Investment decision in oil and gas projects using real option and risk tolerance models

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Abstract: This paper presents a model for valuation and decision making, integrating discounted cash flow, real-options pricing and preference theory, aiming to cover the following questions: i) what is the current value of a project?; ii) what is the optimal investment rule?; iii) what is the optimal working interest? The traditional model suggests that, when the project value is above its investment cost, the corporation should invest immediately and incur in 100% working interest. The real option pricing suggests that the corporation should only invest if the project’s current value is at least 1.85 times investment cost. The preference theory suggests funding only 44.38% working interest, and partners must acquire the remaining 55.62%. These tools must be integrated in order to allow a more realistic treatment of risk. In general, when the uncertainty (volatility) of cash flow components increases, the two models give more divergent results.

Keywords: uncertainty; real options; capital budgeting; preference theory; petroleum exploration; production.

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1 Introduction

The valuation and decision making of capital-intensive projects have been a topic of concern for large corporations and governments in the oil and gas business, in part because the resource is finite and investment is high – for example, irreversible investments in the development phase may reach billions of dollars. Several studies concerning modern investments theory, such as those in the oil and gas industry, have three important characteristics according to Dixit and Pindyck (1994):
1. uncertainty over the future operational cash flow
2. irreversibility of the investment
3. value of the timing or some leeway to implement decisions.

The irreversibility of investment gives rise to implications in the financial status of corporations, depending on capital exposed to risk and the magnitude of budget of the company. For example, consider an investment of US$ 100 million in a risky oil project. For a company with a budget as high as US$ 1 billion, this risky investment might be tolerated. On the other hand, for a firm with a budget of, say, US$ 300 million, managers might postpone this risky project or sell a share of this project, since the potential loss is too high compared to its budget. Since unsuccessful results of an investment may give rise to serious financial impacts on the corporation, the irreversibility of investment is one of the main driving forces of investor’s risk aversion.

The irreversibility of most part of the investment is a frequent characteristic of the capital intensive industries. For example, in case of a crisis in the oil industry, equipments such as rigs, platforms, infrastructure, etc. will have a large reduction of value in a secondary market. Even equipment of more general applications will experience a value reduction due to a decrease of the oil-related assets.

In order to manage a project under a scenario of future uncertainty, coupled with investment irreversibility, the manager needs managerial flexibilities (real options) to adapt the project to new market conditions. This is clearly stated in Trigeorgis (1996) that without this managerial proactive role, uncertainties will tend to generate a symmetric distribution for Net Present Value (NPV), whereas, with active management, the NPV will have an asymmetric distribution to the right. This positive asymmetry in the NPV generated by managerial flexibility has value and can be properly estimated using option pricing methods.

If the management has the option to choose the time to invest, this flexibility has value and must be considered in the model of valuation and decision making. Since the option to invest is alive until the time the decision is implemented, if the corporation invests today, it kills the opportunity of investing in the future, when the market conditions, technology, as well as other market components may be better. Then, by investing today, the corporation incurs in an opportunity cost by not waiting to allocate this capital in the future. In addition, because the corporation invests in a
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capital-intensive project, there is a long period of time (which may reach 20 years or more) of cash-flow exposure to risk, so that firms may prefer to participate with less than 100% in the project. Therefore, in the process of investment valuation and decision making, corporations must consider, in an integrated way, the process of valuation, strategic decision making and tolerance to risk exposure. Three questions are of paramount importance:

1. What is the current cash flow value of the investment?
2. What is the optimal decision rule to invest, that is, should the corporation invest now or in the future?
3. What is the optimal working interest for the project?

The solutions to these questions have been achieved using the traditional model of valuation and decision making, based on the NPV of the project’s cash flows, that is, invest as long as NPV is positive and incur in 100% of working interest.

But, contrarily, McDonald and Siegel (1986), Dixit and Pindyck (1994), Trigeorgis (1996) and Copeland and Antikarov (2001) pointed out that results from the traditional model do not properly account for the role of uncertainty, irreversibility and managerial flexibilities. Alternatively, they suggest the use of option-pricing techniques.

In addition, real-world practice shows that most large projects are financed by a pool of companies, what is different from suggestions from the NPV approach. A model to find the optimal working interest (W) is suggested by Cozzolino (1980), Walls (1995), Walls and Dyer (1996), Lerche and Mackay (1996) and Wilkerson (1998) for petroleum exploration and production projects.

But, these two approaches have been applied in a separate way. Costa Lima (2004) suggests the use of an integrated model considering Monte Carlo simulation, option-pricing and optimal working-interest to give decision makers more realistic information to make their choices. The risk of project is estimated by considering uncertainty in future cash flow inflow and outflow through Monte Carlo simulation. The strategic decision rule is achieved according to the option-pricing model. The optimal working interest in joint venture projects is found according to the optimal working interest from concepts of preference theory.

This paper is structured in three sections. Section 1 presents some details on the integrated methodology for valuation and strategic decision making. Section 2 applies the model to decision making of capital-intensive project for heavy-oil deep-water production. Section 3 presents some discussions and implications.

