Constrained feedback RMPC for LPV systems with bounded rates of parameter variations and measurement errors

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Abstract: For Linear Parameter Varying (LPV) systems with bounded rates of parameter variations and bounded parameter measurement errors, a feedback Robust Model Predictive Control (RMPC) is designed by utilising the information on system parameters. A sequence of feedback control laws is designed based on the model with parameter-incremental uncertainty. Since the sequence of feedback control laws corresponds to the future variations of system parameters and introduces additional freedom, the control performance of RMPC can be improved. The recursive feasibility and closed-loop stability of the proposed RMPC are also proven.

Keywords: feedback RMPC; LPV systems; bounded rates; parameter variations; measurement errors.

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1 Introduction

Due to the capability of handing constraints explicitly, Model Predictive Control (MPC), also known as Receding Horizon Control (RHC), has become a popular technique for industrial process control and attracts much attention, especially robust MPC, such as Kothare et al. (1996) and Li et al. (2009). In some practical applications, the system parameters of LPV systems are often online measurable or vary with known bounded rates. For LPV systems with bounded rates of parameter variations, if the available information on the system parameters can be taken into account during controller design, the control performance is expected to be improved. Considering the measurable parametersss of LPV systems, Lu and Arkun (2000) proposed a quasi-min-max MPC algorithm. For LPV systems with bounded rates of parameter variations and the parameters restricted into the unit simplex, Casavola et al. (2002) developed a feedback Min-Max MPC algorithm. But there is a major problem in its initialisation stage, which is pointed out by Ding and Huang (2007).

For LPV systems with independently varying parameters, Park and Jeong (2004) transformed the system into a system with 'parameter-incremental' uncertainties. Then, by applying the open-loop dual-mode control, i.e., some free control moves followed by a feedback control law, an RMPC algorithm is proposed. But due to the uncertainty of systems, the recursive feasibility of the controller proposed in Park and Jeong (2004) cannot be guaranteed, which directly results in that the closed-loop stability would not be guaranteed.

In practical applications, the parameter measurement error is another issue which must be considered. Therefore, this paper considers the RMPC of LPV systems with both independently varying parameters and parameter measurement errors. In terms of the measurement errors, the error bounds are used to calculate the possible areas where the parameters could belong to in the future, and these areas can be tackled with the parameter variations together. Then the dynamic system model is converted into a sequence of future models with parameter-incremental uncertainty by referring to Park and Jeong (2004), which includes not only the time-varying parameter variations but also the measurement errors. Corresponding to the model sequence, the proposed RMPC adopts a sequence of feedback control laws, instead of open-loop control strategy. The recursive feasibility and closed-loop stability can be guaranteed. Meanwhile, since the feedback control laws are designed according to the future parameter variations, the information on the parameter variations and measurement errors can be utilised in the MPC controller and then better control performance can be expected.

This paper is organised as follows: Section 2 introduces the problem and the issue about the recursive feasibility of RMPC. The feedback RMPC will be introduced with a modified model sequence with parameter-incremental uncertainties in detail in Section 3. Numerical example is given in Section 4 to verify the results proposed in this paper.

Notation: Denote $u(k+i|k)$ and $x(k+i|k)$ as the control input and system state of time $k + i$, predicted at time k. $||x||_Q^2 = x^T Qx$, $x(k|k) = x(k)$. The symbol $*$ induces a symmetric structure, e.g., when L and R are symmetric matrices,

$$\begin{bmatrix} L & * \\ N & R \end{bmatrix} = \begin{bmatrix} L & N^T \\ N & R \end{bmatrix}.$$

2 Background

Consider the discrete-time LPV system

$$x(k+1) = A(\theta(k))x(k) + B(\theta(k))u(k) \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^{n_u}$ and $\theta(k) = \{\theta_1(k), \theta_2(k), \ldots, \theta_L(k)\}$ are the system state, control input and parameter vector, respectively. The parameter vector $\theta(k)$ is assumed measurable with measurement error σ at time k. The measured values of $\theta_i(k)$ is denoted as $\hat{\theta}_i(k)$. Moreover, the real values, measured values and the changes of parameters satisfy the following constraints

$$\theta_j \in \Sigma_j = \left[\underline{\theta}_j, \bar{\theta}_j\right], \tag{2}$$

