Using statistical and interval-based approaches to propagate snow measurement uncertainty to structural reliability

Árpád Rózsás*

TNO Building and Construction Research, PO Box 155, 2600 AD Delft, the Netherlands
Email: arpad.rozsas@tno.nl
*Corresponding author

Miroslav Sýkora

Department of Structural Reliability, Klokner Institute, Czech Technical University of Prague, Prague 16608, Czech Republic
Email: miroslav.sykora@cvut.cz

Abstract: Observations are inevitably contaminated with measurement uncertainty, which is a predominant source of uncertainty in some cases. In present practice probabilistic models are typically fitted to measurements without proper consideration of this uncertainty. Hence, this study explores the effect of this simplification on structural reliability and provides recommendations on its appropriate treatment. Statistical and interval-based approaches are used to quantify and propagate measurement uncertainty in probabilistic reliability analysis. The two approaches are critically compared by analysing ground snow measurements that are often affected by large measurement uncertainty. The results indicate that measurement uncertainty may lead to significant (order of magnitude) underestimation of failure probability and should be taken into account in reliability analysis. Ranges of the key parameters are identified where measurement uncertainty should be considered. For practical applications, the lower interval bound and predictive reliability index are recommended as point estimates using interval and statistical analysis, respectively. The point estimates should be accompanied by uncertainty intervals, which convey valuable information about the credibility of results.

Keywords: measurement uncertainty; snow; structural reliability; interval arithmetic; maximum likelihood; deconvolution.

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Biographical notes: Árpád Rózsás is a research engineer at the Department of Structural Reliability, Netherlands Organization for Applied Scientific Research (TNO). His main research concerns the probabilistic analysis of engineering structures, both from reliability and from uncertainty propagation point of view.

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1 Introduction

1.1 Motivation

Models accounting for all uncertainties are of a considerable interest in structural reliability since they provide the basis of design specifications and thereby affect construction industry with significant impact on economics of countries. Snow is particularly important for light-weight structures for which it is a governing action in most cases. According to the knowledge of the authors, the effect of snow measurement uncertainty on structural reliability has not yet been studied; likewise measurement uncertainty is neglected in modelling of other basic variables in civil engineering. For instance, neither the joint European research on snow actions (Sanpaolesi et al., 1998) nor the Probabilistic Model Code of the Joint Committee on Structural Safety (JCSS, 2000) provide any information on the treatment of measurement uncertainty and its effect. Therefore, this study explores the effect of this simplification on structural reliability and attempts to provide recommendations on its appropriate treatment.

1.2 Problem statement

Observations are inevitably contaminated with measurement uncertainty which is a predominant source of uncertainty in some cases. Uncertainty is understood here as the lack of knowledge (epistemic) and natural variability (aleatory), excluding known systematic biases that are not considered for calibrated standard measurement methods. The distinction between epistemic and aleatory uncertainties may be subjective and is dependent on case-specific conditions. A measurement error is expressing the difference between the real and measured values, it is a realisation of measurement uncertainty.

In the case with snow loads, measurements are often based on snow depth only and estimates of resulting loads are excessively uncertain; for example, measurement error can reach 50% of the measured depth and its coefficient of variation in snow density is around 20% (ISO, 2013). The World Meteorological Organization conducted a comprehensive comparative study on available solid precipitation measurement techniques...
and experimentally confirmed that measurements should be adjusted for wetting loss, evaporation loss, and wind induced undercatch (Goodison et al., 1998). They found that the snow catch ratio of the four most widely used gauges range from 20% to 70% at a 6 m/s wind speed. Even for automated systems, measurement error in solid precipitation can vary from 20% to 50% due to undercatch in windy conditions (Rasmussen et al., 2012). Although these factors mainly contribute to systematic error, they are indicative of random errors in snow measurements as these cannot be exactly corrected. For instance coefficient of determination values vary from 0.40 to 0.80 for the fitted wind correction equations at certain sites; this is associated with about 10% standard error in a catchment ratio. Additional uncertainty may be introduced if no site-specific auxiliary data, e.g. wind speed measurements, are available (Goodison et al., 1998).

