New decomposition method for solving dual fully fuzzy linear systems

Ahmad Jafarian

Department of Mathematics,
Urmia Branch,
Islamic Azad University,
Urmia, Iran
Email: jafarian5594@yahoo.com

Abstract: Various methods are proposed by different authors to solve linear equation for dual fuzzy system. Also, in the dual fully fuzzy linear system (DFFLS) all parameters are considered to be fuzzy numbers. In this manuscript, we have extended symmetric and triangular (ST) decomposition to solve the DFFLS. Since triangular fuzzy numbers is a special case of trapezoidal fuzzy numbers, then on solving the DFFLS with trapezoidal fuzzy numbers is discussed. The ST decomposition method can solve these systems in a smaller computing process. The explicit scheme is given and then the proposed algorithm is illustrated with solving some numerical examples.

Keywords: dual fully fuzzy linear systems; DFFLS; trapezoidal fuzzy numbers; symmetric and triangular decomposition.

Reference to this paper should be made as follows: Jafarian, A. (2016) 'New decomposition method for solving dual fully fuzzy linear systems', *Int. J. Fuzzy Computation and Modelling*, Vol. 2, No. 1, pp.76–85.

Biographical notes: Ahmad Jafarian received his BSc (1997) in Applied Mathematics and MSc in Applied Mathematics from Islamic Azad University Lahijan Branh and Ferdowsi University of Mashhad respectively. He is an Associate Professor in the Department of Mathematics at Islamic Azad University, Urmia Branch, Oumieh, in Iran. His current interest is in artificial intelligence, solving nonlinear problem and fuzzy mathematics.

1 Introduction

One field of applied mathematics that has many applications in various areas of science is solving a system of linear equations. Systems of simultaneous linear equations play a major role in various areas such as operational research, physics, statistics, engineering and social sciences. In many applications, at least one of the system's parameters and measurements are vague or imprecise and we can present them with fuzzy numbers rather than crisp numbers. Hence, it is important to develop mathematical models and numerical procedure that would appropriately treat general fuzzy system and solve them.

Many various methods for solving fuzzy linear systems have been proposed (Karthik and Chandrasekaran, 2013; Nayak and Chakraverty, 2013; Chakraverty and Behera, 2013a, 2013b; Behera and Chakraverty, 2013; Mosleh et al., 2009; Vijayalakshmi, 2011).

Abbasbandy and Jafarian (2006) applied steepest descent method for approximating the unique solution of linear fuzzy equation system. Kumar et al. (2010) proposed a new method to find the solution of fully fuzzy linear systems with trapezoidal fuzzy numbers. A model for solving a fuzzy linear system whose coefficient matrix is fuzzy matrix and the right-hand side column is an arbitrary fuzzy vector was first proposed by Nasseri et al. (2013). Also, Wang (2013) has solved n x n fully fuzzy linear system using Uzawa method.

Recently, dual fuzzy linear systems have attracted much attention and solving these systems have been one of the interesting tasks for mathematicians. Dual fully fuzzy linear systems (DFFLS) have been studied by several authors, like Nikuie (2013), Kiasari and Dogani (2013) and Radhakrishnan and Gajivaradhan (2013), have presented new methods.

The rest of the paper is organised as follows. In Section 2, we recall some main definitions of the DFFLS, fuzzy numbers and arithmetic of trapezoidal fuzzy numbers briefly. Section 3 discusses a new decomposition of a non-singular fuzzy matrix, furthermore in order to find decomposition of a non-singular and non-symmetric matrix, a simple and fast algorithm is proposed. We discuss the model for solving the dual fuzzy linear system $\tilde{A} \otimes \tilde{x} = \tilde{B} \otimes \tilde{x} \oplus \tilde{d}$, in Section 4. In Section 5, the validity of the proposed method is examined with numerical examples. Finally in Section 6 we conclude this paper.

