Development of fuzzy-based autoregressive integrated moving average exogenous input model for filtration process

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Abstract: Development of black-box model for filtration process has been carried out in this work. Data needed for the model development was obtained through experimental study on filtration unit using aqueous calcium carbonate ($CaCO_3$) slurry. The input and output data generated from the experiment consisted of one manipulated variable (feed pressure, P_r), one disturbance variable (concentration of the solute in the feed, C_{A0}) and one controlled variable (suspension concentration, C_A). The data were used to develop a fuzzy-based autoregressive integrated moving average exogenous input (FARIMAX) model structure. Model order selection was carried out based on trial and error while optimal model order was determined using factorial technique. The performance of the developed model was determined using root mean square error (RMSE), coefficient of determination (R^2) and model fit. The model gave optimal order [33 3 3 1], RMSE (1.855×10^{-6}), R^2 (1.000) and fit (99.9980%). These results revealed that the model can be used to represent dynamic behaviours of the system and the model can be subsequently used for model base control design and dynamic optimisation of the process.

Keywords: fuzzy model; FARIMAX model; black-box model; MATLAB; modelling; ARIMAX; filtration process.

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1 Introduction

At present, an important number of industrial processes operate in batch mode. Greater flexibility (multiproduct capabilities), in response to fast changes in market conditions, high quality chemicals or small scale production of fine chemicals are the fundamental reasons that justify batch mode of operation (Georgakis, 1990). Under these requirements, batch processing continues developing parallel to a growing market where demand requires a wider product spectrum. Pharmaceuticals, polymers, biotechnology and the food industry constitute typical areas where batch operation dominates over continuous operation (Nagy and Braatz, 2003). In most of these industries, operations such as drying, fermentation, evaporation, sterilisation and filtration are usually carried out in batches (Bimbenet and Trystram, 1992).

Filtration is a pressure driven separation process that uses membranes to separate components in a liquid solution or suspension based on their size and charge differences. It is a process widely used in industry, especially in medicine, chemical and mining as well as in food and paper industries (Burger et al., 2001). During the filtration process, particles that are larger than the pore size of the membrane are deposited to form a filter cake on the side of the membrane (Ní Mhurchú, 2008). The purpose of the filtration operation is either to increase the solute concentration or for recovery of the liquid content of the suspension. Filtration can be operated with either varying pressure or constant pressure (Konnur et al., 2006). As part of a more general field of solid–liquid separation, filtration has its fundamentals in the flow of fluids in a porous medium and its quantification makes use of the famous Darcy equation (Burger et al., 2001). High quality demand, together with the time varying nature of its dynamics and the uncertainty associated with the filtration process make the process highly complex (Bonvin et al., 2006). These complexities have made the modelling of the filtration process extremely difficult. Thus, the development of a tangible model to represent this process is still a challenge to chemical engineers.

Different researchers have worked on filtration process. Konnur et al. (2006) developed Darcy law based approach for simulation of batch constant pressure filtration of particulate suspensions when the feed solids concentration changes. They used filtration data obtained from feed suspension with a not-very-low solids concentration to propose procedures to predict changes in kinetics of dewatering in the cake formation stage, and transition point between the cake formation and consolidation stages, as the initial solid concentration changes. These two model parameters were used to predict the evolution of dewatering. According to Jelemensky et al. (2013), economically optimal operation of batch diafiltration processes was characterised by complex membrane transport models that adequately described non-ideal membrane characteristics of real-life systems. The optimisation criterion which comprises of two conflicting objectives, a minimisation of processing time and a minimisation of solvent consumption was considered. The problem was treated in the multi-objective form in order to investigate the impact of operational cost factors on optimal operation policy. This economically optimal operation of the process was found in the analytical form through the application of Pontryagin's minimum principle (PMP). The phenomenological theory of sedimentation–consolidation processes of flocculated suspensions was extended to pressure filtration processes by Burger et al. (2001). The local mass and linear momentum balances for the solid and liquid components together with appropriate constitutive assumptions lead to a strongly degenerate (mixed hyperbolic–parabolic) nonlinear partial differential equation for the local solids fraction, which together with initial and boundary conditions determines a dynamic cake filtration process. In the case of a prescribed applied pressure function, a free boundary problem was obtained, in which the piston height has to be determined simultaneously with the solids concentration. A numerical algorithm approximating the physically correct solution, with possible discontinuities such as the cake/suspension interface, was presented and employed to simulate various cake filtration processes. Kovacs et al. (2009) provided useful methodology for the design of batch diafiltration processes. A general mathematical model in a compact form was derived and unified the existing models for constant-volume dilution mode, variable-volume dilution mode and concentration mode operations. A rich representation of the separation process was also presented due to the employment of concentration-dependent solute rejections in the design equations. The use of data-driven and mechanism-driven permeation models allowed the optimisation of the overall diafiltration process.

