A Stackelberg game for ordering and pricing policies in a decentralised dual-channel supply chain

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Abstract: This study explores ordering and pricing policies in a decentralised two-echelon dual-channel supply chain including one manufacturer and one retailer. The manufacturer produces in a lot to supply an integer number of the retailer's order and also to meet the demand of direct sales channel. It is assumed that the manufacturer has more market power than the retailer and he is the leader of a Stackelberg game. The concavity of the manufacturer profit function is proved, which based on it, a solution procedure is proposed by using game theory and Nelder-Mead algorithm to find the optimal pricing and ordering policies for the manufacturer and the retailer. The numerical example is provided to investigate the features of the proposed model. It is shown that the inventory holding costs, setup costs and ordering costs have negligible effects on optimal prices and also by increasing the price elasticity of the demand of one channel to the price of the other channel, both members can achieve more profit.

Keywords: inventory control; dual-channel supply chain; pricing; game theory; Stackelberg game.

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1 Introduction

Increasing development of electronic commerce in the whole world leads many firms to add an online channel for selling their products. The prominent firms such as Nike, Estee Lauder, Mattel, Eastman Kodak, IBM, the former Compaq, Dell computer, Cisco Systems, etc. have added new channel for selling directly (Tsay and Agrawal, 2004). Adding new direct channel creates new opportunities and also new challenges. The dual-channel supply chain can expand the market share and attract new customers for the manufacturer but it can have negative impacts on retail channel. Therefore, optimisation of a dual-channel supply chain needs special attention.

A large body of literature has studied the dual-channel supply chain from different perspectives. Chiang et al. (2003) explored the effects of adding new direct channel and designing a dual-channel supply chain. Chiang and Monahan (2005) presented a two-echelon dual-channel inventory model and the demand is stochastic. Dumrongsiri et al. (2008) studied a dual-channel supply chain in which a manufacturer sells to a retailer as well as to the consumers directly such that consumers can choose a channel to buy the product. Huang and Swaminathan (2009) studied the optimal pricing strategies for a product which is sold through the Internet and a traditional channels and the demand of product depends on prices, the degree of substitution across the channels and the overall market potential. Takahashi et al. (2011) considered a two-echelon dual-channel supply chain with setup of production and delivery in which the demand is stochastic. Panda et al. (2015) explored pricing and replenishment policies for a high-tech product whose unit cost decreases over its short life cycle. Batarfi et al. (2016) investigated the effect of adopting a dual-channel on the performance of a two-level supply chain in which the manufacturer offers customised products in direct channel and also a standard product through retail channel. Modak and Kelle (2019) examined a dual-channel supply chain under price and delivery-time dependent stochastic customer demand.

The demand of the supply chain can be affected by different parameters one of important things is the selling price of the product. Pricing policies is the core of decision making in the supply chain from not only financial perspective but also from inventory control perspective and is usually interconnected with inventory and production policies such that each member can manipulate his received demand by adopting different pricing policies. So it is necessary to consider pricing and ordering policies jointly in the supply chain in order to gain competitive advantage. Chen and Simchi-Levi (2012) reviewed academic research on pricing models in which inventory replenishment is a critical factor. Wang et al. (2015) considered joint pricing and lot-sizing problem for a centralised and decentralised supply chain including one supplier and one retailer in which demand is price sensitive and the supplier's production rate is finite.

Dehghanbaghi and Sajadieh (2017) considered joint production, inventory, transportation and pricing problem for a supply chain including two manufacturers produce two complementary products and the demand of one product is not only dependent to its price but also to its complementary product price. Dhaka et al. (2017) focused on maximisation of the total profit of a retailer of a supply chain including one retailer and one supplier, with respect to pricing and ordering under two-level trade credits. Wei (2020) developed an integrated pricing and inventory control problem for a diary manufacturer who has a fixed price with a weekly replenishment cycle and a periodic-review system. Noori-daryan et al. (2018) studied optimal pricing and replenishment policies for a supply chain including one single manufacturer and multiple retailers in which a composite contract combines quantity and freight discounts and a free shipping contract is also considered in their model. Rastogi and Singh (2018) assumed a production inventory model with price dependent demand under shortage and inflationary environment. Otrodi et al. (2019) investigated joint pricing and lot-sizing problem for a perishable product with multiple demand classes under a two-level trade credit by developing a bi-objective model to jointly maximise the total profit and minimise the total inventory. Khan et al. (2019) formulated the mathematical model for pricing and replenishment decisions in which all units discount environment was assumed for a maximum lifetime-related deteriorating product. Sharma et al. (2019) developed a deterministic inventory model for deteriorating items which have a seasonal demand and the demand is considered as a function of price and time. Giri et al. (2020) studied pricing, shipment and ordering policies for a two-echelon supply chain including a single-vendor single-buyer with price and green sensitive demand under planned backorder in which the buyer has more power than the vendor.