2 Preliminaries

2.1 *The dual fully fuzzy linear systems*

In this paper, we are interested in finding solution of dual fuzzy linear equation systems of the form

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_{11}x_1 + b_{12}x_2 + \ldots + b_{1n}x_n + d_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_{21}x_1 + b_{22}x_2 + \ldots + b_{2n}x_n + d_2 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_{n1}x_1 + b_{n2}x_2 + \ldots + b_{nn}x_n + d_n \end{cases}$$

The matrix form of the above equation is

$$\tilde{A} \otimes \tilde{x} = \tilde{B} \otimes \tilde{x} \oplus \tilde{d},$$

where the coefficient matrix $\tilde{A} = (\tilde{a}_{ij})$, $\tilde{B} = (\tilde{b}_{ij})$, $1 \le i, j \le n$ is a $n \times n$ fuzzy matrix and $\tilde{d}$ and $\tilde{x}$, $1 < i < n$ are fuzzy vectors. The system is called a DFFLS. We define $\tilde{A} = (\tilde{a}_{ij}) = (A_1, B_1, M_1, N_1)$, $\tilde{B} = (\tilde{b}_{ij}) = (A_2, B_2, M_2, N_2)$, $\tilde{x} = (x_i, y_i, z_i, w_i)$, $\tilde{d} = (b_i, g_i, h_i, k_i)$ where $M_1, M_2, N_1, N_2, A_1, A_2, x$ are positive.

2.2 *Fuzzy numbers*

In this section the basic definitions and notations used in fuzzy calculus are introduced.

Definition 1: A fuzzy number is a fuzzy set like: $\mathbb{R}^1 \to I = [0, 1]$ which satisfies:

- u is upper semi-continuous.
- $u(x) = 0$ outside some interval $[a, d]$.
- There is a real numbers b, c: $a \leq b \leq c \leq d$ for which
  - a $u(x)$ is monotonic increasing on $[a, b]$
  - b $u(x)$ is monotonic decreasing on $[c, d]$
  - c $u(x) = 1$, $b \leq x \leq c$.

A more popular equivalent alternative definition of fuzzy number is as follows.

Definition 2: A fuzzy number u is a pair $(\underline{u}(r), \overline{u}(r))$ function $\underline{u}(r)$, $\overline{u}(r)$, $0 \leq r \leq 1$. Which satisfy the following requirements:

- $\underline{u}(r)$ is a bounded monotonically increasing left continuous function
- $\overline{u}(r)$ is a bounded monotonically decreasing left continuous function
- $\underline{u}(r) \leq \overline{u}(r)$, $0 \leq \mathrm{r} \leq 1$.

The set of all these fuzzy numbers is denoted by E which is a complete metric space with Hausdorff distance. A crisp number a is simply represented by $\underline{u}(r) = \overline{u}(r) = a$, $0 \leq r \leq 1$.

Definition 3: Fuzzy number $\tilde{u}$ is said to be positive (negative) denoted by $\tilde{u} > 0$ $(\tilde{u} < 0)$ if the membership function

$$\mu_{\tilde{u}}(x) = 0, \forall x \leq 0 \ (\forall x \geq 0).$$

Definition 4: A fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \dfrac{m-x}{\alpha}, & m - \alpha \leq x \leq m, \alpha > 0, \\ 1 & m \leq x \leq n, \\ 1 - \dfrac{x-n}{\beta}, & n \leq x \leq n + \beta, \beta > 0, \\ 0 & \text{otherwise} \end{cases}$$

Definition 5: A trapezoidal fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be non-negative trapezoidal fuzzy number, i.e., $\tilde{A} \geq 0$ if and only if $m - \alpha \geq 0$.

Definition 6: Two fuzzy numbers $\tilde{A} = (m, n, \alpha, \beta)$ and $\tilde{B} = (x, y, \gamma, \delta)$ are said to be equal if and only if $m = p$, $n = q$, $\alpha = \gamma$ and $\beta = \delta$.

