Novel image watermarking method based on FRWT and SVD

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Abstract: The fractional wavelet transform (FRWT) is a generation of WT associated with the convolution theorem in fractional Fourier transform (FRFT) domain. The FRWT not only inherits the advantages of multi-resolution analysis of the wavelet transform (WT), but also has the capability of image representations in the FRFT domain. In view of the above characteristics, a novel digital image watermarking method based on FRWT and SVD is proposed in this paper. The experimental results show that this method is robust to geometric attacks and image processing attack.

Keywords: digital image watermarking; fractional wavelet transform; FRWT; wavelet transform; singular value decomposition; SVD.


Biographical notes: Zhihai Zhuo is an Assistant Professor at the School of Information and Communication Engineering, Beijing Information Science and Technology University. He received a Doctor’s degree in Electronics Engineering from Beijing Institute of Technology, China. He is interested in signal processing, image processing and information security.

1 Introduction

With the development of science and technology, we have entered the era of digitalisation and networking of information, which brings a lot of convenience to people’s work and life. However, valuable information can easily be copied and distorted. Therefore, how to protect the information security has become a serious problem to be solved. As an effective way to protect the information security, digital watermarking technology arises at the historic moment. The digital watermark technology is a digital protection technology for the purpose of secret transmission, storage and identification; it has become one of the hottest research topics in information security community (Thanh and Tanaka, 2016; Wang et al., 2016; Papakostas et al., 2014; Hu and Hsu, 2017; Sawant and Patil, 2017; Tiwari et al., 2017; Su and Chen, 2017).

Generally speaking, the digital image watermarking method can be divided into spatial domain method and transform domain method. Compared with the spatial
watermarking method, the transform domain watermarking methods have attracted widespread attentions. Because the watermark embedded in the transform domain can not only improve the invisibility, but also improve the robustness. In the transform domain watermarking, discrete wavelet transform (DWT), the discrete fractional Fourier transform (DFRFT), singular value decomposition (SVD) and QR decomposition are widely used in this field (Agarwal et al., 2015; Rawat and Raman, 2012; Lang and Zhang, 2014; Chung et al., 2007; Naderahmadian and Hosseini-Khayat, 2014; Su et al., 2017).

Due to its advantages like multi-resolution representation and better energy compression, the DWT-based image watermarking method can provided high robustness to image processing attacks but low robustness to some geometric attacks. The SVD or QR can be used to extract the geometric features of an image, which is more robust to geometric attacks. Therefore, the image watermarking method based on DWT and SVD or QR is proposed and thus robust against both geometric attacks and image processing attacks (Ye et al., 2014; Ali and Ahn, 2014; Naderahmadian and Hosseini-Khayat, 2010). The fractional wavelet transform (FRWT) is a generation of WT associated with the convolution theorem in fractional Fourier transform (FRFT) domain. The FRWT not only inherits the advantages of multi-resolution analysis of the wavelet transform (WT), but also has the capability of image representations in the FRFT domain (Shi et al., 2012, 2015). These good properties make FRWT become one of the important tools in the image processing. It has been successfully applied on image fusion and image denoising (Xu et al., 2014; Xu and Wang, 2014), it is necessary to study the potential application of FRWT in the image watermarking field. Therefore, based on above facts and the similar reason of the combination DWT and SVD, this paper proposes a novel digital image watermarking method based on FRWT and SVD.

The following contents are arranged as follows: in Section 2, we will introduce some basic knowledge about FRWT and SVD. Then, we propose a novel digital image watermarking method including the watermark embedding and extraction procedures in Section 3. The performance analysis of this novel image watermarking method is shown in Section 4. Finally, conclusions are stated in Section 5.