Modelling of filtration process using the first principle approach which incorporates many assumptions for the development of rigorous theoretical models may not be adequate for a complex process. This is because theoretical models require a large number of equations with a significant number of process variables and unknown parameters. An alternative approach is to develop data-driven model directly from input-output data generated from the laboratory experiment. This type of model is referred to as black-box modelling. Therefore, fuzzy-based autoregressive integrated moving average exogenous input (FARIMAX) model is developed.

2 Methodology

2.1 Data generation

The experimental plant in which the experiments were carried out was a filtration unit set up as shown in Figure 1. The experimental work was carried out in the unit operation laboratory of Chemical Engineering Department, LAUTECH, Ogbomoso, for generation of input and output data. The filtration unit consists of control console, which is used for controlling the pump and the tank stirrer. Feed tank which was constructed from acrylic, for holding feed slurry that is to be filtered, and is capable of holding approximately 13 litres of liquid. Feed pump which is capable of flow rates up to 130 Lh^{-1}. The plate and frame filter of the unit comprises five acrylic sections, two end plates, and two frame plates and one wash plate. The filter medium is a nylon mesh with a 63 μm nominal pore size. Also, the unit is fitted with a thermocouple to monitor temperature of process fluid; two pressure sensors, one to monitor the feed slurry pressure applied to filter during filtration (filtration pressure) and the other to monitor the wash water pressure during filter cake washing, and the optical sensor for measurement of amount of light absorbed whilst passing through the liquid sample.

Figure 1 Experimental set up for filtration system (see online version for colours)

Ten litres solution of 0.5% (w/v) calcium carbonate was fed into the feed tank. The feed tank stirrer was then turned on to maintain a homogenous feed solution. Valve was switched on to allow filtrate to pass into the filtrate vessel. The flowrate of 12 L/h was set for the feed pump and monitored to avoid overshoot. Air was bled from the filter frame plate, and then pump was switched on to begin pumping. Experiment was allowed to continue for 600 seconds (10 mins) by which time cake was retained on the filter surface within the filter. The experiment was then repeated at constant flowrate for 900, 1,200, 1,500, 1,800, 2,100 and 2,400seconds. All the input signals (feed pressure, P_r and concentration of the solute in the feed, C_{A0}) and controlled output (suspension concentration, C_A) were all determined and recorded. The input and output data generated were then used for model development.

2.2 Model structure

Given filtration process, which has, apart from the disturbance, C_{A0}, the feed pressure, P_r as the input and suspension concentration, C_A as output, that is represented as shown in Figure 2. The general black-box model structure of filtration process was formulated as shown Figure 3.

Figure 2 Filtration process

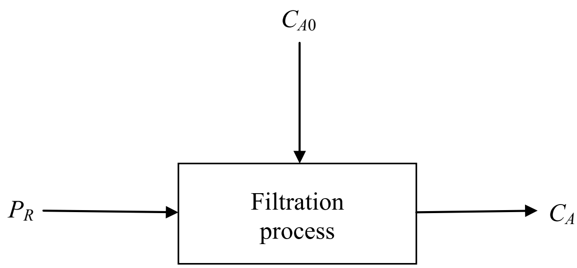
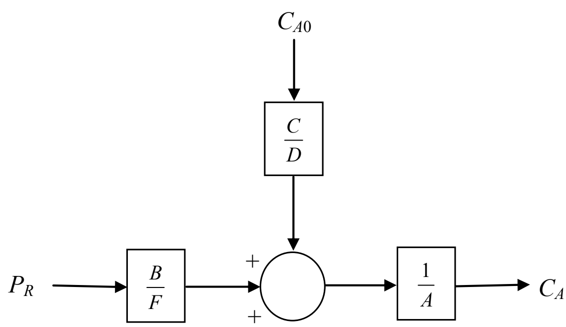


Figure 3 Black-box model structure of filtration process



Due to the non-stationarity in the data generated from the laboratory experiment, an autoregressive integrated moving average exogenous input (ARIMAX) structure was used which has the following formulations (Tangirala, 2015)

$$y(k) = \frac{B\left(q^{-1}\right)}{A\left(q^{-1}\right)F\left(q^{-1}\right)}u(k) + \frac{C\left(q^{-1}\right)}{\nabla^d D\left(q^{-1}\right)}e(k) \tag{1}$$

where

u input signal

y output signal

e white noise

∇^d differencing factor = $1 - q^{-1}$.

