An order quantity scheme for ramp type demand and backlogging during stock out with discount strategy

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Abstract: An offer of reduction in per unit purchasing cost which depends on purchasing quantity attracts and promotes the customers for bulk purchasing. But bulk purchasing of inventory also increase the distinct related costs such as deterioration and holding. In this model we have presented an inventory problem combining all these factors with ramp type demand and quantity discount under partial backlogging. The new launched electronic items and fashionable products follow a ramp-type function for demand. Initially the demand for such product rises with time, and it becomes constant after a certain time. Here, a mathematical model developed to analyse the suitable purchasing amount and to optimise the model. The model is demonstrated numerically, and a sensitivity analysis is also performed to validate the system.

Keywords: service delivery; unit time profit; optimal strategy; ramp type demand; quantity discount; deteriorating products; stock-out; partial backlogging.

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published so many research articles in various national and international journals and dynamically working in the field of inventory modelling and supply chain management.

1 Introduction

Since the quantity discount is a general practice of inventory, but did not receive very much attention by the researchers. It provides the financial advantages for supply chain's entities. The vendor gets the advantage of large quantity sale and can reduce the ordering and setup costs. On the other hand the buyer can reduce the cost in terms of ordering and purchasing cost. Abad (1988) presented an inventory policy to determine lot-size, and the optimum selling price under offered quantity discounts. Tersine and Barman (1991) studied on a classical ordering inventory model with continuous demand in which procurement-cost and the transportation cost received discounts through the incremental quantity. Pantumsinchai and Knowles (1991) described a single-period inventory system considering the typical container amount discounts. Burwell et al. (1997) presented an inventory system with price-sensitive demand and described the algorithms and solution to determine the optimal results for all possible discounts. Darwish (2008) presented a continuous review inventory policy for ordering quantity and reorder point with the discount in quantity and cost of the freight. Chang (2013) explained some issues of Burwell et al. (1997) model and make a note on order quantity inventory model for price influence demand under quantity and freight discounts. Chauhan and Singh (2015) established a replenishment policy with the integrated demand pattern. Pandey et al. (2017) introduced an ordering quantity policy for inventory model with motivational quantity strategy for deteriorating items and partial backlogging. Sebatjane and Adetunji (2019) introduced an order quantity model with increasing quantity discounts.

In the previous available research papers so many inventory models are developed with constant demand rate. This type of papers cannot touch the realistic world. Demand is a multi variate factor and cannot be considered as a constant. For example a high selling price gives a negative impact on demand rate. Stock up to a certain level attracts the customers. Similarly there are season, time, stock and many more factors which can affect the demand. Wee (1995) contributed with an inventory system considering replenishment policy and price simultaneously for deteriorating products for declining market demand. Chang and Dye (1999) presented an EOQ system with time varying demand and partial backlogging for deteriorating inventory. Hsu et al. (2007) introduced an order quantity model with the product having a certain date of expiration and undecided lead time for stock arrival. Giri and Roy (2013) came forward with unequal sized shipment in an integrated system for manufacture model with quantity discount. Tayal et al. (2014a) worked on an inventory system for the products having price sensitive demand with the limited space in warehouse. Taleizadeh et al. (2015) developed an EOQ model with backlogging of shortages and incremental discount. Tayal et al. (2015) considered the non-instantaneous items and developed an economic production quantity model for exponentially increasing demand with time dependent carrying cost. Singh et al. (2016) introduced an inventory system for order-quantity and trade credit period for decaying items under preservation with stock dependent demand and permissible delay in payment. Goyal et al. (2015) discussed EPQ model for stock and selling price dependent demand pattern. Rajput et al. (2019) developed a fuzzy optimisation for an Economic order quantity model with three different demand patterns. Tayal et al. (2019) studied a two warehouses inventory system for deteriorating products, with stock-level dependent rate of demand and backlogging. In this model both the considered warehouses are rented.