The power of each member of the supply chain is determined based on the ability of them for handling the market. One approach to model the structures of the market power is game theory. Moon et al. (2010) presented a dynamic pricing and inventory control problem for a dual-channel supply chain under a Cournot-Nash-Bertrand game. Zhang et al. (2012) investigated the effect of product substitutability and relative channel status on pricing decisions by assuming three game scenarios for market power. Chen et al. (2013) considered pricing policies for a dual-channel supply chain under Nash and Stackelberg game in which the retailer sells a substitutable product from the other manufacturer. Giri et al. (2016) explored the competition in offering trade credit between two manufacturers, produce the same product, to their common retailer. Chen et al. (2017) considered non-price feature such as product quality and explored price and quality decisions in centralised and decentralised dual-channel supply chain. Wang et al. (2020) developed game-theoretic models to discuss three advertising schemes in a dual-channel supply chain with one manufacturer and two competing retailers. Esmaeili et al. (2018) modelled pricing and supplier selection problems for a closed-loop supply chain by game theory approach.

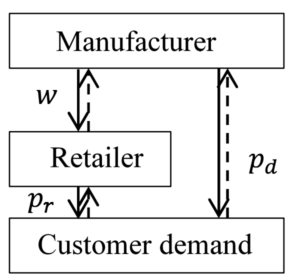
In sum, increasing use of Internet by the customers leads many manufacturers to work on a dual-channel structure. Handling the dual-channel supply chain has some challenges from both the inventory control and the financial perspectives. Up to our knowledge, there is no study which investigates joint ordering and pricing decisions for a two-echelon decentralised dual-channel supply chain so in this paper, we would try to model this problem. In a real supply chain, it may one member has a more market power than the other so he can impose his decision to the other member and the weaker member must make his decision by considering the stronger member's one. Considering inventory and ordering decisions beside the pricing policies make the supply chain more profitable and lead to a comprehensive strategy. We consider a dual-channel supply chain including one manufacturer and one retailer. The manufacturer produces a product in a lot to respond an integer number of retailer's order and also to meet the demand of a direct sales channel. It is assumed that the manufacturer is the leader and the retailer is the follower. Our focus is to derive optimal policies of production, ordering and pricing for each member. We will use Stackelberg game to model leader-follower relationship between the manufacturer and the retailer.

The reminder of the paper is structured as follows. Section 2 includes the problem description, notation and assumptions. Section 3 formulates the Stackelberg game. A solution procedure is presented in Section 4. Section 5 provides a numerical example and some sensitivity analysis. Finally, conclusions with some future researches are discussed in Section 6.

2 Model description

We consider a decentralised two-echelon dual-channel supply chain including one manufacturer and one retailer in which the manufacturer has more market power than the retailer so he has the first move priority in this game and the retailer makes his decision by considering the manufacturer's move. It is assumed that the manufacturer produces a product in a lot for delivering to the retailer in equal-sized batches. The manufacturer produces simultaneously for the retailer and direct channel. The structure of the supply chain can be illustrated as Figure 1.

Figure 1 A dual-channel supply chain



The manufacturer with direct channel and the retailer are both the player of a Stackelberg game to maximise their own profit. In this structure, we suppose that the manufacturer has more power than the retailer and he is the leader and the retailer is the follower. The order quantity for the retailer, number of shipment from the manufacturer to the retailer and the sales price of both channels are decisions obtained from the Stackelberg game approach.

The following notations are used to model this game:

S set-up cost for the manufacturer

c production cost per item for the manufacturer

c_d shipment cost per item for the manufacturer in direct channel

H holding cost per item per time for the manufacturer

A ordering cost for the retailer

h holding cost per item per time for the retailer

Q order quantity for the retailer

n number of shipments

P manufacturer's production rate

D total demand rate

ρ the share of the demand goes to the direct channel

d_d demand rate function in direct channel

d_r demand rate function in retail channel

p_r the price in retail channel

p_d the price in direct channel

w wholesale price of the manufacturer for the retailer

a_1 the coefficient of self-price elasticity of the demand in direct channel

a_2 the coefficient of self-price elasticity of the demand in retail channel

b the coefficient of cross-price elasticity of the demand in one channel to the price of the other channel

T_m manufacturer's cycle

T_r retailer's cycle

TPC_m total production cost for the manufacturer

THC_m total holding cost for the manufacturer

TSC_m total setup cost for the manufacturer

C_m the total cost of the manufacturer

C_r the total cost of the retailer

R_m the total revenue of the manufacturer

R_r the total revenue of the retailer

π_m the total profit of the manufacturer

π_r the total profit of the retailer

The following assumptions will be used in this paper:

- one single product is considered
- shipment lots are equal
- shortage is not allowed.

- the rate of the retailer's holding cost is larger than the manufacturer.
- the production rate is larger than the demand rate
- the demand of each channel is sensitive to the price of this channel and also to the price of the other channel such that:

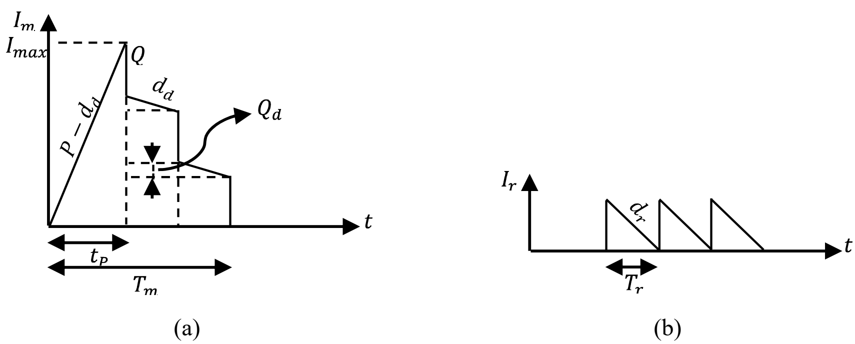
$$d_d = \rho D - a_1 p_d + b p_r \tag{1}$$

$$d_r = (1-\rho)D - a_2 p_r + b p_d \tag{2}$$

The demand of each channel is more sensitive to its own price than the price of the other channel (a_1, $a_2 > b$).