2 The integrated model

2.1 NPV of the project’s cash flow

The first step in the process of investment decision making is the estimation of indicators from the project’s future cash flows. Traditionally, the NPV is the main indicator for investment valuation and decision rule, since it has been recognised in financial literature (Brealey and Myers, 1992) as theoretically correct to measure the creation of value for
stakeholders. If a project generates over time a stream of operational cash flow \((X(t))\) and requires a stream of investments \((I(t))\), its NPV is (Equation (1)):

\[
NPV = \sum_{t=0}^{N} \frac{E[X_t - I_t]}{(1 + \mu)^t}
\]

where \(\mu\) is capital cost of each cash flow. Equation (1) provides the expected value of the NPV under static scenarios for price, cost, production, etc. Since there is uncertainty about the future, the NPV may change as long as fluctuations in price, cost, production rate, etc. take place. Then, NPV is a random variable whose future values will come from a probabilistic distribution. Equation (1) gives the expected value of the NPV, but this is not sufficient to make a decision since this indicator does not account for the impact of uncertainty on future cash flows, except via risk premium of the discount rate. In order to estimate the risk of the project, a Monte Carlo simulation of the NPV may be carried out.

The traditional decision-making model according to the NPV only considers the risk of a project by means of a premium in the discount rate – the riskier the project, the lower the NPV. This model has three main drawbacks:

1. the potential gains from the positive side of uncertainty are not taken into account
2. the value of waiting to invest in the future if current market conditions are bad is ignored
3. the optimal working interest is always 100%, irrespective of magnitude of investment.

These issues will be discussed over the next sections.

2.2 Optimal working interest \((W)\)

In order to discuss the problem of optimal working interest in a risky project, consider an example involving the allocation of US$ 600 million between two alternative investments:

1. Invest all of the money in one large project with an expected NPV of US$ 300 million and a standard-deviation (risk) of US$ 141.42 million.
2. Invest all of the money in five projects, that is, the corporation allocates 20% of its budget to each project. In this case, the corporation’s rate of return will have the following return/risk profile: \(E[\text{NPV}] = \text{US$ 300 million and } \sigma[\text{NPV}] = \text{US$ 63.25 million.}\)

These two alternatives have the same expected return, but different risk levels. Just by increasing the number of non-correlated projects, the return of portfolio remains the same and its global risk drops – this is the remarkable role of diversification. In Table 1, the effects of diversification through different working interest values on risk and return of portfolios are shown.
The choice of working interest depends on the investor’s risk tolerance, since all portfolios give the same return of US$ 300 million. The investor’s risk tolerance will depend on the amount exposed to risk compared to the investor’s total stock of wealth and his risk preference characteristics.

The theoretical foundations of decision making involving choices under uncertainty (such as selecting working interest in a project of high CAPEX) is the preference theory developed by Von Neumann and Morgenstern (1953) which advocates that the usefulness of things determines their attractiveness. Basically, choices under uncertainty or risk attitudes of decision makers are organised into three main groups:

1. risk-neutral
2. risk-averse
3. risk-prone.

According to Luenberger (1998), the risk-neutral individual is one to whom uncertain outcomes are valued as expected values, paying no attention to the potential of gains and losses. A risk-prone individual is one to whom uncertain outcome is valued by more than the expected value. A risk-averse investor has more concern with the potential of loss than that of gains and, therefore, this individual values uncertain outcome by less than expected value.

In this paper, to find the optimal working interest in this model, it is assumed that individuals as well as corporations are risk-averse in capital-intensive projects. This is the case of large heavy-oil projects, where investment irreversibility contributes to motivate management towards limiting risk exposure by taking, for example, less than 100% working interest in the project, depending on the interaction among project risk profile, magnitude of the corporation’s financial budget and decision maker’s risk attitudes (Campbell et al., 2001; Cozzolino, 1980; Nepomuceno et al., 1999; Newendorp and Schuyler, 2000; Walls and Dyer, 1996). For the petroleum industry, Cozzolino (1980) and Walls (1995) suggest the use of the exponential utility function to model the choices made under uncertainty. The complete exponential utility equation is:

$$U(x) = a - b \times e^{-x/T}$$

where $a$ and $b$ are constants, $X$ is the random monetary quantities and $T$ is the corporation’s risk tolerance. The absolute value of utility (positive or negative) is just a number without much significance and is not enough for taking decisions. On the other hand, utility values are suitable for comparisons – for example, if $U(X) = 11$ and $U(Y) = 8$, then $X$ is preferable to $Y$.  

**Table 1** Effects of diversification on portfolio risk and return level

<table>
<thead>
<tr>
<th>Number of projects</th>
<th>Working interest (W)</th>
<th>Portfolio risk (US$ × 10^6)</th>
<th>Portfolio return (US$ × 10^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100.00%</td>
<td>141.42</td>
<td>300</td>
</tr>
<tr>
<td>2</td>
<td>50.00%</td>
<td>100.00</td>
<td>300</td>
</tr>
<tr>
<td>–</td>
<td></td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>–</td>
<td></td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>10</td>
<td>10.00%</td>
<td>44.72</td>
<td>300</td>
</tr>
</tbody>
</table>

