

Return prediction with time varying betas: a research in BIST

Ayca Akyatan*

Turkey

Email: aycaakyatan79@gmail.com

*Corresponding author

Mustafa Koray Cetin

Akdeniz University,

Pınarbasi, 07070 Konyaaltı/Antalya, Turkey

Email: kcetin@akdeniz.edu.tr

Abstract: In the present study, dynamic versions of beta, which is the risk measure of investment instruments, have been employed to predict daily return of 30 random portfolios made of 154 stocks transacted in BIST ALL between dates 02.01.2003 and 29.08.2013. BIST 100 Index has been employed as the market portfolio. The predictions have been made with rolling regression and MGARCH methods. The performance of return predictions of dynamic betas has been compared to the performance of return predictions of traditional beta. Dynamic betas have been estimated with rolling regression, MGARCH DVECH, MGARCH DBEKK, MGARCH CCC and MGARCH DCC. In the study, it has been identified that the return prediction made with dynamic betas performed better than the predictions made with traditional beta. However, the return predictions made with CCC betas have been superior to other dynamic betas in terms of beating the performance of traditional beta.

Keywords: risk; return prediction; conditional covariance; rolling regression; MGARCH; dynamic beta; dynamic conditional correlation; DCC; constant conditional correlation; CCC; diagonal BEKK; DBEKK; BIST.

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Biographical notes: Ayca Akyatan is currently an independent researcher. She completed her BA degree at Istanbul University, and Master's degree at Marmara University, Department of Accounting and Finance. She earned her Doctor's degree at Akdeniz University, in area of Finance. She is an independent researcher and also worked several years in private sector as independent auditor, financial reporter and financial analyst. She is primarily interested in behavioural finance and financial econometrics.

Mustafa Koray Cetin graduated from Bilkent University, Industrial Engineering in 1996. He earned his Master's degree at Akdeniz University, BA Faculty and earned his Doctor's degree at the same faculty specialising in Finance. His primary areas of research are financial markets and investment management.

1 Introduction

The financial asset pricing model is widely used in practice. Its popularity comes from the simplicity of its application. One of the major criticisms of the traditional financial asset pricing model is that the risk premium, and thus the timing of the beta would change. Black et al. (1972) found that the constant term α was not stationary. Fama and MacBeth (1973) suggested that market risk premiums changed with time. Studies on time varying beta is fairly based on studies of Black (1972) and Merton (1980). The efficient portfolio brings maximum return against the minimum risk. Black (1972) states that each efficient portfolio is the weighted average of two basic portfolios. Also, the variance of this portfolio was supposed to be minimised.

Merton (1980) suggests that the expected market risk premium is the multiplication of a constant risk aversion coefficient with market variance. Merton (1980) considered market risk premium as a function of market variance. Thus, the association between market risk premium and market variance was defined as reward to risk ratio and it was constant.

$$\alpha - r = Yg(\sigma^2)$$

Here, $\alpha - r$ is the risk premium; Y is the constant risk aversion coefficient and $g(\sigma^2)$ is the variance function.

The investor has a constant risk aversion utility function. Thus, the reward to risk ratio must be constant and equal to the investor's relative risk aversion (Merton, 1980). As the variance of the change of wealth must be greater than the dividend yield and return differences between alternative bond types with different maturities, the reward to risk ratio has been determined to be constant (Ng, 1991). Supporting this, Pratt (1964) had suggested that when local relative risk aversion was constant global relative risk aversion would be also constant. When the constant in the capital asset pricing model is significant, the time dependent conditional covariances would associate with expected returns of the asset as mentioned by Black (1972).

Frankel's (1982) empirical research null hypotheses were that the risk aversion coefficient was zero and there were not any risk premiums. The risk premium was also allowed to change over time. However, the null hypothesis could not be rejected (Frankel, 1982).

Gibbons and Ferson (1985) are the first implementers of time varying betas. They developed the approach they created by considering a single risk premium asset pricing model to be a multi-risk premium model. According to the results of the tests, the return was consistent with a single risk premium that changed depending on time. Time varying conditional covariances have been developed by Ferson et al. (1987). They tested pricing models that allowed expected risk premiums and market betas change over time. It was suggested that the single index risk premium model would only be valid if risk premium changed with time.

Ng (1991) developed an FVFM model that allows the ratio of the expected market risk premium to the market variance to change over time, with expected excess returns and risk. Bollerslev et al. (1988) calculated a GARCH process for bonds and stocks. Conditional variances are time-varying and have an important role in determining the time-varying risk premium. In the study, the betas varied with time and could be predicted depending on the time. De Santis and Gerard (1997) tested the CAPM model

using the GARCH parameter in a study of the world's largest eight capital markets data. The evidence found supported the majority of the conditional CAPM price constraints. Allowing time variation in market risk improved the model's results. According to Brooks (2002), conditional heteroscedastic models are the best when determining the risk. Fabozzi and Francis (1978) suggested that the beta coefficient is time dependent rather than static. Fabozzi and Francis (1978), Bollerslev et al. (1988), Bodurtha and Mark (1991) and Nelson (1991) calculated time varying betas by the time varying variance method. Gibbons and Ferson (1985), Keim and Stambaugh (1986), Campbell (1987) and Ferson et al. (1987) conducted studies where they examine models that allow returns vary over time (Ng, 1991). Harvey (1989) and Schwert and Seguin (1990) examined and tested time varying CAPM by GMM method and Glejser (1969) weighted least squares method respectively. They both rejected Sharpe-Lintner model. Schwert and Seguin (1990) suggested heteroscedasticity of monthly returns of stocks were closely associated with market and variance and beta was changing with time (Schwert and Seguin, 1990). Harvey (1989) conducted tests of CAPM that allowed conditional covariances to change. He found that high returns were associated with high covariances. According to Harvey (1989), the Sharpe-Lintner model is far from capturing the dynamic movements of the events. Bodurtha and Mark (1991) also concluded that time depending variation was strong. Similarly, Engle (2012) found strong evidence that betas changed with time. Brooks et al. (1998) used a variety of modelling techniques to calculate time-varying betas in their study. In their study with Australian industry portfolios' returns between years 1974 and 1996, they used the techniques multivariate GARCH models, approach of Schwert and Seguin (1990) and Kalman filter techniques. Kalman filter displayed better results than the other techniques. Choudhry and Wu (2008) calculated the time-varying beta values of 20 stocks traded in the FTSE between January 1989 and December 2003. Estimated conditional variances and conditional covariances for finding conditional betas are calculated by applying the multivariate GARCH methods and the Kalman filter. It was suggested that the Kalman filter performed better than the multivariate methods.

In the present study, as a risk measure, performance comparison was made between the methods by estimating the return with time varying variations of betas. As a risk measure, we have estimated the return with dynamic variations of betas, and it has been found that one or more of the methods can provide a better return estimate than the traditional financial asset pricing model.

Numerous studies conducted with different methods regarding time varying risk and risk premium point to time dependent variability. Although there are findings in the opposite direction, this suggestion has been put forward by many important studies.

Dynamic betas have been calculated with two methods: rolling regression and multi variable GARCH methods. DVECH, diagonal BEKK (DBEKK), constant conditional correlation (CCC) and dynamic conditional correlation (DCC) methods were applied. Methods of Bodurtha and Mark (1991), Brooks et al. (1998) and Choudhry and Wu (2008) were applied for Turkish market. It has been found that dynamic methods perform better than static beta, and CCC method yields the lowest error among other dynamic methods when attempting to estimate the return with risk measure beta calculated on the relationship between market and portfolio.

2 Data and methodology

The portfolio series are divided into two parts: the training section and the test section, in another saying, both in-sample and out-of-sample data. The aim of the study is comparing the performance of dynamic betas with the performance of traditional beta. Similar studies were conducted by Choudhry and Wu (2008), Brooks et al. (1998), Morelli (2003) and Bodurtha and Mark (1991).