Deterioration is also an important factor affecting the optimal results. To develop an inventory system without considering the appropriate decaying rate leads the improper results. Ghare (1963) firstly observed that the inventory depletes due to deterioration. Covert and Philip (1973) developed an economic ordering quantity model considering Weibull-distribution as the rate of deterioration. Kang and Kim (1983) considered an inventory system for decaying products, considering price and the level of production. Wee (1995) established a replenishment strategy taking price into account for decaying products with decreasing market demand. Wu et al. (2006) introduced an ordering policy to find the optimal results for an inventory system with non-instantaneous deteriorating product, stock-dependent demand and allowable shortages. Tayal et al. (2014b) presented a multi-item inventory system for deteriorating products with the decreasing demand, dependent on expiration date, and shortages. Chauhan and Singh (2014) established an optimal ordering policy with replenishments and for time dependent demand rate. Tayal et al. (2016) contributed an EPQ model for deteriorating items with permissible delay scheme and also inspected the investment in preservation technology. Goyal (2018) presented a model for multivariate demand rate and Weibull distribution deterioration with effect of inflation and trade credit period.

In this research we have established an inventory system for decaying products with quantity incentive strategy, partial backlogging and ramp-type demand. Subsequently mathematical design of the problem is also demonstrated with the help of an example. Further sensitivity for relevant parameters of system is also made to authenticate the stability of the system.

2 Assumptions and notations

Assumptions

These are the following assumptions, used in the expansion of this model.

- The system is established for finite time horizon.
- Products considered in this model are of decaying nature.
- Constant deterioration rate is considered.
- The demand pattern shown in Figure 1 follows a ramp type function and is specified by

$$\begin{aligned} D(t) &= x + yt \quad 0 \le t \le \mu \\ &= x + y\mu \quad \mu \le t \le T \end{aligned}$$

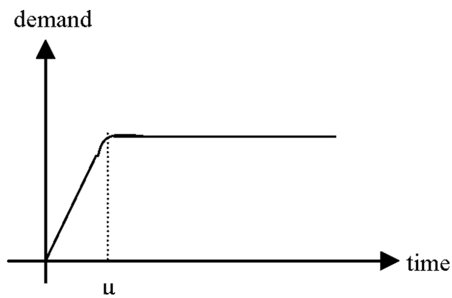
- Lead time is considered negligible.
- Shortages are occurred and backlogged partially.
- Backlogging rate is a constant ratio of occurring shortages.

- The deteriorated products are completely rejected.
- The model is developed for quantity discount strategy in which the material cost per unit is defined as:

$$c_i = \begin{cases} c_1; & 1 \le q_1 < b \\ c_2; & b_1 \le q_2 < b_2 \\ c_3; & b_2 \ge q_3 \end{cases}$$

Here $c_1 > c_2 > c_3$ and b_1, b_2, b_3 denote the incremental boundaries of quantity.

Figure 1 Ramp type demand pattern



Notations:

The notations used in the development of this model are as follow:

c_i: Cost of purchasing per unit

p: Price of selling per unit

q_i: Ordering quantity

K: Deterioration coefficient, $\theta <<< 1$

x: Positive demand coefficient

y: Demand coefficient, $0 < y < 1$

T: Length of cycle

v: Time when stock becomes vanish

Q_1: Initial-stock at the beginning of each cycle

Q_2: Backordered quantity

h: Cost of holding per unit

s: Cost of shortage per unit

l: Lost-sale or opportunity cost per unit

O: Cost of per order

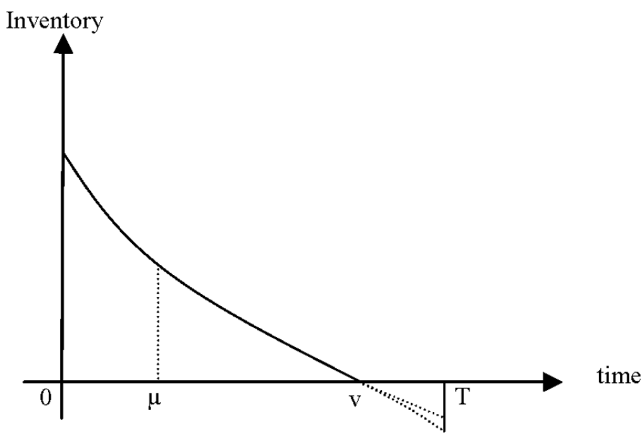
θ: Backlogging rate

$F(v)$: Unit time profit per replenishment cycle.