The choice of working interest depends on the investor’s risk tolerance, since all portfolios give the same return of US$ 300 million. The investor’s risk tolerance will depend on the amount exposed to risk compared to the investor’s total stock of wealth and his risk preference characteristics.
On the other hand, the certainty-equivalent or Risk-Adjusted Value (RAV) is the main concept to estimate the optimal level of financial participation or working interest (W). According to Luenberger (1998), RAV is that value whose utility is equal to the expected value of utility of K possible monetary outcomes (Xi) with probability pi. From Equation (2), if a = 0 and b = 1, the RAV is found from:

\[-e^{-\left(\frac{\text{RAV}}{T}\right)} = \sum_{i=1}^{K} p_i \left(-e^{-\left(W \times X_i / T\right)}\right)\]  

(3)

where T is the corporation’s risk tolerance. From Equation (3), we get:

\[\text{RAV} = -T \times \ln \left(\sum_{i=1}^{K} p_i \times e^{-\left((W \times X_i) / T\right)}\right)\]  

(4)

The model to estimate the optimal working interest is given in Equation (4), that is, the decision-maker should select W that maximise the RAV. In practice, W is found numerically. The use of Equation (4) requires two inputs:

1. corporate risk tolerance (T)
2. probability distribution of Xi – for example, the NPV of the project.

2.3 The optimal investment decision-rule

The optimal investment decision rule must take into account not only the expected future cash flows but also the strategic or operational options that are available for management over the life of a project. As in the case of financial options, these real options are understood as a right, but not as an obligation to be implemented and have value to be added to the traditional NPV. As described by Trigeorgis (1986), the true project value is always equal to or greater than its traditional NPV.

Most modern investments in exploration and production share the following characteristics:

1. future uncertainty in variables such as cost, price, exchange rate, etc.
2. irreversibility of investment, leading to new considerations in the process of the project’s acceptance and management of risk exposure
3. timing or strategic real option to implement decisions.

Dixit and Pindyck (1994) consider that such investments should be analysed using the flexible approach of real-option pricing. Costa Lima and Suslick (2002) pointed out that the interaction of uncertainty and timing may aggregate value to the project, in part because the investor’s loss is limited to the irreversible investment, whereas the upside potential is theoretically infinite.

The first step in valuation and decision-rule from the option-pricing model is the modelling of the dynamics of the asset’s future price. Following Black and Scholes (1973), we assume that the changes in the present value of future cash flows evolve as a risk-neutral geometric Brownian motion:

\[dV = (r - \delta) V dt + \sigma V dZ\]  

(5)
where $V$ is the present value of project’s cash flow, $r$ is the risk-free interest rate, $\delta$ is the dividend rate, $\sigma$ is the volatility and $dZ$ is Wiener’s increment or white noise.

In order to find the optimal investment decision-rule, let $F(V)$ be the value of the option to invest in an oil and gas project. If we assume that option maturity is at least four years, the investment option can be regarded as independent of time. Then, by following standard procedures in financial economics, McDonald and Siegel (1986) and Dixit and Pindyck (1994) showed that, under the assumption of market efficiency and non-arbitrage opportunities, $F(V)$ must satisfy the following ordinary differential equation (PDE)

$$1/2\sigma^2 V^2 F''(V) + (r - \delta)VF'(V) - rF(V) = 0$$

(6)

In order to use Equation (6), the following boundary conditions are considered:

$$F(0) = 0$$

(7)

$$\text{NPV}(V^*) = F(V^*) = V^* - I$$

(8)

$$F'(V) = 1$$

(9)

Equation (7) means that if $V = 0$, $F(0) = 0$, that is, there is no chance of increase in $V$ in the future, which is considered an absorbing property of the GBM. Equation (8) means that the corporation should invest as long as $V$ reaches a trigger value ($V^*$) and not merely $V \geq I$. Equation (9) is the smooth-pasting condition or the last boundary condition for optimisation (see Dixit and Pindyck, 1994, Chapter 4).

The solution of $F(V)$ is:

$$\beta = \frac{1}{2} \left( \frac{r - \delta}{\sigma^2} + \left[ \left( \frac{r - \delta}{\sigma^2} - \frac{1}{2} \right)^2 + 2r \right]^{1/2} \right)$$

(10)

$$V^* = \frac{\beta}{(\beta - 1)} I$$

(11)

$$F(V) = \begin{cases} 
V^* - I & \text{if } V < V^* \\
\left( \frac{V^*}{V^*} \right)^{\beta} & \text{if } V \geq V^* 
\end{cases}$$

(12)

$$F(V) = \text{NPV}(V) = V - I \quad \text{if } V \geq V^*$$

(13)

where $\beta$ is positive root of a quadratic equation derived from the ordinary differential Equation (6), $I$ is the present value of investment cost and $V^*$ is the trigger value, that is, the minimal project value to invest immediately. Equation (12) is the value of the investment option if $V < V^*$ and Equation (13) gives the value of investment option if $V \geq V^*$, that is, Equation (13) has the same value as the traditional NPV. In order to use the set of Equations (10)–(13), as analogous to the case of financial options, the real-options analyst needs those five input parameters of Table 2.
Table 2  Analogy of the determinants of financial and real-option pricing models

<table>
<thead>
<tr>
<th>Financial options</th>
<th>Real options</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Underlying asset</td>
<td>Present value of project’s cash flow ($)</td>
<td>$V$</td>
</tr>
<tr>
<td>Exercise price</td>
<td>Present value of investment cost ($)</td>
<td>$I$</td>
</tr>
<tr>
<td>Financial asset’s future volatility</td>
<td>Future volatility of project’s cash flow (%)</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>Financial asset’s dividend rate</td>
<td>Future dividend from project’s cash flow (%)</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Risk-free interest rate</td>
<td>Risk-free interest rate (%)</td>
<td>$r$</td>
</tr>
</tbody>
</table>

The estimation of these parameters is a condition to valuation and decision making in projects. In the case of financial options, except for future volatility, they can be estimated from past market data. On the other hand, in the case of projects, the procedure is much more complex, since these parameters, especially dividend and volatility, must be estimated by considering the dynamics in future cash flows. For example, Copeland and Antikarov (2001) suggest the use of Monte Carlo simulation to estimate the future volatility of projects, whereas Costa Lima and Suslick (2006) derive an analytical expression for volatility of projects considering two sources of uncertainty.