Tangirala (2015) defined the polynomial coefficients A, B, C, D and F contain in equation (1) in terms of the backward shift operator as.

$$A(q^{-1}) = 1 + aq^{-1} + \cdots + a_{na}q^{-na} \tag{2}$$

$$B(q^{-1}) = b_1 + b_2q^{-1} + \cdots + b_{bn}q^{-nb+1} \tag{3}$$

$$C(q^{-1}) = 1 + cq^{-1} + \cdots + c_{nc}q^{-nc} \tag{4}$$

$$D(q^{-1}) = 1 + dq^{-1} + \cdots + d_{nd} q^{-nd} \tag{5}$$

$$F(q^{-1}) = 1 + fq^{-1} + \cdots + f_{nf} q^{-nf} \tag{6}$$

The nonlinear variance of equation (1) was defined according to:

$$\begin{aligned} y(k+1) = F\big(&y(k)\ldots y(k-na+1), \Delta y(k)\ldots \Delta y(k-nd+1), \\ &u(k)\ldots(k-nb+1), e(k)\ldots e(k-nc+1)\big) \end{aligned} \tag{7}$$

where $y(k)$ represents output, when applied to filtration process under consideration, $y(k)$ represents solid concentration of $CaCO_3$ filtration process, C_A. $u(k)$ is the control signal or process input. There are two input variables, feed pressure of filtration process, P_r and solid concentration into the filtration process, C_{A0}. The constants (na, nb, nc, nd) in equation (7) are the orders of the system which represent number of past output, number of past input, number of past error and number of past differencing factor respectively; and they are used in the regressor vector. The function $F(.)$ was accounted for by using Takagi-Sugeno fuzzy system. The filtration process was approximated by a multi input single output (MISO) Takagi-Sugeno fuzzy system according to:

$$\begin{aligned} &R^i: \text{ IF } Z_i \text{ is } F_1^i \text{ AND}\ldots\text{AND } Z_v \text{ is } F_v^i \\ &\text{THEN } y(k+1) = -a_{i1} y(k) - \cdots - a_{in} y(k-na+1) + d_{i1}\Delta y(k)\ldots + d_{il}\Delta y(k-na+1), \\ &+b_{i1}u(k) + \cdots + b_{im}u(k-nb+1) + c_{i1}e(k)\ldots + c_{ip}e(k-nc+1) \end{aligned} \tag{8}$$

where

- $R^i = i^{\text{th}}$ fuzzy inference rule;
- $a_{i1}, \ldots, a_{in}, d_{i1}, \ldots, d_{il}, b_{i1}, \ldots, b_{im}, e_{i1}, \ldots e_{ip}$ are the Consequent parameters associated with the model
- $z(k)$ is the regressor vector which is given as

  $$\begin{aligned} &z(k) = y(k-1), \ldots, y(k-na), u_1(k-1), \ldots, u_1(k-nb_1), u_2(k-1), \ldots, (k-nb_1), \\ &\Delta y(k-1), \ldots, \Delta y(k-nd), e(k-1), \ldots, e(k-nc) \end{aligned}$$

- $F_j^i\,(j = 1, 2, \ldots, v)$ is the fuzzy set which were represented by membership function, μ_j^i.

Gaussian membership function was used in the characterisation of each fuzzy set. It was adopted to reduce the effect of noise in the signal. It also has low sensitivity to small changes in input parameters than singleton membership function (Nafisi et al., 2011). The two parameters that characterised Gaussian membership function (μ) as shown in equation (9) are the centre, c, and the width, σ.

$$\mu_j^i = \exp\left[-\frac{1}{2}\left(\frac{u_j - c_j^i}{\sigma_j^i}\right)^2\right] \tag{9}$$

2.3 Fuzzy clustering

Fuzzy C-means clustering was adopted, so as to give the best result for overlapped dataset, for the extraction of rules as well as determination of value of membership functions. The premise part of the rules was formed by fuzzy clustering while the defuzzification technique was used to form the consequent part of the rules. The parameters of the premise part of the fuzzy rules were determined by fuzzy C-mean clustering while the parameters of the consequent part were determined by recursive least square (RLS) technique. A crisp output was chosen using the centres of each of the output membership function. Centre of gravity (COG) was employed as defuzzifier to determine the final output of the fuzzy system according to:

$$y = \frac{\sum_{i=1}^{R} \mu\left(z(k)_i\right) g_i\left(z(k)_i\right)}{\sum_{i=1}^{R} \mu\left(z(k)_i\right)} \tag{10}$$

where

$$\begin{aligned} g_i\left(z(k)\right) = {} & a_{i1} y(k) + \cdots + a_{in} y(k - na + 1) + d_{i1} \Delta y(k) \ldots + d_{il} \Delta y(k - nd + 1) \\ & + b_{i1} u(k) + \cdots + b_{im} u(k - nb + 1) + c_{i1} e(k) \ldots + c_{ip} e(k - nc + 1) \end{aligned} \tag{11}$$

The choice of COG as defuzzifier was because smooth and continuous changes in the output parameters were needed (Nafisi et al., 2011; Karray and De Silva, 2004).