Definition 7: A matrix $\tilde{A} = (\tilde{a}_{ij})$ is called a fuzzy matrix, if each element of $\tilde{A}$ is a fuzzy number. $\tilde{A}$ will be positive (negative) and denoted by $\tilde{A} > 0$ $(\tilde{A} < 0)$ if each element of

$\tilde{A}$ be positive (negative). $\tilde{A}$ will be non-positive (non-negative) and denoted by $\tilde{A} \le 0$ $(\tilde{A} \ge 0)$ if each element of $\tilde{A}$ be non-positive (non-negative).

2.3 Arithmetic of trapezoidal fuzzy numbers

In this subsection addition and subtraction and multiplication and scalar multiplication operations between two trapezoidal fuzzy numbers are reviewed (Dubois and Prade, 1978).

- Addition

  $$A + B = (m, n, \alpha, \beta) + (x, y, \gamma, \delta) = (m + x, n + y, \alpha + \gamma, \beta + \delta)$$

- Subtraction

  $$A - B = (m, n, \alpha, \beta) - (x, y, \gamma, \delta) = (m - x, n - y, \alpha - \delta, \beta - \gamma)$$

- Scalar multiplication

  $$\lambda A = \begin{cases} (\lambda m, \lambda n, \lambda \alpha, \lambda \beta) & \lambda \ge 0 \\ (\lambda m, \lambda n, -\lambda \beta, -\lambda \alpha) & \lambda < 0 \end{cases}$$

- Multiplication

  For $A \ge 0$, $B \ge 0$

  $$A \cdot B = (m, n, \alpha, \beta) \cdot (x, y, \gamma, \delta) = (mx, ny, m\gamma + x\alpha, n\delta + y\beta).$$

3 ST decomposition method

In this section the main idea of the ST decomposition is reviewed.

Definition 3.1: In linear algebra, a symmetric matrix is a square matrix that is equal to its transpose. Formally, matrix A is symmetric if $A = A^T$.

Definition 3.2: If the entries on the main diagonal of a triangular matrix are all 1, the matrix is called unit triangular.

Theorem 3.1: For every non-singular and non-symmetric matrix A, whose leading principal sub matrices are non-singular, there exists a decomposition $A = ST$ where S is symmetric matrix and T is unit triangular matrix (Golub and Yun, 2002).

Corollary 3.1: In order to find decomposition of a non-singular and non-symmetric matrix, a simple and fast algorithm is proposed. Properties of the matrices imply

$$A = \begin{pmatrix} A_k & a_{k+1} \\ \tilde{a}_{k+1}^T & \alpha \end{pmatrix}, S = \begin{pmatrix} s_k & s_{k+1} \\ s_{k+1}^T & \mathcal{T} \end{pmatrix} \text{and } T = \begin{pmatrix} T_k & t_{k+1} \\ 0 & 1 \end{pmatrix},$$

where $\alpha \ne 0$ and $\mathcal{T} \ne 0$, S_k is symmetric, T_k is upper triangular and A_k is non-singular. It follows from $A = ST$ that

$$A = ST \Rightarrow \begin{pmatrix} A_k & a_{k+1} \\ \tilde{a}_{k+1}^T & \alpha \end{pmatrix} = \begin{pmatrix} s_k & s_{k+1} \\ s_{k+1}^T & \mathcal{T} \end{pmatrix} \begin{pmatrix} T_k & t_{k+1} \\ 0 & 1 \end{pmatrix}$$

$$A_k = S_k T_k,$$

$$s_{k+1} = T_k^{-T} \tilde{a}_{k+1},$$

$$t_{k+1} = S_k^{-1} \left(a_{k+1} - s_{k+1} \right),$$

$$\mathcal{T} = \alpha - s_{k+1}^T t_{k+1}.$$

The above relations lead to the following algorithm

Algorithm: Set t_{11} such that $s_{11}t_{11} \neq 0$ and $s_{11}t_{11} = a_{11}$

