution is a particular case of the Weibull distribution when its shape parameter takes value 2. The cumulative density function (CDF) of the Rayleigh distribution is given by

$$F(t, \lambda) = 1 - e^{-\left(t^2/2\lambda^2\right)} \tag{1}$$

and the probability density function (PDF) of the Rayleigh distribution is given by

$$f(t, \lambda) = \frac{t}{\lambda^2} e^{-\left(t^2/2\lambda^2\right)} \qquad x > 0, \lambda > 0 \tag{2}$$

where λ is the scale parameter. The mean and variance of the Rayleigh random variable is given by

$$\mu = \left(\lambda/2\right)\Gamma\left(1/2\right) \tag{3}$$

and

$$\sigma^2 = \lambda^2\left[\Gamma(2) - \left(\Gamma\left(\frac{3}{2}\right)\right)^2\right] \tag{4}$$

where $\Gamma(\cdot)$is the gamma function.

2.2 Transformation method of proposed study

The Rayleigh distribution is first transformed into an approximate normal distribution before it can be monitored by the EWMA control chart. In this study, the power transformation of the Weibull distribution where the shape parameter $\beta = 2$ gives a Rayleigh distribution is used. The power transformation is described by Tyagi and Singh (2016) as follows:

Let X follows the Rayleigh distribution which is a special case of the Weibull distribution with scale and shape parameters λ and $\beta = 2$ respectively, i.e., $X \sim W(\lambda, \beta)$. The transformed random variable $Y = X^p$ is an approximate normal random variable such that $Y \sim W(\lambda^p, 2/p)$ where $p = 0.5555$ give an approximate normal distribution (Nelson, 1994). Therefore, we say that the transformed Rayleigh random variable Y is approximately normal if $Y \sim W(\lambda^{0.5555}, 3.6)$.

The mean and variance of the transformed Rayleigh random variable $Y = X^p$ is given as

$$\mu_Y = E(Y) = \lambda^{0.5555}\Gamma\left(1 + \frac{1}{3.6}\right) = 0.090115057\lambda^{0.5555} \tag{5}$$

$$\sigma_Y = \lambda^{0.5555}\sqrt{\left[\Gamma\left(1 + \frac{2}{3.6}\right) - \left(\Gamma\left(1 + \frac{1}{3.6}\right)\right)^2\right]} = 0.2780203\lambda^{0.5555} \tag{6}$$

2.3 Design of the EWMA chart with transformed Rayleigh data

The design of the EWMA control chart for the transformed Rayleigh Data is described in this section.

Using the power transformation Nelson (1994) described by Tyagi and Singh (2016) that $Y = X^p$ follows the approximate normal distribution with mean μ_Y and standard deviation σ_Y. The main procedures for setting up an EWMA control chart with transformed Rayleigh data are given as follows:

Step 1 The power transformation method is used to transform the Rayleigh data to approximately normal distributed data Y_i using $y = x^{0.5555}$.

Step 2 Compute the two-sided EWMA statistic

$$Z_i = \gamma Y_i + (1-\gamma) Z_{i-1} \tag{7}$$

where Y_i is the transformed Rayleigh data, $0 < \gamma \leq 1$ is the smoothing constant. The starting value is the target value μ_0, i.e., $Z_0 = \mu_Y$. The time-varying upper and lower control limits and the centreline of the proposed chart can be computed as

$$UCL = \mu_Y + L\sigma_Y \sqrt{\frac{\gamma}{2-\gamma}\left\{1-(1-\gamma)^{2i}\right\}} \tag{8a}$$

$$CL = \mu_Y$$

$$LCL = \mu_Y - L\sigma_Y \sqrt{\frac{\gamma}{2-\gamma}\left\{1-(1-\gamma)^{2i}\right\}} \tag{8b}$$

where L is the control limit coefficient which is determined to achieved a specified in-control average run length (IC ARL) value, μ_Y and σ_Y can be estimated from the transformed Rayleigh data with

$$\hat{\mu}_Y = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \text{ and } \hat{\sigma}_Y = \sqrt{\frac{1}{n-1}\left\{\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2\right\}} \tag{9}$$

The process is declared as out of control whenever $Z_i > UCL$ or $Z_i < LCL$; otherwise the process is declared as in-control.