$$\Delta\theta_j(k) = \theta_j(k+1) - \theta_j(k) \in \delta_j = \left[\underline{\delta}_j, \bar{\delta}_j\right], \tag{3}$$

$$|\hat{\theta}_j(k) - \theta_j(k)| \leq \sigma_j. \tag{4}$$

System (1) is subjected to the input constraints:

$$|u_j(k)| \leq u_{j,\max}, \quad j = 1, \ldots, m. \tag{5}$$

At each time, the RMPC will calculate the control moves $u(k+i|k)$ by optimising the following optimisation problem

$$\min_{U(k)} \max_{\theta_j \in \Sigma_j, \Delta\theta_j \in \delta_j, j=1,\ldots,L} J_\infty(k) \text{ s.t.}(1) - (5) \tag{6}$$

where $J_\infty(k) = \sum_{i=0}^{\infty} \left[\|x(k+i|k)\|_{Q_1}^2 + \|u(k+i|k)\|_R^2\right]$, $Q_1 \geq 0$ and $R \geq 0$ are weighting matrices.

Remark 1: For LPV systems (1)–(5), although the approach in Kothare et al. (1996) can be directly used to design RMPC, the information on system parameters (2)–(3) is ignored, which may lead to poor performance. In order to improve the control performance, for LPV systems (1)–(5) without parameter measurement errors, Park and Jeong (2004) makes use of the information on parameters to design RMPC by tackling the uncertainty of LPV systems as parameter-incremental uncertainty. However, the open-loop strategy $U(k) = \{u(k|k), u(k+1|k), \ldots, u(k+N|k)\}$ is adopted by Park and Jeong (2004) which is similar to Wan and Kothare (2003). As pointed out by Pluymers et al. (2005), the proof about recursive feasibility of RMPC in Wan and Kothare (2003) is not correct due to the uncertainty of system. The same situation happens in Park and Jeong (2004) when $N > 2$.

For systems (1)–(3), Park and Jeong (2004) suggested the following method to form a system with parameter-incremental uncertainties to make use of the property of parameters. Since the parameter $\theta_j(k)$ can be measured at sample time and the bound rates of parameters variations are also available, the range of $\theta_j(k+i|k)$ can be computed and described as:

$$\theta_j(k+i|k) \in [\max(\theta_j(k) + i \times \underline{\delta}_j, \underline{\theta}_j),\ \min(\bar{\theta}_j, \theta_j(k) + i \times \bar{\delta}_j)].$$

And then the following is defined in Park and Jeong (2004).

$$\mu_j(k+i|k) \triangleq \frac{1}{2}\left[\min(\bar{\theta}_j, \theta_j(k) + i \times \bar{\delta}_j) + \max(\theta_j(k) + i \times \underline{\delta}_j, \underline{\theta}_j)\right],$$
$$\rho_j(k+i|k) \triangleq \frac{1}{2}\left[\min(\bar{\theta}_j, \theta_j(k) + i \times \bar{\delta}_j) - \max(\theta_j(k) + i \times \underline{\delta}_j, \underline{\theta}_j)\right].$$

Park and Jeong (2004) modifies parameter uncertainties into parameter-incremental uncertainties as below,

$$\begin{aligned} A(\theta(k+i|k)) &= A(\mu(k+i|k)) + B_p(k+i|k)\Delta C_q(k+i|k), \\ B(\theta(k+i|k)) &= B(\mu(k+i|k)) + B_p(k+i|k)\Delta D_{qu}(k+i|k), \\ \Delta &= \operatorname{diag}(\eta_1 I, \eta_2 I, \ldots, \eta_L I) \end{aligned}$$

where η_i is a time-varying uncertain variable such that $\|\eta_i\| \leq 1, i = 1, 2, \ldots, p$. Thus, systems (1)–(3) can be transformed into the following structured uncertain system predicted at time k:

$$\begin{aligned} x(k+i+1|k) &= A(\mu(k+i|k))x(k+i|k) + B(\mu(k+i|k))u(k+i|k) \\ &\quad + B_p(k+i|k)p(k+i|k), \\ q(k+i|k) &= C_q(k+i|k)x(k+i|k) + D_{qu}(k+i|k)u(k+i|k), \\ p(k+i|k) &= \Delta q(k+i|k). \end{aligned}$$

Since the recursive feasibility is the precondition of the closed-loop stability for systems with MPC, how to make good use of the information on system parameters (1)–(5) and to guarantee the recursive feasibility of RMPC become key issues to be studied. In the following, we will propose a feedback RMPC to achieve both of them for systems (1)–(5).