3 Analysis of a capital-intensive project

Recently, significant offshore heavy oil discoveries were made in ultra-deep-water of Brazil and West Africa. Among the new technologies required for commercial production of these heavy oil reservoirs in deep water, new artificial lift devices and long horizontal wells length can be detached, completed with efficient sand control mechanisms (Pinto et al., 2003). Besides providing commercial value for the heavy oil wells, it is expected that these new technologies will create a new value for such resources. Indeed, heavy oil reserves can become more available over time in these environments if the cost-reducing effects of new technologies (reduction of CAPEX and OPEX) more than offset the cost-increasing effects of depletion. Offshore heavy oil can be considered good assets for future revenues if new technologies are available to transform the potential reserves of heavy oil into viable projects.

These heavy-oil projects are a good sample to evaluate the performance of the proposed methodology. Most heavy-oil development projects in this type of environment are typically capital-intensive, where irreversibility, uncertainty, timing and risk-aversion are present. In order to discuss some numerical results, consider a heavy-oil project with the geological, economic and financial characteristics shown in Table 3.

The cash flow of this project is in Appendix. For simplicity, we use a simple linear taxation of 50% representing the ‘government take’ of this project based on its specific characteristics, such as water depth, operational conditions, etc. Under a static scenario of price, cost, production and fiscal regime, using Equation (1), we have $E[\text{NPV}] = \text{US$ 391.38 million}$. According to the traditional model of valuation and decision making, this project should be “accepted immediately because it creates value for stockholders and the corporation should incur in 100% of its funding and get 100% of its profits”. On the other hand, over the entire project life, the true values of price, cost, production and tax may differ from those expected ones from the static scenario and the NPV may become more positive or even negative.
Table 3  Geological, technical and economic characteristics of the project

<table>
<thead>
<tr>
<th>Technical and economic characteristics</th>
<th>Properties value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil Reserve (MMbbl)</td>
<td>389.30</td>
</tr>
<tr>
<td>Water depth (metres)</td>
<td>1500.00</td>
</tr>
<tr>
<td>Oil quality</td>
<td>25° API</td>
</tr>
<tr>
<td>Oil production peak (MMbbl/year)</td>
<td>68.50</td>
</tr>
<tr>
<td>CAPEX (US$ MM)</td>
<td>887.02</td>
</tr>
<tr>
<td>OPEX (US$/bbl)</td>
<td>12.00</td>
</tr>
<tr>
<td>Oil spot price (US$/bbl)</td>
<td>30.00</td>
</tr>
<tr>
<td>Discount rate (%)</td>
<td>13.00%</td>
</tr>
<tr>
<td>Government take (%)</td>
<td>50.00%</td>
</tr>
</tbody>
</table>

However, in practice, the real NPV may be quite different and, actually, we will know its true value only when the oil production reaches economic exhaustion, that is, the real NPV is a random variable whose expected value is US$ 391.38 million. Since the NPV is a random variable, the optimal decision making should not be based solely on expected values, but should also consider its uncertainty because even a positive expected NPV may become negative if market conditions change over time. As a result, a natural complement to a static NPV is its risk analysis.

3.1 Estimation of the risk of the project

In finance, broadly speaking there are two types of risk: systematic and unsystematic. There is a consensus that there is no reward for unsystematic risk because it can be eliminated by diversification in well developed markets. It is important to note that this is true for investors in markets with a large menu of assets, what is not always the case in specific domestic markets of the major oil companies. For example, in non-mature financial markets such as in Brazil, the total risk (unsystematic) is what is of concern.

In this paper, we consider risk as the possibility of an unfavourable outcome, such as the chance of a negative NPV. In this sense, a risk analysis consists of finding the probabilistic distribution of the NPV from uncertainty in its primary variables such as price, cost and production, among others.

Although the NPV of this project is highly positive, it has some risk because of a possible change in market conditions, which is a fair and rational motivation for risk analysis. For this project, we use the following assumptions:

- **Future values of oil production cost (OPEX):** this variable is modelled as a triangular distribution, whose most likely value is US$ 12/bbl, whereas the optimistic cost is US$ 6/bbl and the pessimistic cost is US$ 18/bbl.

- **Future values of oil price:** this variable is modelled according to a lognormal distribution where mean price is US$ 30/bbl and standard deviation is US$ 15/bbl.

- **Future values of production:** oil production is expected to reach a yearly peak production of 68.5 million bbl and drop year after year to the ending value of 2.9 million bbl. The uncertainty in oil production is modelled considering that
future production will be distributed as triangular distribution, with pessimistic yearly production equal to 50% of the most likely value, whereas the yearly optimistic production is 50% above the most likely production.

