A directional movement trading strategy using jump-diffusion price dynamics

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Abstract: The purpose of this paper is to forecast stock prices using Merton's jump-diffusion model and develop a directional movement (DM) trading strategy based on the price forecasts. The formula for the probability density function of Merton's daily logarithmic stock return has been simplified. Stock price dynamics of ten companies listed in Bombay Stock Exchange are studied. Both the Black-Scholes and Merton models are compared to fit the historical stock data as well as forecasting stock prices and it is found that the Merton model gives superior in-sample and out-of-sample performances. The adaptive barrier algorithm of Lange is used to find the maximum likelihood estimates of the Merton parameters. Finally, two trading strategies – the buy-and-hold (BH) strategy and the DM strategy are compared. The DM strategy outperforms the BH strategy for most of these ten stocks as well as when traded with a Markowitz minimum variance portfolio of these stocks.

Keywords: jump-diffusion; directional movement; Black-Scholes; adaptive barrier; buy-and-hold; Markowitz.

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1 Introduction and literature review

Black and Scholes (1973) developed a model to find a closed-form solution for the price of a European option. They assumed that the stock prices follow a diffusion process which means that the stock price trajectory is of continuous nature. However, it is observed in the markets that the changes in stock prices often consist of large jumps. Taking that into account, Cox and Ross (1976) considered price dynamics where the stock prices follow a jump process. However, Merton (1976) proposed a hybrid stock price dynamics – the diffusion process which explains the small changes and the jump process which explains sudden large changes in stock prices in a small interval of time, resulting into a jump-diffusion (JD) process. Later, Kou (2002) proposed a double exponential JD option pricing model. Apart from deterministic volatility option pricing models, several stochastic volatility option pricing models have been proposed (Heston, 1993; Bates, 1996; Scott, 1997; Mitra, 2010; Leccadito and Russo, 2016). Mitra (2011) provides a review of some significant deterministic as well as stochastic volatility option pricing models. In this paper, we study the stock price dynamics of the Black-Scholes (BS) model and Merton's JD model and try to fit these models to the historical stock data. We have chosen to study Indian market because studies on estimation of JD parameters were conducted before only for American and European markets, for example, studies by Hanson and Zhu (2004), Synowiec (2008) and Gugole (2016). They have used multinomial maximum likelihood approach and Nelder and Mead (1965) algorithm to estimate the JD model parameters, whereas we use simple maximum likelihood estimation and adaptive barrier algorithm of Lange (1994).

The two major stock exchanges of India are the Bombay Stock Exchange (BSE) and the National Stock Exchange (NSE) of India. These are one of the biggest stock exchanges in the world by market capitalisation, BSE ranked #10 and NSE ranked #11. Both have market capitalisations of over USD2 trillion (https://finance.yahoo.com/news/20-largest-stock-exchanges-world-175549152.html). These exchanges list all the leading Indian firms and most of the trading in the Indian stock market happen in these exchanges. The BSE has 4,200 listed firms as of 28th August 2020 (https://www.bseindia.com/corporates/List_Scrips.aspx) and the NSE has 1795 listed firms as of 31st March 2020 (https://www.nseindia.com/regulations/listing-compliance/nse-market-capitalisation-all-companies). S&P BSE SENSEX is the benchmark stock index of BSE and NIFTY 50 is the benchmark stock index of NSE. Kumar and Nandamohan (2018) studied the randomness and efficiency of the Indian stock markets with respect to efficient market hypothesis and adaptive market hypothesis and found out that there is no uniformity or trend in randomness but efficiency improved in some time periods. They have also observed that efficiency of the Indian markets is time varying.

In the next section, we present with a brief review of BS model and Merton model and study the underlying stock price dynamics. We propose a simplified formula for the probability density function of the daily logarithmic stock return according to Merton model. Next, we do an empirical analysis where we fit the historical closing stock prices of ten different companies listed in BSE with these models and draw a comparison between these models by the quality of fit. We conclude that the Merton model gives better in-sample price estimates than the BS model. Next, we do an out-of-sample forecast of stock prices for one month (23 trading days) for these ten stocks and conclude that the Merton model gives better out-of-sample price estimates than the BS model as well. Next, we discuss and compare two trading strategies – the buy-and-hold (BH) strategy and the directional movement (DM) strategy for trading these stocks during the out-of-sample period. DM have been studied in the past, for example DM of implied volatility by Ahn et al. (2012), directional trade by McKeon (2016), directional changes in market volatility by Atkins et al. (2018). In this paper we propose a DM trading strategy using Merton model estimated prices. Next, we combine Markowitz (1956) minimum variance portfolio theory with DM and form another trading strategy. We conclude our paper by discussing the advantages of the Merton model over the BS model which is evident from the observed market behaviour. We discuss when the DM trading strategy outperforms the BH trading strategy and vice-versa and how to make these strategies profitable. We also show that the DM strategy outperforms the BH trading strategy when done with a Markowitz minimum variance portfolio of stocks. This study will help the risk-taking traders as well as risk-averse investors to follow suitable trading strategies in order to obtain higher profit with lower risk.

The main contributions of our study to the existing literature are:

a deriving a simplified formula for the probability density function of the daily logarithmic stock return according to Merton model

b using simple maximum likelihood approach and Lange algorithm to estimate the Merton model parameters instead of multinomial maximum likelihood approach and Nelder-Mead algorithm which were used in previous studies

c studying an emerging market – Indian stock market instead of developed markets since most of the stock and option price dynamics studies were conducted before on developed markets

d proposing a DM stock trading strategy using a JD option pricing model, which is the first such study ever conducted.

Our aim is to provide a profitable and low-risk trading strategy to the risk-averse investors as well as risk-taking traders. We have shown that our proposed DM trading strategy outperforms the usual BH trading strategy when traded with a Markowitz minimum variance portfolio of stocks.

2 Review of the models

In this section, we briefly discuss the mathematical models, one by Fischer Black and Myron Scholes and the other by Robert C. Merton. Both these models are well-known option pricing models. But in this paper, we will only consider the stock price dynamics portion of these models, not the option price dynamics.

2.1 BS model

Black and Scholes (1973) proposed that the infinitesimal stock price increment follows the stochastic differential equation,

$$\begin{aligned} &dS(t) = S(t)(\mu dt + \sigma dB(t)), \\ &dB(t) \sim \mathcal{N}(0, dt) \text{ for } t > 0, \text{ and } S(0) > 0 \end{aligned} \tag{1}$$

where $S(t)$ is the stock price at time t, $B(t)$ is a standard Brownian motion, μ is the drift rate, and $\sigma > 0$ is the volatility of the stock price.

Using Itô (1944) lemma for diffusion process, it can be shown that

$$S(t) = S(0) \exp\Big\{\sigma B(t) + \Big(\mu - \frac{\sigma^2}{2}\Big)t\Big\} \tag{2}$$

is a solution to equation (1).

