Control method of tool axis vector based on kinematics characteristics for five-axis NC machining

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Abstract: The complexity of tool path in the machining of complex curved surface is often accompanied by the large change of tool axis vector and the large change of tool axis vector can cause larger machining error and is easily beyond the restrictions of angular velocity and angular acceleration for the rotation angle of swing head or turntable of machine tool under a constant feed speed. The optimisation of the planning for the tool axis vector is very important based on the kinematics characteristics. The control method of tool axis vector based on the kinematics characteristics for five-axis NC machining of complex curved surface is studied. Experimental results show that the planning of tool axis vector based on the kinematics characteristics for five-axis NC machining can guarantee the smooth movement of the rotation axis and the machining process for complex curved surface can be more stable and reliable.

Keywords: tool axis vector; kinematics characteristics; five-axis NC machining; angular velocity; curved surface; optimisation.


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1 Introduction

Compared with three-axis NC machine tool, the five-axis NC machine tool has two rotation axis, so the movement of the cutter relative to the workpiece is available in three-dimensional space, which can effectively avoid the interference between cutter and workpiece, make the contact state between cutter and workpiece much better during the processing and improve the machining quality. Therefore, the five-axis NC machine tool, with its advantages in the manufacture of parts with complex curved surface (Liu, 2006; Chen et al., 2004; Rao and Sarma, 2000), is a research hotspot in the field of surface processing in recent years. In the NC machining of complex curved surface, the well contact state between the cutter and the machining surface is one of the key factors to ensure the machining quality. When the cutter axis swings continually, the well contact state between the cutter and the machining surface usually is guaranteed by the geometric condition, so as to obtain the high machining efficiency (Wu et al., 2007). As the control method of tool axis vector is very complex, it restricts the actual application of the five-axis NC machine tool in some degree.

In the machining process of complex curved surface, the tool axis vector should make a real-time adjustment according to the local curvature feature of the machining surface. The complexity of the tool path in the machining of complex curved surface is often accompanied by the large change of tool axis vector, which introduces the large fluctuation of the rotation angle for the swing head or the turntable of the machine tool. Some studies have found that there exists a certain proportion between the machining
error and the angle variation of the tool axis, and the big change of tool axis vector can cause the larger machining error (Hwang and Ho, 2000; Hwang and Liang, 1998), even seriously, the surface of workpiece is destroyed (Ho et al., 2003; Wang and Tang, 2007). Under a constant feed speed, the large change of angle for the tool axis vector is easily beyond the restrictions of angular velocity and angular acceleration for the rotation angle of the swing head or the turntable of the machine tool, and even beyond the power scope of the servo motor, which makes the generated NC codes cannot be applied in the actual production. In the determine process of the tool axis vector, the full consideration of the kinematics characteristics can reach the maximum potential for the function of machine tool. Therefore, the optimisation of the planning for the tool axis vector is very important based on the kinematics characteristics, which has important theoretical and practical value for the machining process of the complex curved surface (Wu et al., 2008).

In recent years, many studies have been carried out on the control and the optimisation for the planning of tool axis vector in the five-axis NC machining, and the main works are focused on the determination of tool axis vector based on the local geometry information of parts with complex curved surface and avoiding the collision interference (Lo, 1999), such as the matching relation of the curvature (Wu et al., 2007). Some scholars analysed the dynamic characteristics of the machine tool, and the relationship between dynamic characteristics and tool axis vector was preliminarily studied (Kim and Sarma, 2002; Pateloup et al., 2004; Tounsi et al., 2003; Ho, 2004). Ho et al. (2003) determined the tool axis vectors at the typical positions that conformed to the kinematics characteristics firstly, and then the quaternion interpolation algorithm was used to calculate the tool axis vector in the middle position and the collision interference was inspected, finally the tool axis vectors which changed uniformly were obtained. Wang and Tang (2007) provided a systematic solution for the optimisation problem for tool axis vector in the five-axis NC machining, however, the realisation process was complex and this method did not consider the influence of the feeding step, also did not real link to the movement of the machine tool. To better control the motion of the machine tool, Jun et al. (2003) defined a smooth curve to determine the attitude of cutter and prevent the big angle change between adjacent tool axes. Castagnetti et al. (2008) provided a method to make the tool path smoothing by controlling the deflection angle of the tool axis, which was used to improve the dynamic characteristics of the machine tool in the milling process. The proposed planning methods of tool axis vector above are applicative in a certain degree, however, the kinematics characteristics of the machine tool is not considered as a constraint condition in the optimisation of the planning for tool axis vector.