The inventory behaviour of the supply chain is depicted in Figure 2.

Figure 2 Inventory behaviour for the (a) manufacturer and the (b) retailer (n = 3)



Q_d is the produced quantity for responding customer demand of direct channel for each retailer's cycle time and t_P is the production cycle time for the manufacturer. I_m is the manufacturer's inventory on hand.

The components of the total profit function are developed in the following.

2.1 The total profit function of the manufacturer

The total profit of the manufacturer includes the revenue and the total cost. The total cost of the manufacturer has three components: setup cost, holding cost and production cost which first they are separately formulated and then the revenue part would be gained.

2.1.1 Setup cost per time unit at the manufacturer

The manufacturer incurs a setup cost S for each production cycle. So the average setup cost per time unit for the manufacturer is:

$$TSC_m = \frac{S}{T_m} \tag{3}$$

That;

$$T_m = nT_r \tag{4}$$

In which

$$T_r = \frac{Q}{d_r}$$

So, equation (3) is rewritten as:

$$TSC_m = \frac{Sd_r}{nQ}$$

2.1.2 Inventory holding cost per time unit at the manufacturer

When the manufacturer produces, production has been stopped and the manufacturer delivers Q units to the retailer for each retailer's cycle time until his inventory finishes. So, the manufacturer's accumulated inventory can be found by calculating the area under the inventory on hand in Figure 1(a).

The holding cost of the inventory held at the manufacturer per time unit can be gained as follows:

$$THC_m = H \times \frac{1}{T_m} \times \left(\frac{I_{\max} \times t_P}{2} + (n-1) \times \frac{Q_d \times T_r}{2} + \left((n-1) + \ldots + 1\right) \times Q \times T_r + \left((n-2) + \ldots + 1\right) \times Q_d \times T_r \right) \quad (5)$$

In which,

$$I_{\max} = nQ + (n-1)\, Q_d \quad (6)$$

$$Q_d = d_d T_r = d_d \frac{Q}{d_r} \quad (7)$$

$$t_P = \frac{I_{\max}}{P - d_d} = \frac{nQ + (n-1)Q_d}{P - d_d} \quad (8)$$

Therefore, by substituting T_m, $I_{\max}$, Q_d and t_P in equation (5), inventory holding cost per time unit at the manufacturer is:

$$THC_m = \frac{1}{2}\frac{Q}{nd_r} H\left(n\left(d_r + d_d\right) - d_d\right)\left(\frac{n\left(d_r + d_d\right) - d_d}{P - d_d} + (n-1)\right) \quad (9)$$

2.1.3 Production cost per time unit at the manufacturer

The manufacturer produces for meeting the demand of both direct channel and retail channel. Since shortage is not allowed, the production quantity is equal to the total demand of both channels. Therefore the production cost at the manufacturer per time unit is:

$$TPC_m = c\left(d_r + d_d\right) \quad (10)$$

The total cost of the manufacturer per time unit is:

$$C_m(n, Q, p_d) = c(d_r + d_d) + \frac{Sd_r}{nQ} + \frac{H}{2n}\frac{Q}{d_r}\left(n(d_r + d_d) - d_d\right)\left(\frac{n(d_r + d_d) - d_d}{P - d_d} + (n-1)\right) + c_d d_d \tag{11}$$

2.1.4 The total revenue of the manufacturer

The manufacturer sells his product in both channels so his revenue can be obtained by multiplying the selling price and the demand for both channels.

$$R_m(w, p_d) = wd_r + p_d d_d \tag{12}$$

Finally, the total profit of the manufacturer is:

$$\begin{aligned}\pi_m(n, Q, p_d, w) &= R_m - C_m \\ &= (w-c)d_r + (p_d - c)d_d - \frac{Sd_r}{nQ} \\ &\quad - \frac{H}{2n}\frac{Q}{d_r}\left(n(d_r + d_d) - d_d\right)\left(\frac{n(d_r + d_d) - d_d}{P - d_d} + (n-1)\right)\end{aligned} \tag{13}$$

2.2 The total profit of the retailer

The total cost of the retailer per time unit can be obtained in similar way of the EOQ model and the total revenue of the retailer would be found by multiplying selling price and the demand of retail channel as follows:

$$C_r(Q) = wd_r + \frac{Ad_r}{Q} + \frac{hQ}{2} \tag{14}$$

$$R_r(p_r) = p_r d_r \tag{15}$$

$$\pi_r(n, Q, p_r) = R_r - C_r = (p_r - w)d_r - \frac{Ad_r}{Q} - \frac{hQ}{2} \tag{16}$$