2.1 Data

Data of the study consists of 30 return portfolios each of which is made up of 15 stocks that were transacted in BIST (Istanbul Stock Exchange) between dates 02.01.2003–29.08.2013. Transactions take place in two session during the working days. First session starts at 9.40 AM and closes 1.00 PM. Second session starts at 1.55 PM and closes at 6.05 PM. During the training and testing period, the market was stable except for the period of mortgage crisis. The decline in market starts with late 2007, hits the bottom in November 2008 and stays there until the end of first quarter of 2009. The market recovered in the end of the same year. More than half of the investors in Turkish markets are foreign. The stock returns in the studied portfolios are all equally weighted. It is suggested that beta calculation of a diversified portfolio is more certain than the beta calculation of one stock [Fama and French, (2004), p.31]. For this reason, it has been preferred to work with portfolios instead of individual stocks. As the financial balance models lean on perpetual time (Merton, 1973; Breeden, 1979), periodical aggregation bias is less in high frequency date. For the sake of big sample, test -statistics get close to asymptotic distribution [Gibbons and Ferson, (1985), p.225]. Thus, daily data was employed in the study.

BIST 100 return has been the benchmark for the market return in the study for the above-mentioned period. Risk-free interest rate is the 3-month term deposit set by Central Bank of Turkish Republic. The stock returns in the portfolio have been calculated over the daily closing prices. In the study, corrected returns have been used. Return corrections have been done according to dividend distribution and splits.

In order to conduct any performance comparison between return forecast performances of traditional CAPM and dynamic CAPM, the data has been split into two as training period and test period. The period between 02.01.2003–31.12.2011 is the training period, and the period between 02.01.2012–29.08.2013 is the test period. The parameters used in the test period have been calculated in the training period.

2.2 Methodology

The study's aim is to compare forecast performances of dynamic betas and traditional beta. In order to calculate the traditional beta, least squares method will be applied. In traditional model, β coefficient is fixed for the period.

$$E(R_i) - R_f = \alpha + \beta(R_m - R_f) + \varepsilon$$

In dynamic approach, two main methods, rolling regression and multivariate GARCH have been applied. Rolling regression was applied with rolling windows of 30, 60, 120, 240 and 360 days. Each day, the regression is renewed with the new day's information.

Rolling regression method assumes that the investor updates beta each day. As in the traditional beta, rolling regressions are conducted by the least square method.

$$E(R_i) - R_f = \alpha + \beta(R_m - R_f) + \varepsilon$$

In the second main method, β is also calculated daily thus the daily return is forecast. In order to calculate the portfolio betas, conditional variance of the market and the conditional covariances between market and the portfolios have been calculated by MGARCH methods.

Bali et al. (2017) stated that static beta is not useful but dynamic betas have significant positive relationship with future returns. In their work, expected return is defined as follows:

$$E(r_{it} | I_{t-1}) = \beta_{it} | I_{t-1} (E(r_{mt} | I_{t-1})) \quad (1.1)$$

The risk factor, conditional beta in equation (1.1) is calculated as follows:

$$\beta_{it} | I_{t-1} = \frac{Cov(r_{it}, r_{mt} | I_{t-1})}{Var(r_{mt} | I_{t-1})}$$

The derivation of conditional beta is as defined:

$$R_{i,d+1} - r_{f,d+1} = \alpha_0^i + \sigma_{i,d+1} \cdot u_{i,d+1} \quad (1.2)$$

$$R_{m,d+1} - r_{f,d+1} = \alpha_0^m + \sigma_{m,d+1} \cdot u_{m,d+1} \quad (1.3)$$

$$E_d [\varepsilon_{i,d+1}^2] \equiv \sigma_{i,d+1}^2 = \beta_0^i + \beta_1^i \sigma_{i,d}^2 u_{i,d}^2 + \beta_2^i \sigma_{i,d}^2 \quad (1.4)$$

$$E_d [\varepsilon_{m,d+1}^2] \equiv \sigma_{m,d+1}^2 = \beta_0^m + \beta_1^m \sigma_{m,d}^2 u_{m,d}^2 + \beta_2^m \sigma_{m,d}^2 \quad (1.5)$$

$$E_d [\varepsilon_{i,d+1} \varepsilon_{m,d+1}] \equiv \sigma_{im,d+1} = \rho_{im,d+1} + \sigma_{i,d+1} \cdot \sigma_{m,d+1} \quad (1.6)$$

$R_{i,d+1} - r_{f,d+1}$ and $R_{m,d+1} - r_{f,d+1}$ denote the day $d + 1$ excess return on stock i and the market portfolio m over risk-free rate, respectively, and E_d denotes that the expectation operator conditional on day d information. $\sigma_{i,d+1}^2$ is the *day-d* expected conditional variance of stock i , $\sigma_{m,d+1}^2$ is the *day-d* expected conditional variance of the market, $\sigma_{im,d+1}^2$ is the *day-d* expected conditional covariance between $R_{i,d+1} - r_{f,d+1}$ and $R_{m,d+1} - r_{f,d+1}$. $u_{i,d} = \frac{\varepsilon_{i,d}}{\sigma_{i,d}}$

and $u_{i,m} = \frac{\varepsilon_{m,d}}{\sigma_{m,d}}$ are standadised residuals for stock i and the portfolio stock m

respectively. $\rho_{im,d+1}$ is the *day-d* expected conditional correlation between $R_{i,d+1} - r_{f,d+1}$ and $R_{m,d+1} - r_{f,d+1}$.

DCC beta is defined as the ratio of equation (1.6) to (1.5):

$$BETA_{i,d+1}^{DCC} = \frac{\sigma_{im,d+1}}{\sigma_{m,d+1}^2}$$

Choudhry and Wu (2008) estimated the conditional betas by the method applied by Bollerslev et al. (1988). They used DVECH and BEKK methods when calculating the conditional variance and conditional covariances. Engle (2012) applied DCC method when calculating the conditional series.

The expected conditional return according to conditional capm is

$$E(r_{it}|I_{t-1}) = \beta_{it}|I_{t-1} (E(r_{mt}|I_{t-1}))$$

The beta risk factor in calculation of expected conditional return is;

$$\beta_{it}|I_{t-1} = \frac{Cov(r_{it}, r_{mt}|I_{t-1})}{Var(r_{mt}|I_{t-1})}$$

Here, conditional covariance, $Cov(r_{it}, r_{mt} | I_{t-1})$ and conditional variance, $Var(r_{mt} | I_{t-1})$ series will be made by MGARCH methods. β_{it} is the beta value of portfolio i at time t . Beta is obtained by dividing the conditional covariance between portfolio and market portfolio to market variance. The slope coefficient in the conditional regression of portfolio return on market return is conditioned information at time $t - 1$ [Lee and Lee, (2006), p.378].

As well as in the studies mentioned above, also in this study, conditional betas were calculated by dividing conditional covariances between portfolio and market returns to market variance. The estimated betas were used in forecasting the portfolio return. In the study, conditional variance and conditional covariances were calculated by MGARCH method. The applied MGARCH methods DVECH method, DBEKK method, CCC method and DCC method. The methods are chosen for their being widely accepted MGARCH methods.

Data is divided into two as training period and test period. The period between 02.01.2003–31.12.2011 is the training period, data between 02.01.2012–29.08.2013 is the test period. In static model, as mentioned before, least squares method has been applied. Returns in the test period were estimated by using the beta obtained in the training period.

Rolling regression method has been applied by windows of 30 days, 60 days, 120 days, 240 days and 360 days. For example, in 30 days window, first 30 days of portfolio samples are put in the regression data, thus the betas of the 30th day are estimated. The beta value of the 31st day is estimated with the data between 2nd day and 31st day. In order to estimate each new day's beta, regression data is rolled one day further.

In MGARCH dynamic models, betas in the testing period were estimated. The parameters of conditional variance and conditional covariances obtained in the training period were used in estimating the conditional variances and conditional covariances in the test period. Thus, the estimated covariances and variances lead to the estimation of dynamic betas. In both traditional and dynamic methods, return estimates in both training and test periods were found by multiplication of market return with estimated betas. Conditional variances and conditional covariances were calculated by MGARCH methods.