As the ball end cutter is often used in the milling of complex curved surface, the adjustment of tool axis vector basically does not affect the machining quality of the complex curved surface. According to the machine tool with the specific structure, the control method of the tool axis vector based on the kinematics characteristics for the five-axis NC machining of the complex curved surface is studied in the paper. The constraint condition of kinematics characteristics for the machine tool is given, the planning of tool axis vector is optimised based on the constraint condition of the kinematics characteristics and the experiment is carried out at last.
2 Analysis of kinematics characteristics for tool axis vector

2.1 Structure of the machine tool

In this study, the structure of the machine tool is shown in Figure 1, and the five-axis NC machine tool has a double swing cutter head. The rotation axis is vertical to the tool axis, the rotation axis $A$ is parallel to $X$ axis and the tool axis is parallel to the $Z$ axis. When the workpiece is fixed on the workbench, the workpiece coordinate system is accordance with the machine coordinate system and the origin of the tool coordinate system coincides with the origin of the workpiece coordinate system.

Figure 1 Five-axis NC machine tool

2.2 Coordinate transformation

The curved surface machined by the five-axis machine tool is shown in Figure 2. As shown in Figure 2, $S$ is the curved surface with $G^1$ continuity, the cutter contacts to the curved surface $S$ at the point $C$, $f$ is the feeding direction, $n$ is the normal vector of the curved surface at the contact point and $b$ is the direction of linewidth with $b = f \times n$. The local coordinate system $CX'L'Y'L'Z'L'$ can be established by setting the $f$, $b$ and $n$ as the directions of $X_L$, $Y_L$ and $Z_L$. Assuming $I$ is the tool axis vector in the machining process, the tool axis rotates an angle $\lambda$ along the $Y_L$ axis, and then rotates an angle $\omega$ along the $Z_L$ axis. When the point $C$, $f$, $n$, $\lambda$ and $\omega$ are given, the attitude of tool axis can be determined completely. In the local coordinate system, the changes of tool axis vector $I$ at the contact point $C$ is decided by $\lambda$ and $\omega$, so the tool axis vector $I$ can be expressed by $\lambda$ and $\omega$. For tool axis vector $I_i$ at each contact point $C_i$, the corresponding coordinate $I_i(\lambda, \omega)$ can be found in a plane established by $\lambda$ and $\omega$. 
The tool axis vector above-mentioned is confirmed in the local coordinate system of \( CX_L Y_L Z_L \) at the contact point \( C \), and then the tool axis vector \( I_i(\lambda, \omega) \) in the machine tool coordinate system \( OXYZ \) can be expressed as:

\[
T_i = M I_i
\]  

(1)

In which, \( M \) is a transformation matrix from \( CX_L Y_L Z_L \) to the coordinate system \( OXYZ \). Based on the five-axis NC machine tool as shown in Figure 1, the rotation angles of rotation axis \( A \) and \( B \) relative to the initial state are \( \alpha, \beta \) respectively (counter clockwise is positive), then the transformation matrix is

\[
M = R_b(\beta) \cdot R_x(\alpha)
\]

(2)

\[
= \begin{bmatrix}
\cos \beta & 0 & \sin \beta \\
0 & 1 & 0 \\
-\sin \beta & 0 & \cos \beta
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \alpha & -\sin \alpha \\
0 & \sin \alpha & \cos \alpha
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\cos \beta & \sin \alpha \sin \beta & \cos \alpha \sin \beta \\
-\sin \beta & \cos \alpha & -\sin \alpha \\
0 & \sin \alpha & \cos \alpha \cos \beta
\end{bmatrix}
\]