This modelling assumes that the components of the project’s cash flow are statistically independent, although in reality, for oil production projects, this assumption is incorrect, since most variables are directly linked to oil price to some degree, apart from their dependency on the non-linearity in tax structure, logistics, etc.

The next step consists of simulating thousands of possible paths for these uncertain variables using the Monte Carlo technique. Then, after carrying out a simulation with 10,000 iterations, we get the histogram of the NPV shown in Figure 1.

![Figure 1](cumulative_frequency_of_the_NPV_and_project_s_risk_level.png)

Simulation of the NPV shows that its values range from US$ −1.3 billion to US$ 2.0 billion, but there is a concentration of values around the mean of US$ 276.07 million. In practice, most of these extreme values of the NPV have little significance, since they occur in very unlikely situations, that is, high cost and low price and vice-versa.

From Figure 1, the probability of a negative NPV is 30.95%. This means that, in the case of unfavourable events, the corporation may not be able to recover its full investment allocated to the project. Since investment is irreversible, the corporation must adopt two complementary policies in order to decide if it should invest in this project or not:

- firstly, the irreversibility of investment, together with the ability to invest in the future in a scenario of less risk, gives rise to a new decision-making rule, which is according to the real-option pricing approach
- secondly, the irreversibility of investment, together with corporation risk aversion, may imply in a working interest of less than 100% in the project.

Another possibility is to make price hedging, but this will also reduce profits because of the cost of hedge. Over the next sections, these two approaches will be applied to the case of an offshore oil project.
3.2 The optimal rule of investment ($F, K, V^*$)

Traditionally, the NPV has been the indicator for static valuation and decision-making. If it is even merely positive, the benefits are in excess over the costs and the project should be accepted. Under the static approach, this logic is correct but, if we take the effect of investment irreversibility and future uncertainty, this approach is no longer right. In Section 2.3, the application of the theory of exercising a financial option to the case of investment decision in projects is proposed. According to this theory, the option is not exercised simply if the value of underlying asset is just above its exercise cost, but only if it is sufficiently high. Analogously, the corporation should exercise its option to invest only when project value is sufficiently above its investment cost. Therefore, there is a critical project value ($V^*$) to trigger investment. The required input parameters of Equations (11)–(13) are shown in Table 4.

<table>
<thead>
<tr>
<th>Input parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project’s cash flow value</td>
<td>$V$ (MM US$)</td>
<td>1278.41</td>
</tr>
<tr>
<td>Project’s investment value</td>
<td>$I$ (MM US$)</td>
<td>887.02</td>
</tr>
<tr>
<td>Project’s dividend rate</td>
<td>$\delta$</td>
<td>11.95%</td>
</tr>
<tr>
<td>Project’ future volatility</td>
<td>$\sigma$</td>
<td>51.00%</td>
</tr>
<tr>
<td>Risk-free interest rate</td>
<td>$r$</td>
<td>5.00%</td>
</tr>
</tbody>
</table>

The optimal decision rule based on the theory of an option-pricing model requires, among other factors, characteristics of project cash flow and of the market interest rate. The risk-free interest rate is assumed to be that paid by government to its bondholders whose maturity is similar to that of the managerial flexibility. It is not easy to find the project’s volatility and dividend rate. The estimation of future project volatility is always a complex task (even for financial assets when there is a long time series available to be used as a proxy) because:

1. There are no marked transactions for project cash flows
2. Petroleum projects are unique, so that each reservoir has its own geological, technical, operational and environmental particularities.

As an alternative solution, in this paper, we use Monte Carlo simulation in order to access the project volatility as the standard deviation of rate of return of the project, as suggested by Copeland and Antikarov (2001). Firstly, we calculate the expected project NPV under static scenarios, which we call $NPV_0$. But, over the project’s entire operational life, there is a chance of an infinite number of different cash flow trajectories. These different cash flow trajectories are the cause of the project’s volatility, due to possible fluctuations in the expected value of the NPV from future cash flows. The expected rate of return of the project ($\mu_v$) at time 1 is:

$$E[\mu_v] = \frac{CF_0(1 + \mu) + NPV}{NPV_0} - 1$$

(14)

The expected project rate of return is very close to the cost of capital (or discount rate), that is, 13%. But, since cash flows in the future are uncertain, there is a chance that thousands of $\mu_v$ may occur. After a simulation, we get a distribution of the rate-of-return
with the following two moments: $E[\mu_r] = 13.00\%$ and $\sigma[\mu_r] = 51.00\%$. Note that expected rate of return is close to the discount rate and the standard deviation of rate of return is the volatility of this project. According to Costa Lima and Suslick (2006), for typical oil projects, if the volatility of oil price is around 20%, the volatility is much higher – around 51.00% as in this paper. These numbers show that project volatility is usually much higher than price volatility. As a result, the common assumption that project’s volatility is equal to price volatility may give rise to significant errors, undervalue projects, and create wrong critical values to exercise the option to invest.