Now, suppose we have $n+1$ historical observations, $S(t)$, $t \in \{0, 1, ..., n\}$. Then the daily logarithmic stock return for $t \in \{1, 2, ..., n\}$,

$$\begin{aligned} R(t) &= \ln \frac{S(t)}{S(t-1)} = \ln \left(\frac{S(0) \exp\Big\{\sigma B(t) + \Big(\mu - \frac{\sigma^2}{2}\Big)t\Big\}}{S(0) \exp\Big\{\sigma B(t-1) + \Big(\mu - \frac{\sigma^2}{2}\Big)(t-1)\Big\}} \right) \\ &= \sigma\Big(B(t) - B(t-1)\Big) + \mu - \frac{\sigma^2}{2} \stackrel{D}{=} \sigma B(1) + \mu - \frac{\sigma^2}{2} \\ &\sim \mathcal{N}\Big(\mu - \frac{\sigma^2}{2}, \sigma^2\Big) \end{aligned} \tag{3}$$

follows normal distribution with mean $\mu - \frac{\sigma^2}{2}$ and variance σ^2. $\stackrel{D}{=}$ means equal in distribution. We assume that the random variables in a sample are independent all through our study.

Next, we can write the likelihood function of the sample $\{R(1), ..., R(n)\}$ as,

$$L_{BS}(\mu, \sigma) = \prod_{t=1}^{n} f_{R(t)}(\tilde{R}(t)) = \prod_{t=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} \exp\Big\{ - \frac{(\tilde{R}(t) - (\mu - \frac{\sigma^2}{2}))^2}{2\sigma^2} \Big\} \tag{4}$$

where $f_{R(t)}(\tilde{R}(t))$ is the probability density function of $R(t)$, and the log-likelihood function of the sample as,

$$logL_{BS}(\mu, \sigma) = \sum_{t=1}^{n} \Big(- \ln \sigma\sqrt{2\pi} - \frac{(\tilde{R}(t) - (\mu - \frac{\sigma^2}{2}))^2}{2\sigma^2} \Big) \tag{5}$$

The maximum likelihood estimates, $\hat{\mu}$ and $\hat{\sigma}$ can be obtained by taking the partial derivatives of $logL_{BS}(\mu, \sigma)$ with respect to μ and σ and then equating them to zero, which gives,

$$\frac{\partial}{\partial \mu}\Big(logL_{BS}(\mu, \sigma)\Big) = 0 \implies \mu - \frac{\sigma^2}{2} = \frac{1}{n}\sum_{t=1}^{n} \tilde{R}(t) = \text{mean}(\tilde{R}(t))$$

Using this relation, we can rewrite equation (5) as,

$$logL_{BS}(\mu, \sigma) = \sum_{t=1}^{n} (- \ln \sigma\sqrt{2\pi}) - \frac{n}{2\sigma^2}\text{var}(\tilde{R}(t))$$

where $\text{var}(\tilde{R}(t)) = \frac{1}{n}\sum_{t=1}^{n}(\tilde{R}(t) - \text{mean}(\tilde{R}(t)))^2$. Therefore,

$$\frac{\partial}{\partial \sigma}\Big(logL_{BS}(\mu, \sigma)\Big) = 0 \implies \text{var}(\tilde{R}(t)) = \sigma^2$$

Thus, the parameter estimates are obtained as,

$$\hat{\sigma} = \sqrt{\operatorname{var}(\tilde{R}(t))},\ \hat{\mu} = \frac{\hat{\sigma}^2}{2} + \operatorname{mean}(\tilde{R}(t)) \tag{6}$$

2.2 Merton's normal JD model

Merton (1976) proposed the first JD option pricing model. According to his model, the infinitesimal stock price increment follows the stochastic differential equation,

$$\begin{aligned} &dS(t) = S(t)((\mu - \lambda k)dt + \sigma dB(t)) + S(t-)dC(t),\\ &dB(t) \sim \mathcal{N}(0, dt),\ C(t) = \sum_{i=1}^{\eta(t)}(D_i - 1),\ D_i = \frac{S(\tau_i)}{S(\tau_i -)} > 0,\ k = E[D_i - 1],\\ &F_i = \ln D_i \sim \mathcal{N}(m, \delta^2)\ \text{for}\ t > 0,\ i \in \mathbb{N}\ \text{, and}\ S(0) > 0 \end{aligned} \tag{7}$$

where $S(t)$ is the stock price at time t and $B(t)$ is a standard Brownian motion. Conditional on no jumps in the stock price trajectory, μ is the drift rate and $\sigma > 0$ is the volatility of the stock price.

The additional assumption of Merton model is the presence of jumps in the stock price trajectory which occur at random times $\{\tau_1, \tau_2, ...\}$, $\tau_i > 0 \forall i$. The number of times jumps occur is mathematically expressed by a Poisson process, $\eta(t) = \{\#i : \tau_i \in (0, t]\}$, which has intensity $\lambda > 0$. This implies, $\eta(t) \sim Po(\lambda t)$ follows Poisson distribution with parameter $\lambda t > 0$. $T_i = \tau_i - \tau_{i-1}$ are the waiting times of the jump events and the expected value of T_i is $\frac{1}{\lambda}$.

We note that there are jumps from $S(\tau_i -)$ to $S(\tau_i)$. The price ratio, D_i is defined as $D_i = \frac{S(\tau_i)}{S(\tau_i -)}$ and that $D_i - 1$ is the relative change in stock price if jump occurs. Therefore, the sum of all such relative changes in stock prices, also called the jump sizes, which we denote by $C(t)$, is a compound Poisson process. Since, $\{D_i\}$ are assumed to be identical and independent log-normal random variables with parameters m and $\delta > 0$, and hence logarithm of the price ratio follows normal distribution, this model is known as normal JD model. k is the expected value of $D_i - 1$, that is, $k = \exp\{m + \frac{\delta^2}{2}\} - 1$.

There are two parts in the infinitesimal stock price increment, $dS(t)$, given in equation (7). One is the diffusion part, $S(t)((\mu - \lambda k)dt + \sigma dB(t))$, that explains the 'normal vibrations in price' and the other is the jump part, $S(t-)dC(t)$, that explains the 'abnormal vibrations in price' which occur because of some incoming 'new information about the stock'. These 'abnormal vibrations in price' have a significant impact on price, which explains the large jumps occurring in the real scenario in the markets. They are modelled by a Poisson process. Merton model becomes the BS model if we exclude the jump process. Another assumption of Merton model is that $B(t)$, $\eta(t)$, and D_i are independent of each other.

Using Itô's lemma for JD process (Cont and Tankov, 2003), it can be shown that,

$$S(t) = S(0) \exp\left\{\sigma B(t) + \left(\mu - \lambda k - \frac{\sigma^2}{2}\right)t\right\} \prod_{i=1}^{\eta(t)} D_i \tag{8}$$

is a solution to equation (7).