3 Feedback RMPC for system with parameter-incremental uncertainty

3.1 The modified model sequence with parameter-incremental uncertainties

For the parameters of systems (1)–(5), the measured values of parameters can be utilised at each time. With consideration of measurement errors (4), the following can be obtained.

$$\theta_j(k) \in \left[\hat{\theta}_j(k) - \sigma_j, \hat{\theta}_j(k) + \sigma_j\right]$$

and for $\hat{\theta}_j(k+1)$, it must satisfied with

$$\hat{\theta}_j(k+1) \in \left[\theta_j(k) + \underline{\delta}_j - \sigma_j, \theta_j(k) + \bar{\delta}_j + \sigma_j\right].$$

Hence, for $\theta_j(k+1)$ we can get

$$\begin{aligned}\theta_j(k+1) &\in \left[\hat{\theta}_j(k+1) - \sigma_j, \hat{\theta}_j(k+1) + \sigma_j\right] \\ &= \left[\hat{\theta}_j(k) - \sigma_j + \hat{\underline{\delta}}_j, \hat{\theta}_j(k) + \sigma_j + \hat{\bar{\delta}}_j\right]\end{aligned}$$

where $\hat{\underline{\delta}}_j = \underline{\delta}_j - 2\sigma_j$ and $\hat{\bar{\delta}}_j = \bar{\delta}_j + 2\sigma_j$. If $\hat{\theta}_j(k) > \bar{\theta}_j$ (or $\hat{\theta}_j(k) < \underline{\theta}_j$), $\hat{\theta}_j(k)$ is forced as $\bar{\theta}_j$ $\left(\text{or } \underline{\theta}_j\right)$ due to (2).

In the same way, we can get that

$$\begin{aligned}\theta_j(k+i) &\in \left[\hat{\theta}_j(k+i) - \sigma_j, \hat{\theta}_j(k+i) + \sigma_j\right] \\ &= \left[\hat{\theta}_j(k) - \sigma_j + i \times \hat{\underline{\delta}}_j, \hat{\theta}_j(k) + \sigma_j + i \times \hat{\bar{\delta}}_j\right] \qquad (7)\end{aligned}$$

By referring to Park and Jeong (2004), we can revise the model with parameter-incremental uncertainties to include the measurement errors for systems (1)–(4). From the measured parameter $\hat{\theta}_j(k)$, the following is defined for $i \geq 0$.

$$\begin{aligned}\tilde{\mu}_j(k+i|k) &\triangleq \frac{1}{2}\left[\min(\bar{\theta}_j, \hat{\theta}_j(k) + \sigma_j + i \times \hat{\bar{\delta}}_j) + \max(\hat{\theta}_j(k) - \sigma_j + i \times \hat{\underline{\delta}}_j, \underline{\theta}_j)\right], \\ \tilde{\rho}_j(k+i|k) &\triangleq \frac{1}{2}\left[\min(\bar{\theta}_j, \hat{\theta}_j(k) + \sigma_j + i \times \hat{\bar{\delta}}_j) - \max(\hat{\theta}_j(k) - \sigma_j + i \times \hat{\underline{\delta}}_j, \underline{\theta}_j)\right].\end{aligned}$$

Then similar to Park and Jeong (2004), systems (1)–(4) can be transformed into the following structured uncertain system predicted at time k:

$$\begin{aligned}x(k+i+1|k) &= A(\tilde{\mu}(k+i|k))x(k+i|k) + B(\tilde{\mu}(k+i|k))u(k+i|k) \\ &\quad + B_p(k+i|k)p(k+i|k), \qquad (8) \\ q(k+i|k) &= C_q(k+i|k)x(k+i|k) + D_{qu}(k+i|k)u(k+i|k), \qquad (9) \\ p(k+i|k) &= \Delta q(k+i|k). \qquad (10)\end{aligned}$$

It is worth to be pointed out that there are also uncertainties for the current system model due to measurement errors. Meanwhile, if there is no measurement error, the above model will be reduced to that in Park and Jeong (2004).