The estimation of a project’s dividend yield is also complex. In an investment in stocks, investors receive dividends as a result of profit distributed by a corporation. The dividend of project can be understood as the cash flows per unit of time (month, year, etc.), but this simple approach has many shortcomings, especially in the case of long-lived projects with irregular cash flows. In this paper, we estimate project dividend rate ($\delta_v$) as a weighted average between cash flows and production:

$$\delta_v = \frac{\sum_{t=0}^{N} Q(t) \times f(t)}{\sum_{t=0}^{N} Q(t)}$$

(15)

where $Q(t)$ is the annual production and $f(t)$ is ratio of the yearly cash flow to the sum of total cash flows. Dividends for exhaustible resources, such as oil projects, can be associated with the reserve’s production flow over time. Using Equation (15), dividend rate is $\delta_v = 11.91\%$ and it means a fraction of the project’s total value that is generated each year, that is, nearly 12.00% of the asset’s value is produced.

Now, we have all inputs necessary to estimate the investment option value, optimal decision rule and value of waiting to invest in the future. From Equation (11), $V^*$ is US$ 1821.09 million. This means that the corporation should exercise its option to invest only if the current value of the project is at least US$ 1821.09 million. Since the current value of this project is US$ 1278.04 million, the optimal decision is to wait in the future. In other words, it means that the “decision to invest immediately requires $V$ equal to at least 2.05 times the investment cost and not merely a positive NPV”.

The value of the option to invest ($F$), with the flexibility of choosing to exercise the right to invest at some moment in the future during option’s maturity, is US$ 491.29 million, which is higher than the traditional NPV ($391.38 million). The value of waiting to invest in the future ($K$) is US$ 100.76 million, which comes from the uncertainty in future cash flows, that is, volatility of projects. According to this new decision rule, even if the NPV is positive today, the act of waiting is able to create more value for shareholders. Thus, the optimal policy is to invest when the flexibility to wait has no more value ($K = 0$), that is, when $F = \text{NPV}$. This condition will hold and be true when the project’s current value is sufficiently above its cost, that is, $V > V^* > I$.

The volatility of projects plays a very important role in both option valuation (Equations (12) and (13)) and decision making (Equation (11)). According to the classic investment theory, project value is represented by its NPV and the optimal decision is: invest as long as $V > I$, that is, a positive NPV. Contrarily, in the real-options model, the investment right is exercised as long as $V \geq V^*$ and both $F$ and NPV have the same value, that is, the strategic value of the option to invest ($F$) and the intrinsic option value (NPV) are equal. Then, $F$ may be considered as the intrinsic option value plus its time value. Another understanding is: when $V = V^*$, we have that $F(V^*) = V^* - I$ and,
consequently, \( V^* = F(V^*) + I \), that is, we can see \( F \) as part of the cost to invest now instead of waiting for a better opportunity in the future. Therefore, because of volatility, we always have \( V > I \) and not \( V^* \geq I \).

From Equation (1), the NPV is US$ 391.38 million and, according to traditional decision making, the corporation should invest right now. On the other hand, from Equation (12), the option value, which captures the value of the flexibility to invest not only now, but also in the future, is US$ 491.29 million and clearly the option value is above the NPV stand alone.

In the real option pricing model, \( F \) (investment option) and \( K \) (value of waiting option) are non-negative functions, whereas the NPV can be negative if \( I > V \). The entire results of both approaches in valuation and decision making can be seen in Figure 2 through a sensitivity analysis of the NPV, \( F \) and \( K \) (option value) to project current value.

As the current value of project increases, Figure 2 shows an increase of NPV, \( F \) and a decrease in \( K \). There are three important regions for decision making:

- **I**: \( 0 \leq V \leq \text{US$ 887.02 million} \). The intrinsic value of the option (NPV) is negative and the corporation will not invest. Meanwhile, \( F \) is positive. Why? Since the future is uncertain – volatility of project is 51%, there is a potential that project value increases to US$ 1821.09 million or more. Then, the investment option is much higher than the intrinsic value.

- **II**: \( \text{US$ 887.02 million} \leq V < \text{US$ 1821.09 million} \). The NPV is positive, but not sufficiently high. Once again, the effect of irreversibility and uncertainty implies that the optimal policy to maximise the option value is to wait until the project value reaches the value of US$ 1821.09 million. The option value is still high, but the option should therefore be exercised in the future.

- **III**: \( V \geq \text{US$ 1821.09 million} \). The investment option must be exercised immediately. Observe Figure 2 to confirm that the option value of waiting (\( K \)) has no more value if \( V \) is above US$ 1821.09 million.

In general terms, the results of valuation and decision making for this project, synthesised in Figure 2, are in agreement with the literature. For example, Costa Lima (2004) has shown that for some oil projects, \( V \) is on the average 2.14 times the
investment cost. These findings are in agreement with Dixit and Pindyck (1994, p.136). In addition, it is worth mentioning that for reasonable input parameters of the real-options model, particularly volatility and dividends, \( V \) may be two times the investment cost or even higher.

### 3.3 The optimal working interest in the project (\( W \))

In many cases, even though the project value is sufficiently above its investment cost, many corporations prefer to develop it in partnership (such as a joint venture), which seems to contradict the optimal decision rule of the option-pricing theory. But this practice in capital-intensive projects has some reasons due to the following characteristics:

1. the magnitude of investment cost
2. technology availability to develop the project alone
3. search for synergy among different business units.

In such situations, a common question is: What is the optimal level of financial participation (working interest) in this project?

