

## **A chance constrained closed-loop supply chain network design considering inventory-location problem**

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**Abstract:** The design of reverse supply chain networks is one of the major solutions for the reduction of solid waste and use of resources for producing product to a lesser extent. The design of a reverse supply chain network leads to reduced costs in addition to reducing environmental detrimental effects. Therefore, this paper seeks to develop a mixed integer linear programming (MILP) model for designing a closed-loop supply chain network (CLSCN) under uncertainty. The study network is multi-product, multi-period and multi-echelon wherein the possibility of storage and facing shortage in the back-order type has been considered. An approach based on chance constrained is applied for controlling uncertainty. In order to investigate the efficiency of the proposed model, we implemented it in an automotive manufacturing industry in Iran where the results of model implementation through real-world data in GAMS software, as well as the results of sensitivity analysis of demand values indicate the precise function and the accuracy of the results.

**Keywords:** closed-loop supply chain; inventory-location problem; mathematical programming model; chance constrained theory.

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## 1 Introduction

Global statistics show that the Earth's population is steadily on the rise, and will reach about 8 billion in 2025. This population growth has been associated with the rapid utilisation of natural resources and has caused serious damage to the environment in such a way that the annual production of carbon dioxide in recent years has been more than 30,000 million tons a year (<http://www.worldometers.info>). This accelerating pace of consumption of natural resources that has caused irreversible damage to the environment gives rise to a critical challenge: "What measures should be taken to stop this destructive process?" In response, one can say what should be done is to move towards the design and implementation of green supply chains (Mina et al., 2014a, 2014b). One of the concepts that can lead to the design of the green supply chain is the use of reverse logistics in the design of the SCN. Reverse supply chain management focuses on collecting products from downstream users and the reuse of them in order to create added

value (Huang et al., 2013). In the reverse/CLSCN, due to the reuse of returned products the amount of produced wastes is also reduced in addition to the reduced consumption of natural resources; thereby, this process leads to the design of a green supply chain.

One of the most important issues in the design of SCNs is the location of centres and the allocation of flow between materials and products (Qazvini et al., 2016; Gholami-Zanjani et al., 2019). Louwers et al. (1999) developed a location-allocation model for collecting, repairing, and reproducing carpets with the aim of minimising investment, operational, and transportation costs. Pishvaei and Shakouri (2009) presented a MILP model to minimise transport costs and establish facilities in order to optimise the return flow of the network through appropriate locating. Soleimani et al. (2017) examined a CLSCN design problem, including suppliers, manufacturers, distribution centres, customers, central warehouses, return centres, and recycling centres. Their chain modelling was designed in order to maximise meeting customer demand, maximise total profits, and minimise the lost working days due to occupational events. Chen et al. (2017) examined the design problem of an integrated CLSCN by taking into account chain costs and environmental concerns in the solar industry from the sustainability perspective. Their proposed model includes practical features, such as flow protection in each production/recycling unit whether in the progressive flow or in the reverse flow, expansion of capacity, and recycled parts. The results of the analysis indicate that a company must adopt a proper recycling strategy or energy-saving technology to achieve an optimal economic efficiency in the case of the application of regulations pertaining to carbon emissions.

Sahebjamnia et al. (2018) developed a multi-objective MILP model for sustainable tire SCN design. The proposed model considers the environmental impacts of setting up centres, tire processing, and transport between each level, as well as social impacts, including job opportunities and occupational injuries in order to optimise overall costs. For the first time, a MILP model was designed to formulate a multi-objective sustainable SCN design by considering the discount estimation in transportation costs by Hajiaghahi-Keshteli and Fard (2018). To date, a vast number of studies have been done on CLSCN design. However, few studies have been conducted on routing/ location-routing problem in this area. Fang et al. (2017) assessed the combination of reverse logistics in a CLSCN with routing problems to reduce carbon emissions. In that study, a MIP model was proposed for the routing problem with reverse logistics with the pickup and delivery possibility. Guo et al. (2018) investigated a location-routing-inventory problem in order to minimise the total costs of the chain in a CLSCN. To this end, they used a mixed integer non-linear programming model. A bi-objective MILP model developed to design green supply chain network considering inventory- location- routing problem under uncertainty by Gholipour et al. (2019a). The aim of their proposed model is to minimise the total cost and shortage simultaneously. Zandkarimkhani et al. (2019) utilised a hybrid approach based on fuzzy analytical hierarchy process, fuzzy TOPSIS, and mathematical programming model for sustainable open-loop SCN design. The model presented by them is a bi-objective one that simultaneously minimises costs and maximises the operation of potential sites with an approach based on fuzzy multi-criteria decision-making methods was developed and the final assessment score was set as the objective function coefficient of potential sites. They used a compromise programming method to solve the bi-objective model and implemented their model regarding the data of a production-distribution chain of PET-based products in Iran. Mardan et al. (2019) represented an accelerated benders decomposition algorithm for designing a bi-objective

green supply chain network. A bi-objective MILP model for designing a green supply chain network was presented by Gholipour et al. (2019b). In their proposed model, the minimisation of total cost and detrimental environmental effects are discussed. Sadeghi et al. (2019) developed a MILP model to solve a location-routing problem in the CLSCN. The routing problem examined by them was a capacitated and multi-depot one wherein vehicles have been considered heterogeneously. In addition, several transportation modes have also been considered for the transfer of raw materials from suppliers to producers where the optimal transportation mode is determined by the model. In order to validate their proposed model, they implemented it in a manufacturing-distribution chain of automotive parts in Iran.