2 Solution strategy

2.1 Adopted approaches

We assume that measurements are obtained by a calibrated method used according to its scope of application and corrected for systematic bias. The following model is assumed to describe the relationship between observed \((Y)\) and real – true \((X)\) values:

\[
\begin{align*}
(h(X,E), (true,real)) \rightarrow Y(\text{observed}).
\end{align*}
\]

The \(h(X,E)\) function represents the mathematical relationship between the true and observed random variables referred hereafter as a reality-observation link. \(E\) covers the unknown processes contributing to measurement uncertainty. The recommended probabilistic models – typically distributions – in the literature are almost exclusively given for the true variable and not for the observed one, potentially contaminated with measurement uncertainty. This is attributable to the following:

- The contamination is commonly site- and measuring technique-dependent, thus no general recommendations can be given for the distribution of \(Y\).
- The model type is often selected based on theoretical arguments considering the physical phenomena generating \(X\), e.g. normal distribution if \(X\) is the result of summation; lognormal if \(X\) is the product of random variables; extreme value distribution if \(X\) is related to extremes.
- Structural reliability is ultimately dependent on \(X\) and not on \(Y\), although observations are limited to \(Y\) only.

When the distribution of \(X\) is known or agreed, the reality-observation link uniquely determines the distribution of \(Y\). Hence, if any measurement uncertainty is present, the distribution of \(Y\) differs from the distribution of \(X\). This is prevalently neglected while fitting distributions in civil engineering – \(Y\) is assumed to be distributed as \(X\). However, this simplification is acceptable in some practical cases.

The distinction between the following approaches, related to different levels of appreciation of the difference between \(X\) and \(Y\), is made:
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Approach 1 Use the probabilistic model of the true random variable \( X \) and treat observations \( y \) – contaminated with measurement uncertainty – as the realisations of the \( X \)-distribution.

Approach 2 Differentiate between the distribution of the true and observed random variables by one of the following sub-approaches:

- **Approach 2a** Representation of measurement uncertainty with intervals at the level of observations and propagating them to the derived parameters via interval analysis. As interval representation naturally contains no information about the reality-observation link, the decontamination of observations is not possible.

- **Approach 2b** Representation of measurement uncertainty with a probability distribution. Use a mathematical model \( h(X,E) \) to describe the relationship between measurement uncertainty \( E \), true phenomenon \( X \), and observed phenomenon \( Y \). Based on this model and on observations \( y \), infer the parameters of the true random variable \( X \). These problems are referred to as measurement error problems (Kondlo, 2010).

Additional assumptions for all the approaches under consideration:

- \( y = \{y_1, y_2, \ldots, y_n\} \) and \( x = \{x_1, x_2, \ldots, x_n\} \) each are independent, identically distributed realisations; \( y \) is contaminated with measurement uncertainty.

- The realisations of the true phenomenon \( x \) and measurement uncertainty \( e \) are mutually independent.

- The true phenomenon \( X \) follows an arbitrary, but known distribution type. An additional assumption is made for Approach 2b: the measurement uncertainty \( E \) follows an arbitrary, but known distribution type, and the reality-observation link is also known.

2.2 Uncertainty representation and propagation

2.2.1 Interval analysis

Interval representation is a possible approach to quantify uncertainty in an observed variable – the width of the interval expresses the related uncertainty (Figure 1). In this concept the true value is assumed to be covered by the interval but we know nothing about how likely it takes a particular value from that. In other words no probability distribution function is assumed over the interval, thus it expresses larger ignorance than probability distributions (Huber, 2010).

Interval variables are defined by the lower and upper endpoints of their interval. The basic objective of interval analysis is to propagate the interval uncertainty of input variables to the outputs. The main challenge is to calculate the interval bounds without excessive overestimation, this commonly occurs if floating point computations are simply replaced by intervals and caused by interval dependency (Moore et al., 2009). Since the operators are often not known explicitly and are non-monotonic, special algorithms are needed to obtain sufficiently narrow approximate interval bounds.
Interval analysis is traditionally used to model floating point truncation error in numerical computations. It has been also successfully applied in various civil engineering issues including structural reliability (Qiu et al., 2008) and system reliability (Qiu et al., 2007). Rao et al. (2015) analysed the effect of incorrect fitting on trusses and frames using mixed interval finite element formulation, using intervals to model fabrication uncertainties. Muhanna et al. (2015) demonstrated the feasibility of the non-linear interval finite element analysis for beam-column structures. Geometric, material and load uncertainties were modelled by intervals.

Figure 1  Interval representation of measurement uncertainty (black) on a sorted random sample (red). The sample is generated from $Q_i$ with the properties given in Table 1 and $V_{i_1} = 0.2$

In this study, the general definition of interval variables is adopted and constrained numerical optimisation is applied to find the interval endpoints. This is motivated by available optimisation algorithms, efficiency of which is sufficient for the study under consideration. For computationally demanding models, more efficient algorithms are available (Zhang et al., 2010; Alibrandi and Koh, 2015; Muhanna et al., 2015). Intervals are herein defined by a midpoint and radius ($\varepsilon_i$) wherein the midpoint is taken as the observed value $y_i$ (Figure 1). The true value is then assumed to be certainly within the interval given the modelling assumptions are valid.