For $k = 1 \ldots N - 1$

$$s_{k+1} = T_k^{-T} \tilde{a}_{k+1},$$

$$\tilde{s}_{k+1} = s_k^{-1} a_{k+1},$$

$$t_{k+1} = \tilde{s}_{k+1} - S_k^{-1} s_{k+1},$$

$$s_{k+1,k+1} = a_{k+1,k+1} - s_{k+1}^T t_{k+1}.$$

End

4 Analysis of the method for DFFLS

We are interested in finding solution of fuzzy linear equation systems of the form

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_{11}x_1 + b_{12}x_2 + \ldots + b_{1n}x_n + d_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_{21}x_1 + b_{22}x_2 + \ldots + b_{2n}x_n + d_2 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_{n1}x_1 + b_{n2}x_2 + \ldots + b_{nn}x_n + d_n \end{cases}$$

The matrix form of the above equation is

$$\tilde{A} \otimes \tilde{x} = \tilde{B} \otimes \tilde{x} \oplus \tilde{d}.$$

where the coefficient matrix $\tilde{A} = (\tilde{a}_{ij})$, $\tilde{B} = (\tilde{b}_{ij})$, $1 \le i, j \le n$ is a $n \times n$ fuzzy matrix and $\tilde{d}$ and $\tilde{x}$, $1 < i < n$ are fuzzy vectors. The system is called a DFFLS. We define $\tilde{A} = (\tilde{a}_{ij}) = (A_1, B_1, M_1, N_1)$, $\tilde{B} = (\tilde{b}_{ij}) = (A_2, B_2, M_2, N_2)$, $\tilde{x} = (x_i, y_i, z_i, w_i)$, $\tilde{d} = (b_i, g_i, h_i, k_i)$ where M_1, M_2, N_1, N_2, A_1, A_2, x are positive. Now we will solve

$$\tilde{A} \otimes \tilde{x} = \tilde{B} \otimes \tilde{x} \oplus \tilde{d}.$$

$$\left(A_1, B_1, M_1, N_1 \right) \otimes (x, y, z, w) = \left(A_2, B_2, M_2, N_2 \right) \otimes (x, y, z, w) \oplus (b, g, h, k)$$

$$\left(A_1 x, B_1 y, M_1 x + A_1 z, B_1 w + N_1 y \right) = \left(A_2 x, B_2 y, M_2 x + A_2 z, B_2 w + N_{2y} \right) \oplus (b, g, h, k)$$

$$\left(A_1x,\, B_1y,\, M_1x + A_1z,\, B_1w + N_1y\right) = \left(A_2x + b,\, B_2y + g,\, M_2x + A_2z + h,\, B_2w + N_{2y} + k\right)$$

$$\begin{cases} A_1x = A_2x + b \\ B_1y = B_2y + g \\ M_1x + A_1z = M_2x + A_2z + h \\ B_1w + N_1y = B_2w + N_2y + k \end{cases}$$

$$\begin{cases} (A_1 - A_2)x = b, \\ (B_1 - B_2)y = g, \\ (M_1 - M_2)x + (A_1 - A_2)z = h, \\ (B_1 - B_2)w + (N_1 - N_2)y = k. \end{cases}$$

We easily have

$$\begin{cases} x = (A_1 - A_2)^{-1} b, \\ y = (B_1 - B_2)^{-1} g, \\ z = (A_1 - A_2)^{-1}\left(h - (M_1 - M_2)x\right), \\ y = (B_1 - B_2)^{-1}\left(k - (N_1 - N_2)y\right). \end{cases}$$

We define $M = M_1 - M_2$, $N = N_1 - N_2$, $A = A_1 - A_2$, $B = B_1 - B_2$ where M, N, A, B are positive. Thus the original problem is transformed to finding a vector $\tilde{x}$ which satisfies in the following systems

$$\begin{cases} x = A^{-1}b, \\ y = B^{-1}g, \\ z = A^{-1}(h - Mx), \\ w = B^{-1}(k - Ny). \end{cases} \tag{1}$$