In traditional model, the return is estimated as below

$$R(r_{it}) = \beta E(r_{mt})$$

In rolling regression model, return is estimated as below

$$E(R_{it}) = \beta_{t-1}(R_{mt} - R_{ft})$$

In MGARCH dynamic method, the return is estimated as below

$$E(r_{it}|I_{t-1}) = \beta_{it}|I_{t-1}E(r_{mt}).$$

In order to identify whether the dynamic models produce better results than traditional CAPM, root mean squares error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE) methods were applied. MSE (Brooks et al., 1998; Avramov; 2002; Cao et al., 2005; Choudhry and Wu, 2008; Guo, 2006), MAE (Cao et al., 2005; Choudhry and Wu, 2008; Guo, 2006) and MAPE were largely used in literature in order to compare performances between methods.

R_i , realised return, e_i , difference between estimated return and realised return

$$RMSE = \sqrt{\frac{\sum e_i^2}{N}}$$

$$MAE = \frac{\sum |e_i|}{N}$$

$$MAPE = \frac{\sum \left| \frac{e_i}{R_i} \right|}{N}.$$

3 Findings and results

In this section, market variance and the covariances between market and portfolios have been estimated. Return has been predicted from risk. Dynamic variations of capital asset pricing models have been applied. It was thought that dynamic betas would produce better results than static betas would do due to its updating facility. Thus, this expectation was tested by various methods.

The results of study which are given in Table 1 shows that the portfolios are stationary at level. T statistic values are minus.

3.1 Results of static CAPM application

The beta coefficients of static capm are presented in Table 2. The coefficients are in the range of 0.67 and 0.78. Those are low risk profile portfolios. The error term autocorrelations were identified by Breusch-Godfrey test. According to test results, except for P7 and P11, the error terms of the portfolios' regression showed autocorrelation. This led us to the idea of searching a better return estimation method.

Table 1 Stationarity test results

<i>Portfolios</i>	<i>T</i>	<i>(p)</i>	<i>AIC</i>	<i>Lag</i>
PM	-12.55039	0.0000	-5.053099	12
P1	-10.63137	0.0000	-5.2494	13
P2	-10.2776	0.0000	-5.432206	13
P3	-11.75356	0.0000	-5.271682	12
P4	-11.4249	0.0000	-5.257071	12
P5	-10.83302	0.0000	-5.255229	13
P6	-10.42074	0.0000	-5.351161	13
P7	-11.44649	0.0000	-5.412086	12
P8	-11.02172	0.0000	-5.2240	12
P9	-10.51166	0.0000	-5.289781	13
P10	-11.879	0.0000	-5.21005	12
P11	-11.45923	0.0000	-5.333882	12
P12	-10.33352	0.0000	-5.263811	13
P13	-11.10925	0.0000	-5.31522	13
P14	-10.45648	0.0000	-5.379704	13
P15	-10.96558	0.0000	-5.310039	12
P16	-10.42856	0.0000	-5.26429	13
P17	-10.31234	0.0000	-5.331718	13
P18	-10.45948	0.0000	-5.387843	13
P19	-11.47682	0.0000	-5.287722	12
P20	-10.83184	0.0000	-5.367565	12
P21	-10.72492	0.0000	-5.368818	13
P22	-10.56998	0.0000	-5.270433	13
P23	-10.55521	0.0000	-5.331981	13
P24	-11.24838	0.0000	-5.277459	12
P25	-10.91612	0.0000	-5.433542	13
P26	-11.03687	0.0000	-5.180166	12
P27	-10.18465	0.0000	-5.186409	13
P28	-9.981622	0.0000	-5.278047	13
P29	-13.71696	0.0000	-5.382725	9
P30	-13.47821	0.0000	-5.253503	9

Note: Series are stationary at $p < 0.0001$.

As seen in Table 2, β coefficients were meaningful. However, α coefficients were not found meaningful.

Table 2 Traditional beta coefficients of the portfolios

	<i>Traditional beta coefficients of the portfolios</i>			<i>Breusch-Godfrey test results</i>	
	β coefficient	<i>T values</i>	<i>P</i>	$X^2 (N \times R^2)$	<i>p</i>
P1	0.7019	57.2594	0.0000	50.0763	0.0002
P2	0.6715	63.9634	0.0000	58.3718	0.0000
P3	0.7129	60.5747	0.0000	42.2176	0.0026
P4	0.7674	74.8668	0.0000	55.9090	0.0000
P5	0.7508	69.3045	0.0000	40.4113	0.0044
P6	0.7054	66.1342	0.0000	40.7169	0.0040
P7	0.7163	78.2568	0.0000	29.8179	0.0729
P8	0.7167	58.4954	0.0000	46.6198	0.0007
P9	0.7092	61.6896	0.0000	38.4952	0.0077
P10	0.7895	76.3330	0.0000	24.2603	0.2312
P11	0.7270	71.9688	0.0000	34.5861	0.0224
P12	0.7851	83.6479	0.0000	55.2594	0.0000
P13	0.7084	64.7814	0.0000	44.4272	0.0013
P14	0.6825	62.8285	0.0000	40.7147	0.0041
P15	0.7484	75.4848	0.0000	47.1458	0.0006
P16	0.6955	56.8573	0.0000	47.3715	0.0005
P17	0.7077	64.9578	0.0000	58.6157	0.0000
P18	0.6721	60.7884	0.0000	44.7369	0.0012
P19	0.7326	67.3676	0.0000	50.7837	0.0002
P20	0.7083	68.2215	0.0000	54.2255	0.0001
P21	0.7330	77.3091	0.0000	44.0754	0.0015
P22	0.7239	63.7479	0.0000	49.5372	0.0003
P23	0.7142	66.9126	0.0000	50.1216	0.0002
P24	0.7450	70.1105	0.0000	58.0954	0.0000
P25	0.6912	70.9891	0.0000	41.1439	0.0036
P26	0.7945	74.9411	0.0000	39.9081	0.0051
P27	0.7398	59.9720	0.0000	56.0254	0.0000
P28	0.7060	60.3434	0.0000	50.1213	0.0002
P29	0.6894	65.6629	0.0000	39.9904	0.0050
P30	0.7841	80.1420	0.0000	38.0304	0.0088

3.2 Results of rolling regression

Rolling regression is the updating of linear regression with each information that arrives each new day in windows of last 30, 60, 120, 240 and 360 days. Rolling regression method is based on presumption that the investor updates beta. Rolling regression is realised by least squares method as in the classical method.

$$E(R_i) - R_f = \alpha + \beta(R_m - R_f) + \varepsilon$$

Return prediction formula is as given below

$$E(R_{it}) = \beta_{t-1}(R_{mt} - R_{ft}).$$

The best results were obtained from 120 days rolling regression window. Time varying betas estimated with 120 daily window rolling regression method are presented in Annex 5.

3.3 MGARCH beta results

Beta calculated on MGARCH is of dynamic quality. The method updates the estimations and does re-modelling with new information.

We can formulise the estimation as below:

$$r_{it} = R_{it} - R_f$$

$$r_{mt} = R_{mt} - R_f$$

$$E(r_{it}|\psi_{t-1}) = \beta_i|\psi_{t-1}(E(r_{mt}|\psi_{t-1}))$$

$$\beta_{it}|\psi_{t-1} = \frac{Cov(R_{it}, R_{mt}|\psi_{t-1})}{Var(R_{mt}|\psi_{t-1})}$$

R_f is the risk-free rate, r_{it} is the excess return of the portfolio. r_{mt} is the excess market return. ψ_{t-1} , is the market information which is thought to affect the market at time t . ($|\psi_{t-1}$), is the expectation conditioned on information at $t - 1$. Conditional model is the change of the risk premium according to three components depending on time. The three components of the conditional model are conditional variance of market portfolio, the conditional covariance between stock and the market and the market risk premium (Morelli, 2003). Expected return is calculated as

$$E(r_{it}) = \beta_i r_{mt}$$

In dynamic models, error terms that were obtained from portfolios' autoregressive processes have been used instead of portfolio returns. Because the portfolios' returns follow an autoregressive process rather than displaying white noise. The portfolios' returns were found to be significantly correlated with their first, sixth, tenth and 13th lags.