However, when X follows the Rayleigh distribution with scale parameter λ and shape $\beta = 2$, the mean and variance of the plotting statistic of the transformed variable $Y = X^p$ is given as

$$\mu_{Z_i} = E\left(Z_i\right) = \lambda^{0.5555}\Gamma\left(1+\frac{1}{3.6}\right) \tag{10}$$

and

$$\sigma_{Z_i}^2 = Var\left(Z_i\right)$$
$$= \frac{\gamma}{2-\gamma}\left\{1-(1-\gamma)^{2i}\right\}\left[\left(\lambda^{0.5555}\right)^2\left[\Gamma\left(1+\frac{2}{3.6}\right)-\left(\Gamma\left(1+\frac{1}{3.6}\right)\right)^2\right]\right] \tag{11}$$

Therefore, the time varying control chart limits of the proposed control chart are given as

$$UCL_{\gamma} = \mu_{Z_i} + L\sqrt{\frac{\gamma}{2-\gamma}\left\{1-(1-\gamma)^{2i}\right\}\left[(\lambda^{0.5555})^2\left[\Gamma\left(1+\frac{2}{3.6}\right)-\left(\Gamma\left(1+\frac{1}{3.6}\right)\right)^2\right]\right]} \tag{12a}$$

$$CL = \mu_{Z_i}$$

$$LCL_{\gamma} = \mu_{Z_i} - L\sqrt{\frac{\gamma}{2-\gamma}\left\{1-(1-\gamma)^{2i}\right\}\left[(\lambda^{0.5555})^2\left[\Gamma\left(1+\frac{2}{3.6}\right)-\left(\Gamma\left(1+\frac{1}{3.6}\right)\right)^2\right]\right]} \tag{12b}$$

where γ is the sensitivity with the range $0 < \gamma \leq 1$ and the control limit coefficient L is a constant specified to give an in-control average run length (IC ARL). The EWMA control chart depends on L and γ. For large values of sample i, the control chart become

$$UCL_{\gamma} = \mu_{Z_i} + L\lambda^{0.5555}\sqrt{\frac{\gamma}{2-\gamma}\left[\Gamma\left(1+\frac{2}{3.6}\right)-\left(\Gamma\left(1+\frac{1}{3.6}\right)\right)^2\right]} \tag{13a}$$

$$CL = \mu_{Z_i}$$

$$LCL_{\gamma} = \mu_{Z_i} - L\lambda^{0.5555}\sqrt{\frac{\gamma}{2-\gamma}\left[\Gamma\left(1+\frac{2}{3.6}\right)-\left(\Gamma\left(1+\frac{1}{3.6}\right)\right)^2\right]} \tag{13b}$$

The process is declared as out-of-control if $Z_i > UCL_{\gamma}$ or $Z_i < LCL_{\gamma}$; otherwise the process is declared as in-control.

3 Performance evaluation of proposed chart

The statistical performance of the proposed EWMA control chart for Rayleigh distribution is measured by the average run length (ARL). The ARL is the expected number of samples required before an out-of-control (OOC) condition is observed (Montgomery, 2013). For an in-control (IC) state, a large IC ARL value is preferred while for out-of-control (OOC) state a small ARL value is required. A common IC ARL value is recommended to compare the performance of two or more control charts. A chart with lower OOC ARL indicates a superior control chart to detect process shifts. In order to calculate the ARL of the EWMA control chart, a Monte Carlo simulation procedure is performed in R program. The R program is given in simulation algorithm as follows:

1 Specify the design parameters (L, γ) and the number of simulations.

2 Generate a random sample from the Rayleigh distribution with parameters $\beta = 2$ and $\gamma = 1$ and transform to normal data using the transformation $Y = X^{0.555}$.