By replacing $A = S_1T_1$, $B = S_2T_2$ we get

$$\begin{cases} x = T_1^{-1}S_1^{-1}b, \\ y = T_2^{-1}S_2^{-1}g, \\ z = T_1^{-1}S_1^{-1}\left(h - MT_1^{-1}S_1^{-1}b\right), \\ w = T_2^{-1}S_2^{-1}\left(k - NT_2^{-1}S_2^{-1}g\right). \end{cases} \tag{2}$$

5 Numerical examples

To illustrate the technique proposed in this paper, consider the following examples.

Example 5.1: Consider the following DFFLS of trapezoidal fuzzy numbers and solve it by proposed method

$$\begin{cases}(6,12,4,5)\otimes\tilde{x}_1\oplus(9,9,5,4)\otimes\tilde{x}_2=(3,6,2,3)\otimes\tilde{x}_1\oplus(5,3,4,2)\otimes\tilde{x}_2\oplus(27,66,26,58),\\(12,6,7,6)\otimes\tilde{x}_1\oplus(8,10,2,3)\otimes\tilde{x}_2=(8,1,6,5)\otimes\tilde{x}_1\oplus(3,2,1,1)\otimes\tilde{x}_2\oplus(35,70,25,55).\end{cases}$$

Thus by before section, we have

$$A_1=\begin{pmatrix}6&9\\12&8\end{pmatrix},\ A_2=\begin{pmatrix}3&5\\8&3\end{pmatrix},\ B_1=\begin{pmatrix}12&9\\6&10\end{pmatrix},\ B_2=\begin{pmatrix}6&3\\1&2\end{pmatrix},$$

$$M_1=\begin{pmatrix}4&5\\7&2\end{pmatrix},\ M_2=\begin{pmatrix}2&4\\6&1\end{pmatrix},\ N_1=\begin{pmatrix}5&4\\6&3\end{pmatrix},\ N_2=\begin{pmatrix}3&2\\5&1\end{pmatrix},$$

$$b=\begin{pmatrix}27\\35\end{pmatrix},\ g=\begin{pmatrix}66\\70\end{pmatrix},\ h=\begin{pmatrix}26\\55\end{pmatrix},\ k=\begin{pmatrix}58\\55\end{pmatrix},$$

$$A=A_1-A_2=\begin{pmatrix}3&4\\4&5\end{pmatrix},\ B=B_1-B_2=\begin{pmatrix}6&6\\5&8\end{pmatrix},$$

$$M=M_1-M_2=\begin{pmatrix}2&1\\1&1\end{pmatrix},\ N=N_1-N_2=\begin{pmatrix}2&2\\1&2\end{pmatrix}.$$

First we obtain ST decomposition for matrices A and B

$$A=\begin{pmatrix}3&4\\4&5\end{pmatrix}=\begin{pmatrix}3&4\\4&5\end{pmatrix}\begin{pmatrix}1&0\\0&1\end{pmatrix}=S_1T_1$$

$$B=\begin{pmatrix}6&6\\5&8\end{pmatrix}=\begin{pmatrix}6&5\\5&43/6\end{pmatrix}\begin{pmatrix}1&1/6\\0&1\end{pmatrix}=S_2T_2$$

Thus, by equation (2) we obtain

$$x=T_1^{-1}S_1^{-1}b=\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}-5&4\\4&-3\end{pmatrix}\begin{pmatrix}27\\35\end{pmatrix}=\begin{pmatrix}5\\3\end{pmatrix}$$

$$y=T_2^{-1}S_2^{-1}g=\begin{pmatrix}1&-1/6\\0&1\end{pmatrix}\begin{pmatrix}48/108&-5/18\\-5/18&1/3\end{pmatrix}\begin{pmatrix}66\\70\end{pmatrix}=\begin{pmatrix}6\\5\end{pmatrix}$$