3 Manufacturer-leader Stackelberge game (MS game)

In this section, a decentralised structure would be assumed for the supply chain. We use game-theoretical approach to investigate joint optimal pricing and ordering decisions. It is assumed that the manufacturer has more market power than the retailer so he first determines optimal values of n, w and p_d and then the retailer as a follower can make his own optimal policy with respect to the leader's decisions. The direct sales price p_d can't be smaller than the wholesale price w otherwise the retailer can buy products from the direct channel in lower price. The problem will be solved by using a procedure based on backward induction. This model can be structured as follows:

$$\begin{cases} \max\limits_{(n,w,p_d)} \pi_m\left(n,\, w,\, p_d\,,\, Q^*\left(n,\, w,\, p_d\right),\, p_r^*\left(n,\, w,\, p_d\right)\right) \\ \quad \text{subject to:} \\ \quad p_d \geq w \\ Q^*\left(n,\, w,\, p_d\right) \text{ and } p_r^*\left(n,\, w,\, p_d\right) \\ \quad \text{are derived from solving the following problem)} \\ \begin{cases} \max\limits_{(n,p_r)} \pi_r\left(Q,\, p_r\right) \\ \quad \text{subject to:} \\ \quad p_r \geq w \end{cases} \end{cases} \tag{17}$$

As the retailer is the follower of the Stackelberg game, he must determine his optimal decisions by considering given values for n, w and p_d. The optimal order quantity for the retailer can be found via equation (18).

$$Q^* = \sqrt{\frac{2Ad_r}{h}} \tag{18}$$

By substituting Q^* in π_r, we have:

$$p_r\left(p_r\right) = \left(p_r - w\right) d_r - \sqrt{2Ahd_r} \tag{19}$$

The first order derivative of π_r with respect to p_r is:

$$\frac{\partial p_r}{\partial p_r} = (1-\rho)D - 2a_2 p_r + bp_d + a_2 w + \frac{Aha_2}{\sqrt{2Ah\left[(1-\rho)D - a_2 p_r + bp_d\right]}} \tag{20}$$

We use the second-order Taylor series approximation to the last phrase of equation (20) in the point of $p_r = 0$ to derive the root of equation (20). So, we have:

$$\frac{\partial \pi_r}{\partial \pi_r} = (1-\rho)D - 2a_2 p_r + bp_d + a_2 w + \frac{Aha_2}{\sqrt{2Ah\left[(1-\rho)D + bp_d\right]}}$$
$$+ \frac{A^2h^2a_2^2}{\left(2Ah\left[(1-\rho)D + bp_d\right]\right)^{\frac{3}{2}}} p_r + \frac{3}{2} \frac{A^3h^3a_2^3}{\left(2Ah\left[(1-\rho)D + bp_d\right]\right)^{\frac{5}{2}}} p_r^2$$

So the optimal value of the retail price is:

$$p_r^*\left(w,\, p_d\right) = \frac{M - \sqrt{M^2 - 6ZN}}{3Z} \tag{21}$$

In which:

$$M = 2a_2 - \frac{A^2h^2a_2^2}{\left(2Ah\left[(1-\rho)D + bp_d\right]\right)^{\frac{3}{2}}} \tag{22}$$

$$N = (1-\rho)D + bp_d + a_2 w + \frac{Aha_2}{2Ah\left[(1-\rho)D + bp_d\right]} \tag{23}$$

$$Z = \frac{A^3 h^3 a_2^3}{\left(2Ah\left[(1-\rho)D + bp_d\right]\right)^{\frac{5}{2}}} \tag{24}$$

At the next step, the manufacturer as a leader in this game knows the optimal decision of the retailer which can be obtained by (21). So, the optimal decisions of the manufacturer would be determined by substituting $p_r^*(w, p_d)$ into the profit of the manufacturer presented in (13). So we have the following objective function for the manufacturer:

$$\begin{aligned}
\pi_m(n, w, p_d) = {} & (w-c)\left[(1-\rho)D - a_2\left(\frac{M-\sqrt{M^2-6ZN}}{3Z}\right) + bp_d\right] \\
& + (p_d - c)\left[\rho D - a_1 p_d + b\left(\frac{M-\sqrt{M^2-6ZN}}{3Z}\right)\right] \\
& - S\left[(1-\rho)D - a_2\left(\frac{M-\sqrt{M^2-6ZN}}{3Z}\right) + bp_d\right] \\
& - \frac{H}{2n}\frac{Q}{\left[(1-\rho)D - a_2\left(\frac{M-\sqrt{M^2-6ZN}}{3Z}\right) + bp_d\right]} \\
& \left(n\left(D - (a_2 - b)\left(\frac{M-\sqrt{M^2-6ZN}}{3Z}\right) - (a_1 - b)p_d\right)\right. \\
& \left. - \left[\rho D - a_1 p_d + b\left(\frac{M-\sqrt{M^2-6ZN}}{3Z}\right)\right]\right) \\
& \left(\frac{n\left(D - (a_2 - b)\left(\frac{M-\sqrt{M^2-6ZN}}{3Z}\right) - (a_1 - b)p_d\right) - d_d}{P - \left[\rho D - a_1 p_d + b\left(\frac{M-\sqrt{M^2-6ZN}}{3Z}\right)\right]} + (n-1)\right)
\end{aligned} \tag{25}$$

For a certain values of w and w_d, it can be proved that the profit function of the manufacturer is concave or decreasing function of n.