3 Compute the EWMA statistic for the transformed data using equation (7). Then calculate the control limits using equation (8).

4 Count the number of samples until an out-of-control process signal is observed. Check if $Z_i > UCL$ or $Z_i < LCL$. Record the number of samples that is OOC as one run length; otherwise go to step 2.

5 Repeat the steps 2–5 (10,000 times) and compute the ARL given by $ARL = \frac{1}{n}\sum_{i=1}^{n} RL_i$, where RL_i is the run length distribution.

Here, we assume that the scale parameter of the Rayleigh distribution is shifted from λ to $\delta\lambda$ where $\delta \geq 1$ is a constant. When $\delta = 1$ the process is said to be IC. Otherwise, it is declared to be OOC. The width of the control limits L and sensitivity parameter γ is selected to achieve an IC ARL value of 200, 370 and 500.

3.1 Determination of control limit constant

The values of constant L required for the optimal EWMA control chart for Rayleigh random variable for $\gamma = 0.10, 0.20, 0.25, 0.5, 0.75$ to satisfy $ARL_0 = 200$, 370 and 500 is presented in Table 1. These values are important in the design of EWMA control chart for the transformed Rayleigh distributed data. The result of Table 1 reveals that

1 L decreases as the values of γ increases for a specified ARL_0 value except for $\gamma = 0.75$. For example when $ARL_0 = 200$, the value of $L = 3.334$ for $\gamma = 0.25$.

2 For a fixed γ the constant L increases as the specified ARL_0 increases from 200 to 500. For example when $\gamma = 0.25$ then $L = 3.243$ if $ARL_0 = 200$, $L = 3.334$ if $ARL_0 = 370$, $L = 3.373$ if $ARL_0 = 500$.

Table 1 Control limit constant for $ARL_0 = 200$, 370 and 500

ARL_0	γ				
	0.10	0.20	0.25	0.5	0.75
200	4.299	3.443	3.243	2.868	2.891
370	4.411	3.537	3.334	2.956	2.988
500	4.456	3.577	3.373	2.997	3.032

3.2 Out-of-control performance

In order to study the OOC performance of the proposed control chart, the ARL_1 value is used for efficient and quick detection of process shifts. For an efficient control chart, it is expected that the ARL_1 value be small for different shift sizes. In this study, the ARL_1 values of the proposed control chart are obtained for $\gamma = 0.1, 0.2, 0.25, 0.5, 0.75$. Here, we use the shift sizes $\delta = 1.0$, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.5, 3.0 based on the Monte Carlo simulation procedure in Section 3 with 10,000 iterations. The results of the OOC performance of the proposed control chart are presented in Tables 2–4. The optimal EWMA control chart with transformed Rayleigh data for detecting specific shifts for the sensitivity parameters γ can be found in Tables 2–4.

Table 2 The ARL values of EWMA chart with transformed Rayleigh data when $ARL_0 = 200$

Shift δ	γ = 0.10		0.20		0.25		0.50		0.75	
	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.0	200.69	168.15	200.22	187.58	199.85	188.44	199.85	191.99	200.85	199.29
1.1	46.41	23.06	38.61	27.29	37.54	28.50	38.07	34.53	40.86	38.96
1.2	26.83	9.11	19.05	9.81	17.44	10.12	15.10	11.83	15.42	13.65
1.3	19.71	5.28	12.80	5.27	11.39	5.32	8.81	6.00	8.42	6.86
1.4	15.90	3.77	9.89	3.45	8.67	3.47	6.17	3.64	5.52	4.11
1.5	13.47	2.98	8.21	2.64	7.09	2.56	4.85	2.61	4.10	2.84
1.6	11.73	2.43	7.06	2.09	6.05	2.04	4.01	2.00	3.27	2.11
1.7	10.46	2.11	6.24	1.81	5.33	1.71	3.45	1.63	2.73	1.70
1.8	9.46	1.89	5.59	1.57	4.78	1.50	3.06	1.38	2.37	1.44
1.9	8.66	1.71	5.11	1.41	4.36	1.34	2.76	1.21	2.10	1.22
2.0	7.99	1.55	4.71	1.29	4.01	1.21	2.51	1.09	1.92	1.07
2.5	5.84	1.16	3.44	0.93	2.95	0.87	1.81	0.78	1.42	0.67
3.0	4.67	0.96	2.78	0.76	2.39	0.71	1.48	0.62	1.24	0.49