Through the application of a standard fuzzy inference method using Gaussian fuzzifier, which reduces the effect of noise in the signal, product fuzzy inference, and COG defuzzifier, fuzzy global dynamic model was obtained as

$$\begin{aligned} y(k+1) = \sum_{i=1}^{w} \xi_i(x) \big[& a_{i1} y(k) + \cdots + a_{in} y\left(k - na + 1\right) + d_{i1} \Delta y(k) \ldots d_{il} \Delta y(k - nd + 1), \\ & + b_{i1} u(k) + \cdots + b_{im} u(k - nb + 1) + c_{i1} e(k) \ldots c_{ip} e(k - nc + 1) \big] \end{aligned} \tag{12}$$

ξ = normalised membership function and was determined by

$$\xi_i(x) = \frac{\mu\left(x_i\right)}{\sum_{i=1}^{R} \mu\left(x_i\right)} \tag{13}$$

Through combination of equations (10) and (11)

$$y = \frac{\sum_{i=1}^{R} a_{i,0} \mu\left(x_i\right)}{\sum_{i=1}^{R} \mu\left(x_i\right)} + \frac{\sum_{i=1}^{R} a_{i,1} \mu\left(x_i\right)}{\sum_{i=1}^{R} \mu\left(x_i\right)} + \cdots + \frac{\sum_{i=1}^{R} a_{i,n} \mu\left(x_i\right)}{\sum_{i=1}^{R} \mu\left(x_i\right)} \tag{14}$$

Equation (12) was expressed in compact form to give

$$y = \theta^T \xi(x) \tag{15}$$

where

$$\begin{aligned} \xi(x) = \big[& \xi_i(x), \xi_2(x), \ldots, \xi_R(x), x_1 \xi_1(x), x_1 \xi_2(x), \ldots, \\ & x_i \xi_R(x), x_n \xi_1(x), x_n \xi_2(x), \ldots, x_n \xi_n(x) \big]^T \end{aligned}$$

and

$$\theta = \left[a_{1,0}, a_{2,0}, \ldots, a_{R,0}, a_{1,1}, a_{2,0}, \ldots, a_{R,1}, \ldots a_{1,n}, a_{2,n} \ldots a_{R,n}\right]^T$$

$\xi(x)$ denotes vector of regressor variables, R is the number of rules.

2.4 Parameter estimation

The estimations of the unknown model parameters were carried out in MATLAB Environment according to equation (16). These unknown parameters were determined by minimising the sum of the square of difference between the actual and predicted output values with possible weighting that measures degree of precision (Duan et al., 2012).

$$J_k = \sum_{i=1}^{k} \left[y(i) - \Theta^T(k)\Phi(i)\right]^2 \tag{16}$$

$y(i)$ is the process output in i^{th} step, $\Theta^T(k)\Phi(i)$ is the predicted process output, $\Theta^T(k)$ is the vector of unknown parameters, $\Phi(i)$ is the vector of measurements or regressor vector.

Recursive least squares (RLS) algorithm was employed in the determination of these unknown parameters of the model. This is because it gives faster convergence and smaller error (Xin et al., 2002; Paleologu et al., 2008).

The new parameter estimate was computed as

$$\widehat{\Theta}(k+1) = \widehat{\Theta}(k) + P(k+1)\Phi(k)\varepsilon(k+1) \tag{17}$$

$$\Theta^T = \left(a_{i1}, \ldots, a_{in}, d_{i1}, \ldots d_{il}, b_{i1}, \ldots, b_{im}, c_{i1}, \ldots c_{ip}\right)$$

$$\begin{aligned}\Phi(k) = \big(&y(k-1), y(k-2), y(k-3), u_1(k-1), u_1(k-2),\\ &u_1(k-3), u_2(k-1), u_2(k-2), u_2(k-3)\big)\end{aligned}$$

$$\widehat{\Theta}(k+1) = \widehat{\Theta}(k) + \frac{P(k)\Phi(k)}{1+\Phi^T(k)P(k)\Phi(k)}\varepsilon(k+1) \tag{18}$$

$$\varepsilon(k+1) = \left[y(k) - \widehat{\Theta}^T(k+1)\Phi(k)\right] \tag{19}$$

P is the covariance matrix or adaptation gain matrix.