Now, suppose we have $n+1$ historical observations, $S(t)$, $t \in \{0, 1, ..., n\}$. Then the daily logarithmic stock return for $t \in \{1, 2, ..., n\}$,

$$\begin{aligned}
R(t) &= \ln \frac{S(t)}{S(t-1)} \\
&= \ln \left(\frac{S(0) \exp\left\{\sigma B(t) + \left(\mu - \lambda k - \frac{\sigma^2}{2}\right)t\right\} \prod_{i=1}^{\eta(t)} D_i}{S(0) \exp\left\{\sigma B(t-1) + \left(\mu - \lambda k - \frac{\sigma^2}{2}\right)(t-1)\right\} \prod_{i=1}^{\eta(t-1)} D_i} \right) \\
&= \sigma(B(t) - B(t-1)) + \mu - \lambda k - \frac{\sigma^2}{2} + \sum_{i=\eta(t-1)+1}^{\eta(t)} F_i \\
&\overset{D}{=} \sigma(B(t) - B(t-1)) + \mu - \lambda k - \frac{\sigma^2}{2} + \sum_{i=1}^{\eta(t)-\eta(t-1)} F_i \\
&\overset{D}{=} \sigma B(1) + \mu - \lambda k - \frac{\sigma^2}{2} + \sum_{i=1}^{\eta(1)} F_i.
\end{aligned} \tag{9}$$

We have used the fact that the increments of $B(t)$ and $\eta(t)$ are stationary.

Next, the cumulative distribution function (cdf) of $R(t)$ can be expressed as [similar to Hanson and Westman (2002) and Synowiec (2008)],

$$\begin{aligned}
P(R(t) < \tilde{R}(t)) &= \sum_j P(R(t) < \tilde{R}(t) | \eta(1) = j) P(\eta(1) = j) \\
&= \sum_j P(\sigma B(1) + \mu - \lambda k - \frac{\sigma^2}{2} + \sum_{i=1}^{j} F_i \\
&< \tilde{R}(t)) P(\eta(1) = j) \\
&= \sum_j P(H(t) < \tilde{R}(t)) P(\eta(1) = j)
\end{aligned} \tag{10}$$

where $H(t) = \sigma B(1) + \mu - \lambda k - \frac{\sigma^2}{2} + \sum_{i=1}^{j} F_i \sim \mathcal{N}\left(\mu - \lambda k - \frac{\sigma^2}{2} + jm,\, \sigma^2 + j\delta^2\right)$.

In the next step, we simplify the formulae for the cumulative distribution function and the probability density function of the daily logarithmic stock return. We have assumed discrete time in our study, hence we can express the Poisson process at time $t = 1$ as,

$$\eta(1) = \{\#i : \tau_i \in (0, 1]\} = \{\#i : \tau_i = 1\} = 1 \text{ or } 0.$$

Hence, equation (10) can be expressed as a finite sum.

$$P(R(t) < \tilde{R}(t)) = \sum_{j=0}^{1} P(H(t) < \tilde{R}(t)) P(\eta(1) = j). \tag{11}$$

Clearly, the cdf of $R(t)$ is differentiable, which implies the probability density function of $R(t)$ can be expressed as,

$$\begin{aligned} f_{R(t)}(\tilde{R}(t)) &= \sum_{j=0}^{1} f_{H(t)}(\tilde{R}(t))P(\eta(1)=j) \\ &= \sum_{j=0}^{1} \frac{1}{h_2\sqrt{2\pi}} e^{-\frac{(\tilde{R}(t)-h_1)^2}{2{h_2}^2}} e^{-\lambda} \frac{\lambda^j}{j!} \\ &= \sum_{j=0}^{1} \frac{\lambda^j}{j! h_2 \sqrt{2\pi}} \exp\Big\{ -\frac{(\tilde{R}(t)-h_1)^2}{2{h_2}^2} - \lambda \Big\} \end{aligned} \tag{12}$$

where $h_1 = \mu - \lambda k - \frac{\sigma^2}{2} + jm$ and $h_2 = \sqrt{\sigma^2 + j\delta^2}$.

Next, we can write the likelihood function of the sample $\{R(1), ..., R(n)\}$ as,

$$\begin{aligned} L_{JD}(\mu, \sigma, m, \delta, \lambda) &= \prod_{t=1}^{n} f_{R(t)}(\tilde{R}(t)) \\ &= \prod_{t=1}^{n} \sum_{j=0}^{1} \frac{\lambda^j}{j! h_2 \sqrt{2\pi}} \exp\Big\{ -\frac{(\tilde{R}(t)-h_1)^2}{2{h_2}^2} - \lambda \Big\} \end{aligned} \tag{13}$$

and the log-likelihood function of the sample as,

$$\begin{aligned} &logL_{JD}(\mu, \sigma, m, \delta, \lambda) \\ &= \sum_{t=1}^{n} \ln \left(\sum_{j=0}^{1} \frac{\lambda^j}{j! h_2 \sqrt{2\pi}} \exp\Big\{ -\frac{(\tilde{R}(t)-h_1)^2}{2{h_2}^2} - \lambda \Big\} \right) \end{aligned} \tag{14}$$

In the next section, we do an empirical analysis of the stock market data. We use the Merton model log-likelihood function of the sample to obtain the parameter estimates using maximum likelihood estimation and Lange (1994) adaptive barrier algorithm which is discussed in details in the section, *Merton model parameter estimates*.

3 Empirical analysis

We consider ten different companies listed in BSE which are some of the largest companies and whose stocks are one of the most actively traded. These companies are also components of S&P BSE SENSEX, which is the benchmark stock index of BSE. The daily closing stock prices for a period of eight years, starting from January 2011 to December 2018 are taken for these companies. The trading occurred between 3rd January 2011 and 31st December 2018 for a total of 1981 days. The initial closing price, $S(0)$ is taken to be the closing price on 3rd January 2011 for each of these stocks. The dataset is taken from the website of BSE, https://www.bseindia.com/markets/equity/EQReports/StockPrcHistori.aspx?flag=0 where the prices are available in INR. In our study, we have considered stocks of these companies, Housing Development Finance Corp. Ltd. (HDFC), Bajaj Finance Limited (BAJFINANCE), HDFC Bank Ltd

(HDFCBANK), Infosys Ltd. (INFY), Kotak Mahindra Bank Ltd. (KOTAKBANK), Asian Paints Ltd. (ASIANPAINT), Axis Bank Ltd. (AXISBANK), HCL Technologies Ltd. (HCLTECH), Bharti Airtel Ltd. (BHARTIARTL), and Tata Consultancy Services Ltd. (TCS). Time, t is measured in trading days, hence t varies from $t = 0$ to $t = n$, where $n = 1980$. The timespan of eight years taken for historical prices may contain some regular days during which trading has not occurred, but since we have only considered trading days, $\Delta t = t + 1 - t = 1$, which denotes the difference between two consecutive trading days and the difference is also measured in trading days only.

Now, the observed daily logarithmic stock returns $\{R_{obs}(t)\}$ form a weakly stationary time series with 1% level of significance since by augmented Dickey-Fuller test, using R package *tseries* and function *adf.test* (Trapletti et al., 2019), the p-values are found to be less than 0.01.

3.1 BS model parameter estimates

The historical drift rate of the stock price, $\tilde{\mu}_{BS}$, is calculated as the arithmetic mean of the observed daily logarithmic stock returns and the historical volatility of the stock price, $\tilde{\sigma}_{BS}$, is calculated as the sample standard deviation of the observed daily logarithmic stock returns. Using equation (6), the parameter estimates for BS model are obtained (see Table 1).

Using these estimates, we can rewrite equation (3) as,

$$\hat{S}_{BS}(t) \stackrel{D}{=} \exp\left\{ \ln \hat{S}_{BS}(t-1) + \hat{\sigma}_{BS} B(1) + \hat{\mu}_{BS} - \frac{\hat{\sigma}_{BS}^2}{2} \right\} \tag{15}$$

where $\hat{S}_{BS}(0) = S(0)$.