2 Preliminaries

2.1 Fractional Fourier transform

The FRFT with real parameter angle \( \alpha \) of signal \( f(t) \) is defined as (Tao et al., 2009)

\[
F_\alpha \{f(t)\}(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) K_\alpha (t, u) dt
\]

with the kernel

\[
K_\alpha (t, u) = \begin{cases} 
\frac{1-i\cot \alpha}{2\pi} e^{\frac{i}{2} \cot \alpha (t^2 + u^2) - 2 \cot \alpha tu} & \alpha \neq k\pi/2 \\
\delta(t-u) & \alpha = k\pi \\
\delta(t+u) & \alpha = k\pi + \pi/2
\end{cases}
\]
where $\alpha$ denotes the rotation angle of FRFT, and $F_\alpha(u)$ is the FRFT of $f(t)$. The inverse FRFT is the FRFT with order $-\alpha$. Whenever $\alpha = \pi/2$, the FRFT reduces to the Fourier transform.

At present, some well-known concepts and theorems associated with FRFT have been derived, such as uncertainty principle, sampling theorem and the convolution theorem (Tao et al., 2009). The fractional convolution of the FRFT for functions $f(t)$ and $h(t)$ can be defined as follows:

$$f(t)\Theta_\alpha h(t) = \int_{-\infty}^{+\infty} f(\tau)e^{-2\cot(\alpha)(t^2-\tau^2)}h^*(t-\tau)d\tau$$

(3)

### 2.2 Fractional wavelet transform

Since WT can be written as a classical convolution form as following:

$$W_f(a, b) = \langle f(t), \psi_{a,b}(t) \rangle = f(t) * \left( \frac{1}{\sqrt{a}} \psi \left( \frac{t}{a} \right) \right)$$

(4)

where $a \in R^+$, $b \in R$ and $\psi_{a,b}(t)$ is a continuous affine transformation of the mother wavelet $\psi(t)$, i.e., $\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi \left( \frac{t-b}{a} \right)$, and * denotes the classical convolution operator and $\langle \cdot, \cdot \rangle$ indicates the inner product in $L^2(R)$.

The FRWT is a generation of WT associated with the convolution theorem in FRFT domain, which is defined as follows:

$$W_f^\theta(a, b) &= \int_{-\infty}^{+\infty} f(t)e^{\frac{\theta^2}{2}\cot^2} \psi_{a,b}(t)dt$$

(5)

The definition of 2-D FRWT is given by following equation:

$$W_f^{\theta_1, \theta_2}(a, b_1, b_2) = e^{\frac{\theta_1^2}{2}(\cot^2(x/a) + \cot^2(y/a))} \int_{G^2} f(x, y)e^{\frac{\theta_2^2}{2}(y^2 \cot^2(x/a) + x^2 \cot^2(y/a))} \frac{1}{a} \psi \left( \frac{x}{a} \right) \psi \left( \frac{y}{a} \right) dxdy$$

(6)

The authors have researched the discrete FRWT algorithm, given the decomposition and reconstruction of FRWT (Xu et al., 2014). They also pointed out that FRWT is the discrete fractional convolution process in different scale, the coefficients of decomposition and reconstruction process is a process of Mallat algorithm with coefficient modulation. Similar to DWT, LCWT decomposes image into four subbands: fractional domain low frequency subband LL, horizontal fractional domain high frequency subband HL, vertical fractional domain high frequency subband LH, fractional domain high frequency subband HH.
2.3 **Singular value decomposition**

The SVD is related to the theory of diagonalising a symmetric matrix in linear algebra. Suppose $A$ is an $m \times n$ matrix whose entries come from the field $\mathbb{K}$, which is either the field of real numbers or the field of complex numbers. Then there exists a factorisation, called a SVD of $A$, i.e.,

$$A = U \Sigma V^T$$

(7)

where $U$ is a $m \times m$ unitary matrix, (if $\mathbb{K} = \mathbb{R}$, unitary matrices are orthogonal matrices), $\Sigma$ is a diagonal $m \times n$ matrix with non-negative real numbers on the diagonal, and $V$ is a $n \times n$ unitary matrix over $\mathbb{K}$. The diagonal entries $\sigma_i$ of $\Sigma$ are known as the singular values of $A$.