The initial value of covariance matrix determined the influence of initial parameter estimations on the identification and it was updated as

$$P(k) = \frac{1}{\lambda}\left(I - P(k-1)\Phi(k)\left(\lambda I + \Phi^I(k)P(k-1)\Phi^{-1}(k)\right)\Phi^T(k)\right)P(k-1) \tag{20}$$

I is identity matrix; λ is forgetting factor.

2.5 Selection of model orders

The selection of appropriate model orders (*na*, *nb*, *nc*, *nd*) is very important when developing any black-box model. During the model development, the optimum values of these model orders are necessary to be determined in order to avoid under-fitting or over-fitting of the developed model. Many criteria are available in the literature for the optimum selections of these model orders. In this work, model order selection was carried out based on trial and error while optimal model order was determined using factorial technique. Thereafter, the optimal model order was used to develop the ARIMAX model for the filtration process.

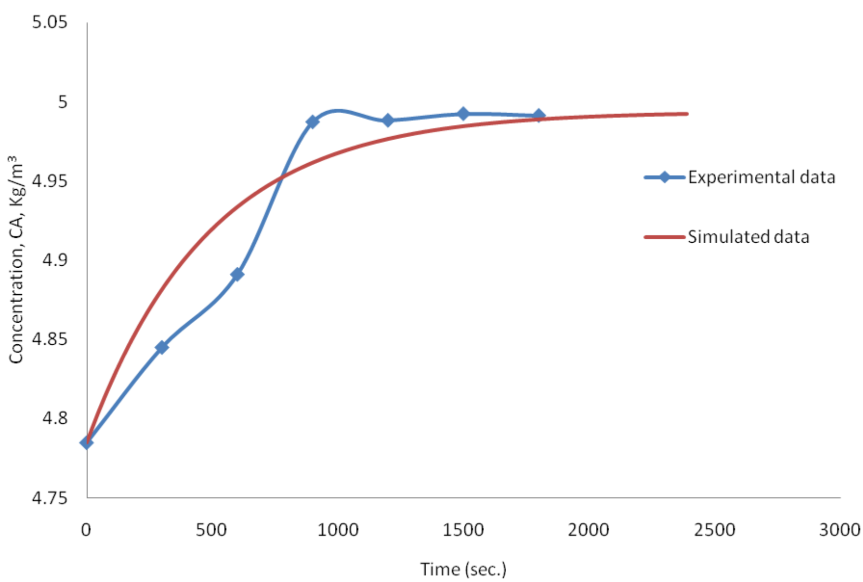
2.6 Model validation

Validation of FARIMAX model was performed through the evaluation of its performance using the second data points that were not used for training of the model. The effectiveness of the model developed was determined using statistical measures such as standard deviation, root mean square error (RMSE), root mean square (RMS) and model fit.

3 Results and discussion

The concentration responses of data obtained from the laboratory experiment and that of simulation to a step change are shown in Figure 4. It is observed in Figure 4 that there is a deviation between the experimental and simulation responses. This is as a result of nonlinearity of the filtration process.

Figure 4 Simulation and experimental concentration responses to a step change (see online version for colours)

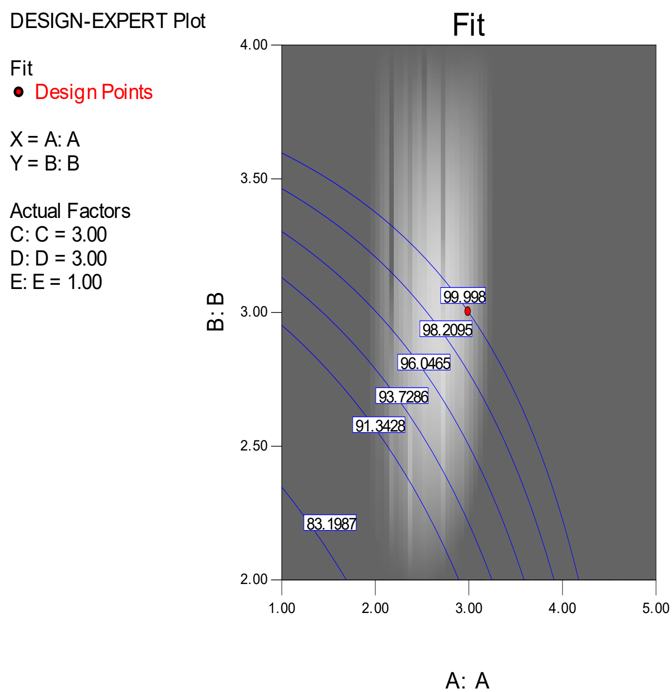


Using the data acquired from the experiment, the optimal model orders were estimated by factorial technique as shown in Figure 5. The optimum model orders obtained were $n_a = 3$, $n_d = 3$, $n_{b1} = 3$, $n_{b2} = 3$, and $n_c = 1$. This implies, three past output, $y(k-1)$, $y(k-2)$, $y(k-3)$, three past differencing operation, $\Delta y(k-1)$, $\Delta^2 y(k-1)$, $\Delta^3 y(k-1)$, three past input 1, $u_1(k-1)$, $u_1(k-2)$, $u_1(k-3)$ three past input 2, $u_2(k-1)$, $u_2(k-2)$, $u_2(k-3)$, and one past error, $e(k - 1)$ were used to predict current output. The optimal model order was then used to develop the ARIMAX model for the filtration process as given below.