Table 3 The ARL values of EWMA chart with transformed Rayleigh data when $ARL_0 = 370$

Shift δ	γ = 0.10		0.20		0.25		0.50		0.75	
	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.0	370.63	338.25	371.22	355.51	369.28	359.09	369.37	364.56	371.12	363.47
1.1	57.58	32.63	51.71	39.45	51.63	41.82	55.76	51.51	63.03	60.91
1.2	30.01	11.08	22.20	12.26	20.75	12.79	19.24	15.80	20.56	18.72
1.3	21.28	6.00	14.15	6.11	12.76	6.31	10.30	7.28	10.27	8.71
1.4	16.91	4.11	10.69	3.93	9.41	3.95	6.95	4.26	6.45	4.99
1.5	14.18	3.14	8.74	2.90	7.59	2.83	5.29	2.94	4.59	3.26
1.6	12.30	2.59	7.45	2.27	6.42	2.21	4.32	2.20	3.60	2.40
1.7	10.93	2.22	6.54	1.89	5.61	1.86	3.69	1.79	2.99	1.88
1.8	9.86	1.98	5.86	1.67	5.02	1.58	3.24	1.49	2.54	1.56
1.9	9.01	1.79	5.33	1.48	4.55	1.41	2.91	1.28	2.24	1.34
2.0	8.29	1.63	4.90	1.34	4.18	1.26	2.64	1.15	2.02	1.16
2.5	6.03	1.20	3.56	0.96	3.04	0.89	1.87	0.81	1.47	0.71
3.0	4.81	0.98	2.86	0.79	2.46	0.72	1.53	0.65	1.26	0.52

Table 4 The ARL values of EWMA chart with transformed Rayleigh data when $ARL_0 = 500$

Shift δ	*γ* 0.10		0.20		0.25		0.50		0.75	
	ARL	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*
1.0	501.13	466.17	500.87	486.24	500.04	486.40	500.24	497.87	501.08	498.64
1.1	63.47	38.09	59.44	46.57	60.05	50.27	67.59	62.87	78.17	75.66
1.2	31.46	11.99	23.74	13.53	22.47	14.36	21.70	18.14	24.01	22.39
1.3	22.02	6.42	14.83	6.53	13.41	6.80	11.13	8.05	11.28	9.64
1.4	17.31	4.27	11.04	4.08	9.78	4.20	7.35	4.63	6.95	5.44
1.5	14.46	3.23	8.96	2.99	7.81	2.95	5.53	3.12	4.86	3.50
1.6	12.54	2.64	7.62	2.35	6.58	2.28	4.49	2.34	3.79	2.57
1.7	11.12	2.27	6.67	1.95	5.74	1.91	3.79	1.85	3.09	1.97
1.8	10.01	2.01	5.97	1.69	5.12	1.63	3.33	1.54	2.63	1.62
1.9	9.13	1.81	5.42	1.51	4.62	1.44	2.98	1.32	2.31	1.40
2.0	8.40	1.66	4.98	1.37	4.24	1.29	2.71	1.18	2.07	1.19
2.5	6.10	1.22	3.61	0.97	3.08	0.91	1.90	0.82	1.49	0.73
3.0	4.86	1.00	2.90	0.80	2.49	0.72	1.55	0.66	1.27	0.53