Equation (15) estimates the closing prices for $t \in \{1, 2, ..., n\}$ using BS model.

3.2 Merton model parameter estimates

In our study, the jump events are taken as the outlier events. Conditional on no jumps in the stock price trajectory, the historical drift rate of the stock price, $\tilde{\mu}_{JD}$, is calculated as the arithmetic mean of those observed daily logarithmic stock returns which are not outliers and the historical volatility of the stock price, $\tilde{\sigma}_{JD}$, is calculated as the sample standard deviation of those observed daily logarithmic stock returns which are not outliers. Next, we calculate historical m as the arithmetic mean of those observed daily logarithmic stock returns which are outliers and historical δ as the sample standard deviation of those observed daily logarithmic stock returns which are outliers. Finally, the historical λ is calculated as the multiplicative inverse of *arithmetic mean of historical waiting times of the jump events*. We obtain the parameter estimates using maximum likelihood estimation and Lange (1994) adaptive barrier algorithm using *stats4* R-package and *constrOptim* function. In the *constrOptim* function, we take the negative of log-likelihood function of the sample in equation (14) as the objective function, and the constraints, $\sigma > 0$, $\delta > 0$, and $\lambda > 0$ as the linear inequality constraints, and obtain the parameter estimates (see Table 2).

Using these estimates, we can rewrite equation (9) as,

$$\hat{S}_{JD}(t) \stackrel{D}{=} \exp\left\{ \ln \hat{S}_{JD}(t-1) + \hat{\sigma}_{JD} B(1) + \hat{\mu}_{JD} - \hat{\lambda}\hat{k} - \frac{\hat{\sigma}_{JD}^2}{2} + \sum_{i=1}^{\eta(1)} F_i \right\} \tag{16}$$

where $\hat{k} = \exp\{\hat{m} + \frac{\hat{\delta}^2}{2}\} - 1$ and $\hat{S}_{JD}(0) = S(0)$.

Table 1 BS model parameter estimates

Stock \ Parameter	$\hat{\mu}_{BS}$	$\hat{\sigma}_{BS}$
HDFC	0.000632864	0.01612379
BAJFINANCE	0.002206417	0.05548494
HDFCBANK	0.0006725949	0.03827953
INFY	–0.0003275072	0.0318851
KOTAKBANK	0.0007735485	0.02289367
ASIANPAINT	0.001087147	0.05403946
AXISBANK	0.0004769039	0.04187144
HCLTECH	0.0006621006	0.02379566
BHARTIARTL	0.0001166102	0.01931403
TCS	0.0004908703	0.0220537

Notes: $\hat{\mu}_{BS}$ is the BS drift rate estimate and $\hat{\sigma}_{BS}$ is the BS volatility estimate.

Table 2 Merton model parameter estimates

Stock \ Parameter	$\hat{\mu}_{JD}$	$\hat{\sigma}_{JD}$	$\hat{m}$	$\hat{\delta}$	$\hat{\lambda}$
HDFC	0.0006353737	0.0143505595	0.0018786326	0.0303349168	0.0584864062
BAJFINANCE	0.002196933	0.019329425	–0.067464328	0.487882044	0.011154407
HDFCBANK	0.0004680556	0.0125925556	–0.4480710705	0.6968045937	0.0019467992
INFY	–0.0003614767	0.0135736923	–0.0877093422	0.2078064571	0.0164385535
KOTAKBANK	0.0007724813	0.0154527938	–0.0184123148	0.1511323884	0.0123116209
ASIANPAINT	0.0008070287	0.0149885211	–0.4171211547	0.8609583384	0.0029545907
AXISBANK	0.0003901196	0.0202124905	–0.2154644775	0.5352069534	0.0040474616
HCLTECH	0.0006550052	0.0160371432	–0.0576722550	0.1586933761	0.0108717177
BHARTIARTL	0.000116165	0.016194844	0.009086206	0.034567245	0.087228102
TCS	0.0004873563	0.0142765962	–0.0394923570	0.1618729354	0.0101862285

Notes: $\hat{\mu}_{JD}$ and $\hat{\sigma}_{JD}$ are the Merton diffusion parameters (drift rate and volatility) estimates; $\hat{m}$, $\hat{\delta}$, and $\hat{\lambda}$ are the Merton jump parameters estimates.

Equation (16) estimates the closing prices for $t \in \{1, 2, ..., n\}$ using Merton model.

3.3 In-sample performance: BS vs. Merton

To determine which model is a better fit for the stock prices, we calculate the root-mean-square error (RMSE) which measures how much the estimated values deviate from the observed values. Mathematically, it is expressed as,

$$RMSE = \sqrt{\frac{1}{n+1}\sum_{t=0}^{n}(S_{est}(t) - S_{obs}(t))^2} \tag{17}$$

where $\{S_{obs}(t)\}$ are the observed stock prices and $\{S_{est}(t)\}$ are the estimated stock prices.

For BS model, $S_{est}(t) = \hat{S}_{BS}(t)$ and for Merton's JD model, $S_{est}(t) = \hat{S}_{JD}(t)$. The RMSE values for both the models for each of the ten stocks are thus obtained (see Table 3).

Table 3 In-sample RMSE values (in INR) for BS and Merton model

Model / *Stock*	*Black-Scholes*	*Merton*
HDFC	614.0331	218.2584
BAJFINANCE	6,551.809	3413.715
HDFCBANK	3053.206	593.196
INFY	1,159.352	719.0132
KOTAKBANK	649.3662	507.0624
ASIANPAINT	4,152.864	861.3841
AXISBANK	1,176.116	685.9484
HCLTECH	633.18	569.8936
BHARTIARTL	116.152	80.61038
TCS	869.1622	746.7614

Notes: The values in columns, *BS* and *Merton* are the RMSE values.

Table 4 AIC values for BS and Merton model

Model / *Stock*	*Black-Scholes*	*Merton*
HDFC	–10,721.74	–10,780.04
BAJFINANCE	–5,827.912	–9,670.251
HDFCBANK	–7,297.849	–11,615.42
INFY	–8,021.645	–10,972.27
KOTAKBANK	–9,333.508	–10,610.34
ASIANPAINT	–5,932.445	–10,891.6
AXISBANK	–6,942.682	–9,689.513
HCLTECH	–9,180.481	–10,483.84
BHARTIARTL	–10,006.82	–10,095.91
TCS	–9,481.533	–10,956.36

Notes: The values in columns, *BS* and *Merton* are the AIC values.

We observe that RMSE for Merton model is coming to be less than that for BS model for each of these stocks. Hence, we conclude that the in-sample performance of Merton model is better than that of BS model.

For measuring the goodness of fit of these models, apart from RMSE, we can also use the Akaike information criterion (AIC) (Akaike, 1974). AIC is usually used for model selection and it is especially useful when we do not have the out-of-sample data. Mathematically, it is expressed as,

$$\begin{aligned} AIC = & -2\ln(\text{maximum likelihood}) \\ & + 2(\text{\# independently adjusted parameters within the model}). \end{aligned} \tag{18}$$

For BS model, maximum likelihood = $logL_{BS}(\hat{\mu}_{BS}, \hat{\sigma}_{BS})$ and # independently adjusted parameters within the model = 2; for Merton model, maximum likelihood =

$logL_{JD}(\hat{\mu}_{JD}, \hat{\sigma}_{JD}, \hat{m}, \hat{\delta}, \hat{\lambda})$ and # independently adjusted parameters within the model = 5. The AIC values for both the models for each of the ten stocks are thus obtained (see Table 4).