$$z=T_1^{-1}S_1^{-1}\left(h-MT_1^{-1}S_1^{-1}b\right)$$
$$=\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}-5&4\\4&-3\end{pmatrix}\left(\begin{pmatrix}26\\25\end{pmatrix}-\begin{pmatrix}2&1\\1&1\end{pmatrix}\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}-5&4\\4&-3\end{pmatrix}\begin{pmatrix}27\\35\end{pmatrix}\right)=\begin{pmatrix}3\\1\end{pmatrix}$$

$$w=T_2^{-1}S_2^{-1}\left(k-NT_2^{-1}S_2^{-1}g\right)$$
$$=\begin{pmatrix}1&-1/6\\0&1\end{pmatrix}\begin{pmatrix}48/108&-5/18\\-5/18&1/3\end{pmatrix}\left(\begin{pmatrix}58\\55\end{pmatrix}-\begin{pmatrix}2&2\\1&2\end{pmatrix}\begin{pmatrix}1&-1/6\\0&1\end{pmatrix}\begin{pmatrix}48/108&-5/18\\-5/18&1/3\end{pmatrix}\begin{pmatrix}66\\70\end{pmatrix}\right)$$
$$=\begin{pmatrix}3\\3\end{pmatrix}$$

Hence we have the fuzzy solution $\tilde{x}_1=(5,6,3,3)$ and $\tilde{x}_2=(3,5,1,3)$. To check the validity of the solution $\tilde{x}_1=(5,6,3,3)$ and $\tilde{x}_2=(3,5,1,3)$ for upper DFFLS, we substitute $\tilde{x}_1$ and $\tilde{x}_2$ into the original system by arithmetic of trapezoidal fuzzy numbers Section 2.3.

We have

- The left hand side of the first equation as

$$(6, 12, 4, 5) \otimes (5, 6, 3, 3) \oplus (9, 9, 5, 4) \otimes (3, 5, 1, 3) = (57, 117, 62, 113).$$

- The right hand side of the first equation as

$$(3, 6, 2, 3) \otimes (5, 6, 3, 3) \oplus (5, 3, 4, 2) \otimes (3, 5, 1, 3) \oplus (27, 66, 26, 58) = (57, 117, 62, 113).$$

- The left hand side of the second equation as

$$(8, 6, 7, 6) \otimes (5, 6, 3, 3) \oplus (6, 10, 2, 3) \otimes (3, 5, 1, 3) = (58, 86, 71, 99).$$

- The right hand side of the second equation as

$$(4, 1, 6, 5) \otimes (5, 6, 3, 3) \oplus (1, 2, 1, 1) \otimes (3, 5, 1, 3) \oplus (35, 70, 25, 55) = (58, 86, 71, 99).$$

So we see that the fuzzy solution of proposed DFFLS is true.

Example 5.2: Consider the following DFFLS of trapezoidal fuzzy numbers and solve it by proposed method

$$\begin{cases} (6, 9, 5, 6) \otimes \tilde{x}_1 \oplus (7, 8, 3, 3) \otimes \tilde{x}_2 = (3, 3, 3, 4) \otimes \tilde{x}_1 \oplus (3, 2, 2, 1) \otimes \tilde{x}_2 \oplus (27, 66, 54, 70), \\ (9, 4, 7, 8) \otimes \tilde{x}_1 \oplus (14, 6, 9, 5) \otimes \tilde{x}_2 = (8, 2, 1, 6) \otimes \tilde{x}_1 \oplus (10, 1, 7, 1) \otimes \tilde{x}_2 \oplus (17, 37, 58, 55). \end{cases}$$