2.3 Angular velocity of tool axis vector

For a given curved surface \( S \), the cutting tool moves from point \( C_i \) to the next point \( C_{i+1} \) along the tool path, and the euclidian distance between the \( C_{i+1} \) and \( C_i \) is \( D_{i+1} \), then the tool axis vector transforms from the \( I_i(\lambda_i, \omega_i) \) to \( I_{i+1}(\lambda_{i+1}, \omega_{i+1}) \). For the planning of the contact points, the distribution of feeding step length between adjacent contact points is usually uneven. At the same time, the variation of two angles corresponding to the tool axis vector can be expressed as the following,

\[
\begin{align*}
\delta_\lambda &= \frac{v_i |\lambda_{i+1} - \lambda_i|}{D_{i+1}}, i = 1, 2, \ldots, n-1 \\
\delta_\omega &= \frac{v_i |\omega_{i+1} - \omega_i|}{D_{i+1}}
\end{align*}
\]

(3)
In which, \( v_i \) is the feeding speed corresponding to the current contact point. The feeding speed of cutting tool is keeping constant in the machining process, so the numerator in Equation (3) can be written as \( |\dot{\lambda}_{i+1} - \dot{\lambda}_i| \) and \( |\omega_{i+1} - \omega_i| \). In this way, \( \delta_\lambda \) and \( \delta_\omega \) in Equation (3) can be as the angular velocity of \( \lambda \) and \( \omega \) in the machining process. Assuming the constraint values of the angular velocity for the angle \( \lambda \) and \( \omega \) are \( \phi_\lambda \) and \( \phi_\omega \), the tool axis vectors \( I(\lambda, \omega), i = 1, 2, \ldots, n \), should be found for the orderly contact points \( C = \{C_1, C_2, \ldots, C_n\} \) in the tool path, which satisfy the constraint conditions of angular velocity, just as the following,

\[
\begin{align*}
\left| \frac{\lambda_{i+1} - \lambda_i}{D_{\lambda i}} \right| &< \phi_\lambda, \\
\left| \frac{\omega_{i+1} - \omega_i}{D_{\omega i}} \right| &< \phi_\omega, \\
&i = 1, 2, \ldots, n-1
\end{align*}
\]

(4)

If the point \( P(P_{\lambda}, P_{\omega}) \) on the plane \( \lambda \omega \) meets the following constraint conditions,

\[
\begin{align*}
\left| \frac{P_{\lambda} - \dot{\lambda}_i}{D_{\lambda i}} \right| &< \phi_\lambda, \quad \left| \frac{P_{\omega} - \dot{\omega}_i}{D_{\omega i}} \right| < \phi_\omega
\end{align*}
\]

(5)

It indicates that the movement from \( P \) to \( C_i \) meets the constraints of kinematics characteristics of the machine tool.

2.4 Angular acceleration of tool axis vector

\( a_\lambda \) is the angular acceleration of \( \lambda \) in the machining process. According to the angular velocity of \( \lambda \) and the machining time \( t \), \( a_\lambda \) can be obtained as

\[
a_\lambda = \frac{\delta_\lambda}{t}
\]

(6)

In a similar way, the angular acceleration \( a_\omega \) of \( \omega \) is

\[
a_\omega = \frac{\delta_\omega}{t}.
\]

(7)

3 Smoothing method of tool axis vector based on kinematics characteristics

In order to guarantee the planning of the tool axis vector satisfying the kinematics characteristics of five-axis NC machining, the smoothing method of tool axis vector based on kinematics characteristics is proposed in this section. For the orderly contact points \( C = \{C_1, C_2, \ldots, C_n\} \) in the tool path, the corresponding tool axis vector is \( I(\lambda, \omega), i = 1, 2, \ldots, n \). Based on equation (1), the rotation angles of the rotation axis \( A \) and \( B \) corresponding to the discrete points \( C = \{C_1, C_2, \ldots, C_n\} \) can be obtained as
Control method of tool axis vector based on kinematics characteristics

\[ T_\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_n\} \]
\[ T_\beta = \{\beta_1, \beta_2, \ldots, \beta_n\} \]

(8)

To smooth the variation of the tool axis vector based on kinematics characteristics, the optimisation model proposed in this study for the rotation angles of the rotating coordinates is as following,