A common convention is to list the singular values in descending order.

In the special, yet common case when $A$ is an $m \times m$ real square matrix with positive determinant, $U$, $V^T$, and $\Sigma$ are real $m \times m$ matrices as well, $\Sigma$ can be regarded as a scaling matrix, and $U$, $V^T$ can be viewed as rotation matrices. Thus the expression $U \Sigma V^T$ can be intuitively interpreted as a composition of three geometrical transformations: a rotation or reflection, a scaling, and another rotation or reflection. SVD has been successfully applied on solving homogeneous linear equations, total least squares minimisation, low-rank matrix approximation and signal processing fields (Klema and Laub, 1980; Wall et al., 2003; Golub and Reinsch, 1970).

3 **Proposed digital image watermark method**

3.1 **Watermark embedding**

In this subsection, the watermark embedding procedures are introduced. Assume the cover image $X_{512 \times 512}$ is a grey level image. The watermark image $W_{64 \times 64}$ is a gray image. The embedding procedures can be summarised as follows:

1. Perform one level FRWT to $X$ to obtain four subbands LL, HL, LH and HH, where the fractional approximate subband LL of size $256 \times 256$ will be used in following steps.

2. LL is divided into non-overlapping blocks $l_{ij}$ ($1 \leq i, j \leq 64$) of size $4 \times 4$.

3. Apply SVD on each block $l_{ij}$ to obtain three components $U_{ij}$, $\Sigma_{ij}$ and $V_{ij}$, i.e.,

$$l_{ij} = U_{ij} \Sigma_{ij} V_{ij}^T$$

(8)

4. The $\Sigma_{ij}(1, 1)$ is selected to embed watermark as following:

$$\Sigma_{ij}(1, 1) = \Sigma_{ij}(1, 1) + \lambda \cdot w(i, j)$$

(9)

where $R_y(1, 1)$ denotes the first row and first column of $R_y$, $\Sigma_{ij}$ denotes the watermarked block. $\lambda$ denotes watermark embedding strength.
Perform inverse SVD to each block to obtain watermarked block, i.e., $l_{ij}^w = U_{ij} \Sigma_{ij} V_{ij}^T$.

Merge all these watermarked blocks together to obtain $LL^w$ as opposite to Step 2, where $LL^w$ is watermarked fractional approximate subband.

Perform inverse LCWT on $LL^w$ and other three subbands HL, LH and HH obtained in Step 1, watermarked cover image $X^w$ can be obtained.

3.2 Watermark extraction

The extraction procedures can be summarised as following:

1. Apply one level FRWT to the watermarked cover image $X^w$, four subbands $LL'$, $HL'$, $LH'$ and $HH'$ can be obtained, where the watermarked fractional approximate subband $LL'$ of size 256×256 will be used in following steps.

2. $LL'$ is divided into non-overlapping blocks $l_{ij}'$ of size 4×4, the number of all blocks are $64^2$.

3. Apply SVD on each block $l_{ij}'$ to obtain $U_{ij}'$, $\Sigma_{ij}'$ and $V_{ij}'$.

4. Watermark image is extracted from the first row and first column of $\Sigma_{ij}'(1,1)$, which is given by following equation:

$$w(i, j) = \frac{\Sigma_{ij}'(1,1) - \Sigma_{ij}(1,1)}{\lambda}$$  \(10\)

4 Results and analysis

In this section, the invisibility and robustness of the proposed image watermark method are tested by experiments. All simulation experiments are conducted on a personal computer having Intel dual core 3.2 GHz CPU with 8.0 GB RAM, and using MATLAB version R2010a under the Windows 8 environment. Meanwhile, some frequently used images in image watermarking field are selected as the cover images, which are shown in Figure 1(a)–1(c) respectively. A 64×64 image is used as watermark image, which is shown in Figure 1(d). The parameter settings of FRWT are $\alpha = 0.9$, watermark strength is $\lambda = 0.2$.