DVECH method did not produce parameters for the conditional variance and conditional covariance for the approach of iteration due to nonlinear form of the model, log-likelihood of the DVECH equation is maximised through iteration (Oztek, 2013). The estimation produced parameters only for the 20th portfolio. The other portfolios could not produce parameters that maximised the log-likelihood. According to DVECH method, being h_{11} the conditional variance of the 20th portfolio, h_{22} the conditional variance of the market portfolio and h_{12} being the conditional covariace between the first portfolio and market portfolio,

$$h_{11,t} = c_1 + a_{11}\varepsilon_{1,t-1}^2 + b_{11}h_{11,t-1}$$

$$h_{12,t} = c_2 + a_{12}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + b_{12}h_{12,t-1}$$

$$h_{22,t} = c_3 + a_{22}\varepsilon_{2,t-1}^2 + b_{22}h_{22,t-1}$$

When we write the equations of those at time t , putting the parameters in place, we will obtain the following:

$$h_{11,t} = 0.0000281 + 0.168487\varepsilon_{1,t-1}^2 + 0.726668h_{11,t-1}$$

$$h_{12,t} = 0.0000231 + 0.125635\varepsilon_{1,t-1}\varepsilon_{2,t-1} + 0.774735h_{12,t-1}$$

$$h_{22,t} = 0.0000252 + 0.101266\varepsilon_{2,t-1}^2 + 0.826654h_{22,t-1}$$

The parameters are presented in Annex 1.

In the application of DBEKK method, the maximisation could not be reached in the estimation of the covariance between P6 and PM. Being h_{11} the conditional variance of the first portfolio, h_{22} the conditional variance of the market portfolio and h_{12} being the conditional covariance between the first portfolio and market portfolio, we may display the equations as follows:

$$h_{11,t} = c_{11}^2 + a_{11}^2\varepsilon_{1,t-1}^2 + b_{22}^3h_{11,t-1}$$

$$h_{22,t} = c_{11}^2 + c_{22}^2 + a_{22}^2\varepsilon_{2,t-1}^2 + b_{22}^2h_{22,t-1}$$

$$h_{12,t} = c_{11}c_{21} + a_{11}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + b_{11}b_{22}h_{12,t-1}$$

When we write the equations of those at time t , putting the parameters in place, we will obtain the following:

$$h_{11,t} = (0.0000275)^2 + (0.353511)^2\varepsilon_{1,t-1}^2 + (0.879728)^2h_{11,t-1}$$

$$h_{22,t} = (0.0000173)^2 + (0.000017)^2 + (0.258158)^2\varepsilon_{2,t-1}^2 + (0.93944)^2h_{22,t-1}$$

$$h_{12,t} = (0.0000275)(0.0000173) + (0.353511)(0.258158)\varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ + (0.879728)(0.93944)h_{12,t-1}$$

The parameters of portfolios according to DBEKK method are presented in Annex 2. The conditional betas according to DBEKK method are presented in Annex 6. The first 2,247 observation belong to the training period and the subsequent observations belong to the test period.

According to CCC method estimation, conditional betas are presented in Annex 7. The first 2,247 observations belong to the training period and the subsequent observations belong to the test period. Parameters of CCC method are given in Annex 3.

In the application of CCC method, the maximisation could not be reached in the estimation of the covariance between P24 and PM. Being h_{11} the conditional variance of the first portfolio, h_{22} the conditional variance of the market portfolio and h_{12} being the conditional covariance between the first portfolio and market portfolio, we may display the equations as follows:

$$h_{11,t} = \omega_1 + \alpha_{p1}\varepsilon_{1,t-1}^2 + \beta_{p1}h_{11,t-1}$$

$$h_{22,t} = \omega_{pm} + \alpha_{pm}\varepsilon_{2,t-1}^2 + \beta_{pm}h_{22,t-1}$$

$$h_{12,t} = \rho_{12} (h_{1,t}/h_{22,t})^{1/2}$$

When we write the equations of those at time t , putting the parameters in place, we will obtain the following:

$$h_{11,t} = 0.0000298 + 0.127542\varepsilon_{1,t-1}^2 + 0.764919h_{11,t-1}$$

$$h_{22,t} = 0.0000205 + 0.074243\varepsilon_{2,t-1}^2 + 0.865517h_{22,t-1}$$

$$h_{12,t} = 0.737358\sqrt{h_{11,t}}\sqrt{h_{22,t}}$$

As the CCC method has been the best performing method among the methods applied in the study, return estimations calculated by this method were presented as graphics (see Annex 8).

The parameters obtained in DCC method are presented in Annex 4. The parameters of 30 portfolios estimated by DCC method are constant (ω), coefficient of error terms (α) and the effect of the previous variance (β).

According to the DCC method, the conditional variance of the first portfolio (P1), conditional variance of the market portfolio and the conditional covariance between those two are denominated with h_{11} , h_{22} and h_{12} , respectively.

$$h_{11,t} = \omega_1 + \alpha_{p1}\varepsilon_{1,t-1}^2 + \beta_{p1,t-1}$$

$$h_{22,t} = \omega_{pm} + \alpha_{pm}\varepsilon_{2,t-1}^2 + \beta_{pm,t-1}$$

When we write the equations of those at time t , putting the parameters in place, we will obtain the following:

$$h_{11,t} = 0.0000187 + 0.163837\varepsilon_{1,t-1}^2 + 0.78421h_{11,t-1}$$

$$h_{22,t} = 0.0000152 + 0.106041\varepsilon_{2,t-1}^2 + 0.855014h_{22,t-1}$$

$$h_{12,t} = R_t\sqrt{h_{11,t}}\sqrt{h_{22,t}}$$

$$R_t = (\text{Diag}(Q_t))^{-1/2} Q_t (\text{Diag}(Q_t))^{-1/2}$$

$$Q_t = (1 - \lambda_1 - \lambda_2)\bar{Q} + \lambda_1 v_{t-1} v'_{t-1} + \lambda_2 Q_{t-1}$$

$$Q_t = (1 - 0.033548 - 0.928764)0.727555 + 0.033548v_{t-1}v'_{t-1} + 0.928764Q_{t-1}$$

Conditional variance and conditional covariance parameters estimated by DCC method are presented in Annex 4. Conditional betas according to DCC estimation method are presented in Annex 9. The first 2,247 observation belong to the training period and the subsequent observations belong to the test period.

3.4 Performance comparison between traditional and dynamic CAPM

The performances have been compared by RMSE, MAE and MAPE methods. The results are presented in Table 3–Table 8. In sample and out of sample comparisons were made separately.