Thus by before section, we have

$$A_1 = \begin{pmatrix} 6 & 7 \\ 9 & 14 \end{pmatrix}, A_2 = \begin{pmatrix} 3 & 3 \\ 8 & 10 \end{pmatrix}, B_1 = \begin{pmatrix} 9 & 8 \\ 4 & 6 \end{pmatrix}, B_2 = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix},$$

$$M_1 = \begin{pmatrix} 5 & 3 \\ 7 & 9 \end{pmatrix}, M_2 = \begin{pmatrix} 3 & 2 \\ 1 & 7 \end{pmatrix}, N_1 = \begin{pmatrix} 6 & 3 \\ 8 & 5 \end{pmatrix}, N_2 = \begin{pmatrix} 4 & 1 \\ 6 & 1 \end{pmatrix},$$

$$b = \begin{pmatrix} 66 \\ 37 \end{pmatrix}, g = \begin{pmatrix} 66 \\ 37 \end{pmatrix}, h = \begin{pmatrix} 54 \\ 58 \end{pmatrix}, k = \begin{pmatrix} 70 \\ 55 \end{pmatrix},$$

$$A = A_1 - A_2 = \begin{pmatrix} 3 & 4 \\ 1 & 4 \end{pmatrix}, B = B_1 - B_2 = \begin{pmatrix} 6 & 6 \\ 2 & 5 \end{pmatrix},$$

$$M = M_1 - M_2 = \begin{pmatrix} 2 & 1 \\ 6 & 2 \end{pmatrix}, N = N_1 - N_2 = \begin{pmatrix} 2 & 2 \\ 2 & 4 \end{pmatrix}.$$

First we obtain ST decomposition for matrices A and B

$$A = \begin{pmatrix} 3 & 4 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = S_1 T_1$$

$$B = \begin{pmatrix} 6 & 6 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 2 & 11/3 \end{pmatrix} \begin{pmatrix} 1 & 2/3 \\ 0 & 1 \end{pmatrix} = S_2 T_2$$

Thus, by equation (2) we obtain

$$x = T_1^{-1} S_1^{-1} b = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -3/8 & 1/8 \\ -1/8 & -3/8 \end{pmatrix} \begin{pmatrix} 27 \\ 17 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$y = T_2^{-1} S_2^{-1} g = \begin{pmatrix} 1 & -2/3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 11/54 & -1/9 \\ -1/9 & 1/3 \end{pmatrix} \begin{pmatrix} 66 \\ 37 \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$$

$$\begin{aligned} z &= T_1^{-1} S_1^{-1} \left(h - M T_1^{-1} S_1^{-1} b \right) \\ &= \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -3/8 & 1/8 \\ -1/8 & -3/8 \end{pmatrix} \left(\begin{pmatrix} 54 \\ 58 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ 6 & 2 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -3/8 & 1/8 \\ -1/8 & -3/8 \end{pmatrix} \begin{pmatrix} 27 \\ 17 \end{pmatrix} \right) = \begin{pmatrix} 19/2 \\ 25/8 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} w &= T_2^{-1} S_2^{-1} \left(k - N T_2^{-1} S_2^{-1} g \right) \\ &= \begin{pmatrix} 1 & -2/3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 11/54 & -1/9 \\ -1/9 & 1/3 \end{pmatrix} \left(\begin{pmatrix} 70 \\ 55 \end{pmatrix} - \begin{pmatrix} 2 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & -2/3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 11/54 & -1/9 \\ -1/9 & 1/3 \end{pmatrix} \begin{pmatrix} 66 \\ 37 \end{pmatrix} \right) \\ &= \begin{pmatrix} 17/3 \\ 7/3 \end{pmatrix} \end{aligned}$$

Hence we have the fuzzy solution $\tilde{x}_1 = (5, 6, 19/2, 17/3)$ and $x_2 = (3, 5, 25/8, 7/3)$.