3 Development of mathematical model

Due to the demand and decay, inventory level decreases, and it has been shown in Figure 2. The stock is replenished at $t = 0$ and depletes in the duration $[0,v]$ due to the result of deterioration as well as demand, where demand is considered as ramp type function. In the duration $[0, \mu]$, the demand increases with time and after this, demand for the item becomes constant. The system has been depicted by the following equations:

Figure 2 Inventory time graph of the system



$$\frac{dI_1(t)}{dt} = -K\, I_1(t) - (x + yt) \quad 0 \le t \le \mu \tag{1}$$

$$\frac{dI_2(t)}{dt} = -KI_2(t) - (x + y\mu) \quad \mu \le t \le v \tag{2}$$

$$\frac{dI_3(t)}{dt} = -(x + y\mu) \quad v \le t \le T \tag{3}$$

with boundary conditions

$$I_2(v) = I_3(v) = 0 \text{ and } I_1(\mu) = I_2(\mu) \tag{4}$$

Using the boundary conditions given in equation (4), the solution of equations (1)–(3) will be:

$$I_1(t) = \frac{y}{K^2}(1 - e^{K(\mu - t)}) - \frac{(x + yt)}{K} + \frac{(x + y\mu)}{K} e^{K(v - t)} \quad 0 \le t \le \mu \tag{5}$$

$$I_2(t) = \frac{(x + y\mu)}{K}(e^{K(v - t)} - 1) \quad \mu \le t \le v \tag{6}$$

$$I_3(t) = (x + y\mu)(v - t) \quad v \le t \le T \tag{7}$$

3.1 Different associated cost

Acquiring cost

If c_i is the acquiring cost per unit then the acquiring cost per replenishment cycle will be:

Acquiring Cost (A.C.) = $(Q_1 + Q_2)c_i$

Here Q_1 is the initial ordering quantity and is given by:

$$Q_1 = \left(\frac{y}{K^2}(1-e^{K\mu}) - \frac{x}{K} + \frac{(x+y\mu)}{K}\right)e^{Kv}$$

Then the backordered quantity Q_2 is given by:

$$Q_2 = \theta\int_v^T D(t)\,dt$$

$$Q_2 = \theta(x+y\mu)(T-v)$$

So, the acquiring cost per cycle will be:

$$P.C. = \left\{\left(\frac{y}{K^2}(1-e^{K\mu}) - \frac{x}{K} + \frac{(x+y\mu)}{K}\right)e^{Kv} + \theta(x+y\mu)(T-v)\right\}c_i \tag{8}$$

Cost of holding

If 'h' denotes the cost of holding per unit per unit time, then the cost of holding per replenishment cycle during positive inventory will be:

$$H.C. = h\left\{\int_0^\mu I_1(t)\,dt \int_\mu^v I_2(t)\,dt\right\}$$

$$\begin{aligned} H.C. = h\Bigg[&\Bigg\{xv\mu + \frac{1}{2}xKv^2\mu - \frac{x\mu^2}{2} + yv\mu^2 - \frac{1}{2}xKv\mu^2 + \frac{1}{2}yKv^2\mu^2 - \frac{2y\mu^3}{3} \\ &+ \frac{1}{6}xK\mu^3 - \frac{1}{2}yKv\mu^3 + \frac{1}{6}yK\mu^4\Bigg\} + \frac{1}{6}(3+K(v-\mu)(v-\mu)^2(x+y\mu))\Bigg] \end{aligned} \tag{9}$$

Cost of ordering

The cost of ordering per cycle will be

$$O.C. = c_o \tag{10}$$

Cost due to shortage

The shortages occur for the duration [v, T]. If 's' be the shortage cost per unit then the total shortage-cost per replenishment cycle will be:

$$S.C. = s\int_v^T D(t)dt$$

$$S.C. = s(x+y\mu)(T-v) \tag{11}$$

Lost-sale or opportunity cost

The lost sale occurs during the stock out period. Since during the stock-out, some customers are impatient for the arrival of stock and make their purchases from other place. So it will be a loss opportunity in sale. If 'l' be the cost of lost-sale per unit then the lost-sale cost associated with it will be:

$$L.S.C. = l(1-\theta)\int_v^T (x+y\mu)\,dt$$

$$L.S.C. = l(1-\theta)(x+y\mu)(T-v) \tag{12}$$

The sales revenue for this cycle will be as follow:

$$S.R. = \left\{\left(\frac{y}{K^2}(1-\mathrm{e}^{K\mu}) - \frac{x}{K} + \frac{(x+y\mu)}{K}\right)\mathrm{e}^{Kv} + \theta(x+y\mu)(T-v)\right\} p \tag{13}$$