Table 3 In sample performance comparison according to RMSE

	<i>RMSE – in sample</i>									
	<i>Traditional</i>	<i>DBEKK</i>	<i>CCC</i>	<i>DCC</i>	<i>Rolling (30 days)</i>	<i>Rolling (60 days)</i>	<i>Rolling (120 days)</i>	<i>Rolling (240 days)</i>	<i>Rolling (360 days)</i>	
P1	0.0112	0.0111	0.0111	0.0111	0.0114	0.0112	0.0113	0.0114	0.0114	0.0114
P2	0.0096	0.0095	0.0095	0.0095	0.0098	0.0097	0.0096	0.0098	0.0098	0.0099
P3	0.0108	0.0106	0.0106	0.0106	0.0109	0.0108	0.0108	0.0110	0.0110	0.0109
P4	0.0094	0.0092	0.0092	0.0093	0.0095	0.0094	0.0094	0.0093	0.0093	0.0093
P5	0.0100	0.0098	0.0099	0.0099	0.0101	0.0099	0.0113	0.0102	0.0102	0.0102
P6	0.0098	-	0.0097	0.0097	0.0100	0.0099	0.0098	0.0100	0.0100	0.0098
P7	0.0084	0.0082	0.0082	0.0083	0.0085	0.0085	0.0085	0.0087	0.0087	0.0087
P8	0.0113	0.0111	0.0112	0.0111	0.0114	0.0113	0.0113	0.0114	0.0114	0.0115
P9	0.0105	0.0104	0.0104	0.0104	0.0107	0.0106	0.0106	0.0105	0.0105	0.0104
P10	0.0094	0.0092	0.0093	0.0093	0.0095	0.0094	0.0094	0.0095	0.0095	0.0094
P11	0.0093	0.0092	0.0091	0.0092	0.0095	0.0093	0.0093	0.0095	0.0095	0.0095
P12	0.0086	0.0085	0.0085	0.0085	0.0088	0.0087	0.0087	0.0088	0.0088	0.0088
P13	0.0100	0.0098	0.0098	0.0098	0.0102	0.0101	0.0100	0.0101	0.0101	0.0099
P14	0.0100	0.0097	0.0098	0.0097	0.0101	0.0100	0.0099	0.0100	0.0100	0.0099
P15	0.0091	0.0089	0.0091	0.0089	0.0092	0.0092	0.0092	0.0092	0.0092	0.0091
P16	0.0112	0.0109	0.0110	0.0110	0.0113	0.0113	0.0112	0.0114	0.0114	0.0115
P17	0.0100	0.0098	0.0098	0.0098	0.0101	0.0101	0.0101	0.0102	0.0102	0.0102
P18	0.0101	0.0101	0.0100	0.0101	0.0103	0.0101	0.0101	0.0102	0.0102	0.0102
P19	0.0100	0.0098	0.0098	0.0098	0.0100	0.0099	0.0099	0.0100	0.0100	0.0101
P20	0.0095	0.0189	0.0093	0.0094	0.0097	0.0097	0.0096	0.0097	0.0097	0.0097
P21	0.0087	0.0086	0.0086	0.0086	0.0088	0.0087	0.0087	0.0088	0.0088	0.0088
P22	0.0104	0.0102	0.0103	0.0103	0.0106	0.0105	0.0105	0.0106	0.0106	0.0106
P23	0.0098	0.0096	0.0096	0.0095	0.0099	0.0099	0.0099	0.0100	0.0100	0.0100
P24	0.0098	0.0094	-	0.0094	0.0098	0.0098	0.0097	0.0098	0.0098	0.0097
P25	0.0089	0.0086	0.0087	0.0120	0.0090	0.0089	0.0088	0.0090	0.0090	0.0090
P26	0.0098	0.0097	0.0097	0.0098	0.0100	0.0098	0.0098	0.0098	0.0098	0.0098
P27	0.0113	0.0111	0.0111	0.0111	0.0114	0.0113	0.0114	0.0115	0.0115	0.0115
P28	0.0107	0.0105	0.0105	0.0105	0.0109	0.0108	0.0107	0.0108	0.0108	0.0108
P29	0.0096	0.0094	0.0094	0.0094	0.0098	0.0096	0.0096	0.0097	0.0097	0.0096
P30	0.0090	0.0088	0.0088	0.0088	0.0091	0.0089	0.0089	0.0089	0.0089	0.0089

Table 4 Out-of-sample performance comparison according to RMSE

	<i>RMSE – out of sample</i>									
	<i>Traditional</i>	<i>DBEKK</i>	<i>CCC</i>	<i>DCC</i>	<i>Rolling (30 days)</i>	<i>Rolling (60 days)</i>	<i>Rolling (120 days)</i>	<i>Rolling (240 days)</i>	<i>Rolling (360 days)</i>	
P1	0.0086	0.0089	0.0084	0.0088	0.0094	0.0090	0.0088	0.0089	0.0088	
P2	0.0086	0.0088	0.0086	0.0087	0.0093	0.0090	0.0087	0.0087	0.0087	
P3	0.0087	0.0093	0.0085	0.0089	0.0095	0.0094	0.0092	0.0091	0.0091	
P4	0.0071	0.0074	0.0069	0.0071	0.0078	0.0076	0.0075	0.0075	0.0076	
P5	0.0077	0.0086	0.0076	0.0081	0.0087	0.0084	0.0088	0.0081	0.0081	
P6	0.0095	-	0.0073	0.0077	0.0086	0.0084	0.0080	0.0080	0.0079	
P7	0.0065	0.0068	0.0064	0.0066	0.0069	0.0068	0.0066	0.0066	0.0066	
P8	0.0082	0.0083	0.0076	0.0082	0.0086	0.0084	0.0081	0.0082	0.0083	
P9	0.0080	0.0083	0.0078	0.0082	0.0085	0.0085	0.0083	0.0084	0.0085	
P10	0.0080	0.0081	0.0078	0.0080	0.0082	0.0080	0.0079	0.0080	0.0081	
P11	0.0085	0.0085	0.0082	0.0085	0.0093	0.0088	0.0086	0.0086	0.0087	
P12	0.0068	0.0066	0.0065	0.0066	0.0069	0.0069	0.0067	0.0068	0.0068	
P13	0.0089	0.0088	0.0083	0.0087	0.0099	0.0096	0.0094	0.0095	0.0095	
P14	0.0076	0.0078	0.0076	0.0079	0.0080	0.0080	0.0077	0.0078	0.0079	
P15	0.0076	0.0078	0.0076	0.0076	0.0079	0.0076	0.0075	0.0075	0.0075	
P16	0.0097	0.0096	0.0094	0.0097	0.0106	0.0104	0.0100	0.0101	0.0102	
P17	0.0083	0.0088	0.0081	0.0085	0.0089	0.0087	0.0086	0.0087	0.0088	
P18	0.0080	0.0080	0.0076	0.0080	0.0087	0.0085	0.0085	0.0085	0.0084	
P19	0.0081	0.0086	0.0080	0.0083	0.0088	0.0086	0.0084	0.0083	0.0083	
P20	0.0078	0.0086	0.0075	0.0079	0.0087	0.0084	0.0080	0.0081	0.0081	
P21	0.0087	0.0088	0.0085	0.0087	0.0097	0.0089	0.0088	0.0089	0.0089	
P22	0.0080	0.0082	0.0078	0.0082	0.0086	0.0085	0.0081	0.0083	0.0083	
P23	0.0080	0.0083	0.0078	0.0080	0.0087	0.0086	0.0083	0.0084	0.0084	
P24	0.0080	0.0081	-	0.0079	0.0084	0.0082	0.0080	0.0080	0.0080	
P25	0.0068	0.0079	0.0066	0.0078	0.0074	0.0074	0.0072	0.0071	0.0071	
P26	0.0074	0.0073	0.0071	0.0078	0.0074	0.0071	0.0070	0.0071	0.0071	
P27	0.0106	0.0103	0.0101	0.0104	0.0118	0.0112	0.0108	0.0108	0.0107	
P28	0.0095	0.0096	0.0092	0.0095	0.0101	0.0100	0.0097	0.0097	0.0098	
P29	0.0076	0.0076	0.0073	0.0075	0.0080	0.0078	0.0078	0.0079	0.0080	
P30	0.0068	0.0072	0.0067	0.0070	0.0073	0.0072	0.0069	0.0069	0.0069	