From Tables 2–4, it can be observed that

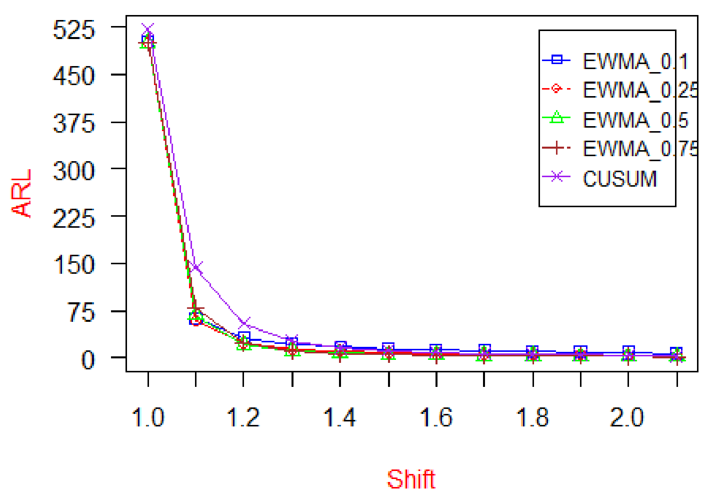
1. The EWMA control charts with larger γ are more sensitive to detect shifts in the process parameters. For example, when the shift $\delta = 1.7$, the EWMA chart with $\gamma = 0.75$ and $L = 3.032$ is the optimal EWMA control chart design with the smallest out-of-control $ARL = 3.09$ for a fixed $ARL_0 = 500$.
2. For a fixed γ the values of ARL_1 decreases as the shift sizes increase. For example when ARL_0, $\gamma = 0.25$ then $ARL_1 = 8.67$ for a shift size $\delta = 1.4$ while $ARL_1 = 4.01$ when shift size $\delta = 1.4$ for the same design parameter (see Table 1).
3. For fixed ARL_0, the ARL_1 decreases as the values of γ increases from 0.1 to 0.75 (see Tables 2–4). For example when $ARL_0 = 370$, then $ARL_1 = 14.18$ for a shift size $\delta = 1.5$ when $\gamma = 0.10$, $ARL_1 = 7.59$ when $\gamma = 0.25$ for a shift size $\delta = 1.5$, $ARL_1 = 5.29$ when $\gamma = 0.5$ for a shift size $\delta = 1.5$ and $ARL_1 = 4.59$ when $\gamma = 0.75$ for a shift size $\delta = 1.5$.
4. For fixed γ the ARL_1 values increases as the values of ARL_0 increases from 200 to 500.

To compare the performance of the proposed control chart with existing control charts for transformed distributed data, Table 5 and Figure 1 present the average run length comparison of the proposed EWMA and the recent CUSUM chart for transformed Rayleigh distributed data proposed by Tyagi and Singh (2016) for $ARL_0 = 500$. From Figure 1, we observed that the run length curve of the CUSUM control chart are always higher than those of the EWMA control chart for monitoring transformed Rayleigh distributed data indicating that the proposed control chart is performing better than the CUSUM control chart in this study.

Table 5 ARL comparison of the transformed Rayleigh data for EWMA and CUSUM control charts when ARL=500

Shift δ	*EWMA*				*CUSUM*
	ψ				*K = 0.76*
	0.1	*0.25*	*0.5*	*0.75*	*H = 3.02*
1.0	501.13	500.04	500.24	501.08	521.80
1.1	63.47	60.05	67.59	78.17	143.27
1.2	31.46	22.47	21.70	24.01	54.35
1.3	22.02	13.41	11.13	11.28	26.54
1.4	17.31	9.78	7.35	6.95	16.14
1.5	14.46	7.81	5.53	4.86	11.35
1.6	12.54	6.58	4.49	3.79	8.70
1.7	11.12	5.74	3.79	3.09	6.90
1.8	10.01	5.12	3.33	2.63	5.85
1.9	9.13	4.62	2.98	2.31	5.06
2.0	8.40	4.24	2.71	2.07	4.42
2.5	6.10	3.08	1.90	1.49	2.89