Rotation axis \( A \):

\[
T(\alpha) = \begin{cases} 
\min \sum \left( \frac{\alpha_{i+1} - \alpha_i}{D_{i+1}} \left| \frac{\alpha_i - \alpha_{i-1}}{D_{i+1}} \right| \right)^2 & i = 2, \ldots, n - 1 \\
\min \sum \left( \frac{\alpha_{i+1} - \alpha_i}{D_{i+1}} \right)^2 & i = 1, \ldots, n - 1 
\end{cases}
\]

s.t. \( \alpha_i \in [\alpha_{\text{min}}, \alpha_{\text{max}}] \)

(9)

Rotation axis \( B \):

\[
T(\beta) = \begin{cases} 
\min \sum \left( \frac{\beta_{i+1} - \beta_i}{D_{i+1}} \left| \frac{\beta_i - \beta_{i-1}}{D_{i+1}} \right| \right)^2 & i = 2, \ldots, n - 1 \\
\min \sum \left( \frac{\beta_{i+1} - \beta_i}{D_{i+1}} \right)^2 & i = 1, \ldots, n - 1 
\end{cases}
\]

s.t. \( \beta_i \in [\beta_{\text{min}}, \beta_{\text{max}}] \)

(10)

In which, \([\alpha_{\text{min}}, \alpha_{\text{max}}]\) and \([\beta_{\text{min}}, \beta_{\text{max}}]\) are the mapping interval of tool axis vectors with no interference to the range of rotation angles of rotation axis for the five-axis NC machine tool. In the optimisation process, two end points of tool axis vectors are fixed as \( T(\alpha_1) = \alpha_1, T(\alpha_n) = \alpha_n, T(\beta_1) = \beta_1, T(\beta_n) = \beta_n \).

By combining the recursive method and the quadratic optimisation algorithm, the equations (9)–(10) can be solved and the rotation angle series after optimised for the rotation axis can be obtained as,

\[ O_\alpha = \{O\alpha_1, O\alpha_2, \ldots, O\alpha_n\} \]
\[ O_\beta = \{O\beta_1, O\beta_2, \ldots, O\beta_n\} \]

(11)

4 Simulation and experiment

Simulation and experiment are carried out to verify the proposed control method of tool axis vector based on kinematics characteristics for the five-axis NC machining in this section.

The curved surface selected is a hyperbolic paraboloid, and its parametric equation is

\[ S(u, v) = \left\{ u, v, -\frac{u^2}{100} + \frac{v^2}{100} \right\}, \quad u \in [-50, 50], \quad v \in [-50, 50]. \]

A tool path is obtained by setting \( u = \xi, v = 15 \). Then the parametric equation of the curve for tool path is

\[ r(\xi) = \left\{ \xi, -15, -\frac{\xi^2}{100} + \frac{9}{4} \right\}, \xi \in [-50, 50] \]

(12)
The initial planning of tool axis vector based on the geometric condition is shown in Figure 3. The angular velocity of the rotation axis based on the initial planning of tool axis vector is calculated as shown in Figure 4. In Figure 4, $\omega_A$ is the angular velocity of the rotation axis $A$ and $\omega_B$ is the angular velocity of the rotation axis $B$. The actual measurement of angular velocity for the rotation axis $A$ is shown in Figure 5 and the actual measurement of angular velocity for the rotation axis $B$ is shown in Figure 6. The angular acceleration of the rotation axis based on the initial planning of tool axis vector is calculated as shown in Figure 7, in which $a_A$ is the angular acceleration of the rotation axis $A$ and $a_B$ is the angular acceleration of the rotation axis $B$. 

Figure 3 Planning of tool axis vector (see online version for colours)

![Planning of tool axis vector](image)

Figure 4 Angular velocity of rotation axis based on initial planning of tool axis vector (see online version for colours)

![Angular velocity of rotation axis](image)
Figure 5  Actual measurement of angular velocity for the rotation axis $a$ (see online version for colours)

![Graph showing angular velocity for rotation axis a](image1.png)

Figure 6  Actual measurement of angular velocity for the rotation axis $B$ (see online version for colours)

![Graph showing angular velocity for rotation axis B](image2.png)

Figure 7  Angular acceleration of rotation axis based on initial planning of tool axis vector (see online version for colours)

![Graph showing angular acceleration](image3.png)
For the rotation axis $A$, it should be noticed that there exists a phase difference of $\pi$ between the computation and the measurement of the angular velocity from Figure 4 and Figure 5. Through the approximation is carried out in the computation of angular velocity for the rotation axis $A$ and $B$, the change trends of the calculated angular velocity are the same as the measured angular velocity from Figure 4, Figure 5 and Figure 6. So the calculation method of angular velocity is effective. Meanwhile, it can be obvious found that there exist large fluctuations of kinematics characteristics for the rotation axis of five-axis NC machine tool and the angular acceleration changes dramatically at the same time as shown in Figure 7. The variety of the kinematics characteristics will cause the vibration of machine tool easily, which is serious for the machining of curved surface.