In addition, the original LPV system (1)–(2) can be converted into a structured feedback uncertain system as follows.

$$\begin{aligned}x(k+1) &= \tilde{A}(\alpha)x(k) + \tilde{B}(\alpha)u(k) + \tilde{B}_p p(k), \qquad (11) \\ q(k) &= \tilde{C}_q x(k) + \tilde{D}_{qu} u(k), \qquad (12) \\ p(k) &= \tilde{\Delta} q(k), \qquad (13)\end{aligned}$$

where $\alpha \triangleq \frac{1}{2}(\bar{\theta}_j + \underline{\theta}_j)$, $\beta \triangleq \frac{1}{2}(\bar{\theta}_j - \underline{\theta}_j)$ and $A(\theta(k+i)) \subseteq \tilde{A}(\alpha) + \tilde{B}_p\tilde{\Delta}\tilde{C}_q$, $B(\theta(k+\ i)) \subseteq \tilde{B}(\alpha) + \tilde{B}_p\tilde{\Delta}\tilde{D}_{qu}$, $\tilde{\Delta} = \mathrm{diag}\,(\eta_1 I, \eta_2 I, \ldots, \eta_L I)$.

To simplify the presentation, systems (8)–(10) at time k is denoted as $\Sigma_{k,i}$ and systems (11)-(13) is denoted as Ψ in the following. Obviously, it can be concluded that $\Sigma_{k,i} \subseteq \Sigma_{k,i+1}$ and $\Sigma_{k,i} \subseteq \Psi$.

3.2 The feedback robust MPC

From the analysis in the last section, the system model will vary along the sequence $\{\Sigma_{k,0}, \Sigma_{k,1}, \Sigma_{k,2}, \ldots, \Sigma_{k,N-1}\}$ and $\Sigma_{k,N-1} = \Psi$, which is denoted as $\Sigma(k)$. To avoid the difficulty to guarantee the recursive feasibility and make use of the information on system parameters, a closed-loop strategy should be adopted and the varying feedback control law F_i at each time should correspond to $\Sigma_{k,i}$. Hence, the control strategy $\pi := \{u(k), F_1, F_2, \ldots, F_{N-1}\}$ is adopted, where F_i is the feedback control gain at the ith step and after the Nth step the feedback control gain is always F_{N-1}.

For $\Sigma(k)$ with $i > 0$, consider the following quadratic function:

$$V(i,k) = x(k+i|k)^T P(i,k) x(k+i|k),$$

where $P(i,k) = P(N-1,k)$ when $i > N-1$.

From time $k+i$ to $k+i+1$ ($i \geq 1$), the following robust stable condition is imposed on $V(i,k)$:

$$V(i+1,k) - V(i,k) \leq -\|x(k+i|k)\|^2_{Q_1} - \|u(k+i|k)\|^2_R,$$

which is equivalent to

$$\begin{aligned}&||(A(\tilde{\mu}(k+i|k)) + B(\tilde{\mu}(k+i|k))F_i)x(k+i|k)\\&\quad +B_p p(k+i|k)||^2_{P(i+1,k)} - ||x(k+i|k)||^2_{P(i,k)}\\&\qquad \leq -[||x(k+i|k)||^2_{Q_1} + ||u(k+i|k)||^2_R].\end{aligned} \tag{14}$$

By summing (14) from $i = 1$ to $i = \infty$, it follows

$$\sum_{i=1}^{\infty} \left[\|x(k+i|k)\|^2_{Q_1} + \|u(k+i|k)\|^2_R\right] \leq V(1,k). \tag{15}$$

Suppose there exists a non-negative parameter γ such that $V(1,k) \leq \gamma$. Then, we can get

$$J_\infty(k) \leq \|x(k)\|^2_{Q_1} + \|u(k)\|^2_R + \gamma. \tag{16}$$

To guarantee (14) and $V(1,k) \leq \gamma$, the following lemma is given.