2.2.2 Statistical analysis

Alternatively, a classical statistical approach representing measurement uncertainty by a probability distribution is applied (Figure 2). The likelihood function depends on the reality-observation link (equation 1). This connection is assumed to be a known relationship for the sake of clarity whilst a possible treatment of this uncertainty is discussed in Section 5. The algebra of random variables can be used to obtain the likelihood function reflecting the distribution of involved random variables and the reality-observation link:

$$L(\theta_x, \theta_\varepsilon \mid y) = \prod_{i=1}^{n} f(y_i \mid \theta_x, \theta_\varepsilon),$$ (2)
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where $L(.)$ denotes the likelihood of distribution parameters $\theta_x, \theta_E$ given observed $y$. The observations ($y$) are connected with the parameters through the reality-observation link (equation 1). In Approach 1, this means no additional complication since the observations are assumed to be distributed as the true random variable and the reality-observation link is neglected. In Approach 2b, the likelihood function should be constructed to remove the effect of measurement uncertainty ($E$) on a variable of interest ($X$).

Figure 2 Illustration of probability distribution representation of measurement uncertainty (black) on a sorted random sample (red). The sample is generated from $Q_1$ with properties given in Table 1 and $V_{\theta_1} = 0.2$

The measurement error problem arises in many applications where only contaminated values are accessible, but the inference of true, uncontaminated values is of great interest. Among others, these areas include astronomy, econometrics, biometrics, medical statistics, and image reconstruction (Stefanski and Bay, 2000; Koen and Kondlo, 2009; Meister, 2009). A straightforward solution is to construct the likelihood function (equation 2) and to infer the parameters of $X$ by a selected method. It seems that this approach has not been applied in civil engineering yet.

The maximum likelihood method is used herein to infer the parameters in the statistical formulation of the measurement uncertainty problem. This method is a widespread technique that favours parameters at which the data are most likely generated by the adopted model. The maximum likelihood method is an asymptotically efficient and consistent method (Casella and Berger, 2001).

Additive and multiplicative reality-observation links are considered. For the former, $Y = X + E$, the density function of the sum of two independent, continuous random variables is obtained by convolution:

$$f_Y(y) = (f_x \ast f_E)(y) = \int_{-\infty}^{\infty} f_X(y-x) \cdot f_E(x) \cdot dx.$$  \hspace{1cm} (3)

Note that an additive model for $E$ is preferred when measurement uncertainty is independent of the magnitude of the true variable; see the related discussion on model uncertainties and additive and multiplicative formats by Holický et al. (2016).
For the multiplicative relationship, \( Y = X \cdot E \), the density function of the product of two independent, continuous random variables is obtained by computing the following integral:

\[
 f_Y(y) = \int_{-\infty}^{\infty} f_X(x) \cdot f_E\left(\frac{y}{x}\right) \cdot \frac{1}{|x|} \, dx.
\]  

(4)

3 Example 1: reliability of a generic structural member

3.1 Model description

Reliability of a structural member is analysed considering a generic limit state function:

\[
g = R - (G + Q_{50}).
\]  

(5)

It represents a structure subjected to permanent \((G)\) and variable \((Q_{50})\) actions; the subscript 50 indicates maxima over a 50-year reference period – common design working life. The probabilistic models are based on the recommendations of JCSS (2000); see Table 1. For the sake of clarity, only the variable action is deemed to be affected by measurement uncertainty. Extensions to more variables affected by measurement uncertainty are straightforward.

<table>
<thead>
<tr>
<th>Variable name (symbol)</th>
<th>Distribution</th>
<th>Mean</th>
<th>Coefficient of variation (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance ((R))</td>
<td>Lognormal</td>
<td>*</td>
<td>0.10</td>
</tr>
<tr>
<td>Permanent action ((G))</td>
<td>Normal</td>
<td>8</td>
<td>0.10</td>
</tr>
<tr>
<td>Variable action ((Q))†</td>
<td>Normal, Lognormal, Gumbel</td>
<td>10</td>
<td>[0.20, 0.40, 0.60]</td>
</tr>
</tbody>
</table>

Notes: * Set to reach \(\beta = 3.8\) for each combination of inputs.
† The specified parameters are used to generate a 50-element sample. From the sample, the parameters of the probabilistic model are then inferred using Approaches 1, 2a and 2b.

A \(Q_{50}\) distribution is derived from a distribution of annual maxima considering they are independent in subsequent winter seasons:

\[
F_{Q_{50}}(q) = F_{1}(q)^{50},
\]  

(6)

where \(F(.)\) is the cumulative distribution function.