We observe that AIC for Merton model is coming to be less than that for BS model for each of these stocks. Hence, we conclude that the Merton model is better than that of BS model for forecasting future stock prices in the out-of-sample period. We arrive at the same conclusion when we find out that the out-of-sample performance of Merton model is better than that of BS model based on RMSE values.

3.4 Out-of-sample performance: BS vs. Merton

We have taken in-sample period from January 2011 to December 2018, and based on the historical data we have estimated the parameters of these models. Now we do a out-of-sample forecast of stock prices for the month of January 2019 which has 23 trading days, that is n = 23. For out-of-sample forecasting, we take the initial closing price, $S(0)$ as the closing price on 31st December 2018 for each of these stocks. The RMSE values for both the models for each of the ten stocks are thus obtained (see Table 5).

We observe that RMSE for Merton model is coming to be less than that for BS model for each of these stocks. Hence, we conclude that the out-of-sample performance of Merton model is better than that of BS model.

Based on two measures of goodness of fit, RMSE and AIC, our empirical analysis concludes that the Merton's JD model performs better than the BS model when estimating stock prices. One can use any of the two measures to select the best available model among a given number of models for forecasting.

3.5 Trading strategies

In this section, we discuss and compare two trading strategies.

Table 5 Out-of-sample RMSE values (in INR) for BS and Merton model

Stock \ *Model*	*Black-Scholes*	*Merton*
HDFC	76.4522	48.86823
BAJFINANCE	279.807	124.127
HDFCBANK	193.0651	81.36389
INFY	111.3285	95.27267
KOTAKBANK	54.76692	54.21209
ASIANPAINT	208.6506	30.91859
AXISBANK	109.8098	29.04598
HCLTECH	55.25868	41.05978
BHARTIARTL	25.63055	15.07525
TCS	116.9464	53.67831

Notes: The values in columns, *BS* and *Merton* are the RMSE values.

3.5.1 BH strategy

Using this trading strategy, first, we buy one unit of each stock at the end of the in-sample (training) period, 31 December 2018. We hold these stocks for the entire out-of-sample (forecast) period and sell these stocks at the end of the forecast period, 31 January 2019. BSE charges a fee of 0.003% of total turnover as transaction charges on equity and delivery trading. From our data of observed prices, we calculate the profits if we follow this strategy (see Table 6).

Table 6 Profit/loss (in INR) for BH trading strategy

Stock	*Price on 31 December 2018*	*Price on 31 January 2019*	*Transaction costs*	*Profit*
HDFC	1,970	1,923.7	0.12	–46.42
BAJFINANCE	2,641.15	2,570.35	0.16	–70.96
HDFCBANK	2,122.45	2,081.15	0.13	–41.43
INFY	659.85	749.6	0.04	89.71
KOTAKBANK	1,254.75	1,253.25	0.08	–1.58
ASIANPAINT	1,373.7	1,412.65	0.08	38.87
AXISBANK	619.8	722.95	0.04	103.11
HCLTECH	962.55	1,005.2	0.06	42.59
BHARTIARTL	312.9	307.15	0.02	–5.77
TCS	1,893.55	2,014.6	0.12	120.93

Notes: In the column, *profit*, positive values indicate profit and negative values indicate loss.

Calculating profit for stock BAJFINANCE using BH trading strategy.

Since, the stock is bought/sold on 31 December 2018 and 31 January 2019, i.e., two days, the transaction cost is calculated as,

$$TC_{BH} = \frac{0.003}{100}(2{,}641.15 + 2{,}570.35) \approx 0.16 \text{ INR}.$$

Therefore the profit is calculated as,

$$P_{BH} = -2{,}641.15 + 2{,}570.35 - 0.16 = -70.96 \text{ INR}.$$

The profits for the other stocks are calculated in a similar way.

3.5.2 DM strategy using Merton model

According to this daily trading strategy, we buy or sell a stock today based on next day's Merton model estimated price going up or down. We start this strategy at the end of the training period and we assume no short-selling. For stocks whose estimated prices go up (becoming more than the previous day's observed price) right after the training period, that is on 1 January 2019, we buy one unit of each of these stocks on 31 December 2018. For stocks whose estimated prices go up on some other days within the forecast period, we buy one unit of each of those stocks on the previous days. After buying the stock, we hold it as long as next day's estimated price does not go below current day's observed price. Once next day's estimated price goes below current day's observed price, we sell the stock on the current day. The observed and Merton model estimated prices during the forecast period are given in Tables 7 and 8.

Table 7 Observed (obs) and Merton model estimated (est) prices (in INR) of HDFC, BAJFINANCE, HDFCBANK, INFY, KOTAKBANK, and ASIANPAINT during January 2019

Date	*HDFC obs*	*HDFC est*	*BAJFINANCE obs*	*BAJFINANCE est*	*HDFCBANK obs*	*HDFCBANK est*
31 December 2018	1,970.00	1,970.000	2,641.15	2,641.150	2,122.45	2,122.450
1 January 2019	2,009.60	1,971.309	2,656.20	2,645.862	2,147.45	2,124.544
2 January 2019	1,979.00	2,008.666	2,616.50	2,716.027	2,125.95	2,160.702
3 January 2019	1,935.80	2,014.800	2,587.95	2,729.626	2,112.25	2,167.365
4 January 2019	1,971.15	1,961.928	2,579.25	2,635.923	2,117.75	2,118.229
7 January 2019	1,971.00	1,978.618	2,552.65	2,668.538	2,120.00	2,134.894
8 January 2019	1,959.25	1,999.732	2,535.05	2,709.368	2,102.85	2,155.740
9 January 2019	1,993.30	1,970.889	2,525.10	2,558.691	2,116.20	2,129.290
10 January 2019	1,979.00	1,989.557	2,515.85	2,593.688	2,109.55	2,147.842
11 January 2019	1,987.65	2,011.183	2,511.00	2,634.070	2,112.15	2,169.190
14 January 2019	1,969.30	2,001.932	2,540.75	2,620.087	2,100.85	2,161.303
15 January 2019	1,990.80	2,033.480	2,599.25	2,678.230	2,121.05	2,192.045
16 January 2019	1,974.35	2,026.759	2,572.95	2,668.679	2,120.00	2,186.567
17 January 2019	2,004.25	1,990.809	2,535.60	2,607.431	2,129.05	2,153.365
18 January 2019	2,008.05	2,033.667	2,540.85	2,685.700	2,131.20	2,194.874
21 January 2019	2,003.30	2,046.041	2,588.90	2,710.138	2,146.55	2,207.478
22 January 2019	1,982.60	2,017.284	2,616.65	2,733.536	2,134.35	2,219.487
23 January 2019	1,957.65	1,985.348	2,633.05	2,677.782	2,109.75	2,189.506
24 January 2019	1,969.05	1,939.189	2,631.50	2,596.567	2,101.90	2,145.637
25 January 2019	1,977.60	1,983.397	2,596.35	2,678.990	2,093.95	2,189.383
28 January 2019	1,948.60	2,006.464	2,456.20	2,723.456	2,083.55	2,212.602
29 January 2019	1,918.80	2,034.664	2,511.90	2,787.136	2,058.10	2,245.777
30 January 2019	1,885.50	2,001.207	2,596.35	2,728.001	2,032.55	2,214.231
31 January 2019	1,923.70	1,990.220	2,570.35	2,710.251	2,081.15	2,204.449