The design of the SCN in the real world is highly used so that wealth of articles has evaluated it with actual data in order to measure the efficiency of their proposed model. Automotive industry is among the other industries that have received attention in this area in recent years (Chavez and Sharma, 2018). Today, with the growth of the population and the increasing demand for automobile, the need for using reverse supply chain design in the automotive industry is strongly felt. Hence, in this research, a MILP model is developed to design a CLSCN by considering the inventory-location problem for the automotive parts industry under uncertainty. Inventory planning to escape from facing shortage, storage for future periods, production and transportation planning, supplier selection and order allocation are some of crucial decisions that will be considered in the design of under study network.

In the following, the problem definition and proposed model are introduced in the second part. The deterministic equivalent formulation is presented in the third section. The forth part is dedicated to the implementation of the proposed model in the real world. Next section is allocated to sensitivity analysis and the final section is dealt with the conclusion section.

## **2 Problem definition and proposed model**

The current chain in this paper is simultaneously a forward and reverses one and includes such levels as suppliers, plants, central warehouses, retailers, collection centres, recycling centres, and disposal centres. The plant purchases raw materials from the selected suppliers and, then, it dispatches the manufactured products to central warehouses. Then, the products are distributed by central warehouses among retailers. Defective products are delivered from retailers to by collection centres and resent to recycling centres after inspection for reuse or are transferred to disposal centres. Recyclable products enter into the recycling cycle through the central warehouse facility after experiencing the recycling process. In the following, the assumptions of the proposed model have been presented.

Assumptions:

- the intended chain has been considered multi-product and multi-period
- supplier selection and location of recycling centres, disposal centres, collection centres and central warehouses are determined by the model
- there is only one plant for the manufacturing of products
- centres are capacitated

- the return rate of products from retailers is definitive
- there is the possibility of an encounter with shortage in the warehouse of retailers
- there is the possibility of storage in the warehouse of retailers
- the shortage has been considered to be of a back-order nature
- in the end of last time period, all back-order demands will be responded to
- this study considers the uncertainties for capacities of suppliers, central warehouses, collection centres, recycling centres and disposal centres.

### *Indices*

- i* raw material  
*p* product  
*a* supplier  
*b* central warehouse  
*c* retailer  
*d* collection centre  
*e* recycling centre  
*f* disposal centre  
*t* time period.

### *Parameters*

- $CA_{at}$  fixed ordering cost to supplier *a* at time period *t*  
 $CB_b$  cost of setting up central warehouse *b*  
 $CD_d$  cost of setting up collection centre *d*  
 $CE_e$  cost of setting up recycling centre *e*  
 $CF_f$  cost of setting up disposal centre *f*  
 $PA_{iat}$  cost of purchasing per unit of raw material *i* from supplier *a* at time period *t*  
 $PB_{pbt}$  cost of processing per unit of product *p* in central warehouse *b* at time period *t*  
 $PD_{pdt}$  cost of processing per unit of product *p* in collection centre *d* at time period *t*  
 $PE_{pet}$  cost of processing per unit of product *p* in recycling centre *e* at time period *t*  
 $PF_{pft}$  cost of processing per unit of product *p* in disposal centre *f* at time period *t*  
 $TA_{iat}$  cost of transferring per unit of raw material *i* from supplier *a* to plant at time period *t*  
 $TB_{pbt}$  cost of transferring per unit of product *p* from plant to central warehouse at time period *t*

$TBC_{pbct}$	cost of transferring per unit of product $p$ from central warehouse $b$ to retailer $c$ at time period $t$
$TCD_{pcdt}$	cost of transferring per unit of product $p$ from retailer $c$ to collection centre $d$ at time period $t$
$TDE_{pdet}$	cost of transferring per unit of product $p$ from collection centre $d$ to recycling centre $e$ at time period $t$
$TDF_{pdfi}$	cost of transferring per unit of product $p$ from collection centre $d$ to disposal centre $f$ at time period $t$
$TEB_{pebt}$	cost of transferring per unit of product $p$ from recycling centre $e$ to central warehouse $b$ at time period $t$
$CAPA_{iat}$	capacity of supplier $a$ for raw material $i$ at time period $t$
$CAPB_{pbt}$	capacity of central warehouse $b$ for product $p$ at time period $t$
$CAPD_{pdt}$	capacity of collection centre $d$ for product $p$ at time period $t$
$CAPe_{pet}$	capacity of recycling centre $e$ for product $p$ at time period $t$
$CAPF_{pfi}$	capacity of disposal centre $f$ for product $p$ at time period $t$
$DMND_{pct}$	demand of retailer $c$ for product $p$ at time period $t$
$WC_{pct}$	rate of returned product $p$ from retailer $c$ at time period $t$
$WM_{ip}$	amount of required raw material $i$ for producing product $p$
$WD_{pdt}$	rate of disposed product $p$ by collection centre $d$ at time period $t$
$HLD_{pct}$	cost of holding per unit of product $p$ at time period $t$
$PNLTY_{pct}$	shortage cost for per unit of product $p$ in warehouse of retailer $c$ at time period $t$
$M$	a big number.