Coefficient of variations 0.2, 0.4, and 0.6 for \(Q_{50}\) provide realistic estimates for annual ground snow load maxima for mountains, highlands, and lowlands, respectively, in the Carpathian Region (Rózsás, 2016).

A lognormal model for annual snow maxima is typically adopted in the USA (ASCE, 2010) while the Gumbel model is widespread in Europe (Sanpaolesi et al., 1998; JCSS, 2001). The international standard (ISO, 2013) recommends a Gumbel distribution for regions where an annual maximum results from accumulation during a long part of the winter season; a lognormal distribution is then appropriate in regions with an extreme
snow load resulting from one or a few snowfalls. For the sake of comparison, a normal distribution is included in the analysis as a candidate model. Appropriate models for Germany and Italy are discussed by Formichi et al. (2016).

In the presented numerical analysis, light-tailed Gumbel and normal distributions are adopted for annual maxima along with a heavy-tailed lognormal distribution. The adopted distributions and the parameter ranges given in Table 1 may represent also other variable actions such as wind and thermal actions, thus the results can be readily generalised. The time-invariant components affecting roof snow loads, such as shape, exposure and thermal factors, are not considered to keep a focus on measurement uncertainty modelling.

3.2 Interval and reliability analysis

To model the effect of measurement uncertainty by Approach 1, 50 random observations of $Q_i$ are treated as observed ($Y$) values since the reality-observation link is unknown (Figure 3). The intervals are centred at observations and various interval radiuses are taken into account. The distribution of $Q_i$ is fitted by the method of moments that is a widely used approach in civil engineering (Sanpaolesi et al., 1998) and was proved to be robust e.g. for modelling hydrological extremes (Madsen et al., 1997). The hybrid interval-probabilistic reliability problem is solved by optimisation and the First Order Reliability Method (FORM); an outcome of the analysis is an interval reliability index.

The upper bound is of low importance for reliability analysis and the lower bound is often considered in practical applications (Qiu et al., 2007). This can be justified by consideration of a distinct feature of intervals and representation of uncertainty: the real value can be any within the interval, but all points within the interval are not equally likely (principle of indifference). Hence we chose the recommended, conservative engineering approach and use the lower endpoint of the reliability index interval as a representative value.

3.2.1 Full and approximate propagation of interval uncertainty

As measurement uncertainty is expressed at the level of individual observations, its full propagation leads to two distinct 50-dimensional constrained optimisation problems. These can be computationally demanding if each iteration step involves fitting a distribution function and solving a reliability problem. The computational demands can be considerably reduced by a two-step approximate technique, consisting of the following steps:
The distribution parameters are fitted to the interval observations. The intervals for distribution parameters are used in reliability analysis. Thus, the optimisation coupled with reliability analysis is limited to a two-dimensional search space. Moreover, numerical experience of the authors suggest that the optimum be at the bounds and can be found by considering only the possible permutations of the parameter bounds. The accuracy of full and two-step approximate uncertainty propagations are compared using Gumbel distributed $Q_1$. The resulting reliability indices are presented in Figure 4. The interval uncertainty is expressed as the ratio of interval radius and mean of annual maxima ($Q_1$). 0–10% range is covered and it is assumed that all observations are contaminated with measurement uncertainty described by a constant radius.

**Figure 4**  Reliability index intervals as the function of normalised measurement uncertainty radius ($\varepsilon_r / \mu_{Q_1}$) with the full and approximate propagation of interval uncertainty

For each coefficient of variation under consideration, the mean of the resistance is set to reach the target reliability index of 3.8 by the analysis that is based on:

- Assumption of no measurement uncertainty ($\varepsilon_r = 0$).
- The parameters given in Table 1, hence with no regard to sampling variability.

The calculated upper and lower reliability index endpoints are presented in the plots by solid and dashed lines for two-step and full propagations, respectively. Also shown in Figure 4 is the reliability index obtained by Approach 1 – a dotted line, unaffected by measurement uncertainty.

Figure 4 shows that the approximate technique slightly overestimates the reliability intervals based on full propagation of measurement uncertainty. The difference is observed particularly for $V_{Q_1} = 0.2$ and increasing measurement uncertainty. The practical impact of such overestimation is, however, small and the approximate technique is thus used hereafter. Also displayed in Figure 4 is a sensitivity factor of 50-year maxima of snow load ($\alpha_{Q_{50}}$) as obtained by the analysis without measurement
uncertainty and based on the parameters in Table 1. The values close to one suggest a dominating influence of $Q_{50}$ on structural reliability.