The total average profit function for the system will be:

$$\begin{aligned} F(v) = \frac{1}{T}[&\text{Sales Revenue} - \text{Acquiring Cost} - \text{Ordering Cost} \\ &- \text{Holding Cost} - \text{Shortage Cost} - \text{Lost-Sale Cost}] \end{aligned} \tag{14}$$

4 Solution procedure

- We know that unit time profit is a function of the variable v. To find out the optimal solution we put the partial derivatives

  $$\partial F(v)/\partial v = 0. \tag{15}$$

- To reach the optimality it is necessary to find the value of 'v' from the above step and take $v = v^*$.

5 Algorithm to achieve optimality

The ordering quantity q_i* which maximize the unit time profit can be achieved with the help of following steps.

Step 1: Starting with a known value of T, solve the equation (15) to find the optimal values of v for every given price break.

Step 2: Evaluate the value of q_i corresponding to every value of v.

Step 3: Find out a valid quantity $q_i = q_i^*$.

Step 4: Calculate the unit time profit $F_i^*(v)$ corresponding to this value of q_i^*.

Step 5: Calculate the unit time profit for all price break quantities which is greater than q_i^*.

Step 6: Now compare all the values of $F_i(v)$ and pick the optimum solution which provides the maximum value of unit time profit.

6 Numerical example

These following are the numerical values used as input variable for solving the numerical example:

$$T = 90 \text{ days}, \quad x = 250 \text{ unit}, \quad y = 0.3, \quad \mu = 20, \quad K = 0.005, \quad \theta = 0.8,$$
$$h = 1 \text{ rs/unit}, \quad s = 12 \text{ rs/unit}, \quad l = 12 \text{ rs/unit}, \quad c_o = 500 \text{ rs/order}, \quad c = 75 \text{ rs/unit}$$

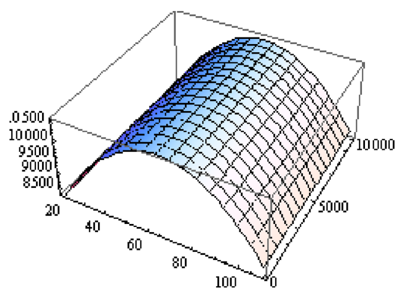
$$c_i = \begin{cases} 80 \text{ rs/unit} & 1 \le q_1 < 10{,}000 \\ 75 \text{ rs/unit} & 10{,}000 \le q_2 \le 20{,}000 \\ 70 \text{ rs/unit} & q_3 > 20000 \end{cases}$$

With the help of these numerical values we get

$$v = 66.01 \text{ days}, \quad F^*(v) = 10{,}573 \text{ rs}, \quad q^* = 17340.8 \text{ unit}$$

Figure 3 shows the optimality of the system graphically.

Figure 3 Concavity of the unit time profit function (see online version for colours)



7 Sensitivity analysis

A sensitivity analysis is performed with respect to various relevant parameters and the effect in selling price, critical time and unit-time profit has been observed.

7.1 Observation

With the help of the sensitivity analysis it has been observed that unit time profit is maximum for the second slot quantity and purchasing price. Although the purchasing price is less in the third slot, but the cost of deterioration and holding will be more in comparison of the increased profit. So to buy the quantity q^* is more profitable for the buyer.

These are the observations:

- It is observed that an increase in initial demand rate 'x' shows a decrease in critical time and an increase in unit time profit by Table 1. It is also shown with the help of Figure 4.
- From Table 2 and Figure 5, it is observed that critical time and unit time profit is highly effected by increase in demand parameter b.

- Table 3 and Figure 6 list the effect in deterioration factor 'K'. From this table we observed that when the deterioration factor increases, critical time (v) and unit time profit (F) follow the pattern of decrease.
- From Table 4 and Figure 7, it has been observed that as the holding cost parameter 'h' increases, critical time (v) and unit time profit (F) show the reverse effect.
- An increase in backlogging parameter 'θ' shows the positive effect of increment on critical time (v) and unit time profit (F), also shown graphically by Figure 8 and by Table 5.