The answer cannot be found through pure NPV analysis since, from this approach, the corporation should incur in 100% of investment and revenue as long as the NPV is positive. An alternative solution can be found using the theory of finding the optimal working interest to maximise the RAV in Equation (4). This equation requires two main inputs:

1. the probabilistic distribution of NPV
2. the corporation risk tolerance (\( T \)).

The corporation risk tolerance is difficult to estimate. Wilkerson (1988) argues that \( T \) can be estimated as a fraction of, for example, corporation market value, budget and other strategic variables. Walls (1995) shows empirically that \( T \) is around 25% of the capital allocated to petroleum explorations. Meanwhile, for other phases in the E&P chain, \( T \) may assume different values.

In this paper, it is assumed that the corporation budget is US$ 300 million and its risk tolerance is US$ 120 million, that is, \( T = 40\% \) of the budget. From Equation (4), the analyst can estimate the RAV for this project, considering different assumptions and characteristics of the corporation and of the project. As shown in Equation (4), RAV depends on utility function, corporation risk tolerance and project risk profile – in some cases, the RAV is an increasing function of \( W \), whereas, in others, it is a decreasing or even constant function. The main objective is to estimate \( W \), numerically or analytically, in order to maximise the RAV, given the corporation’s profile. For this case study of oil project, the sensitivity of RAV to \( W \) is shown in Figure 3.

As shown in Figure 3, an increase in \( W \) will increase the satisfaction of decision maker up to around 44.38%. Higher values of \( W \) reduce the satisfaction of decision maker through the decrease in RAV. In addition, if \( W \) is higher than 93%, the RAV becomes negative. Over the interval between 0 and 100%, the RAV is a non-linear and concave function of \( W \). The theory of maximising the RAV shows that, contrarily to the traditional view, the “corporation will not take 100% working interest in the project’s cost and revenue, but only 44.38% because this value gives the highest RAV”.
Note that the curve of the RAV to $W$ takes into account the uncertainty in price, production and operational cost together with the corporation’s utility function. Nevertheless, it is important to observe that, in order to get realistic results, it is important to search for the best estimation of $T$ because of its impact on $W$, that is, an increase in $T$ tends to increase $W$ in the project and vice-versa. In Figure 4, an extended sensitivity of the RAV to $W$ for different values of $T$ is presented.

Note that, in the base case, $T =$ US$ 120$ million and $W = 44.38\%$. If $T$ increases to US$ 200$ million, $W$ is also increased to 75%. Analogously, if $T$ decreases to US$ 80$ million, $W$ is also reduced to only 30%. In terms of valuation and strategic decision making, by taking $W = 44.38\%$, the company will spend around US$ 393.66$ million in the investment and receive a NPV of only US$ 173.69$ million. If a negative outcome occurs, the amount under risk is limited to that fraction of the investment, that is, US$ 393.66$
million. This policy of reduction in the project’s NPV may be understood as the price to limit risk exposure or as a hedge scheme against possible catastrophic losses.

The result from preference theory and risk-adjusted analysis intends to complement the results of the NPV and option pricing because of two main reasons:

1. the investment cost may be quite high if compared to the corporation’s budget
2. even in the case of a project with a current value equal to two or more times its cost, it may become profitless since volatility over the future may, for example, generate a scenario of high cost and low prices.

In this case, a rational policy intended to reduce the risk of losses and, consequently, of financial troubles, may be the choice of a small fraction of the project investment and revenues.

This practice is very common in the oil exploration and production business and this framework may be a valuable tool for the decision process in capital-intensive projects. Finally, by taking a working interest of less than 100% in the project, the corporation may invest in more than one project and, consequently, reduce the corporation’s portfolio risk.

4 Discussions and implications

The proposed model provides a complement to results of the classic NPV in valuation and decision making. It integrates valuation, risk quantification, strategic flexibilities and decision making, using theories of preference and real option pricing. Although more sophisticated, this integrated model requires two input parameters that are difficult to estimate: corporation risk tolerance ($T$) and project volatility ($\sigma$).

In the present case study, we have pointed out the need to complement results of the traditional cash flow when analysing investment in capital-intensive projects, such as those of oil and gas industry. Table 5 presents a comparison between the traditional and the proposed valuation and decision model.

Table 5

<table>
<thead>
<tr>
<th>Subjects</th>
<th>Main indicators</th>
<th>Traditional approach</th>
<th>Proposed model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valuation and risk</td>
<td>$V$ (million US$)</td>
<td>1278.41</td>
<td>1278.41</td>
</tr>
<tr>
<td></td>
<td>$I$ (million US$)</td>
<td>887.02</td>
<td>887.02</td>
</tr>
<tr>
<td></td>
<td>NPV (million US$)</td>
<td>391.38</td>
<td>276.07</td>
</tr>
<tr>
<td></td>
<td>$F$ (million US$)</td>
<td>–</td>
<td>452.34</td>
</tr>
<tr>
<td>Decision making</td>
<td>Risk (%)</td>
<td>30.95%</td>
<td>30.95%</td>
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<tr>
<td></td>
<td>NPV (million US$)</td>
<td>391.38</td>
<td>491.29</td>
</tr>
<tr>
<td></td>
<td>$V^*$ (million US$)</td>
<td>887.02</td>
<td>1821.09</td>
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<tr>
<td></td>
<td>$W$ (%)</td>
<td>100.00%</td>
<td>44.38%</td>
</tr>
<tr>
<td>Optimal decision</td>
<td>$VF$</td>
<td>Since NPV is positive, invest now and take 100% working interest</td>
<td>Even though NPV is positive, invest in the future (when $V$ is US$1821.09 million) and incur in a working interest of 44.38%</td>
</tr>
</tbody>
</table>
According to the traditional cash flow model, the NPV is the indicator for valuation and decision making. The project NPV is US$ 276.07 million and the corporation should invest immediately and incur in 100% of the project. The proposed model has a complementary structure, where $F$ is the valuation indicator and the decisions are made according to the results of $V$ and $W$.