Figure 1 The test covers images and watermark, (a) Lena (b) boat (c) Elaine (d) watermark
The embedded watermark should be invisible to human eyes in order to ensure the watermark safety, so watermark invisibility is an important index to measure the ability of image watermarking method. First, peak signal to noise ratio (PSNR) is commonly used as a index to measure the quality of reconstruction watermarked cover image, which is defined as (Huynh-Thu and Ghanbari, 2008)

$$\text{PSNR}(X, X^w) = 10 \log_{10} \frac{MN \cdot [\max(X(m, n))]^2}{\sum_{m=1}^{M} \sum_{n=1}^{N} [X(m, n) - X^w(m, n)]^2}$$

(11)

where $X$ is original cover image and $X^w$ is watermarked cover image. In addition, NC is used to measure the similarity between the extracted watermark and the original watermark. The larger NC value means the higher similarity between the extracted watermark and the original watermark. The definition of NC is shown as following (Shieh et al., 2004):

$$\text{NC}(w, w') = \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} w(m, n)w'(m, n)}{\sqrt{\sum_{m=1}^{M} \sum_{n=1}^{N} [w(m, n)]^2} \sqrt{\sum_{m=1}^{M} \sum_{n=1}^{N} [w'(m, n)]^2}}$$

(12)

where $M$ and $N$ are the size of original watermark image $w$ and extracted watermark image $w'$.

4.1 Invisibility

Based on three test cover images, the experiments are conducted to intuitively show the relationship between PSNR and the watermark embedding strength $\lambda$, the results are shown in Figure 2. It can be seen from the Figure 2 that the PSNR overall decrease with the increasing of watermark embedding strength $\lambda$. In order to guarantee watermark invisibility (i.e., PSNR $\geq 30$ dB), the watermark strength $\lambda$ is thus selected as $\lambda = 0.2$ in following experiments.

Based on watermarking embedding procedures, the watermarked cover images and corresponding PSNR values are listed in Table 1. It can be seen from the Table 1 that there are no visual quality differences between watermarked cover images and original cover images. Generally speaking, human eye cannot distinguish the difference between original and watermarked cover images when the PSNR is larger than 30 dB. It can be found that the PSNR listed in Table 1 are all larger than 30 dB. This implies that our method has better watermark invisibility. Moreover, when the watermarked cover images suffer no attack, the corresponding extracted watermark images and NC values are also listed in Table 1 using watermarking extraction procedures. All NC values equal to one mean that the proposed method can extract the watermark correctly.
Figure 2  The relationship between PSNR and watermark embedding strength $\lambda$ (see online version for colours)

Table 1  The watermarked cover image and extracted watermark

<table>
<thead>
<tr>
<th>Image</th>
<th>Lena</th>
<th>Boat</th>
<th>Elaine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Watermarked cover image</td>
<td><img src="image" alt="Lena" /></td>
<td><img src="image" alt="Boat" /></td>
<td><img src="image" alt="Elaine" /></td>
</tr>
<tr>
<td>PSNR (dB)</td>
<td>32.1002</td>
<td>32.4637</td>
<td>32.0157</td>
</tr>
<tr>
<td>Extracted watermark</td>
<td><img src="image" alt="USTB" /></td>
<td><img src="image" alt="USTB" /></td>
<td><img src="image" alt="USTB" /></td>
</tr>
<tr>
<td>NC</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

4.2 Robustness

The watermarked cover image is often destroyed by various attacks, so the robustness is an important index to show the performance of the image watermarking method. In this subsection, the robustness of the proposed image watermarking method is tested by performing several attacks. In this paper, the proposed method is compared with the
similar image watermark methods based on DWT and QR decomposition (abbreviated as DWT+QR) (Naderahmadian and Hosseini-Khayat, 2010) and watermark method based on QR decomposition (abbreviated as QR) (Naderahmadian and Hosseini-Khayat, 2014).