$$\begin{aligned} C_A(k) &= a_0 + a_1 C_A(k-1) + a_2 C_A(k-2) + a_3 C_A(k-3) + d_1 \Delta y(k-1) \\ &+ d_2 \Delta^2 y(k-1) + d_3 \Delta^3 y(k-1) + b_{11} P_r(k-1) + b_{21} P_r(k-2) \\ &+ b_{31} P_r(k-3) + b_{12} C_{A0}(k-1) + b_{22} C_{A0}(k-2) + b_{32} C_{A0}(k-3) + ce(k-1) \end{aligned} \tag{21}$$

C_A is the model output; P_r is the manipulated variable; C_{A0} is the disturbance variable.

Figure 5 Optimised result of model order (see online version for colours)



3.1 *Developed FARIMAX model*

Three linguistic rules were obtained for FARIMAX model. The output set of the model which is dynamic model is a linear combination of the inputs. The FARIMAX model developed is a linear relationship between the input and output variables. It comprises of two parts, the premise part (If-part) and the consequent part (then-part). In the premise part of the model, each rule was assigned 13 membership functions and contains 13 parameters while the consequent part contains 14 parameters.

The general form of linguistic rules for FARIMAX model is given as:

$$Rule_i: \text{If } y(k-1) \text{ is } \mu^i_{yk-1} \text{ and } y(k-2) \text{ is } \mu^i_{yk-2} \text{ and } y(k-3) \text{ is } \mu^i_{yk-3} \text{ and } \Delta y(k-1) \text{ is } \mu^i_{\Delta yk-1} \text{ and } \Delta^2 y(k-1) \text{ is } \mu^i_{\Delta^2 yk-1} \text{ and } \Delta^3 y(k-1) \text{ is } \mu^i_{\Delta^3 yk-1} \text{ and } u_1(k-1) \text{ is } \mu^i_{u1k-1} \text{ and } u_1(k-2) \text{ is } \mu^i_{u1k-2} \text{ and } u_1(k-3) \text{ is } \mu^i_{u1k-3} \text{ and } u_2(k-1) \text{ is } \mu^i_{u2k-1} \text{ and } u_2(k-2) \text{ is } \mu^i_{u2k-2} \text{ and } u_3(k-1) \text{ is } \mu^i_{u2k-3} + e(k-1) \text{ is } \mu^i_{ek-1}$$

Then

$$y(k) = a_{0i} + a_{1i}y(k-1) + a_{2i}y(k-2) + a_{3i}y(k-3) + d_{1i}\Delta y(k-1) + d_{2i}\Delta^2 y(k-1) + d_{3i}\Delta^3 y(k-1) + b_{1i}u_1(k-1) + b_{2i}u_1(k-2) + b_{3i}u_1(k-3) + b_{1i}u_2(k-1) + b_{2i}u_2(k-2) + b_{3i}u_2(k-3) + c_i e(k-1)$$

$$i = 1, \ldots, 3$$

μ is the membership function.

The centre (c) and width (σ) of the membership function, which are the estimates of the membership function associated with each rule are presented in Table 1.

Table 1 Estimates of membership function

	Centre, C			*Sigma, σ*		
	Rule 1	*Rule 2*	*Rule 3*	*Rule 1*	*Rule 2*	*Rule 3*
$y(k-1)$	0.3904	0.5775	0.4390	13.5712	14.0926	13.2571
$y(k-2)$	0.6473	0.4111	0.5134	14.0904	13.3720	13.2800
$y(k-3)$	0.4994	0.6108	0.5446	13.3493	13.1189	13.7840
$\Delta y(k-1)$	0.5471	0.6222	0.6443	–0.4082	–0.51365	0.5574
$\Delta y^2(k-1)$	0.5529	1.2423	0.6647	0.1287	–1.2196	0.5987
$\Delta y^3(k-1)$	1.6264	1.6544	0.8405	1.3903	–1.6932	0.0984
$u_1(k-1)$	1.7527	1.8087	3.5101	10.0996	10.1279	17.6246
$u_1(k-2)$	1.5245	2.8529	3.0475	10.0361	19.7396	12.6866
$u_1(k-3)$	3.0609	1.7852	1.4262	19.6886	14.8599	9.9871
$u_2(k-1)$	0.6329	0.5772	0.4615	2.2659	2.2587	2.6474
$u_2(k-2)$	0.6451	0.5925	0.4640	2.2541	2.259279	2.2628
$u_2(k-3)$	0.6683	0.5915	0.4777	2.2540	2.2588	2.2460
$e(k-1)$	1.7300E-05	1.5100E-05	1.3000E-05	2.6100E-05	2.1300E-05	2.4600E-05