Date	*INFY obs*	*INFY est*	*KOTAKBANK obs*	*KOTAKBANK*	*ASIANPAINT obs*	*ASIANPAINT est*
31 December 2018	659.85	659.8500	1,254.75	1,254.750	1,373.70	1,373.700
1 January 2019	664.65	660.4128	1,251.35	1,256.041	1,371.85	1,375.226
2 January 2019	669.30	672.3948	1,240.90	1,282.091	1,381.55	1,403.039
3 January 2019	667.55	674.4880	1,237.35	1,286.711	1,385.15	1,408.101
4 January 2019	660.75	702.1851	1,246.70	1,250.780	1,385.30	1,370.099
7 January 2019	671.15	707.9926	1,247.55	1,262.636	1,394.80	1,382.850
8 January 2019	669.85	715.2968	1,232.25	1,277.551	1,401.25	1,398.848
9 January 2019	675.85	705.6927	1,238.90	1,258.114	1,402.90	1,378.355
10 January 2019	679.75	712.1728	1,221.35	1,271.348	1,397.55	1,392.572
11 January 2019	683.70	619.2712	1,221.25	1,286.638	1,402.20	1,408.972
14 January 2019	700.90	616.7147	1,210.45	1,280.667	1,388.40	1,402.787

Notes: The values in the *obs* columns are the observed closing stock prices; values in the *est* columns are the Merton model estimated closing stock prices.

Table 7 Observed (obs) and Merton model estimated (est) prices (in INR) of HDFC, BAJFINANCE, HDFCBANK, INFY, KOTAKBANK, and ASIANPAINT during January 2019 (continued)

Date	*INFY obs*	*INFY est*	*KOTAKBANK obs*	*KOTAKBANK*	*ASIANPAINT obs*	*ASIANPAINT est*
15 January 2019	726.55	626.0438	1,212.15	1,302.820	1,406.05	1,426.477
16 January 2019	736.55	624.2261	1,205.10	1,298.589	1,389.00	1,422.144
17 January 2019	733.40	613.8859	1,220.10	1,274.203	1,389.45	1,396.389
18 January 2019	731.00	626.5190	1,237.35	1,304.172	1,401.30	1,428.395
21 January 2019	742.75	630.2654	1,267.30	1,313.130	1,420.65	1,438.071
22 January 2019	744.35	633.8286	1,291.60	1,321.662	1,406.55	1,447.295
23 January 2019	731.45	624.4733	1,276.20	1,299.551	1,396.05	1,423.965
24 January 2019	732.80	610.8685	1,267.95	1,267.442	1,400.25	1,389.982
25 January 2019	730.20	584.2343	1,265.15	1,298.990	1,372.20	1,423.687
28 January 2019	728.20	590.7915	1,261.15	1,315.676	1,379.20	1,441.585
29 January 2019	727.35	600.2190	1,250.95	1,339.682	1,392.50	1,467.255
30 January 2019	724.85	591.0117	1,221.50	1,316.387	1,388.75	1,442.664
31 January 2019	749.60	588.0740	1,253.25	1,309.017	1,412.65	1,434.990

Notes: The values in the *obs* columns are the observed closing stock prices; values in the *est* columns are the Merton model estimated closing stock prices.

Table 8 Observed (obs) and Merton model estimated (est) prices (in INR) of AXISBANK, HCLTECH, BHARTIARTL, and TCS during January 2019

Date	*AXISBANK obs*	*AXISBANK est*	*HCLTECH obs*	*HCLTECH est*
31 December 2018	619.80	619.8000	962.55	962.5500
1 January 2019	627.50	620.3251	962.00	963.8082
2 January 2019	618.90	636.8892	945.85	984.7965
3 January 2019	607.95	639.5734	947.95	988.7152
4 January 2019	619.85	616.0064	933.30	960.3062
7 January 2019	637.45	623.3448	939.70	969.9870
8 January 2019	650.90	632.6775	945.10	982.1148
9 January 2019	670.05	619.8075	940.10	966.8430
10 January 2019	663.00	628.0361	935.85	977.6334
11 January 2019	666.50	637.6159	940.50	990.0747
14 January 2019	659.40	633.4324	937.55	985.5421
15 January 2019	660.10	647.4797	947.05	1,003.4801
16 January 2019	663.65	644.4098	938.15	1,000.3376
17 January 2019	676.30	628.3133	954.80	981.0827
18 January 2019	664.30	647.3902	964.50	1,005.2814
21 January 2019	660.25	652.8871	962.60	1,012.6901
22 January 2019	661.80	658.1128	941.60	1,019.7626
23 January 2019	661.40	643.4285	944.90	1,002.3029
24 January 2019	663.95	622.4032	947.20	976.8470

Notes: The values in the *obs* columns are the observed closing stock prices; values in the *est* columns are the Merton model estimated closing stock prices.

Table 8 Observed (obs) and Merton model estimated (est) prices (in INR) of AXISBANK, HCLTECH, BHARTIARTL, and TCS during January 2019 (continued)

Date	*AXISBANK obs*	*AXISBANK est*	*HCLTECH obs*	*HCLTECH est*
25 January 2019	667.75	642.4239	971.25	1,002.3319
28 January 2019	655.95	652.9139	975.95	1,015.9404
29 January 2019	660.80	668.2065	988.10	1,035.4316
30 January 2019	690.90	652.7243	1017.10	1,016.9958
31 January 2019	722.95	647.6251	1,005.20	1,011.3282
Date	*BHARTIARTL obs*	*BHARTIARTL est*	*TCS obs*	*TCS est*
31 December 2018	312.90	312.9000	1,893.55	1,893.550
1 January 2019	319.55	312.7248	1,902.35	1,895.290
2 January 2019	312.80	319.0025	1,923.15	1,931.516
3 January 2019	313.15	319.6830	1,896.45	1,937.883
4 January 2019	322.60	309.8257	1,873.95	1,887.773
7 January 2019	324.45	312.3922	1,896.65	1,904.239
8 January 2019	328.85	315.7428	1,893.05	1,924.949
9 January 2019	335.15	310.2017	1,887.80	1,897.816
10 January 2019	337.25	313.1089	1,888.15	1,916.191
11 January 2019	334.05	316.5375	1,841.95	1,937.411
14 January 2019	331.90	314.4826	1,814.40	1,929.041
15 January 2019	337.65	306.8431	1,864.20	1,959.787
16 January 2019	333.35	305.2985	1,870.10	1,953.845
17 January 2019	332.30	298.8027	1,895.10	1,919.860
18 January 2019	310.95	305.6711	1,900.40	1,961.479
21 January 2019	310.45	307.3679	1,905.80	1,973.860
22 January 2019	304.25	308.9546	1,900.35	1,985.642
23 January 2019	304.05	303.0432	1,879.75	1,954.871
24 January 2019	301.60	294.7177	1,901.75	1,910.143
25 January 2019	307.05	301.9152	1,919.05	1,953.965
28 January 2019	304.90	305.4802	1,951.95	1,977.081
29 January 2019	307.10	310.8145	1,983.15	2,010.321
30 January 2019	303.35	304.6535	1,977.85	1,977.942
31 January 2019	307.15	302.3703	2,014.60	1,967.645

Notes: The values in the *obs* columns are the observed closing stock prices; values in the *est* columns are the Merton model estimated closing stock prices.