It appears that the interval of $\beta$ shrinks with $V_{Q1}$. This is attributable to the decreasing effect of interval uncertainty with respect to overall uncertainty of $Q_{1}$, i.e. aleatory uncertainty dominates. The shrinking intervals are also shown in Figure 5, which is based on 50 random realisations; the same pattern is observed for other sets of random realisations.

Figure 5  Shrinking uncertainty interval with an increasing coefficient of variation (constant measurement uncertainty interval)

### 3.2.2 Effect on reliability index and required resistance

Equation (5) is solved for normal, lognormal and Gumbel distributed variable action ($Q_{1}$) using the two-step approximation technique. The results are summarised in Figure 6; they share the same basis with Figure 4. The light grey bands show the widening reliability interval with increasing measurement uncertainty based on 1000 random samples, each with 50 realisations as is typical for climatic loads records in Europe. These are indicative of the effect of sampling variability – parameter estimation uncertainty in this case. The results show that sampling variability has substantial effect on reliability. It is dominating over measurement uncertainty for small interval radiiuses and is comparable for larger radiiuses. The thick black lines denote the median of the 1000 sample sets. At the intersection of these lines is the median reliability index for “no measurement uncertainty”. The difference between this reliability level and the lower bound is of primary interest here as it indicates the impact of neglecting measurement uncertainty. The difference is deemed significant if it is larger than 0.5 (six-fold increase in failure probability for $\beta = 3.8$; 0.5 being approximately the difference amongst subsequent consequence classes in EN 1990:2004 and ISO 2394:2015) – this level is indicated by a dashed horizontal line and the significant range by a red half-line.
Figure 6  Reliability index intervals as a function of the normalised measurement uncertainty radius ($\epsilon_i / \mu_{\epsilon_i}$). The grey regions represent 75% confidence bands obtained from 1000 random samples, indicating sampling variability. The black lines are the median lower and upper interval endpoints of the reliability index. The red half-line indicates the range where the lower endpoint of the reliability interval is significantly lower (>0.5) than the reliability calculated without measurement uncertainty ($\epsilon_i = 0$).

The results in Figure 6 suggest that moderate ±4% measurement uncertainty can lead to significant reduction of the reliability level for mountains and highlands ($V_{\hat{\theta}_1} = 0.2$ – 0.4). It is also observed that even a small ±2% measurement uncertainty can lead to an order of magnitude change in failure probability estimates – see the lognormal distribution and $CV_{\hat{\theta}_1} = 0.2$. However, for lowlands ($CV_{\hat{\theta}_1} = 0.6$) and measurement uncertainty up to ±10%, the estimates based on the Gumbel model lie in the region for which the effect of measurement uncertainty on structural reliability is of low practical significance.

Measurement uncertainty thus seems to have a substantial effect on structural reliability. The practical question then arises: what are its implications on design and how it should be accounted for in a practical and sufficiently accurate way? To examine this, we calculated the mean resistance required to reach the target reliability with the lower bound of the reliability interval (Approach 2a). Such a value is then compared to the mean resistance required to reach the target reliability for the “no measurement uncertainty” alternative (Approach 1). The ratios of the mean values (with interval measurement uncertainty/without measurement uncertainty) are illustrated in Figure 7. These indicate how large adjustment might be needed in representative resistance values.
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to meet target reliability in the presence of measurement uncertainty. The plots are structured and have the same rationale as Figure 6. The ratio over 1.1 is deemed practically significant. This level is indicated by a dashed horizontal line while the significant range by a red half-line. The region of practical significance is reached for all the distributions under consideration. The small effect of sampling variability for the normal distribution is due to the small sensitivity factor of $Q_0$. Considerable effect is observed particularly for the lognormal distribution for which the ratio increases with $V_{Q_1}$. For the Gumbel distribution and $V_{Q_1} = 0.2$ (mountains), moderate ±4% measurement uncertainty can lead to significant mean resistance ratio. For the lowlands ($V_{Q_1} = 0.6$), the ratio is over the selected threshold only for excessive measurement uncertainty ±9% which suggests that measurement uncertainty can be neglected for large values of $V_{Q_1}$.

Figure 7 Mean resistance ratio for the variable action with and without measurement uncertainty as the function of the normalised measurement uncertainty radius ($\epsilon_1 / \mu_{Q_1}$). The red half-line indicates the significant range where the ratio is larger than 1.1.