Lemma 1: *For the uncertain system $\Sigma(k)$ without input constraints, the policy $\pi = \{u(k), F_1, F_2, \ldots, F_{N-1}\}$, which guarantees* (14) *and $V(1,k) \leq \gamma$, is given by $u(k) = u_k$ and $F_i = Y_i X_i^{-1}$ with $P(i,k) = \gamma X_i^{-1}$, if there exists $\gamma > 0$, $X_i \in \mathbb{R}^{n\times n}$,*

$X_i > 0$, $Y_i \in \mathbb{R}^{n_u \times n}$ $(i = 1, 2, \ldots, N-1)$ *and positive-definite diagonal matrices* $\Lambda_j \in \mathbb{R}^{n \times n}$, $(j = 0, 1, 2, \ldots, N-1)$, *satisfying the following conditions.*

$$\begin{bmatrix} 1 & * & * \\ \Xi_1(k) & \Lambda_0 & * \\ \Xi_2(k) & 0 & \Xi_3(k) \end{bmatrix} \geq 0 \tag{17}$$

$$\begin{bmatrix} X_i & * & * & * & * \\ R^{1/2} Y_i & \gamma I & * & * & * \\ Q_1^{1/2} X_i & 0 & \gamma I & * & * \\ \Xi_1(k+i) & 0 & 0 & \Lambda_i & * \\ \Xi_2(k+i) & 0 & 0 & 0 & \Xi_3(k+i) \end{bmatrix} \geq 0, \tag{18}$$

where $\Xi_1(k) = C_q(k)x(k) + D_{qu}(k)u_k$, $\Xi_2(k) = A(\tilde{\mu}(k))x(k) + B(\tilde{\mu}(k))u_k$, $\Xi_3(k) = X_1 - B_p(k)\Lambda_0 B_p^T(k)$, $\Xi_1(k+i) = C_q(k+i|k)X_i + D_{qu}(k+i|k)Y_i$, $\Xi_2(k+i) = A(\tilde{\mu}(k+i|k))X_i + B(\tilde{\mu}(k+i|k))Y_i$, $\Xi_3(k+i) = X_{i+1} - B_p(k+i|k)\Lambda_i B_p^T(k+i|k)$ *and* $X_N = X_{N-1}$.

Proof: From (8)–(10) (or (11)–(13)), we can get

$$\begin{aligned} p(k+i|k)^T p(k+i|k) \leq\ & x(k+i|k)^T (C_q(k+i|k) \\ & + D_{qu}(k+i|k))F_i)^T (C_q(k+i|k) \\ & + D_{qu}(k+i|k))F_i) x(k+i|k) \end{aligned}$$

Condition (14) holds if the following condition can be guaranteed.

$$\chi^T \begin{bmatrix} ||F_i||_R^2 + Q_1 - P(i,k) + ||A(\mu(k+i|k)) & \\ +B(\mu(k+i|k))F_i||^2_{P(i+1,k)} & * \\ B_p^T(k+i|k)P(i+1,k)(A(\mu(k+i|k)) \times & \\ +B(\mu(k+i|k))F_i) & \mathscr{B} \end{bmatrix} \chi \leq 0,$$

where $\chi = \begin{bmatrix} x(k+i|k) \\ p(k+i|k) \end{bmatrix}$, $\mathscr{B} = ||B_p(k+i|k)||^2_{P(i+1,k)}$. Then, by S-procedure, the above condition can be guaranteed if there exists $\Lambda_i' = \operatorname{diag}\left(S_{(1,i)}, \ldots, S_{(L,i)}\right)$, $S_{(l,i)} \geq 0$, $l = 1, 2, \ldots, L$ such that

$$\begin{bmatrix} \mathscr{A}_1 & * \\ \mathscr{A}_2 & \mathscr{B}_1 \end{bmatrix} \leq 0,$$

where $\mathscr{A}_1 = ||A(\tilde{\mu}(k+i|k)) + B(\tilde{\mu}(k+i|k))F_i||^2_{P(i+1,k)} - P(i,k) + ||F_i||^2_R + Q_1 + ||C_q(k+i|k) + D_{qu}(k+i|k)F_i||^2_{\Lambda_i'}$, $\mathscr{A}_2 = B_p^T(k+i|k)P(i+1,k)(A(\tilde{\mu}(k+i|k)) + B(\tilde{\mu}(k+i|k))F_i)$, $\mathscr{B}_1 = ||B_p(k+i|k)||^2_{P(i+1,k)} - \Lambda_i'$.

Let $P(i,k) = \gamma X_i^{-1}$, $Y_i = F_i X_i$ and $\Lambda_i = \gamma(\Lambda_i')^{-1}$. By using Schur complement, it can be concluded that the above condition is equivalent to (18).