### Table 2

The results under some image processing attacks

<table>
<thead>
<tr>
<th>Image processing operation</th>
<th>FRWT+SVD</th>
<th>DWT+QR</th>
<th>QR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression JPEG = 30</td>
<td><img src="image1.png" alt="Image" /> 0.9871</td>
<td><img src="image2.png" alt="Image" /> 0.8828</td>
<td><img src="image3.png" alt="Image" /> 0.8302</td>
</tr>
<tr>
<td>NC</td>
<td>0.9969</td>
<td>0.9806</td>
<td>0.9031</td>
</tr>
<tr>
<td>Compression JPEG = 60</td>
<td><img src="image4.png" alt="Image" /> 0.9964</td>
<td><img src="image5.png" alt="Image" /> 0.9022</td>
<td><img src="image6.png" alt="Image" /> 0.8141</td>
</tr>
<tr>
<td>Smooth Gaussian low-pass filter</td>
<td><img src="image7.png" alt="Image" /> 0.9964</td>
<td><img src="image8.png" alt="Image" /> 0.9022</td>
<td><img src="image9.png" alt="Image" /> 0.8141</td>
</tr>
<tr>
<td>Average filter</td>
<td>0.9736</td>
<td>0.8791</td>
<td>0.7813</td>
</tr>
<tr>
<td>Noise Gaussian noise</td>
<td><img src="image10.png" alt="Image" /> 0.9603</td>
<td><img src="image11.png" alt="Image" /> 0.6933</td>
<td><img src="image12.png" alt="Image" /> 0.6873</td>
</tr>
<tr>
<td>Speckle noise</td>
<td>0.9874</td>
<td>0.8776</td>
<td>0.8728</td>
</tr>
</tbody>
</table>

The image processing attacks robustness is explored when the watermarked cover image are attacked by compression, smooth and noise. In order to make the results more intuitively, we present the extracted watermark images and corresponding NC values in Table 2 using three methods. Through analysis and comparison, it can be found that the NC values of proposed method are higher than other two methods. From both subjective perception and objective perception, the proposed method has a better robustness of image processing attacks compared with other two methods.
The geometry attack robustness are explored when the watermarked cover image are attacked by geometry attack like rotation and resize. The extracted watermark images and corresponding NC values are listed in Table 3 based on three methods. Through analysis and comparison, it can be found that the NC values of proposed method are higher than other two methods. The subjective perceptions are in agreement with the meaning of the data. The extracted watermark images are clearer than other two methods at rotation and resize. In brief, compared with the watermark method based on DWT+QR and QR, the proposed method is more robust to geometry attacks.

Table 3  The results under some geometry attacks

<table>
<thead>
<tr>
<th>Image processing operation</th>
<th>FRWT+SVD</th>
<th>DWT+QR</th>
<th>QR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotate Angle = 45</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NC</td>
<td>0.9860</td>
<td>0.8796</td>
<td>0.8823</td>
</tr>
<tr>
<td>Angle = 60</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NC</td>
<td>0.9866</td>
<td>0.8956</td>
<td>0.8933</td>
</tr>
<tr>
<td>Resize 1/2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NC</td>
<td>0.9893</td>
<td>0.9491</td>
<td>0.8330</td>
</tr>
<tr>
<td>1/4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NC</td>
<td>0.9760</td>
<td>0.7527</td>
<td>0.7720</td>
</tr>
</tbody>
</table>

5 Conclusions

In this paper, a novel image watermarking method based on FRWT and SVD is proposed. The invisibility and robustness of the image watermarking method are detected. The experimental results show that the proposed method can extract the watermark image correctly and is more robust to both geometric and image processing attacks. Compared with the watermark method proposed in Naderahmadian and Hosseini-Khayat (2010, 2014), the performance of the proposed method is better.
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References


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