3.3 Statistical and reliability analysis

This section presents the statistical approach to quantify and propagate measurement uncertainty (Approach 2b). To model the effect of measurement uncertainty, 50 random observations are generated from $Q_1$, representing true ($X$) values. Considering an adopted reality-observation link, the $X$ values are then contaminated with measurement uncertainty. This is generated from a known distribution of ($E$) that is statistically
independent from \((X)\). The algorithm is outlined in Figure 8. Additive and multiplicative reality-observation links are assumed and the measurement uncertainty is taken as normally distributed unbiased with zero (unity) mean for additive (multiplicative) formats. When data are contaminated, the information about the parameters of the underlying generating models – with the exception of the zero (unity) mean of measurement uncertainty – is disregarded and the maximum likelihood method is applied to decouple true values from measurement uncertainty. This means that the inferred parameters are the parameters of \(X (\theta)\) and the standard deviation of \(E (\sigma_e)\). Finally, the model of decontaminated observations is used in reliability analysis. The sampling variability is assessed by 200 samples.

Figure 8  Algorithm of analysing the effect of measurement uncertainty on reliability using statistical technique

3.3.1 Decontamination of observations

To illustrate the technique and the effect of decontamination, random realisations are generated from a Gumbel distribution – with parameters given in Table 1 – and contaminated with measurement uncertainty (Figure 8). First, the additive format is assumed and Approach 1 and Approach 2b are used to infer the model parameters. The maximum likelihood method is used to obtain point estimates and the delta method is applied to construct 90% confidence intervals to illustrate parameter estimation uncertainty (Coles, 2001). The results for the Gumbel distribution, three alternative values \(V_0 = 0.2, 0.4\) and 0.6 and two levels of measurement uncertainty with varying standard deviation are discussed only. The realisations and the fitted models are shown in Figure 9. It comprises return value–return period plots transformed to the Gumbel space. Though the plots are corresponding to a particular set of realisations, they convey reliably the trends and expected differences: (i) Approach 1 typically overestimates the fractiles and leads to lower reliability levels; (ii) the difference between the Approaches increases with an increasing return period. Due to the small sample size, the difference is affected by large sampling variability.

The calculations are repeated with the multiplicative format of measurement uncertainty. The results correspond well with those obtained for the additive format; wider confidence intervals of Approach 2b compared with Approach 1 are observed. This is attributable to the larger model space (\(\sigma_e\) is also a parameter to be inferred) where the same sample size allows less certain inference. This effect is less pronounced for the additive model.
Figure 9  Gumbel distributions fitted to random realisations contaminated with additive measurement uncertainty using Approach 1 and Approach 2b. The point estimates (dashed lines) are accompanied by 90% confidence intervals (dotted lines).

For both the formats, Approach 1 is inherently biased while Approach 2b asymptotically converges to the true model. Thus, from a theoretical point of view – Approach 2b is preferable. However, Approach 1 seems to be generally conservative for the considered reality-observation links. This latter aspect is analysed in more detail in the following section focusing on reliability index as a quantity of practical interest.

3.3.2 Effect on reliability index

The effect of measurement uncertainty on reliability index is analysed for the additive relationship considering normally, lognormally and Gumbel distributed true values and with coefficient of variation ranging from 0.2 to 0.6. The measurement uncertainty has a normal distribution with known zero mean and varying standard deviation. Consistently with the interval analysis in Section 3.2, the mean resistance is determined to reach the target reliability without measurement and parameter estimation uncertainty. The
algorithm presented in Figure 8 is then applied to generate contaminated observations, to filter out the effect of measurement uncertainty and to calculate reliability index using inferred parameters. The calculations are repeated for 20 samples with a sample size of 50 annual maxima. The resulting reliability indices are displayed in Figure 10 and in Figure 11. Light grey and light blue areas depict the 75% confidence bands of reliability indices obtained for the 200 samples, and the corresponding thick solid lines are the median reliability indices. The width of the confidence bands illustrates the extent of statistical uncertainties. For the normal distribution, this effect is small compared to the lognormal and Gumbel models for which the width increases with standard deviation of measurement uncertainty. For Approach 2b, the width is larger than Approach 1 as the larger model space amplifies the effect of parameter estimation uncertainty. Additionally, the salient large scatter of Approach 2b might be partially attributed to the unstable maximum likelihood estimators for small samples (Hosking et al., 1985, Martins and Stedinger, 2000). Figure 10 and in Figure 11 show that the reliability index can be considerably overestimated when parameter estimation uncertainty is neglected.

Figure 10  Reliability indices as the function of the normalised standard deviation of measurement uncertainty (\(\sigma_{\mu_0} / \mu_0\)). The filled regions represent 75% confidence bands obtained from 200 random samples, indicating sampling variability. The solid lines are the median of the reliability indices.
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Figure 11  Reliability indices as the function of the normalised standard deviation of measurement uncertainty ($\sigma_E / \mu_Q$)

For the normal and Gumbel distribution, Approach 1 leads to systematically lower reliability indices than Approach 2b. For the normal distribution, Approach 1 seems to be overly conservative as the median is well below the target reliability level.