In addition, by Schur complement, (17) is equivalent to

$$\begin{bmatrix} \mathscr{A}_1(0) & * \\ \mathscr{A}_2(0) & \mathscr{B}_1(0) \end{bmatrix} \leq 0$$

where $\mathscr{A}_1(0) = ||A(\tilde{\mu}(k))x(k) + B(\tilde{\mu}(k))u_k||^2_{X_1^{-1}} - 1 + ||C_q(k)x(k) + D_{qu}(k))u_k||^2_{\Lambda_0^{-1}}, \mathscr{A}_2(0) = B_p^T(k)X_1^{-1}(A(\tilde{\mu}(k))x(k) + B(\tilde{\mu}(k))u_k)$ and $\mathscr{B}_1(0) = ||B_p(k)||^2_{X_1^{-1}} - \Lambda_0^{-1}$.

Left- and right-multiplying the above inequality by $[1\ \ p(k)]$ and $[1\ \ p(k)]^T$, respectively, and then by (8)–(10) and using S-procedure, we can get

$$||A(\tilde{\mu}(k))x(k) + B(\tilde{\mu}(k))u_k + B_p(k)p(k)||^2_{X_1^{-1}} \leq 1.$$

That is, $V(1,k) \leq \gamma$ holds if (17) is satisfied according to $P(1,k) = \gamma X_1^{-1}$. Therefore, the lemma is proven. □

From $V(1,k) \leq \gamma$ and (14), it is obvious that $V(i,k) \leq \gamma$. Then constraints (5) can be satisfied if the following lemma holds, whose proof can be easily obtained by the similar procedure in Kothare et al. (1996) or Li et al. (2009) and is omitted here.

Lemma 2: *The input constraints* (5) *can be satisfied if there exists* $\gamma > 0$, $X_i \in \mathbb{R}^{n\times n}$, $X_i > 0$, $Y_i \in \mathbb{R}^{n_u\times n}$, $Z_i \in \mathbb{R}^{n_u\times n_u}$ $(i = 1, 2, \ldots, N-1)$ *and positive-definite diagonal matrices* $\Lambda_j \in \mathbb{R}^{n\times n}$, $(j = 0, 1, 2, \ldots, N-1)$, *satisfying conditions* (17) and (18), *and also satisfying the following conditions:*

$$|(u_k)_l| \leq u_{l,\max}, \quad l = 1, 2, \ldots, m \tag{19}$$

$$\begin{bmatrix} Z_i & * \\ Y_i & X_i \end{bmatrix} \geq 0, (Z_i)_{ll} \leq u^2_{l,\max} \quad i = 1, 2, \ldots, N-1, \quad l = 1, 2, \ldots, m. \tag{20}$$

Lemma 2 can be proven in a similar way to the proof of the constraints in Kothare et al. (1996). Therefore, it is omitted here.

Based on Lemmas 1 and 2, the optimisation problem of feedback RMPC for $\Sigma(k)$ can be formulated as below.

Algorithm 1: *Let* $x(k) = x(k|k)$ *be the state of the uncertain system* $\Sigma(k)$ *measured at sampling time* k, *and the input constraints are described as in* (5). *Then the policy* $\pi = \{u(k), F_1, F_2, \ldots, F_{N-1}\}$ *that minimises the upper bound on the robust performance objective function at sampling time* k *is given by*

$$u(k) = u_k, F_i = Y_i X_i^{-1}$$

where $\gamma > 0$, $X_i \in \mathbb{R}^{n\times n}$, $X_i > 0$, $Y_i \in \mathbb{R}^{n_u\times n}$, $Z_i \in \mathbb{R}^{n_u\times n_u}$ $(i = 1, 2, \ldots, N-1)$ *and positive-definite diagonal matrices* $\Lambda_j \in \mathbb{R}^{n\times n}$, $(j = 0, 1, 2, \ldots, N-1)$, *are obtained from the solution (if it exists) of the following linear objective minimisation problem*

$$\min_{\gamma_0,\gamma,u_k,X_i,Y_i,Z_i,\Lambda_j} \gamma + \gamma_0 \tag{21}$$

$$s.t. \quad (17) - (20) \tag{22}$$

$$\begin{bmatrix} \gamma_0 & x^T(k) & u_k^T \\ x(k) & Q_1^{-1} & 0 \\ u_k & 0 & R^{-1} \end{bmatrix} \geq 0. \tag{23}$$

The current control input is $u(k) = u_k$.