Table 5 In sample performance comparison according to MAE

	MAE – in sample									
	Traditional	DBEKK	CCC	DCC	Rolling (30 days)	Rolling (60 days)	Rolling (120 days)	Rolling (240 days)	Rolling (360 days)	
P1	0.0083	0.0081	0.0081	0.0081	0.0084	0.0082	0.0080	0.0076	0.0071	0.0071
P2	0.0072	0.0071	0.0070	0.0071	0.0073	0.0071	0.0069	0.0066	0.0066	0.0063
P3	0.0079	0.0077	0.0076	0.0077	0.0080	0.0078	0.0076	0.0073	0.0073	0.0068
P4	0.0070	0.0068	0.0068	0.0068	0.0070	0.0069	0.0067	0.0062	0.0062	0.0059
P5	0.0073	0.0072	0.0072	0.0072	0.0074	0.0072	0.0080	0.0067	0.0067	0.0063
P6	0.0072	-	0.0071	0.0071	0.0073	0.0072	0.0069	0.0066	0.0066	0.0061
P7	0.0062	0.0060	0.0060	0.0060	0.0062	0.0061	0.0060	0.0057	0.0057	0.0053
P8	0.0083	0.0081	0.0081	0.0081	0.0084	0.0082	0.0080	0.0076	0.0076	0.0072
P9	0.0082	0.0077	0.0077	0.0077	0.0079	0.0078	0.0076	0.0071	0.0071	0.0066
P10	0.0071	0.0069	0.0069	0.0069	0.0071	0.0069	0.0067	0.0064	0.0064	0.0059
P11	0.0068	0.0067	0.0067	0.0067	0.0070	0.0068	0.0066	0.0063	0.0063	0.0060
P12	0.0064	0.0063	0.0063	0.0063	0.0065	0.0063	0.0061	0.0059	0.0059	0.0056
P13	0.0074	0.0071	0.0071	0.0072	0.0075	0.0074	0.0071	0.0067	0.0067	0.0062
P14	0.0073	0.0071	0.0071	0.0071	0.0074	0.0073	0.0071	0.0066	0.0066	0.0062
P15	0.0069	0.0068	0.0068	0.0068	0.0069	0.0068	0.0067	0.0063	0.0063	0.0058
P16	0.0082	0.0080	0.0080	0.0080	0.0082	0.0081	0.0079	0.0076	0.0076	0.0071
P17	0.0074	0.0072	0.0072	0.0073	0.0075	0.0074	0.0071	0.0068	0.0068	0.0064
P18	0.0074	0.0073	0.0073	0.0073	0.0075	0.0073	0.0071	0.0068	0.0068	0.0064
P19	0.0074	0.0072	0.0072	0.0072	0.0074	0.0073	0.0071	0.0067	0.0067	0.0064
P20	0.0069	0.0119	0.0068	0.0068	0.0069	0.0069	0.0067	0.0064	0.0064	0.0060
P21	0.0064	0.0063	0.0062	0.0063	0.0064	0.0063	0.0061	0.0058	0.0058	0.0055
P22	0.0076	0.0074	0.0075	0.0075	0.0077	0.0075	0.0073	0.0070	0.0070	0.0065
P23	0.0073	0.0071	0.0071	0.0071	0.0074	0.0073	0.0071	0.0067	0.0067	0.0063
P24	0.0072	0.0069	-	0.0070	0.0073	0.0071	0.0069	0.0066	0.0066	0.0061
P25	0.0065	0.0063	0.0064	0.0089	0.0066	0.0065	0.0062	0.0059	0.0059	0.0056
P26	0.0073	0.0073	0.0073	0.0073	0.0075	0.0073	0.0071	0.0067	0.0067	0.0062
P27	0.0083	0.0081	0.0081	0.0081	0.0084	0.0082	0.0080	0.0076	0.0076	0.0072
P28	0.0079	0.0076	0.0077	0.0076	0.0079	0.0078	0.0075	0.0071	0.0071	0.0067
P29	0.0072	0.0070	0.0070	0.0070	0.0073	0.0071	0.0069	0.0065	0.0065	0.0061
P30	0.0067	0.0065	0.0066	0.0066	0.0068	0.0066	0.0064	0.0061	0.0061	0.0057

Table 6 Out-of-sample performance comparison according to MAE

	MAE – out of sample									
	Traditional	DBEKK	CCC	DCC	Rolling (30 days)	Rolling (60 days)	Rolling (120 days)	Rolling (240 days)	Rolling (360 days)	
P1	0.0066	0.0063	0.0063	0.0063	0.0065	0.0064	0.0063	0.0065	0.0065	0.0065
P2	0.0066	0.0064	0.0064	0.0066	0.0067	0.0065	0.0064	0.0064	0.0064	0.0065
P3	0.0066	0.0063	0.0062	0.0065	0.0065	0.0064	0.0063	0.0063	0.0063	0.0065
P4	0.0053	0.0054	0.0052	0.0054	0.0055	0.0054	0.0054	0.0054	0.0054	0.0055
P5	0.0058	0.0058	0.0056	0.0059	0.0058	0.0058	0.0063	0.0058	0.0058	0.0058
P6	0.0055	-	0.0053	0.0056	0.0056	0.0056	0.0054	0.0055	0.0055	0.0055
P7	0.0050	0.0047	0.0048	0.0050	0.0048	0.0047	0.0047	0.0048	0.0048	0.0049
P8	0.0064	0.0059	0.0058	0.0064	0.0060	0.0058	0.0058	0.0059	0.0059	0.0061
P9	0.0060	0.0059	0.0058	0.0063	0.0060	0.0058	0.0058	0.0059	0.0059	0.0060
P10	0.0063	0.0058	0.0060	0.0061	0.0059	0.0059	0.0059	0.0060	0.0061	0.0061
P11	0.0056	0.0055	0.0054	0.0056	0.0058	0.0055	0.0054	0.0055	0.0055	0.0056
P12	0.0052	0.0049	0.0050	0.0050	0.0050	0.0049	0.0048	0.0049	0.0049	0.0050
P13	0.0065	0.0063	0.0061	0.0065	0.0067	0.0065	0.0063	0.0064	0.0064	0.0066
P14	0.0058	0.0057	0.0057	0.0061	0.0060	0.0058	0.0057	0.0057	0.0057	0.0058
P15	0.0060	0.0058	0.0058	0.0059	0.0058	0.0057	0.0057	0.0057	0.0057	0.0058
P16	0.0073	0.0071	0.0070	0.0074	0.0076	0.0073	0.0071	0.0072	0.0072	0.0073
P17	0.0062	0.0060	0.0059	0.0060	0.0061	0.0060	0.0060	0.0060	0.0060	0.0062
P18	0.0060	0.0059	0.0057	0.0062	0.0060	0.0061	0.0060	0.0060	0.0060	0.0062
P19	0.0060	0.0061	0.0059	0.0062	0.0061	0.0060	0.0060	0.0060	0.0060	0.0061
P20	0.0056	0.0060	0.0054	0.0058	0.0058	0.0057	0.0055	0.0056	0.0056	0.0056
P21	0.0057	0.0055	0.0055	0.0056	0.0058	0.0054	0.0054	0.0055	0.0055	0.0056
P22	0.0063	0.0058	0.0060	0.0063	0.0060	0.0059	0.0059	0.0061	0.0061	0.0062
P23	0.0060	0.0058	0.0058	0.0060	0.0060	0.0059	0.0059	0.0060	0.0060	0.0061
P24	0.0062	0.0060	-	0.0060	0.0060	0.0059	0.0058	0.0058	0.0058	0.0058
P25	0.0050	0.0052	0.0048	0.0058	0.0050	0.0050	0.0050	0.0051	0.0051	0.0052
P26	0.0059	0.0052	0.0055	0.0061	0.0053	0.0051	0.0051	0.0052	0.0052	0.0054
P27	0.0072	0.0069	0.0069	0.0071	0.0075	0.0072	0.0070	0.0070	0.0070	0.0070
P28	0.0070	0.0068	0.0068	0.0071	0.0068	0.0068	0.0067	0.0068	0.0068	0.0070
P29	0.0058	0.0056	0.0056	0.0057	0.0059	0.0056	0.0056	0.0057	0.0057	0.0058
P30	0.0052	0.0050	0.0050	0.0052	0.0049	0.0049	0.0048	0.0049	0.0049	0.0050