For all the considered distributions, Approach 1 is reasonably conservative with the exception of the lognormal distribution and coefficient of variation of 0.6. Though too conservative reliability estimates are obtained for the normal distribution, the currently prevalent Approach 1 appears to be safely applicable to measurement uncertainty problems in the case with the additive reality-observation relationship. Approach 2b is sound from theoretical point of view; however its median overestimates reliability level, thus the predictive reliability index – incorporating parameter estimation uncertainty (Der Kiureghian, 1989) – should be used to avoid underestimation of failure probability. Associated larger parameter estimation uncertainty can lead large reduction in reliability index for small sample sizes.
4 Example 2: turbine hall of a nuclear power plant

4.1 Fragility function

The turbine hall of a nuclear power plant is selected to demonstrate the effect of measurement uncertainty in reliability assessment of a safety-critical structure. The examined critical frame is shown in Figure 12.

Figure 12 Selected frame of turbine hall (right) and its fragility curves (left) for governing failure modes

Reliability of the steel structure (Figure 12) is crucial for the performance of the whole power plant. System reliability is governed by failures of multiple frames. Reliability of each of the frames is then affected by reliability of multiple truss members. To keep a sharp focus on the effect of measurement uncertainty, reliability analysis of a single frame with its governing failure mode is discussed hereafter only. This characterises well the whole hall system and is deemed sufficient for illustration. The main dimensions with the governing limit states are illustrated in Figure 12 where component and system (series) level fragility curves are also presented. The $g_2$ limit state function is used in all analyses of the turbine hall. For the purpose of illustration, it is sufficient to know the fragility curves; further details on how these were obtained were provided by Vigh et al. (2014).

4.2 Snow action

The probabilistic model of ground snow load is inferred from the snow water equivalent data of CarpatClim database (Szalai et al., 2013). The observations are available for 49 full winter seasons; the time trend of annual maxima is statistically insignificant (Rózsás et al., 2016) and is neglected. The basic statistical parameters of the annual (winter season) maxima are given in Table 2. A reference period of one year is used in all calculations related to the turbine hall. This is consistent with the commonly accepted practice in probabilistic risk assessments of nuclear power plants.
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<table>
<thead>
<tr>
<th>Table 2</th>
<th>Basic statistical characteristics of annual ground snow maxima</th>
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</thead>
<tbody>
<tr>
<td><strong>Statistics</strong></td>
<td><strong>Value</strong></td>
</tr>
<tr>
<td>Sample size</td>
<td>49</td>
</tr>
<tr>
<td>Mean</td>
<td>0.35 kN/m²</td>
</tr>
<tr>
<td>Coefficient of variation (bias corrected)</td>
<td>0.83</td>
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<tr>
<td>Skewness (bias corrected)</td>
<td>1.34</td>
</tr>
<tr>
<td>Characteristic value*</td>
<td>1.10 kN/m²</td>
</tr>
<tr>
<td>GEV shape coefficient (ξ)*</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Notes: *0.98 fractile of Gumbel distribution fitted with method of moments in line with Sanpaolesi et al. (1998).† Obtained using maximum likelihood method and belongs to Fréchet family.

4.3 Interval and reliability analysis

The interval approach is used to represent measurement uncertainty and the two-step approximation (Section 3.2.1) is applied to propagate interval uncertainty. Annual ground snow maxima related to the location are contaminated with measurement uncertainty whilst the other random variables are represented by probability distributions. The results in terms of reliability indices and failure probabilities are given in Figure 13.

**Figure 13** Reliability index and failure probability intervals as the function of normalised measurement uncertainty radius (ε1/μQ1). Comparison of Approach 1 and Approach 2a for the turbine hall of a nuclear power plant

The widest reliability index interval is 0.2, which is obtained for the largest measurement uncertainty radius. This practically insignificant difference is in agreement with the results obtained in the previous example (Figure 6). It is attributed to the large coefficient of variation of annual ground snow load that is dominating over measurement uncertainty.
5 Discussion

One can distinguish two components of measurement uncertainty: the reality-observation link and the nature of the contamination $E$. The three approaches considered in this study differ in treatment of these two components:

- The present prevalent approach (Approach 1) neglects both components, thus entirely ignores the possibility that real values are greater or smaller than the observed due to measurement uncertainty.

- The interval approach (Approach 2a) expresses full ignorance with respect to a reality-observation link and represents measurement uncertainty by intervals. Therefore, no decoupling of true values from measurement uncertainty is possible. Intervals should be used with caution as by definition, values outside of the interval are impossible. However, this assumption is rarely met in civil engineering. Measurement uncertainty is often described on the basis of expert judgement and wide intervals are applied to almost surely capture real, unobserved values.