Preposition 1. The profit function of the manufacturer is concave or decreasing function of n.

$$\begin{aligned}
\frac{\partial \pi_m}{\partial n} = {} & \frac{Sd_r}{n^2 Q} - \frac{HQd_d}{2d_r n^2}\left(\frac{n(d_r + d_d) - d_d}{P - d_d} + (n-1)\right) \\
& - \left(\frac{P + d_r}{P - d_d}\right)\frac{H}{2n}\frac{Q}{d_r}\left(n(d_r + d_d) - d_d\right)
\end{aligned}$$

$$\frac{\partial^2 \pi_m}{\partial n^2} = -2\frac{Sd_r}{n^3 Q} + \frac{HQd_d}{d_r n^3}\left(\frac{n(d_r + d_d) - d_d}{P - d_d} + (n-1)\right)$$

There are two cases for $\frac{\partial^2 \pi_m}{\partial n^2}$:

- $\frac{\partial^2 \pi_m}{\partial n^2} \le 0$

  So the profit function is concave with respect to *n*.

- $\frac{\partial^2 \pi_m}{\partial n^2} \ge 0$

  We can rewrite the first order derivative with respect to *n* as:

  $$\frac{\partial \pi_m}{\partial n} = -2n\left(\frac{\partial^2 \pi_m}{\partial n^2}\right) - \left(\frac{P+d_r}{P-d_d}\right)\left(\frac{H}{2n}\frac{Q}{d_r}\left(n\left(d_r+d_d\right)-d_d\right)\right)$$

The first phrase of $\frac{\partial \pi_m}{\partial n}$ is negative and the second one is positive so it can be proved that $\frac{\partial \pi_m}{\partial n}$ would be negative so π_m is a decreasing function of *n*. □

The model is obviously so complicated that an optimal solution cannot be obtained by any analytical approach. Therefore, we propose a solution procedure using Nelder-Mead algorithm for deriving optimal decisions. The Nelder-Mead algorithm is a direct search and derivative free approach which was proposed by Nelder and Mead (1965).

4 Solution procedure

We assume that the manufacturer is the leader so he first determines the values of *n*, *w* and p_d then the retailer as a follower obtains the optimal values for *Q* and P_r by considering *w** and p_d^*. We use the following solution procedure:

1. Set π_m^*, $n^* = 0$
2. Set $n = 1$
3. Find the optimal values of *w* and p_d by using Nelder-Mead algorithm as explained in Appendix.
4. If $\pi_m(n) \ge \pi_m^*$ then set $\pi_m^* = \pi_m$, $n^* = n$ and go to Step 5, else the current solution is optimal and stop.
5. Add *n* by 1 and go to Step 3.
6. By considering the optimal values of *w**, p_d^* and *n**, the optimal value of p_r is gained by using equation (21) and π_r^* can be calculated by equation (19).

5 Numerical example

In this section, we present a numerical example to explore the optimal decisions for a Stackelberg game in a dual-channel supply chain. The following input parameters are used:

Table 1 Input parameters

P	D	S	A	H	h	c	c_d	ρ	a_1	a_2	b
2,000	1,000	200	10	4	8	20	2	0.1	4	5	2

The optimal decisions for the retailer and the manufacturer are presented in Table 2.

Table 2 Optimal decisions for the channel members

	Retailer			*Manufacturer*			
	Q^*	p_r^*	π_r^*	n^*	w^*	p_d^*	π_m^*
Decentralised structure	22.88	171.46	8,271.13	5	129.25	83.38	28,921.74

In the following, we will explore the effects of different parameters on optimal decisions in Stackelberg game. One of important parameters is ρ because each channel makes his financial and operational decisions with respect to the received demand.

Table 3 The sensitivity analysis of the share of the demand for direct channel

ρ	*Retailer*			*Manufacturer*			
	Q^*	p_r^*	π_r^*	n^*	w^*	p_d^*	π_m^*
0.1	22.88	171.46	8,271.13	5	129.25	83.38	28,921.74
0.2	21.46	160.22	6,364.25	4	123.06	92.64	28,886.59
0.3	19.95	148.98	4,727.25	4	116.79	102.05	29,923.12
0.4	18.32	137.73	3,337.10	4	110.50	111.47	32,018.69
0.5	16.47	126.59	2,151.28	3	104.50	120.70	35,192.76
0.6	14.45	115.33	1,253.80	3	98.19	130.12	39,434.26
0.7	11.82	104.54	537.95	2	92.90	139.31	44,752.01
0.8	8.77	93.35	142.73	2	86.69	148.74	51,182.34
0.9	0	82.57	0	1	82.57	157.87	59,078.45

As can be found from Table 3 and Figure 3, by increasing the value of ρ, the profit of the manufacturer will be increased and vice versa the profit of the retailer decreases because the market share of the manufacturer would be increased and the retailer's share is decreased. The optimal sales price in retail channel is decreased to absorb more demand on the other hand the optimal sales price for direct channel would be increased because of plenty of demand also it can be gained that the wholesale price is decreased because the demand of retail channel has been dropped.