Table 7 In sample performance comparison according to MAPE

	MAPE – in sample									
	Traditional	DBEKK	CCC	DCC	Rolling (30 days)	Rolling (60 days)	Rolling (120 days)	Rolling (240 days)	Rolling (360 days)	
P1	35.1179	24.7029	26.8227	21.9402	16.5511	22.6013	28.4616	25.0948	30.9812	
P2	5.1953	5.3731	5.1042	5.4470	5.4570	5.5376	5.4566	5.0305	4.8461	
P3	6.5275	6.0201	5.8762	6.0885	6.6651	6.7816	6.5939	5.6917	5.3038	
P4	4.2776	3.8669	3.9430	3.8089	4.0442	4.0959	4.1843	3.9784	3.8778	
P5	2.5054	2.1873	2.2554	2.1883	2.1757	2.2404	28.4616	1.8252	1.7049	
P6	3.1016	-	2.9198	2.8563	2.7247	2.9571	2.8234	2.8138	2.7443	
P7	2.9788	2.6815	2.8007	2.7273	2.5020	2.5163	2.7741	2.5414	2.4733	
P8	3.0547	2.5628	2.4954	2.4722	2.9433	2.8700	2.7540	2.6622	2.4948	
P9	2.8088	2.5320	2.6064	2.5648	2.5126	2.6038	2.6493	2.5549	2.4798	
P10	4.3011	4.0688	4.1358	4.1470	5.8471	6.3786	5.6133	4.1640	4.0993	
P11	2.9175	2.7293	2.7577	2.7372	2.8305	2.8175	2.5857	2.4764	2.3539	
P12	2.6451	2.5031	2.5344	2.5489	2.4683	2.5010	2.5910	2.4980	2.2955	
P13	4.0669	3.5061	3.6843	3.4922	3.3377	3.8836	3.8153	3.5872	3.4507	
P14	2.5438	2.2718	2.3535	2.3027	2.3994	2.3049	2.3872	2.3745	2.2541	
P15	3.6049	3.3721	3.3864	3.4413	2.9057	2.5795	2.9579	2.9063	2.8075	
P16	4.0281	3.7390	3.8410	3.8343	3.6315	3.6862	3.1727	3.2518	3.1906	
P17	6.0470	5.1518	5.4142	5.1348	5.3310	5.0040	5.8206	5.7320	5.7367	
P18	9.7045	8.9889	8.8790	9.0245	7.8744	9.3487	8.5358	8.9320	9.3749	
P19	2.5690	2.3316	2.4174	2.3424	2.3616	2.4758	2.4771	2.3015	2.2214	
P20	3.5455	1.1873	3.0874	3.0095	2.9867	3.1892	3.3098	3.1955	3.0725	
P21	4.0459	3.6208	3.7169	3.6213	3.7649	3.5243	3.6132	3.6697	3.6430	
P22	6.4357	5.5808	5.8656	5.5159	5.2146	5.6761	5.8220	5.7972	5.9696	
P23	3.2123	2.9499	2.9971	2.9590	2.9575	3.0772	2.8769	2.6408	2.5457	
P24	3.4853	3.1430	-	3.1415	3.2623	3.1823	3.1281	2.9021	2.8337	
P25	3.4151	3.0235	3.1405	5.0570	3.2286	3.4915	3.2952	3.1569	3.0060	
P26	2.0299	1.9049	1.9154	1.8998	1.9186	1.9962	1.9283	1.7851	1.7812	
P27	3.5177	3.1982	3.3627	3.3851	2.9733	3.0692	3.4541	3.2133	3.3821	
P28	5.0412	4.4201	4.5523	4.3966	4.9526	4.6131	4.3889	4.4444	4.1798	
P29	3.3350	3.1531	3.1862	3.1705	3.2078	3.2248	3.1758	3.1150	2.9648	
P30	3.1331	2.9463	2.9583	2.9541	2.7390	2.7341	2.8073	2.2003	1.9516	

Table 8 Out of sample performance comparison according to MAPE

Traditional	MAPE – out of sample									
	DBEKK	CCC	DCC	Rolling (30 days)	Rolling (60 days)	Rolling (120 days)	Rolling (240 days)	Rolling (360 days)		
P1	2.3696	1.5807	2.1483	2.3581	1.8463	1.9173	2.1091	2.2261		
P2	2.2509	1.6940	2.1742	2.2870	1.7099	1.8824	2.0236	2.1560		
P3	2.8329	1.8387	2.5478	2.7110	1.9959	2.0549	2.3659	2.6440		
P4	8.8457	5.0448	7.7608	7.7465	6.8164	8.7653	7.8482	9.6497		
P5	3.7655	3.2403	3.6115	4.0878	3.2592	1.9173	3.5422	3.6918		
P6	2.5011	-	2.3822	2.5678	1.9896	2.0645	2.2789	2.4484		
P7	3.0486	2.1005	2.7896	2.9444	2.4220	2.4062	2.6198	2.8770		
P8	2.3558	1.6897	1.9915	2.4095	1.7238	1.7546	1.9313	2.0877		
P9	2.4063	1.6910	2.2341	2.5626	1.8967	2.0038	2.2270	2.3788		
P10	5.1268	3.5565	4.7428	4.8816	5.4541	3.7015	4.0306	4.4409		
P11	3.1870	2.4563	3.0462	3.3067	2.4796	2.8471	3.1998	3.3171		
P12	3.5244	2.3362	3.2646	3.3062	2.3544	2.5936	2.6812	2.8609		
P13	3.2078	2.1885	2.8866	3.1277	2.6606	3.0915	3.2080	3.3365		
P14	2.4285	1.5291	2.2787	2.5155	1.8356	1.9128	2.0888	2.3908		
P15	2.0043	1.3853	1.8463	1.8703	1.4023	1.5811	1.6346	1.7489		
P16	3.0242	1.8311	2.8597	3.2200	2.1013	2.0365	2.7581	2.9151		
P17	2.9149	2.0529	2.6170	2.5637	2.0804	2.2428	2.3152	2.5296		
P18	7.3805	8.5191	7.2295	8.9898	9.7708	8.4161	7.7838	7.7813		
P19	2.2346	1.7273	2.1187	2.2867	1.9120	2.0349	2.1415	2.2528		
P20	2.5232	2.1171	3.2245	2.6393	2.1572	2.2216	2.3117	2.4364		
P21	2.2132	1.6882	2.1970	2.2062	1.6683	1.8444	1.9577	2.0546		
P22	9.0651	6.7053	8.7276	9.9801	6.8859	8.4463	8.7622	9.8052		
P23	17.5611	15.4988	16.6467	18.7507	16.9184	18.7357	19.2842	18.8601		
P24	3.2853	2.0751	-	3.0001	2.2758	2.5711	2.5876	2.7497		
P25	7.3815	2.6426	6.5612	8.2531	5.0570	5.6544	6.0370	6.9713		
P26	3.9609	1.9925	3.7238	4.1773	2.2838	2.4784	2.9410	3.3112		
P27	3.4650	2.0471	3.1528	3.5246	2.3384	2.4719	2.7855	2.9986		
P28	4.4055	2.3859	3.9363	4.0127	2.2120	3.1535	3.6057	4.4085		
P29	3.0017	1.9995	2.7542	2.7991	2.1127	2.3256	2.4912	2.7076		
P30	2.9158	2.0733	2.7063	2.8575	2.1517	2.2444	2.4172	2.6040		

3.4.1 *In sample and out-of-sample performance comparison according to RMSE*

According to RMSE comparison method results, DBEKK, CCC and DCC performed better than traditional method in sample. In comparison to traditional method, DBEKK method performed better except for P18 and P20. Similarly, CCC method performed better in all portfolios except for P15. DCC method also performed better than the traditional method except for P18, P25 and P26. In sample, 30 days window rolling regression method could not beat the traditional method in any of the portfolios. 60 days window rolling regression method performed better than the traditional in P5, P19 and P30. 120 days window rolling regression method produced better results than the traditional method in P14, P19, P24, P25 and P30. The 240 days window rolling method produced better results than the traditional method in P4 and P30. 360 days window rolling regression produced better results in P4, P9, P13, P14, P24 and P30. In sample, DBEKK, CCC and DCC methods produced approximate results according to RMSE method.