The plots of the estimates of membership function for each linguistic rule are shown in Figures 6–8. Each plot has 13 membership functions for each rule. The membership functions have similar pattern. An individual permissible variation limit was defined for each of the input and output signal. The permissible variation limit for changes in output, y, for each rule is from 10 to 20 kg/m^3. The membership functions of these parameters were defined such that increase or decrease from the minimum or maximum permissible limits can be specified in the fuzzy controller. For the first input, μ_1, for all the 3 rules the changes from 0 to 20 bar are permissible while in second input, μ_2, changes from 0 to 4 kg/m^3 are permissible for all the three rules.

Figure 6 Input membership function for rule 1 (see online version for colours)

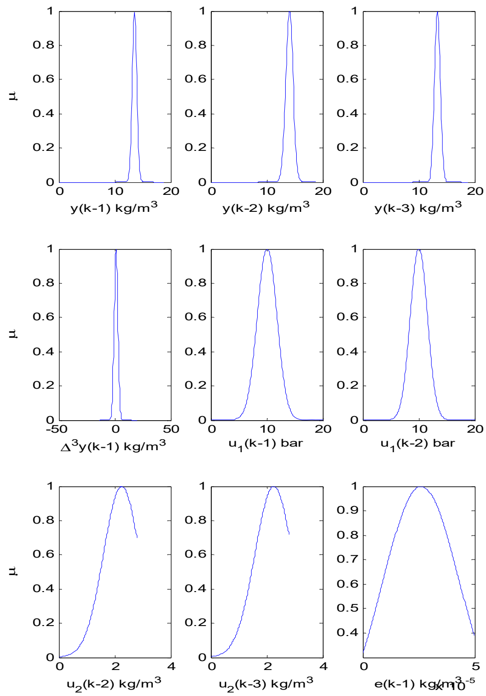


Figure 7 Input membership function for rule 2 (see online version for colours)

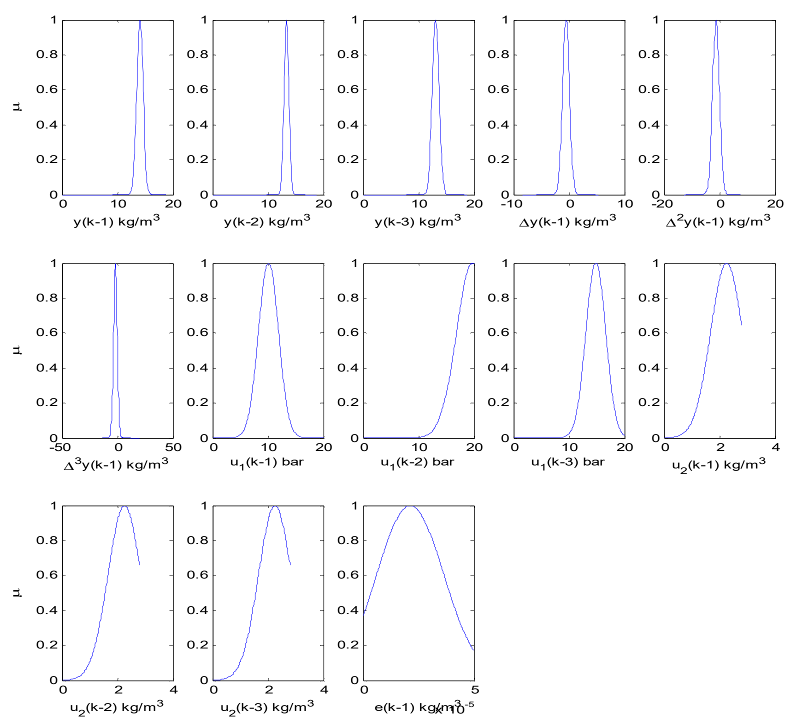
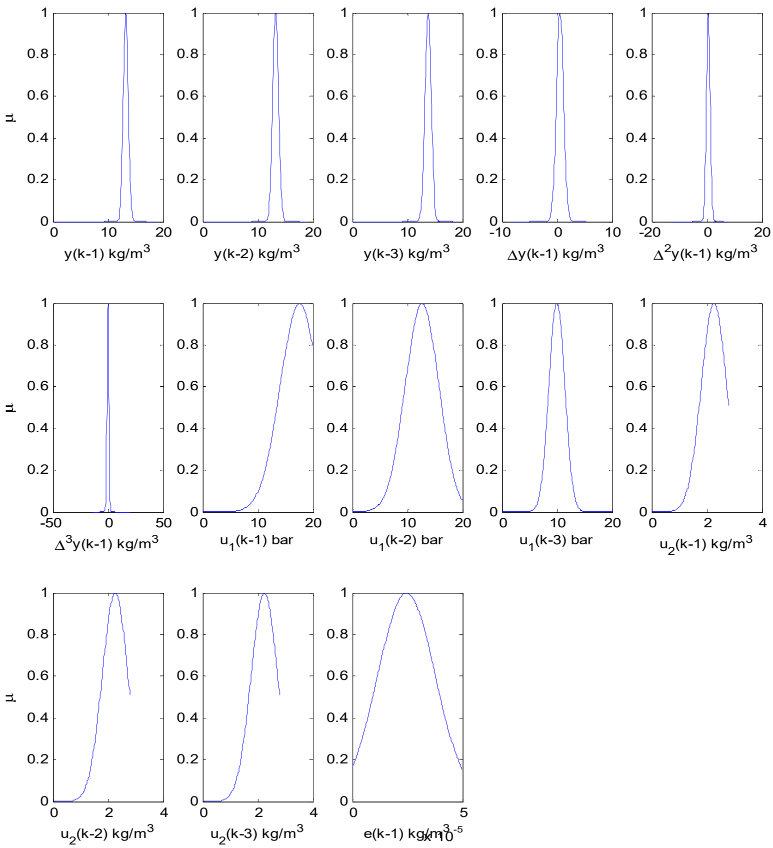