Figure 1 ARL comparison of proposed chart and CUSUM chart for transformed Rayleigh distribution (see online version for colours)



Also, we compare the performance of the EWMA chart with transformed Rayleigh data and the non-transformed Rayleigh EWMA chart. The non-transformed EWMA chart is designed to achieve in-control ARL of 500 for detecting upward shifts in the scale parameter. The OOC ARL of the proposed EWMA chart with transformed Rayleigh data and the non-transformed Rayleigh EWMA chart are presented in Table 6. We observed that the EWMA chart with transformed Rayleigh data is more sensitive than the non-transformed Rayleigh EWMA chart in detecting small and moderate shifts in process parameter, but slightly worse in detecting large shifts when $\gamma \leq 0.20$. Hence, the EWMA chart with transformed Rayleigh data outperforms the EWMA chart for the non-transformed Rayleigh data for small and moderate shifts in the process parameter.

Table 6 The ARL comparison of transformed Rayleigh EWMA (TRE) and Rayleigh EWMA (RE) chart when $ARL_0 = 500$

	γ							
Shift δ	*0.10*		*0.20*		*0.25*		*0.50*	
	RE	*TRE*	*RE*	*TRE*	*RE*	*TRE*	*RE*	*TRE*
1.0	503.49	501.13	501.03	500.87	500.08	500.04	502.04	500.24
1.1	99.93	63.47	107.43	59.44	113.93	60.05	134.22	67.59
1.2	34.73	31.46	39.41	23.74	42.08	22.47	52.74	21.70
1.3	18.19	22.02	19.95	14.83	21.31	13.41	26.89	11.13
1.4	11.30	17.31	12.62	11.04	13.29	9.78	16.18	7.35
1.5	8.34	14.46	8.98	8.96	9.05	7.81	11.05	5.53
1.6	6.42	12.54	6.68	7.62	6.95	6.58	8.11	4.49
1.7	5.11	11.12	5.41	6.67	5.48	5.74	6.25	3.79
1.8	4.29	10.01	4.55	5.97	4.62	5.12	5.21	3.33
1.9	3.75	9.13	3.85	5.42	3.91	4.62	4.39	2.98
2.0	3.26	8.40	3.44	4.98	3.48	4.24	3.81	2.71
L	2.8289	4.456	3.0195	3.577	3.0938	3.373	3.3142	2.997

4 Illustrative example

In this section, we present a real-life and a simulated data to demonstrate the application of the proposed control chart for monitoring transformed Rayleigh data.

4.1 Real-life application

This section illustrate the application of the EWMA control chart for the monitoring of transformed Rayleigh based on data taken from Smith and Naylor (1987) and also used by Hossain et al. (2020). The data is the recorded strength of 46 samples of 15 cm glass fibres. In order to assess the performance of the proposed chart in detecting a shift in the process, we intentionally increase the shift by 50% after the 25th sample and the data is then transformed using the power transformation of Liu et al. (2007) as discussed in Section 2. Thereafter the EWMA statistics is computed based on the transformed Rayleigh and the non-transformed Rayleigh data. The proposed EWMA control chart using the transformed Rayleigh data is designed using $\gamma = 0.25$, $L = 3.373$ and $ARL_0 = 500$ obtained from Table 1 and the EWMA chart for the non-transformed Rayleigh data is designed using $\gamma = 0.25$, $L = 3.0938$ and $ARL_0 = 500$ obtained from Table 6. The asymptotic lower and upper control limits of the proposed EWMA control chart for the transformed Rayleigh and non-transformed Rayleigh data are given as LCL = 0.8163, UCL = 1.2859 and LCL = 0.888, UCL = 1.856, respectively. Figure 2 display the EWMA control chart for the transformed Rayleigh data of 15-cm glass fibres while Figure 3 display the EWMA chart for the non-transformed Rayleigh data. The control chart in Figure 2 detects 16 OOC samples with the first out-of-control signal on the 29th sample, which is the fourth sample after the shift whereas the control chart for the non-transformed Rayleigh data failed to detect OOC samples. Hence, confirming the superiority of the EWMA chart with transformed Rayleigh data.