Table 1 Variation of demand parameter 'x'

% change in x	*x*	*v*	*F*
–20%	200	69.862	9390.2
–15%	212.5	68.723	9682.9
–10%	225	67.715	9977.8
–5%	237.5	66.818	10275
0%	250	66.013	10573
5%	262.5	65.288	10872
10%	275	64.631	11173
15%	287.5	64.032	11474
20%	300	63.4854	11776.4

Figure 4 Unit time profit vs. demand parameter 'x'

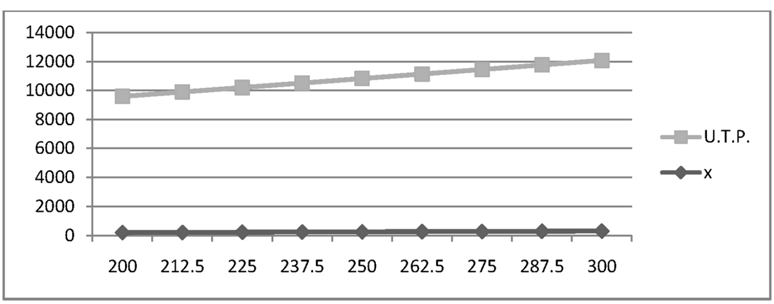


Table 2 Variation of demand parameter 'y'

% change in y	*y*	*v*	*F*
–20%	0.24	62.984	9662.4
–15%	0.255	63.737	9888.1
–10%	0.27	64.493	10115
–5%	0.285	65.252	10343
0%	0.3	66.013	10573
5%	0.315	66.777	10803
10%	0.33	67.544	11035
15%	0.345	68.314	11269
20%	0.36	69.0867	11503.2

Figure 5 Unit time profit vs. demand parameter ‘*y*’

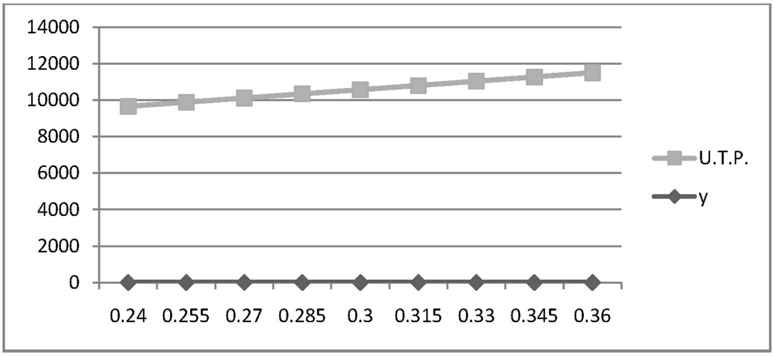


Figure 6 Unit time profit vs. deterioration parameter ‘*k*’

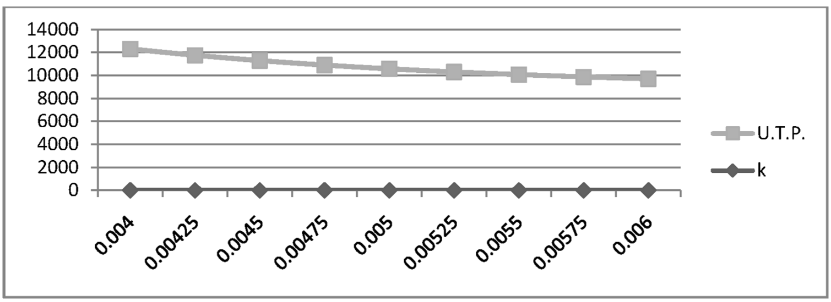


Figure 7 Unit time profit vs. holding cost parameter ‘*h*’

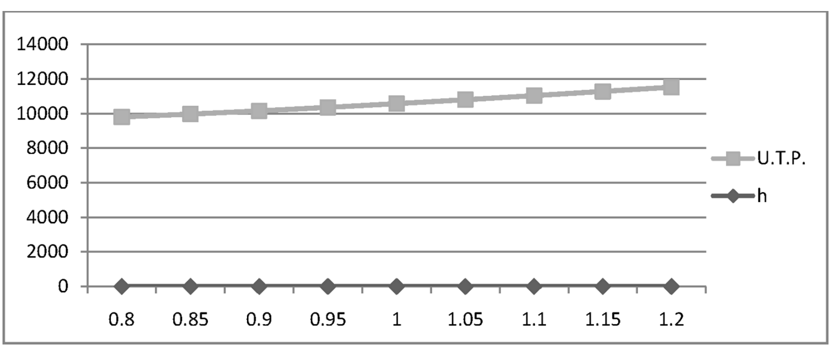


Figure 8 Unit time profit vs. backlogging parameter ‘*θ*’