Out-of-sample, DBEKK performed better than traditional method in P12, P13, P16, P26 and P27 according to RMSE. CCC produced lower errors in portfolios other than P2, P14 and P15. DCC method produced lower root mean squared errors compared to traditional method in P12, P13, P24, P27, and P29. According to RMSE, out of sample, 30 days window rolling regression method performed poor compared to traditional method out-of-sample. 60 days, 120 days, 240 days and 360 days window rolling regression produced better results in P26; P8, P10, P12, P15, P26; P8, P15, P26; P15 and P26, respectively. CCC method produced best results out of sample according to RMSE.

3.4.2 *In sample and out-of-sample performance comparison according to MAE*

In sample, according to MAE, betas calculated by DBEKK, CCC and DCC methods produced better results than traditional method in portfolios other than P20 and P26; P26; P25 and P26, respectively. 30 days window rolling regression method did not produce better results than the traditional method in any of the portfolios. 60 days window rolling regression performed better in portfolios other than P6, P9, P11, P13, P14, P17, P20, P23, P25 and P26 in sample. 120 days window rolling regression method produced lower MAEs than traditional method in portfolios except for P5. In sample, rolling regression method of both 240 and 360 days performed better than the traditional method in all portfolios.

Out of sample, only CCC method could perform better than the traditional method in all portfolios. DBEKK method produced better results than the traditional method in portfolios other than P4, P5, P6, P19, P20 and P25. DCC method became more successful out-of-sample in portfolios except for P1, P3, P10, P12, P15, P17, P24, P27 and P29. 30 days rolling regression method produced better results than the traditional in P1, P3, P7, P8, P10, P12, P15, P17, P22, P24, P26, P28 and P30. 60 days rolling regression method also performed better in the same portfolios as 30 days window. However, 60 days window also succeeded in portfolios P2, P9, P11, P21, P23 and P29. 120 days window rolling regression method produced lower MAEs except for P4, P5, P18, P19 and P25. In rolling regression method, until 120 days, the performance increased gradually as the number of days increased. Performance decreased in 240 days window. This window could not beat the traditional method in portfolios P4, P5, P6, P18, P19,

P20, P21, P23 and P25. In 360 days rolling regression method, performance decreased further and displayed better results than the traditional method only in P1, P2, P3, P7, P8, P10, P12, P15, P22, P24, P26 and P27 out of sample. CCC method performed better than other methods in P3, P4, P5, P6, P13, P16, P17, P18, P19, P20 and P25. Against this, 120 days window rolling regression method produced better results than other methods in P12, P28 and P30, out of sample. In other portfolios, CCC, 60 days and 120 days window rolling regression methods produced approximate MAEs. To sum up, CCC method estimated returns more successfully than other methods according to both RMSE comparison and MAE comparison.

3.4.3 In sample and out-of-sample performance comparison according to MAPE

In sample, according to MAPE, CCC produced better results in all portfolios other than P24, for which it did not produce parameters. Similarly, DBEKK produced better results than the traditional method except for P2 and P6 for which it did not produce any parameter.

DCC method produced better results in all portfolios other than P2 and P25 than the traditional method in sample. 30 days and 60 days window rolling regression methods produced better results than the traditional method except for P2, P3 and P10; P2, P3, P10 and P25, respectively. 120 days window rolling regression method could not outperform the traditional method in P2, P3, P5 and P10. 240 days and 360 days window rolling regression methods have been proved to be better than the traditional method in all portfolios in the study.

Out-of-sample, CCC produced lower mean percentage errors than the traditional method in all portfolios except for P24, for which it could not produce parameters. On the other hand, DBEKK produced lower errors in the current study's portfolios except for P18 and P6, for which it could not produce parameters. DCC method has been less successful compared to CCC and DBEKK out-of-sample. It performed better than the traditional method in P1, P3, P4, P7, P10, P12, P13, P15, P17, P21, P24 and the last three portfolios. 30 days window rolling regression method produced better results than the traditional method in P10 and P18 out-of-sample, 60 days rolling window produced lower errors in all portfolios except for P18. Similarly, 120 days and 240 days window rolling regressions could not outperform traditional method in P18 and P23. 240 days window rolling regression method did not produce better results for P11 and P13 either. 360 days window rolling regression method performed worse than other methods. This method outperformed the traditional method only in P4, P11, P13, P18, P19, P22, P23 and P28.

DVECH method, which produced meaningful parameters only in P20, did not perform better than the traditional method. It produced root mean squared errors of 0.0094 and 0.0201, in-sample and out-of-sample, respectively. In sample, a MAE of 0.0068 and a mean percentage error of 3.0569 were calculated respectively. Out-of-sample, the MAE and mean percentage error are 0.0087 and 4.3126, respectively. Those results indicate that DVECH produced better results than traditional method when compared out-of-sample.

4 Discussion

Time varying betas' return predictions proved to be better than traditional method. The results of the study are consistent with Harvey (1989) who concluded that higher returns were associated with higher covariances and the traditional method is not able to capture the returns' dynamic movements. The study results are also consistent with the results of Bodurtha and Mark (1991) suggesting that time varying variation is strong. The findings are in line with Engle (2012) who suggested that DCC produced better results than the traditional method. Godeiro (2013) who suggested that beta was time dependent and Bollerslev et al. (1988) supported our evidences.

In current study, the performance of the time dependent betas outperformed the performance of static betas. According to RMSE comparison method, DBEKK, CCC and DCC methods predicted returns better in sample than the static method did. In sample, 120 days window rolling regression produced better results than other day windows. Out of sample, both MGARCH and rolling regression betas produced better results than static betas. However, the errors produced by CCC method are smaller than the errors produced by other MGARCH methods. According to MAE method, all dynamic betas except for the ones calculated by 30 days and 60 days window rolling regression method produced better results than traditional method in sample. Out of sample, CCC method performed better than static method in some portfolios where other dynamic methods did not. According to MAPE method, in sample, MGARCH methods and rolling regression method except for 30 days and 60 days windows displayed approximate performances against traditional method. Out of sample, CCC, DBEKK and rolling regression methods except for 360 days window displayed approximate success against traditional method.

In the study, risk has been studied through the time varying variations of capital asset pricing model. The first and second dynamic estimations have been with rolling regression and MGARCH methods, respectively. DVECH, DBEKK, CCC and DCC methods have been applied. While DVECH and DBEKK estimates the covariance directly, CCC and DCC methods do the estimation indirectly. Although it seems that DVECH and DBEKK methods are superior as those calculate the covariances directly, those methods have a disadvantage of parameter and operation abundance. Another disadvantage of DVECH model against DBEKK model is that the variance-covariance matrix may not be positive definite. CCC and DCC methods are superior for fewer operations and parameters, also variance-covariance matrices are always positive definite. It was unknown for Turkish capital market, which of those methods would predict returns better. An answer to this question was sought in this study conducted with 30 portfolios each randomly made of 15 different stocks transacted in BIST ALL. Out-of-sample, CCC method proved to be better not only against the traditional method but also the alternative methods. This shows that CCC method does a better covariance and variance estimation than the other MGARCH methods in Turkish market. Among the rolling regression methods, 120 days window rolling regression proved to be better than other windows.

For the corporate governors, the beta values are significant while deciding not only on the capital structure, but also on investment valuation. For this reason, it is important that the betas' return estimation deviation from real returns would be as small as possible. The findings of the study are leading to the suggestion that the investors prefer MGARCH CCC method or 120 days window rolling regression method and thus calculate dynamic beta instead of using static beta values to predict future returns.

Appendices/Supplementary materials are available on request by emailing the corresponding author.

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