The numerical analysis summarised in Table 5 shows that, although the NPV is positive, the option to invest is not deep-in-the-money. Therefore, the optimal decision is to wait for an increase in $V$ to, at least, US$ 2.10 billion. This may happen, for example, due to oscillation in price, costs and production, as well as in other strategic variables. By choosing only projects whose $V$ is at least equal to $V^*$, the corporation follows a more conservative and risk-limiting policy, which is an advance and is radically different from the traditional NPV. Nevertheless, this procedure does not assure that the option exercise is profitable, because the result of the exercise of real options, contrary to financial options, is not immediate, but depends on a cash flow from many years of operations.

In this context, since the project’s investment is high compared to the corporation’s risk tolerance, the corporation’s optimal working interest is 44.38%, whereas other partners must fund the remaining 55.62% of the investment. This framework derives from the chosen parameters, which may not be constant over time. If some of them undergo oscillations, for example, volatility or operational cost, global output may be very different, requiring that the analysis must be updated continuously in order to improve the corporation’s valuation and decision making in the search for an efficient resource allocation and value creation for stockholders.

5 Conclusions

This paper presented an integrated framework, combining theories of real-options and preference, useful for valuation and decision making under uncertainty for capital-intensive projects. The outputs of the real-options model may be used to complement those of the traditional NPV, especially by incorporating the value of uncertainty and irreversibility.

The results from the preference theory allow the corporation to estimate its optimal level of financial participation in a risky project that is compatible with the utility function of the decision maker. This integrated model represents a significant gain when compared with the traditional one (based solely on expected values) by considering the relationship among irreversible investment, risk tolerance, timing and risk-aversion.

In addition, findings from this new model is that its results tend to diverge from the traditional valuation and decision-making model as the level of uncertainties increases as in the case of heavy-oil in deep water projects where the timing and frequency of the unknown technologies for cost reductions are speculative and not fully dominated by oil industry.

Acknowledgements

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References


Notes

1The model assumes that a corporation has enough credit to finance the project, irrespective of capital limitations.

2The expected NPV is: $E[\text{NPV}] = 0.2 \times 300 + 0.2 \times 300 + 0.2 \times 300 + 0.2 \times 300 + 0.2 \times 300 = US$ 300 million. The risk of NPV is: $\sigma[\text{NPV}] = [(0.2)^2 \times 2 \times 10^{16} + (0.2)^2 \times 2 \times 10^{16} + (0.2)^2 \times 2 \times 10^{16} + (0.2)^2 \times 2 \times 10^{16} + (0.2)^2 \times 2 \times 10^{16}]^{1/2} = US$ 63.25 million.

3Petrobras is the most traded asset in the Sao Paulo Stock Exchange and is present in the majority of institutional and private portfolios of assets.

4These assumptions considering an oscillation of 50% are intended to show how to make decisions in a high environment of uncertainty when dealing with heavy-oil projects in deep waters. In practice, according to American Association of Cost Engineers (AACE) recommendation, making decision with and oscillation range of no more than 10%.

5In practice, a petroleum engineer knows that oil production declines exponentially, at least when approaching exhaustion or higher production costs due to heavy-oil production profile.

6The simulated expected value will differ from the expected value because as predicted by the Jensen’s inequality (see Dixit and Pindyck, 1994).

7This high project volatility may also reflect the cost structures of the project.

8The total discounted cash flow is $1,278,416,393.97 and the first operational cash flow is 268,177,684.38. Then the first dividend is: $f(1) = 268,177,684.38/1,278,416,393.97 = 20.98\%$. The other values of dividends are estimated accordingly. An estimation of the true dividend of the project is found using Equation (15) which is a weighted average.
<table>
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<tr>
<th>Years</th>
<th>Invest.</th>
<th>Prod.</th>
<th>Cost</th>
<th>Price</th>
<th>EBIT</th>
<th>EATI</th>
<th>YCF</th>
<th>Div.  (%)</th>
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Note: This is a simplified cash flow and its purpose is just to serve as an input for the optimal decision-rule from option pricing techniques and preference theory. For this reason, several elements of a real cash flow have been ignored such as depreciation, detailed tax treatment, etc. The inclusion of these elements may have a large impact on net cash flows, but in any way, this will not invalidate our integrated model.
**Investment decision in oil and gas projects**

**Nomenclature**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>EBTI</td>
<td>Earnings Before Tax and Interest</td>
</tr>
<tr>
<td>EATI</td>
<td>Earnings After Tax and Interest</td>
</tr>
<tr>
<td>NCF</td>
<td>Net Cash Flow</td>
</tr>
<tr>
<td>Div</td>
<td>Dividends</td>
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</table>