Figure 3 The manufacturer's and the retailer's average profit vs. ρ

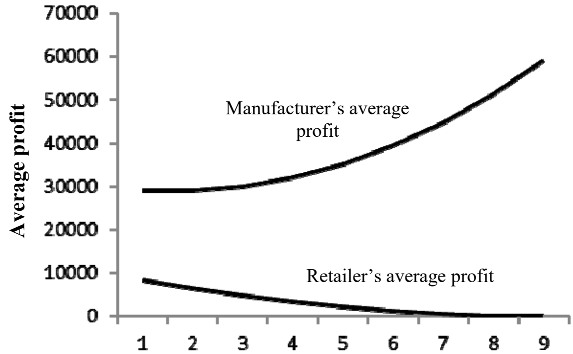


Table 4 The sensitivity analysis of the cost parameters

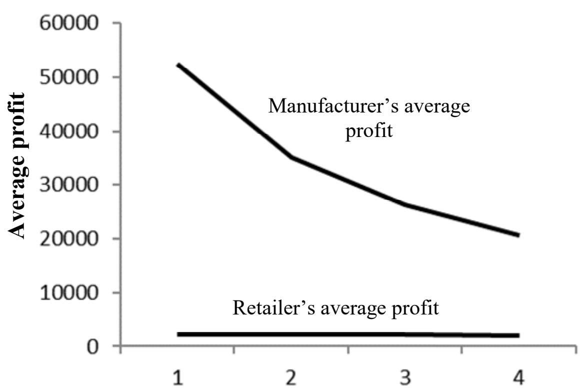
Cost parameters		*Retailer*			*Manufacture*			
		Q^*	p_r^*	π_r^*	n^*	w^*	p_d^*	π_m^*
H	2	22.91	171.34	8,301.82	7	129.05	83.28	29,101.01
	4	22.88	171.46	8,271.13	5	129.25	83.38	28,921.74
	6	22.84	171.55	8,205.40	4	129.51	83.21	28,795.23
h	6	26.42	171.44	8,194.09	4	129.29	83.29	28,945.01
	8	22.88	171.46	8,271.13	5	129.25	83.38	28,921.74
	10	20.46	171.47	8,305.92	5	129.25	83.33	28,903.46
S	100	22.89	171.32	8,283.65	3	129.07	83.10	29,142.50
	200	22.88	171.46	8,271.13	5	129.25	83.38	28,921.74
	300	22.87	171.57	8,250.69	6	129.41	83.53	28,749.73
	400	22.86	171.66	8,236.65	7	129.54	83.67	28,603.78
A	5	16.19	171.45	8,091.49	7	129.31	83.44	28,914.41
	10	22.88	171.46	8,271.13	5	129.25	83.38	28,921.74
	15	28.02	171.46	8,341.98	4	129.21	83.32	28,927.61

From Table 4 we can see that the cost parameters have negligible effects on optimal prices. It can also be inferred that by increasing the holding cost of the retailer, the optimal value of Q would be decreased and the optimal value of n increased to reduce the total holding cost of the retailer. Increasing the ordering cost of the retailer increases the optimal value of Q and decreases the optimal value of n to reduce the total ordering cost of the retailer. Also, the analysis shows that increasing the value of holding cost of the manufacturer can cause to decreasing the optimal value of n and increasing the value of setup cost of the manufacturer decreases the optimal value of n.

Table 5 The sensitivity analysis of the coefficient of self-price elasticity of the demand in direct channel

a_1	ρ	*Retailer*			*Manufacture*			
		Q^*	p_r^*	π_r^*	n^*	w^*	p_d^*	π_m^*
3	0.1	22.88	184.52	8,272.02	5	142.31	116.06	36,305.08
	0.5	16.47	146.47	2,151.19	3	124.39	170.42	52,467.36
	0.9	0	224.64	0	1	109.35	224.64	90,335.28
4	0.1	22.88	171.46	8,271.13	5	129.25	83.38	28,921.74
	0.5	16.47	126.59	2,151.28	3	104.50	120.70	35,192.76
	0.9	0	82.57	0	1	82.57	157.87	59,078.45
5	0.1	22.88	164.62	8,269.78	5	122.41	66.26	25,117.12
	0.5	16.46	116.18	2,150.83	3	94.08	94.65	26,204.93
	0.9	0	68.54	0	1	68.54	122.94	42,763.42
6	0.1	22.88	160.41	8,268.23	5	118.20	55.72	22,826.51
	0.5	16.46	109.77	2,150.08	3	87.68	78.62	20,723.05
	0.9	0	101.38	0	1	59.87	101.38	32,769.95

Figure 4 The manufacturer's and the retailer's average profit vs. $a_1 = 0.5$



From Table 5 and Figure 4, we can find that, for a certain value of ρ, increasing the value of the price sensitivity in direct channel causes the optimal direct price decreases to keep the customers in direct channel and the wholesale price also decreases to retain the retail channel in supply chain. So, by decreasing the wholesale price, the retailer can decreases his price and attract more demand on his channel. By increasing the manufacturer's market share, he decides to increase the sales price in direct channel and obtains more profit.