Figure 2 EWMA control chart for real life transformed Rayleigh distributed data (see online version for colours)

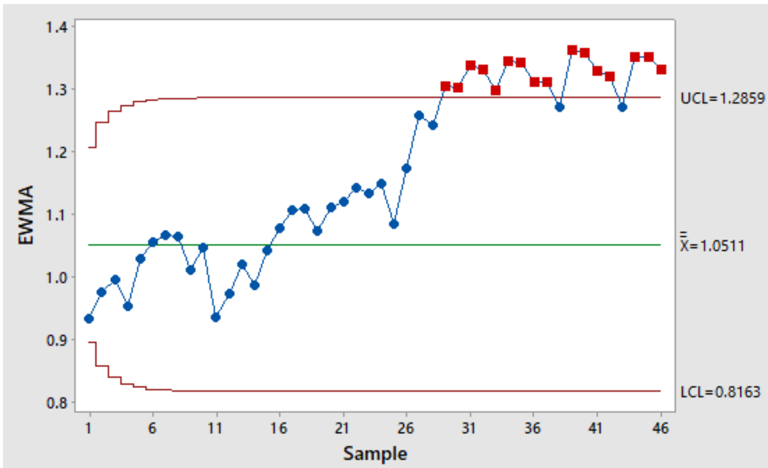
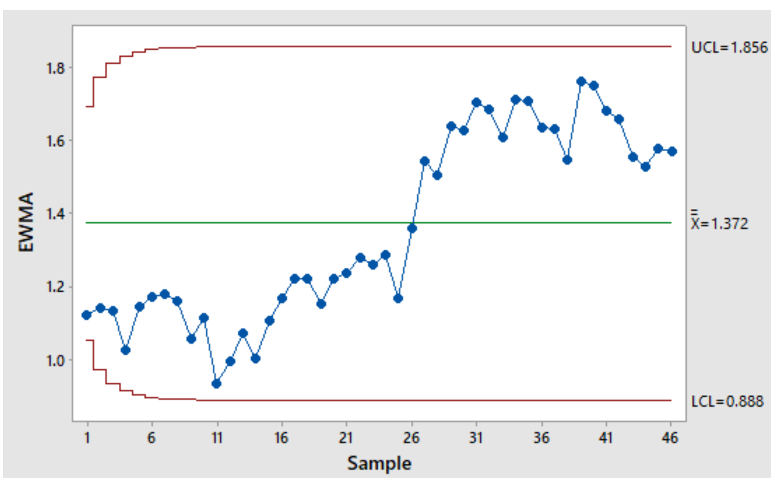


Figure 3 EWMA control chart for real life non-transformed Rayleigh data (see online version for colours)