6 Conclusions

Fuzzy number arithmetic is widely applied and useful in computation of linear system whose parameters are all or partially represented by fuzzy numbers. We offered a proper method for solving DFFLS of the form $\tilde{A} \otimes \tilde{x} = \tilde{B} \otimes \tilde{x} \oplus \tilde{d}$ in the case of trapezoidal fuzzy numbers. We conclude that any dual fuzzy linear system of equations in the form of trapezoidal fuzzy matrices can be decomposed into the form such that $A = ST$ where S is symmetric matrix and T is unit triangular matrix. Furthermore, the result of proposed method is more precise and efficient to implement for solving DFFLS.

References

Abbasbandy, S. and Jafarian, A. (2006) ‘Steepest descent method for system of fuzzy linear equations’, *Applied Mathematics and Computation*, Vol. 175, No. 1, pp.823–833.

Behera, D. and Chakraverty, S. (2013) ‘Solution method for fuzzy system of linear equations with crisp coefficients’, *Fuzzy Information and Engineering*, Vol. 5, No. 2, pp.205–219.

Chakraverty, S. and Behera, D. (2013a) ‘Fuzzy centre based solution of fuzzy complex linear system of equations’, *International Journal of Uncertainty Fuzziness and Knowledge-Based Systems*, Vol. 21, No. 4, pp.629–642.

Chakraverty, S. and Behera, D. (2013b) ‘Fuzzy system of linear equations with crisp coefficients’, *Journal of Intelligent and Fuzzy Systems*, Vol. 25, No. 2, pp.201–207.

Dubois, D. and Prade, H. (1978) ‘Operation on fuzzy numbers’, *International Journal of Systems Science*, Vol. 9, No. 6, pp.613–626.

Golub, G.H. and Yun, J. (2002) ‘Symmetric-triangular decomposition and its applications’, *Swets & Zeilinger*, Vol. 42, No. 4, pp.814–822.

Karthik, N.J. and Chandrasekaran, E. (2013) 'Solving fully fuzzy linear systems with trapezoidal fuzzy number matrices by singular value decomposition', *Intern. J. Fuzzy Mathematical Archive*, Vol. 3, No. 5, pp.16–22.

Kiasari, S.M.K. and Dogani, M.H. (2013) 'Solution of fuzzy dual linear system with trapezoidal variable', *World Applied Sciences Journal*, Vol. 25, No. 3, pp.1373–1377.

Kumar, A., Neetu, B. and Bansal, A. (2010) 'A new method to solve fully fuzzy linear system with trapezoidal fuzzy numbers', *Canadian Journal Science Engineering Mathematics*, Vol. 1, No. 12, pp.45–56.

Mosleh, M., Otadi, M. and Khanmirzaie, A. (2009) 'Decomposition method for solving fully fuzzy linear systems', *Iranian Journal of Optimization*, Vol. 1, No. 1, pp.188–198.

Nasseri, S.H., Behmanesh, E. and Sohrabi, M. (2013) 'A new method for system of fully fuzzy linear equations based on a certain decomposition of its coefficient matrix', *Annals of Fuzzy Mathematics and Informatics*, Vol. 6, No. 1, pp.135–140.

Nayak, S. and Chakraverty, S. (2013) 'A new approach to solve fuzzy system of linear equations', *Journal of Mathematics and Computer Science*, Vol. 7, No. 3, pp.205–212.

Nikuie, M. (2013) 'The effect of Drazin inverse in solving singular dual fuzzy linear systems', *Mathematical Sciences and Applications E-Notes*, Vol. 1, No. 5, pp.150–157.

Radhakrishnan, S. and Gajivaradhan, P. (2013) 'A new method for solving dual fully fuzzy linear system', *International Journal of Scientific Research in Knowledge (IJSRK)*, Vol. 1, No. 5, pp.100–105.

Vijayalakshmi, M. (2011) 'ST decomposition method for solving fully fuzzy linear systems using Gauss Jordan for trapezoidal fuzzy matrices', *International Mathematical Forum*, Vol. 6, No. 45, pp.2245–2254.

Wang, K. (2013) 'Uzawa method for fuzzy linear system', *Journal of Fuzzy Set Valued Analysis*, Vol. 2013, pp.1–7.