Remark 2: For RMPC based on Algorithm 1, if the control input $u(k)$ in control strategy π is removed and N is chosen as 1, the controller will be simplified to the design in Kothare et al. (1996). The added freedom in Algorithm 1 makes it possible to utilise the information on system parameters, which is helpful to improve the control performance. To simplify the presentation, let $\mathbb{Q}_i := (X_i, Y_i, Z_i, \Lambda_i)$.

In terms of the recursive feasibility and closed-loop stability of Algorithm 1, the following theorem can be given.

Theorem 3: *If there is a feasible solution of Algorithm* 1 *at time* k *with system state* $x(k)$*, there will also exist a feasible solution for Algorithm* 1 *at next time, and the closed-loop system is asymptotically stable.*

Proof: Since Algorithm 1 is feasible at time k, suppose $\Gamma^*(k) = \{\gamma_0^*(k), \gamma^*(k), u_k^*, \Lambda_0^*, \mathbb{Q}_1^*, \ldots, \mathbb{Q}_{N-1}^*\}$ as the optimal solution for the current state $x(k)$. That implies that $\Gamma^*(k)$ satisfies (17)–(20).

At time $k+1$, for Algorithm 1, we construct a solution

$$\begin{aligned}\Gamma(k+1) = \{&\|x(k+1)\|_{Q_1}^2 + \left\|Y_1^*(X_1^*)^{-1}x(k+1)\right\|_R^2,\\ &a\gamma^*(k), Y_1^*(X_1^*)^{-1}x(k+1), a\Lambda_1^*, a\mathbb{Q}_2^*, \ldots a\mathbb{Q}_{N-1}^*, a\mathbb{Q}_{N-1}^*\}\end{aligned}$$

with definition $a := V(1,k+1)/\gamma^*(k)$ where $V(1,k+1) = x^T(k+2|k+1)\gamma^*(X_2^*)^{-1}x(k+2|k+1)$, $x(k+2|k+1) = [A(\theta(k+1)) + B(\theta(k+1))Y_1^*(X_1^*)^{-1}]$ $x(k+1)$. The above definition means that$V(1,k+1) = a\gamma^*(k)$.

From the model with parameter incremental uncertainty, $\Sigma_{k+1,i} \subseteq \Sigma_{k,i+1}$ and $\Sigma_{k,i} \subseteq \Psi$. We observe that conditions (18) and (20) are affine in the matrices $(\gamma, \mathbb{Q}_1^*, \mathbb{Q}_2^*, \ldots, \mathbb{Q}_{N-1}^*)$. Multiplying them by parameter a respectively, we can see that $\Gamma(k+1)$ satisfies (20) and (18) when $i = 1, 2, \ldots, N-1$.

Let $u_{k+1} = Y_1^*(X_1^*)^{-1}x(k+1)$ and $\gamma_0(k+1) = \|x(k+1)\|_{Q_1}^2 + \left\|Y_1^*(X_1^*)^{-1}x(k+1)\right\|_R^2$. It is obvious that (19) and (23) are satisfied by $\Gamma(k+1)$. Furthermore, since $\mathbb{Q}_1^*(k)$ satisfies (18) at time k, it implies that (17) holds at time $k+1$ with the constructed solution $\Gamma(k+1)$ due to the definition of a. That means, the constructed solution is a feasible solution of Algorithm 1 at time $k+1$. Hence, the recursive feasibility of Algorithm 1 can be established.

In addition, from Lemma 1, it can be concluded that $\|x(k+1)\|_{Q_1}^2 + \left\|Y_1^*(X_1^*)^{-1}x(k+1)\right\|_R^2 + V(1,k+1) \leq V(1,k) \leq \gamma^*(k)$, i.e., $\gamma_0(k+1) + \gamma(k+1) \leq \gamma^*(k)$. Therefore, it can be obtained that $\gamma_0^*(k+1) + \gamma^*(k+1) \leq \gamma_0(k+1) + \gamma(k+1) < \gamma_0^*(k) + \gamma^*(k)$ when $x(k) \neq 0$. That is, the closed-loop system is asymptotically stable. □