Throughout the trading period, we trade with only one unit of each stock. The trade details are given below:

- 31 December 2018: Buy 1 unit of HDFC, 1 unit of BAJFINANCE, 1 unit of HDFCBANK, 1 unit of INFY, 1 unit of KOTAKBANK, 1 unit of ASIANPAINT, 1 unit of AXISBANK, 1 unit of HCLTECH, and 1 unit of TCS.
- 1 January 2019: Sell 1 unit of HDFC.
- 2 January 2019: Buy 1 unit of HDFC and 1 unit of BHARTIARTL.
- 3 January 2019: Sell 1 unit of ASIANPAINT, 1 unit of BHARTIARTL, and 1 unit of TCS.

- 4 January 2019: Buy 1 unit of TCS.
- 7 January 2019: Buy 1 unit of ASIANPAINT and sell 1 unit of AXISBANK.
- 8 January 2019: Sell 1 unit of ASIANPAINT.
- 9 January 2019: Sell 1 unit of HDFC.
- 10 January 2019: Buy 1 unit of HDFC and 1 unit of ASIANPAINT.
- 11 January 2019 to 22 January 2019: No trade.
- 23 January 2019: Sell 1 unit of HDFC, 1 unit of BAJFINANCE, 1 unit of KOTAKBANK, and 1 unit of ASIANPAINT.
- 24 January 2019: Buy 1 unit of HDFC, 1 unit of BAJFINANCE, 1 unit of KOTAKBANK, 1 unit of ASIANPAINT, and 1 unit of BHARTIARTL.
- 25 January 2019: Sell 1 unit of BHARTIARTL.
- 28 January 2019: Buy 1 unit of AXISBANK and 1 unit of BHARTIARTL.
- 29 January 2019: Sell 1 unit of AXISBANK, 1 unit of BHARTIARTL, and 1 unit of TCS.
- 30 January 2019: Sell 1 unit of HCLTECH.
- 31 January 2019: Sell 1 unit of HDFC, 1 unit of BAJFINANCE, 1 unit of HDFCBANK, 1 unit of KOTAKBANK, and 1 unit of ASIANPAINT.

Using this trading strategy, we calculate the profits (see Table 9).

Calculating profit for stock BAJFINANCE using DM trading strategy.

The observed stock price on 31 December 2018 is 2,641.15 INR. The observed and estimated stock prices during the forecast period are given in Table 7.

The transaction cost is calculated as,

$$TC_{DM} = \frac{0.003}{100}(2{,}641.15 + 2{,}633.05 + 2{,}631.5 + 2{,}570.35) \approx 0.31 \text{ INR}.$$

Therefore the profit is calculated as,

$$P_{DM} = -2{,}641.15 + 2{,}633.05 - 2{,}631.5 + 2{,}570.35 - 0.31 = -69.56 \text{ INR}.$$

The profits for the other stocks are calculated in a similar way.

Our results show that the DM strategy outperforms the BH strategy for five stocks, HDFC, BAJFINANCE, KOTAKBANK, HCLTECH and BHARTIARTL, and for four stocks, INFY, ASIANPAINT, AXISBANK, and TCS, the BH strategy outperforms the DM strategy. Both the trading strategies incur same amount of loss in trading HDFCBANK. We notice that for each of the stocks whose observed price at the end of the forecast period is less than its initial observed price, $S(0)$, the BH strategy fails to outperform the DM strategy. Hence, we conclude that the DM strategy is a better strategy for trading low performing stocks (stocks whose closing prices go down for a considerable amount of time, say few months).

Using this information, a trader can take a decision about which trading strategies he shall follow for which stocks during the next forecast period (say, for example, February 2019) to make profit. The share turnover ratio of a stock for a particular day is calculated as the ratio of total number of shares of the stock that were bought and sold on that day and the total number of shares that have been issued to investors and are available for purchase on that day. It is a measure of stock liquidity. The mean share turnover ratios of the ten stocks during the forecast period (calculated as the average of daily share turnover ratios over the forecast period) are given in Table 10. The low ratios suggest that the stocks are illiquid and that our trading strategy works well for illiquid stocks.

Table 9 Profit/loss (in INR) for DM trading strategy using Merton model

Stock	*Transaction costs*	*Profit*
HDFC	0.47	–13.27
BAJFINANCE	0.31	–69.56
HDFCBANK	0.13	–41.43
INFY	0.04	19.86
KOTAKBANK	0.15	6.6
ASIANPAINT	0.33	28.47
AXISBANK	0.08	22.42
HCLTECH	0.06	54.49
BHARTIARTL	0.06	7.94
TCS	0.23	111.87

Notes: In the column, *profit*, positive values indicate profit and negative values indicate loss.

Table 10 Mean share turnover ratios

Stock	*Mean share turnover ratio (in %)*
HDFC	3.69
BAJFINANCE	4.14
HDFCBANK	3.33
INFY	1.61
KOTAKBANK	2.09
ASIANPAINT	2.96
AXISBANK	1.18
HCLTECH	3.73
BHARTIARTL	0.9
TCS	4.42

Note: Low mean share turnover ratios suggest illiquid stocks.

3.6 Markowitz minimum variance portfolio

Now, suppose we as risk-averse investors want to invest our money on these stocks with minimum risk. This can be achieved by forming a Markowitz minimum variance portfolio of these stocks.

Let $\{r_1, r_2, ..., r_{10}\}$ be the daily stock returns (percent returns) for ten stocks and $r = (r_1, ..., r_{10})^T$ be the transpose of the return vector. Let $e = (e_1, ..., e_{10})^T$ be the transpose of the expected return vector. We can represent the return of the portfolio as

$$r_p = w_1 r_1 + ... + w_{10} r_{10} \tag{19}$$

where $\{w_i\}$ are the weights of the stocks.

The value of the portfolio can be represented as,

$$P = w_1 S_1 + ... + w_{10} S_{10} \tag{20}$$

Let e_p be the expected return of the portfolio, σ_p^2 be the variance of the portfolio, and $V = (\sigma_{ij})$ be the covariance matrix where $\sigma_{ij} = (r_i, r_j)$. Let $w = (w_1, ..., w_{10})^T$, $1 = (1, ..., 1)^T$, and $0 = (0, ..., 0)^T$. By Markowitz portfolio theory, the optimal weights which minimise σ_p, given a value of e_p, can be obtained as

$$w_p = g + h e_p \tag{21}$$

where $A = e^T V^{-1} 1$, $B = e^T V^{-1} e$, $C = 1^T V^{-1} 1$, $D = BC - A^2$, $g = \frac{1}{D}(BV^{-1}1 - AV^{-1}e)$, and $h = \frac{1}{D}(CV^{-1}e - AV^{-1}1)$. The minimum portfolio variance is obtained as $\frac{1}{C}$ when $e_p = \frac{A}{C}$, i.e., $e_{mvp} = \frac{A}{C}$ and $\sigma_{mvp} = \frac{1}{C}$ (Pärna, 2016).