Figure 8  Angular velocity of the rotation axis after smoothing (see online version for colours)

![Angular velocity graph](image)

Figure 9  Angular acceleration of the rotation axis after smoothing (see online version for colours)

![Angular acceleration graph](image)

According to the proposed control method of tool axis vector, the planning of tool axis vector is optimised based on the kinematics characteristics for the five-axis NC machining as shown in Figure 3. The angular velocity of the rotation axis after smoothing is shown in Figure 8 and the angular acceleration of the rotation axis after smoothing is
shown in Figure 9. After the smoothing for the planning of tool axis vector, the large fluctuations of kinematics characteristics for the rotation axis are avoided. As the ball end cutter is often used in the milling of curved surface, the adjustment of tool axis vector basically does not affect the machining quality of the curved surface. However, the planning of tool axis vector based on the kinematics characteristics for the five-axis NC machining can guarantee the smooth movement of the rotation axis, and the machining process for the curved surface can be more stable and reliable, meanwhile, the larger feeding speed can be chosen for machining, which can improve the machining quality and efficiency for the curved surface based on five-axis NC machine tool.

Next, another complex curved surface with rapidly varied geometric feature is selected to verify the proposed control method of tool axis vector based on kinematics characteristics for the five-axis NC machining. The curved surface is a quadric surface, and its parametric equation is \( S(u, v) = ((180u - 90)(v + 1/3), 50v, 20v - (60u - 30)^2 / 18, u \in [0, 1], v \in [0, 1] \). By using the plane \( x = 25 \) to cut the curved surface, the parametric equation of the curve for tool path can be obtained as:

\[
\mathbf{r}(\xi) = \left( \frac{5}{6}(180\xi - 90), 25, 10 - (60\xi - 30)^2 / 18 \right), \quad \xi \in [0, 1] \tag{13}
\]

The initial planning of tool axis vector based on the geometric condition is shown in Figure 10. The angular velocity of the rotation axis based on the initial planning of tool axis vector is calculated as shown in Figure 11. In Figure 11, \( \omega_A \) is the angular velocity of the rotation axis \( A \) and \( \omega_B \) is the angular velocity of the rotation axis \( B \). It can be obvious found that there exist large fluctuations of kinematics characteristics for the rotation axis of five-axis NC machine tool. According to the proposed control method of tool axis vector, the planning of tool axis vector is optimised based on the kinematics characteristics for the five-axis NC machining as shown in Figure 10. The angular velocity of the rotation axis after smoothing is shown in Figure 12. After the smoothing for the planning of tool axis vector, the large fluctuations of kinematics characteristics for the rotation axis are avoided, which also proves the availability of the proposed control method of tool axis vector.

**Figure 10** Planning of tool axis vector for another curved surface (see online version for colours)
5 Conclusions

In the five-axis NC machining of complex curved surface, the complexity of the tool path in the machining of complex curved surface is often accompanied by the large change of tool axis vector, which will introduce the large fluctuation of the rotation angle for the swing head or the turntable of the machine tool and cause larger machining error. In this study, a control method of tool axis vector based on kinematics characteristics for the five-axis NC machining of curved surface is proposed. According to the five-axis NC machine tool with specific structure, the planning of tool axis vector is optimised based on the obtained constraint conditions of kinematics characteristics. Experimental results show that the planning of tool axis vector based on the kinematics characteristics for the five-axis NC machining can guarantee the smooth movement of rotation axis and the
machining process for the curved surface can be more stable and reliable, which has important theoretical and practical value for the machining process of the complex curved surface based on five-axis NC machine tool. Meanwhile, the proposed method is more convenient and more timesaving than changing the internal control parameters of the NC machine tool.

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