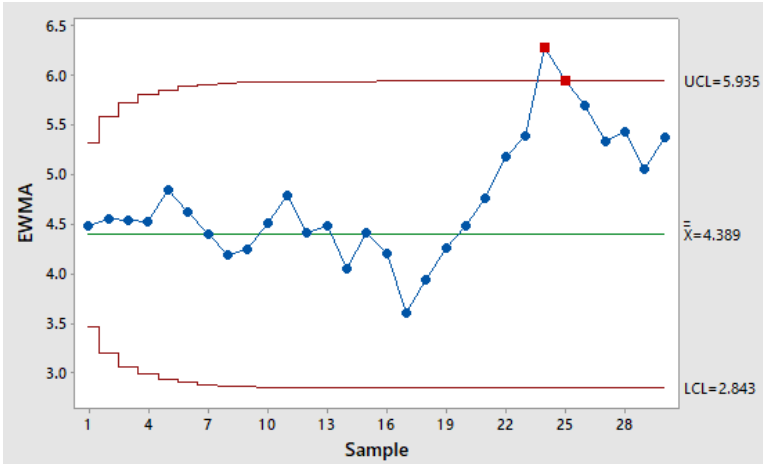
4.2 Simulation study

A simulated study is conducted to demonstrate the application of the EWMA control chart with transformed Rayleigh data in detecting small shifts in the process data. A sample of thirty observations is generated from the Rayleigh distribution. The first twenty observations are generated from Rayleigh distribution with mean 10 and the last ten observations with mean 18, a shift of δ = 1.8. Table 7 presents the summary of the computations of the proposed control chart. The in-control ARL is set equal to 500. The starting value Z_0 is the mean of the first twenty observations and the design parameters are given as γ = 0.20 and L = 3.537. Figure 4 presents the EWMA control chart with transformed Rayleigh data. The control chart signals an out-of-control on the 24th sample.

Table 7 Simulated data of EWMA transformed Rayleigh distribution

Sample number	*Observation (X_i)*	*Transformed data (Y_i)*	*EWMA statistic (Z_i)*	*UCL*	*LCL*
1	16.9208	4.8127	4.4739	5.3167	3.4617
2	16.9307	4.8143	4.5420	5.5769	3.2015
3	15.1879	4.5323	4.5401	5.7170	3.0614
4	14.5994	4.4339	4.5189	5.7994	2.9790
5	26.0926	6.1217	4.8395	5.8496	2.9288
6	10.5182	3.6956	4.6107	5.8809	2.8975
7	9.5005	3.4925	4.3871	5.9006	2.8778
8	9.0591	3.4014	4.1900	5.9131	2.8653
9	14.5581	4.4270	4.2374	5.9210	2.8574
10	22.1585	5.5905	4.5080	5.9261	2.8524
11	24.3431	5.8902	4.7844	5.9293	2.8491
12	6.6986	2.8763	4.4028	5.9313	2.8471
13	16.5486	4.7536	4.4730	5.9326	2.8458
14	4.4853	2.3018	4.0388	5.9335	2.8449
15	24.3065	5.8853	4.4081	5.9340	2.8444
16	8.8385	3.3552	4.1975	5.9344	2.8440
17	1.3963	1.2037	3.5987	5.9346	2.8438
18	20.0739	5.2919	3.9373	5.9347	2.8437
19	21.7114	5.5275	4.2553	5.9348	2.8436
20	20.6564	5.3767	4.4796	5.9349	2.8435
21	23.9045	5.8310	4.7499	5.9349	2.8435
22	32.1707	6.8769	5.1753	5.9349	2.8435
23	26.7112	6.2019	5.3806	5.9350	2.8434
24	61.7509	9.8788	6.2802*	5.9350	2.8434
25	15.4104	4.5691	5.9380	5.9350	2.8434
26	16.2439	4.7048	5.6914	5.9350	2.8434
27	11.4605	3.8760	5.3283	5.9350	2.8434
28	23.7982	5.8166	5.4236	5.9350	2.8434
29	9.6722	3.5274	5.0444	5.9350	2.8434
30	30.4008	6.6641	5.3683	5.9350	2.8434

Figure 4 EWMA control chart for simulated transformed Rayleigh distributed data (see online version for colours)



5 Conclusions

In this paper, a control chart using the transformed Rayleigh distributed data based on the EWMA statistics is proposed. The power transformation of Nelson (1994) has been used for constructing the proposed control chart. The performance of the proposed control chart is evaluated in terms of ARL and SDRL and compared with the recent CUSUM control chart for transformed Rayleigh data. The result of study reveals that the proposed control chart outperforms the CUSUM control chart for transformed Rayleigh distributed data. The mixed EWMA-CUSUM control chart may be developed to monitor the transformed Rayleigh distributed data as future research.

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