- The statistical approach (Approach 2b) requires the knowledge of the reality-observation link and represents measurement uncertainty with a distribution function. This is the only approach that can decontaminate the observations and can directly infer the variable of interest – the true variable on which structural reliability is dependent. The statistical and interval analysis based approaches are conceptually different, which makes their quantitative comparison difficult; uncertainty representation is inherently distinct and no equivalence between interval and distribution-based representations exists.

Additionally, it must be emphasised that another type of uncertainty – statistical uncertainty in parameter estimation and selection of distribution function – often needs to be taken into account in reliability analysis as it may be even more important than measurement uncertainty (Rózsás and Sýkora, 2015a,b).

Bayesian paradigm is a natural choice to incorporate this uncertainty and has been recommended for practical applications. For example, a reality-observation link cannot be established with certainty in practical applications. Yet, this uncertainty can be captured by using multiple models and averaging them with respect to their fit to the data by the Bayesian model averaging (Hoeting et al., 1999).

Although this study is limited by the considered distribution types, reality-observation functions, and parameter ranges, it covers many practical situations. The presented approach and algorithms can be easily used for other distribution types and measurement uncertainty structures. An additional limitation consists in considering measurement uncertainty affecting a dominant variable action only. However, it is anticipated that the effect is mostly reduced for other random variables due to their smaller sensitivity factors. Moreover, measurement uncertainty is much smaller for other than climatic actions such as resistance and permanent actions. Furthermore, the effect of sample size should be analysed in the future. It is believed that the outcomes would be similar for sample sizes ranging from 20 to couple of hundreds, which cover the majority of cases in civil engineering. In general, larger databases allow for better model identification.
6 Conclusions

The current practice in engineering and structural reliability fields treats observed data contaminated with measurement uncertainty as realizations of the true distribution, thus neglecting the contamination mechanism. Statistical and interval-based analyses are thus conducted to investigate the effect of this simplification on structural reliability. Extensive parametric analyses reveal that:

1 Interval representation of measurement uncertainty:
   - Sampling variability (parameter estimation uncertainty) has significant effect on reliability; it is dominating over measurement uncertainty for small interval radiiuses and it is comparable for large radiiuses.
   - For mountains and highlands, moderate $\pm 4\%$ measurement uncertainty – relative to a value of an observed variable – can lead to significant reduction of reliability level. For lowlands, even a large $\pm 10\%$ measurement uncertainty has an insignificant effect. The effect is deemed significant when six-fold increase in failure probability is observed when compared to Approach 1 (neglect of measurement uncertainty).
   - Reliability interval ranges indicate that a small $\pm 2\%$ measurement uncertainty can lead to reduction of 0.6 in reliability index. For larger measurement uncertainties, the width of the reliability intervals can exceed 2.0.
   - The effect of measurement uncertainty is more pronounced for low variability random variables where its contribution to the total uncertainty increases.
   - Parameter ranges where Approach 1 overestimates the reliability index are identified in Figure 6.

2 Representation of measurement uncertainty by probabilistic distributions:
   - The statistical approach can be used to decontaminate the observations and to access the variable of interest.
   - If parameter estimation uncertainty is disregarded, reliability index can be considerably overestimated.
   - Approach 1 is reasonably conservative in most cases when an additive measurement uncertainty format is taken into account.

3 Practical recommendations:
   - Figure 6 and Figure 7 can be used to identify cases when Approach 1 significantly overestimates the reliability index. In these cases, interval analysis could be used when no or very limited information on measurement uncertainty is available; it is advised to base reliability assessment on a lower bound of a reliability index interval.
   - If the reality-observation link is known, the statistical approach is recommended. For small and moderate sample sizes (say, less than 100 measurements), the effect of parameter estimation uncertainty should be taken into account, for
example through the predictive reliability index. Approach 1 is conservative for an additive measurement uncertainty format.

- Point estimates such as median reliability index should be accompanied by intervals to indicate uncertainty in reliability level estimates (parameter estimation uncertainty).
- For ground snow load extremes at lowlands, Approach 1 provides a reasonable approximation, thus the effect of measurement uncertainty can be neglected. For highlands and mountains measurement uncertainty effect increases and the use of a more advanced analysis is recommended.

Assessment of measurement uncertainty should be region- and case-specific accounting for measuring techniques, and applied correction equations. This is why involvement of meteorologists, analysts, or other experts is beneficial. Moreover, a selected approach to propagate measurement uncertainty should always be based on the particular issue in question, acknowledging “the degree of precision to which the nature of the subject admits”.

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