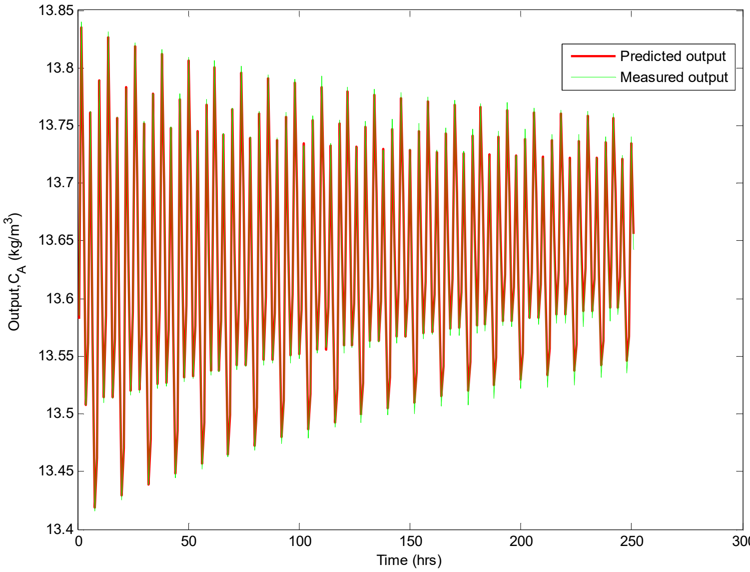
Figure 8 Input membership function for rule 3 (see online version for colours)



3.2 *Validation of FARIMAX model*

The predicted model output is shown together with the measured validation data in Figure 9. Statistical measurement such as RMSE was used to test for the performance of the model. The RMSE obtained for FARIMAX model was 1.855×10^{-6}. Due to the small value of RMSE obtained, it can be said that FARIMAX model developed gave good fit. The fit value refers to the percentage of the data that model could account for. The higher the value of the fit, the better the model. The best fit of the developed model is 99.9980%, and then it means that the ARIMAX model developed could account for 99.9980% of the experimental data. Due to the small value of RMSE obtained, it can be said that FARIMAX model developed gave good fit.

Figure 9 The predicted and measured output responses (see online version for colours)



4 Conclusions

This research work has considered FARIMAX Input model to characterise nonlinear behaviours of filtration process. The nonlinearity, coupling between inputs and error were approximated by Takagi-Sugeno fuzzy model whose structure is assumed to be known but the parameters are unknown. The model consists of nonlinear fuzzy part and linear part. The input and output data for the model identification were generated using filtration unit. Parameter estimations were carried out by using recursive least square method. Due to the high fit value and low mean squared error, FARIMAX model developed can be used to represent the behaviour of the filtration process successfully. It can also be used for the development of control scheme for the filtration system.

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