We take w_1 = weight of HDFC, w_2 = weight of BAJFINANCE, w_3 = weight of HDFCBANK, w_4 = weight of INFY, w_5 = weight of KOTAKBANK, w_6 = weight of ASIANPAINT, w_7 = weight of AXISBANK, w_8 = weight of HCLTECH, w_9 = weight of BHARTIARTL, and w_{10} = weight of TCS.

3.6.1 BH minimum variance portfolio (BHMVP) strategy

Now instead of trading individual stocks, suppose if we form a Markowitz minimum variance portfolio of these stocks and follow the BH trading strategy, that is we buy the mean variance portfolio at the end of the training period, hold the portfolio for the entire forecast period and sell it at the end of the forecast period. By Markowitz portfolio theory, the optimal weights of the stocks are calculated (by considering the observed stock prices and the corresponding daily stock returns during the training period) as

$$\begin{aligned} &w_1 \approx 17\%, w_2 \approx 4\%, w_3 \approx 8\%, w_4 \approx 7\%, w_5 \approx 8\%, \\ &w_6 \approx 9\%, w_7 \approx 2\%, w_8 \approx 12\%, w_9 \approx 16\%, w_{10} \approx 17\%, \end{aligned}$$

that is, if we buy 100 units of these stocks (17 units of HDFC, 4 units of BAJFINANCE, ..., and 17 units of TCS) at the end of the training period, hold them throughout the forecast period and sell them, we have a profit of

$$\begin{aligned} P_{BHMVP} &= -138{,}041.4 + 140{,}291.5 - \frac{0.003}{100}(138{,}041.4 + 140{,}291.5) \\ &= 2{,}241.75 \text{ INR.} \end{aligned}$$

3.6.2 DM minimum variance portfolio (DMMVP) strategy

In this strategy, we form a Markowitz minimum variance portfolio of stocks each day we buy these stocks according to the DM strategy using Merton model. Each day we can buy 100 units of these stocks forming a Markowitz minimum variance portfolio and sell the stock units which we already hold based on the DM strategy.

Using our observed and estimated prices during the forecast period (see Tables 7 and 8), we form the trading strategy as:

- 31 December 2018: Buy 23 units of HDFC, 5 units of BAJFINANCE, 10 units of HDFCBANK, 8 units of INFY, 9 units of KOTAKBANK, 10 units of ASIANPAINT, 3 units of AXISBANK, 13 units of HCLTECH, and 19 units of TCS. Cashflow = –160,398.8 – $\frac{0.003}{100}$(160,398.8) = –160,403.6 INR.
- 1 January 2019: Sell 23 units of HDFC. Cashflow = 46,219.41 INR.
- 2 January 2019: Buy 62 units of HDFC and 38 units of BHARTIARTL. Cashflow = –134,588.4 INR.
- 3 January 2019: Sell 10 units of ASIANPAINT, 38 units of BHARTIARTL, and 19 units of TCS. Cashflow = 61,781.9 INR.
- 4 January 2019: Buy 100 units of TCS. Cashflow = –187,400.6 INR.
- 7 January 2019: Buy 100 units of ASIANPAINT and sell 3 units of AXISBANK. Cashflow = –137,571.9 INR.
- 8 January 2019: Sell 100 units of ASIANPAINT. Cashflow = 140,120.8 INR.
- 9 January 2019: Sell 62 units of HDFC. Cashflow = 123,580.9 INR.
- 10 January 2019: Buy 76 units of HDFC and 24 units of ASIANPAINT. Sell 8 units of INFY. Cashflow = –178,512.9 INR.
- 11 January 2019 to 22 January 2019: No trade.
- 23 January 2019: Sell 76 units of HDFC, 5 units of BAJFINANCE, 9 units of KOTAKBANK, and 24 units of ASIANPAINT. Cashflow = 206,931.4 INR.
- 24 January 2019: Buy 35 units of HDFC, 9 units of BAJFINANCE, 17 units of KOTAKBANK, 13 units of ASIANPAINT, and 26 units of BHARTIARTL. Cashflow = –140,204.5 INR.
- 25 January 2019: Sell 26 units of BHARTIARTL. Cashflow = 7,983.061 INR.
- 28 January 2019: Buy 30 units of AXISBANK and 70 units of BHARTIARTL. Cashflow = –41,022.73 INR.
- 29 January 2019: Sell 30 units of AXISBANK, 70 units of BHARTIARTL, and 100 units of TCS. Cashflow = 239,628.8 INR.
- 30 January 2019: Sell 13 units of HCLTECH. Cashflow = 13,221.9 INR.
- 31 January 2019: Sell 35 units of HDFC, 9 units of BAJFINANCE, 10 units of HDFCBANK, 17 units of KOTAKBANK, and 13 units of ASIANPAINT. Cashflow = 150,939.3 INR.

Using this trading strategy, we have a profit of

$$\begin{aligned} P_{DMMVP} = & -160{,}403.6 + 46{,}219.41 - 134{,}588.4 + 61{,}781.9 - 187{,}400.6 \\ & - 13{,}7571.9 + 140{,}120.8 + 123{,}580.9 - 178{,}512.9 + 206{,}931.4 \\ & - 140{,}204.5 + 7{,}983.061 - 41{,}022.73 + 239{,}628.8 + 13{,}221.9 \\ & + 150{,}939.3 = 10{,}702.84 \text{ INR.} \end{aligned}$$

Our results show that the DMMVP strategy clearly outperforms the BH minimum variance portfolio strategy.

4 Conclusions

In this paper, we have simplified the formula for the probability density function of the daily logarithmic stock return according to Merton model and used the adaptive barrier algorithm of Lange to find the maximum likelihood estimates of the Merton model parameters. Using the parameter estimates of BS and Merton models, we estimated the stock prices for both the training and forecast periods. Both the in-sample and out-of-sample RMSE values as well as AIC values conclude that Merton's JD model is a better fit of historical stock prices as well as give superior forecasts of stock prices than the BS model. The abnormal rise and fall in stock prices has been better explained with a jump (compound Poisson) process rather than with a diffusion process. This paper proposed a DM trading strategy using Merton model estimated prices which can be useful and profitable to traders and investors. We have shown that this strategy can be a better trading strategy than the usual BH strategy for trading low performing stocks. When traded with a Markowitz minimum variance portfolio of stocks, the DM strategy is being shown to outperform the BH strategy. We have proposed a trading strategy which will be profitable and less risky to the risk-taking traders and risk-averse investors. We have shown that this trading strategy works well for trading in an illiquid stock market like India. Our proposed DM trading strategy is developed using Merton (1976) normal JD model and the scope for future research is that similar DM strategies can also be developed using other option pricing models such as Kou (2002) double exponential JD model, Heston (1993) stochastic volatility model, and Bates (1996) and Scott (1997) stochastic volatility JD models.

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