Table 6 The sensitivity analysis of the coefficient of self-price elasticity of the demand in retail channel

a_2	ρ	*Retailer*			*Manufacture*			
		Q^*	p_r^*	π_r^*	n^*	w^*	p_d^*	π_m^*
4	0.1	23.18	222.86	11,024.48	5	168.82	103.18	42,072.92
	0.5	16.89	164.64	3,037.66	3	135.73	136.32	44,992.10
	0.9	0	109.10	0	1	109.10	169.33	65,121.33
5	0.1	22.88	171.46	8,271.13	5	129.25	83.38	28,921.74
	0.5	16.47	126.59	2,151.28	3	104.50	120.70	35,192.76
	0.9	0	82.57	0	1	82.57	157.87	59,078.45
6	0.1	22.58	139.83	6,451.17	5	105.51	71.50	21,427.25
	0.5	16.03	103.30	1,574.73	3	85.76	111.33	29,462.08
	0.9	0	66.41	0	1	66.41	151.00	55,469.99
7	0.1	22.28	118.38	5,164.34	5	89.69	63.58	16,628.08
	0.5	15.59	87.57	1,175.14	3	73.28	105.08	25,721.61
	0.9	0	55.52	0	1	55.52	146.40	53,072.59

Figure 5 The manufacturer's and the retailer's average profit vs. a_2 = 0.5

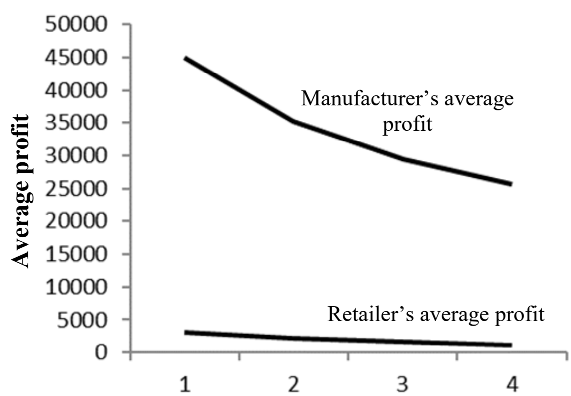


Table 6 and Figure 5 show that for a certain value of ρ, as the sensitivity of demand to the price in retail channel increases the optimal retail price decreases to keep the customers in this channel and the optimal sales price in direct channel will be decreased to retain the customers in this channel also the wholesale price decreases to allow the retailer to reduce his price and retain sensitive customers in his channel. We can see that the optimal profits of the retailer and the manufacturer reduce as the customers get more sensitive.

Table 7 The sensitivity analysis of the coefficient of cross-price elasticity of the demand

b	ρ	*Retailer*			*Manufacture*			
		Q^*	p_r^*	π_r^*	n^*	w^*	p_d^*	π_m^*
1	0.1	22.56	148.95	7,808.48	5	107.89	48.32	18,743.55
	0.5	16.02	97.52	1,923.76	3	76.55	90.26	21,322.24
	0.9	0	45.63	0	1	45.63	132.06	46,024.01
2	0.1	22.88	171.46	8,271.13	5	129.25	83.38	28,921.74
	0.5	16.47	126.59	2,151.28	3	104.50	120.70	35,192.76
	0.9	0	82.57	0	1	82.57	157.87	59,078.45
3	0.1	23.18	231.11	8,714.44	4	187.82	156.84	57,719.81
	0.5	16.90	193.03	2,390.32	3	169.84	193.15	68,889.30
	0.9	0	157.09	0	1	157.09	229.18	94,433.47
4	0.1	23.50	554.96	9,189.15	4	510.58	523.92	220,078.39
	0.5	17.34	535.03	2,638.25	3	510.78	573.84	248,519.56
	0.9	0	518.57	0	1	518.57	623.50	289,441.79

Figure 6 The manufacturer's and the retailer's average profit vs. $b = 0.5$

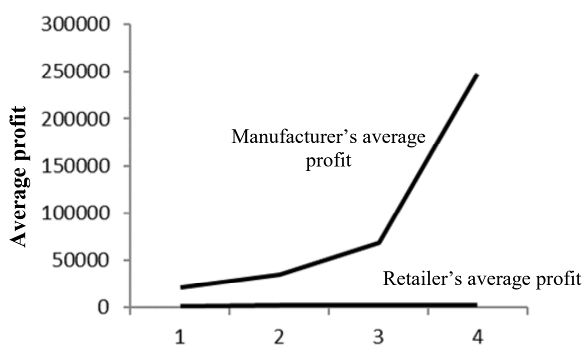


Table 7 and Figure 6 show that, for a certain value of ρ, by increasing the cross-price elasticity of demand, the total profit of the manufacturer and the total profit of the retailer are increased because each member can use this sensitivity of the customers to the price of the other member to increase his price and gain more profit. Also, as the customers' sensitivity of one channel to the price of the other channel increases the opportunity of increasing price would be prepared in this channel and the optimal retail price of direct channel and retail channel get closer to each other.