4 Numerical example

Consider the following system:

$$x(k+1) = [\theta(k)A_1 + (1-\theta(k))\,A_2]\,x(k) + [\theta(k)B_1 + (1-\theta(k))\,B_2]\,u(k)$$

where $A_1 = \begin{bmatrix} 1 & 0 \\ -0.3 & 1.4 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ -0.1 & 1.1 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}$, $B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $|u| \leq 1$, $\theta(k) \in \begin{bmatrix} 0 \; 1 \end{bmatrix}$, $|\Delta\theta| \leq \delta$, $|\hat{\theta}(k) - \theta(k)| \leq \sigma$ and

$$\theta(k+1) = \begin{cases} 0, \theta(k) + \delta \sin(k-1) \leq 0 \\ \theta(k) + \delta \sin(k-1) \\ 1, \theta(k) + \delta \sin(k-1) \geq 1 \end{cases}$$

$$\hat{\theta}(k) = \begin{cases} 0, \theta(k) + \sigma \leq 0 \\ \theta(k) + \sigma \\ 1, \theta(k) + \sigma \geq 1 \end{cases}.$$

The initial state is chosen as $x(0) = [1, 1]^T$ and the weighting matrices are chosen as $Q_1 = \mathrm{diag}(1, 0.1)$, $R = 0.001$. First, let us verify the recursive feasibility of Park and Jeong (2004). Investigate an extreme case for RHC in Park and Jeong (2004) with $N = 3$, i.e., the case with $\theta(k) = 1$, $\delta = 1$ and no measurement errors. For $x(k) = [1, 1]^T$ and $k = 1$, RHC in Park and Jeong (2004) optimises the control inputs $u(k), u(k+1), u(k+2)$ to steer $x(k)$ to a terminal invariant set. The states $x(k|k), x(k+1|k), x(k+2|k), x(k+3|k)$ are shown in Figure 1, which is a partial enlarged drawing. From Schuurmans and Rossiter (2000), $x(k+i), i > 1$ is a state constructed by the linear combination from the states $x(k+i|k)$ corresponding to the model vertices $\{A1, B1\}, \{A2, B2\}$. Thus, if Lemma 2 in Park and Jeong (2004) is correct, there must be a $u(k+3|k+1)$ with $u(k+1|k+1) = u(k+1|k), u(k+2|k+1) = u(k+2|k)$ steering $x(k+1|k+1)$ into the terminal set computed at time k. In Figure 1, states $x(k+1|k+1), x(k+2|k+1), x(k+3|k+1)$ are marked by a cycle. By computing, it is found that this $u(k+3|k+1)$ does not exist. That is, the RHC in Park and Jeong (2004) cannot guarantee the recursive feasibility.

Figures 2 and 3 show the state responses from $x(0) = [1, 1]^T$ with $\theta(0) = 0.6$, $\delta = 0.15, \sigma = 0$ and $\theta(0) = 0.6, \delta = 0.15, \sigma = 0.01$, respectively, where $N = 3$ for RHC in Park and Jeong (2004) and Algorithm 1. The results by using the techniques in Lu and Arkun (2000) and Kothare et al. (1996) are also included in Figures 2 and 3, respectively to make a comparison. From Figure 2, the performance of RMPC with Algorithm 1 is best, where the cost value of Algorithm 1 is 34.56, better than 42.69 of Lu and Arkun (2000) and 42.94 of Park and Jeong (2004). For the case with measurement errors $\sigma = 0.01$, the results are compared between the proposed Algorithm 1 and the technique in Kothare et al. (1996), which is the technique capable of dealing with LPV systems with measurement error in the previous literatures. The state response is shown in Figure 3 and the cost value of Algorithm 1 is 28.73 and that of Kothare et al. (1996) is 38.44. Therefore, it reflects that Algorithm 1 can achieve better control performance than the design in Kothare et al. (1996).

The above results verify the effectiveness of utilises the information of system parameters in Algorithm 1.

5 Conclusions

This paper presents a new approach to RMPC for LPV systems with bounded rates of parameter variations and bounded parameter measurement errors. By adopting a sequence of feedback control laws corresponding to the parameter variation of LPV systems, the information on system parameters can be made use of, which is helpful to reduce the design conservativeness and then improve the control performance of RMPC. The recursive feasibility and closed-loop stability of the proposed MPC can be guaranteed by the proposed RMPC.

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