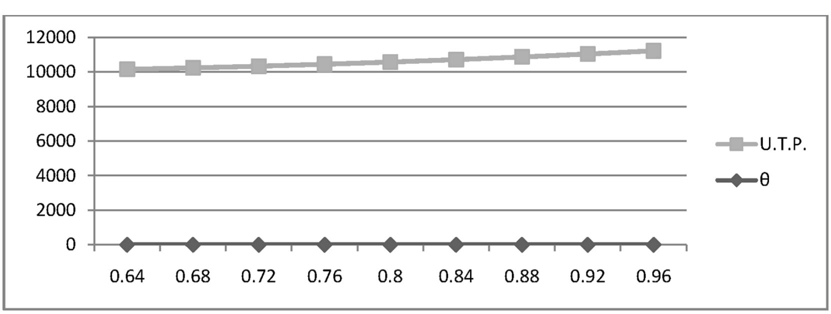


Table 3 Variation of deterioration parameter 'k'

% change in k	*k*	*v*	*F*
–20%	0.004	65.89	12306
–15%	0.00425	65.79	11749
–10%	0.0045	65.785	11286
–5%	0.00475	65.862	10899
0%	0.005	66.013	10573
5%	0.00525	66.229	10297
10%	0.0055	66.505	10063
15%	0.00575	66.836	9863.6
20%	0.006	67.2166	9694.16

Table 4 Variation of holding cost parameter 'h'

% change in h	*h*	*v*	*F*
–20%	0.8	77.792	9800.3
–15%	0.85	74.174	9967.1
–10%	0.9	71.068	10154
–5%	0.95	68.374	10357
0%	1	66.013	10573
5%	1.05	63.928	10799
10%	1.1	62.074	11035
15%	1.15	60.413	11278
20%	1.2	58.9179	11527.6

Table 5 Variation of backlogging parameter 'θ'

% change in θ	θ	*v*	*F*
–20%	0.64	77.792	9800.3
–15%	0.68	74.174	9967.1
–10%	0.72	71.068	10154
–5%	0.76	68.374	10357
0%	0.8	66.013	10573
5%	0.84	63.928	10799
10%	0.88	62.074	11035
15%	0.92	60.413	11278
20%	0.96	58.9179	11527.6

8 Conclusion

In this paper, we have developed a deteriorating inventory model with a very realistic and practical demand rate. In present scenario, where market trends change to a large extent, it is crucial that more than one trend in account is taken while considering customer's demand. The proposed model is very useful in the present market situation as almost every item can be identified as having a time, stock and selling price dependent demand rate. The inventory is allowed to deteriorate during the period it is stored and during this time period it undergoes Weibull deterioration rate. This deterioration rate has the advantage as it is being able to account for more than one factor affecting deterioration. These different factors might be humidity, temperature, lack of proper lighting, etc. Weibull rate can easily account for these different kinds of factors. However, in real life, most of goods would have a span of maintaining quality and the original condition and during that period, no deterioration occurs. The item is allowed a definite life time since no article in real life can be expected to start deteriorating as soon as it is produced. Deterioration sets in afterwards and it has been made more realistic and practical by taking two or three parameter Weibull distribution function. Shortfalls are allowed and partially backlogged. Further cases for stochastic demand and in more realistic conditions can be developed.

In the discussed paper, we have established an inventory system with quantity incentive strategy. A quantity discount policy fascinates the customers to purchase the more quantity in a single lot which decreases also the cost of ordering with the purchasing cost. But increased amount of inventory leads to the more cost of deterioration and holding stock. So the bulk purchasing is not always profitable. It has been shown with the help of numerical, that unit time profit is not maximum for higher quantity and less purchasing cost but it is maximum for a amount between the middle slot of example. This paper is developed to show the realistic feature of quantity discount and ramp type demand rate. It has a further possibility of extension in the field of permissible delay and cash discount.

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