6 Conclusions

Increasing use of Internet of customers leads manufacturers to add an online channel to the retail one. Our contribution in this paper is that focuses on joint ordering and pricing decisions for a two-echelon decentralised dual channel supply chain including one retailer and one manufacturer. We used game theoretical approach to model market power structure which helps both members to determine best possible prices based on their market share.

In this study, the manufacturer has more market power than the retailer so he is the leader of a Stackelberg game and each member makes his optimal decisions with respect to their market power and the customer demand is assumed to be sensitive to the price of both channels such as two channels can be considered as competitors of each other. So, we focused how the manufacturer maximises his profit as a leader and then the retailer, as a follower, makes his optimal decisions by considering the manufacturer's decisions. We also studied the impacts of different parameters on optimal decisions of the retailer and the manufacturer. It was shown that the production costs including inventory holding cost, setup cost and ordering cost have negligible effects on optimal prices. It is observed from numerical example that when customers of one channel are more sensitive to the price of the other channel both members can achieve more profit and the retail prices of both channels get close to each other to control the sensitivity of customers.

As managerial insights which can be gained from our developed model, is that as the share of direct channel demand have been increased the total profit of the manufacturer would be increased and the necessity of adding an online channel is proved and sensitivity of customer demand to the sale price can impressively affect on optimal decisions.

This study can be extended in several directions. Our analysis is based on certain demand function but there are lots of uncertain parameters which can influence these findings so one of important extension can be studying other form of demand function in a fuzzy or stochastic environment. It is assumed that both members have perfect information from each other costs and demand but in real world problem, these kinds of information are secret thus it would be interesting to explore this model in asymmetric form of information as well as considering other kind of power structure and other contracts between manufacturer and retailer such as revenue sharing.

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Appendix

The Nelder-Mead algorithm

The Nelder-Mead algorithm is a direct search and derivative free method for minimisation of an n-dimensional function f using by only the values of it which was developed by Nelder and Mead (1965) and its convergence behaviour was explored by Lagarias et al. (1998). In two dimensions, Lagarias et al. (2012) showed for any nondegenerate starting simplex and any twice-continuously differentiable function with positive definite Hessian and bounded level sets, the algorithm always converges to the minimiser. This search method is based on replacement of worst vertex with a new point and this new point is obtained through four operations: reflection, expansion, contraction and shrinkage and this process is repeated until the convergence is reached. The search procedure is structured as follows:

1 At the first step, four parameters should be determined as reflection (α), expansion (β), contraction (γ) and shrinkage (σ) coefficients. The standard values for these coefficients are $\alpha = 1$, $\beta = 2$ and $\gamma = \sigma = 0.5$.

2 Then $n + 1$ vertices are selected as an initial point and ordered according to function values at these vertices as:

$$f(x_1) \le f(x_2) \le f(x_{n+1})$$

and the centroid of the best n points $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ is gained.

3 Reflection: the worst point x_{n+1} is reflected through $\bar{x}$ by a reflection coefficient of α, $x_r = \bar{x} + \alpha(\bar{x} - x_{n+1})$, if $f(x_1) \le f(x_r) \le f(x_{n+1})$ then x_{n+1} is replaced with x_r and the iteration terminate.

4 Expansion: if $f(x_r) \le f(x_1)$, the reflection operation is useful so the reflected point is expanded $x_e = \bar{x} + \beta(x_r - \bar{x})$. If $f(x_e) \le f(x_r)$, then the worst point x_{n+1} is replaced with x_e otherwise we have a failed expansion operation and x_{n+1} is replaced with x_r and the iteration terminate.

5 Contraction: if $f(x_r) \le f(x_n)$, then the distance between vertices should be reduced and contracted. There are two possible cases:

- If $f(x_r) \geq f(x_{n+1})$, then the new point is calculated as $x_{ci} = \bar{x} + \gamma(\bar{x} - x_{n+1})$. If $f(x_{ci}) < f(x_{n+1})$ the worst point is replaced with x_{ci} else the shrinkage operation is done.
- If $f(x_r) \geq f(x_{n+1})$, the new point is obtained as $x_{co} = \bar{x} + \gamma(x_r - \bar{x})$. If $f(x_{co}) \leq f(x_r)$ then x_{n+1} is replaced with x_{co} otherwise the shrinkage operation would be started.

6 Shrinkage: the new vertices are calculated from $v_i = x_1 + \sigma(x_i - x_1)$ for $i = 2, 3, \ldots, n + 1$ and the next iteration starts.

For finding optimal values, we first suppose a certain value for n and an initial point for w and p_d, then we can start searching to find optimal values for w and p_d using Nelder-Mead algorithm. We have two variables so we need to have three vertices so we use the starting point $\begin{bmatrix} w \\ p_d \end{bmatrix} = \begin{bmatrix} 0 & 0 & c \\ 0 & c & c \end{bmatrix}$ for performing Nelder-Mead algorithm.

Then the manufacturer's profit for each vertex is calculated and all vertices are sorted based on their profit. Then, reflection, expansion, contraction and shrinkage operations are done. This process is repeated until three vertices converge and the optimal values of w and p_d for that certain value of n are obtained.