*Variables*

$XA_{iat} \begin{cases} 1 \\ 0 \end{cases}$	Binary	If supplier $a$ is selected for purchasing product $p$ at time period $t$ Otherwise
$XB_b \begin{cases} 1 \\ 0 \end{cases}$	Binary	If central warehouse $b$ is set up Otherwise
$XD_d \begin{cases} 1 \\ 0 \end{cases}$	Binary	If collection centre $d$ is set up Otherwise
$XE_e \begin{cases} 1 \\ 0 \end{cases}$	Binary	If recycling centre $e$ is set up Otherwise
$XF_f \begin{cases} 1 \\ 0 \end{cases}$	Binary	If disposal centre $f$ is set up Otherwise
$ZA_{ait}$	Positive	Amount of purchased raw material $i$ from supplier $a$ at time period $t$
$ZB_{pbt}$	Positive	Amount of transferred product $p$ from plant to central warehouse $b$

$ZBC_{pbct}$	Positive	Amount of transferred product $p$ from central warehouse $b$ to retailer $c$ at time period $t$
$ZCD_{pcdt}$	Positive	Amount of transferred product $p$ from retailer $c$ to collection centre $d$ at time period $t$
$ZDE_{pdet}$	Positive	Amount of transferred product $p$ from collection centre $d$ to recycling centre $e$ at time period $t$
$ZDF_{pdfi}$	Positive	Amount of transferred product $p$ from collection centre $d$ to disposal centre $f$ at time period $t$
$ZEB_{pebt}$	Positive	Amount of transferred product $p$ from recycling centre $e$ to central warehouse $b$ at time period $t$
$STRP_{pct}$	Positive	Amount of stored product $p$ in warehouse of retailer $c$ at time period $t$
$STRN_{pct}$	Positive	Amount of shortage for product $p$ in warehouse of retailer $c$ at time period $t$
$STR_{pct}$	Positive	Amount of Inventory (auxiliary variable)

## 2.1 Mathematical model

### 2.1.1 Objective function

The objective function minimises total costs. These costs are as follows: Fixed ordering cost to suppliers, cost of setting up central warehouses, collection centres, recycling centres and disposal centres, cost of purchasing raw materials, cost of processing product in central warehouses, collection centres, recycling centres and disposal centres, transportation costs between levels, cost of holding products in warehouse of retailers and finally cost of facing shortage.

$$\begin{aligned}
 Z^{Cost} = & \sum_{i,a,t} CA_{iat} \times XA_{iat} + \sum_b CB_b \times XB_b + \sum_d CD_d \times XD_d + \sum_e CE_e \times XE_e \\
 & + \sum_f CF_f \times XF_f + \sum_{i,a,t} PA_{iat} \times ZA_{iat} + \sum_{p,b,t} PB_{pbt} \times ZB_{pbt} + \sum_{p,c,d,t} PD_{pdt} \times ZCD_{pcdt} \\
 & + \sum_{p,d,e,t} PE_{pet} \times ZDE_{pdet} + \sum_{p,d,f,t} PF_{pft} \times ZDF_{pdfi} + \sum_{i,a,t} TA_{iat} \times ZA_{iat} \\
 & + \sum_{p,b,t} TB_{pbt} \times ZB_{pbt} + \sum_{p,b,c,t} TBC_{pbct} \times ZBC_{pbct} + \sum_{p,c,d,t} TCD_{pcdt} \times ZCD_{pcdt} \\
 & + \sum_{p,d,e,t} TDE_{pdet} \times ZDE_{pdet} + \sum_{p,d,f,t} TDF_{pdfi} \times ZDF_{pdfi} + \sum_{p,e,b,t} TEB_{pebt} \times ZEB_{pebt} \\
 & + \sum_{p,c,t} HLD_{pct} \times STRP_{pct} + \sum_{p,c,t} PNLTY_{pct} \times STRN_{pct}
 \end{aligned} \tag{1}$$

Subjected to:

- Capacity constraints: Maximum capacity limitations of suppliers, central warehouses, collection centres, recycling centres and disposal centres are provided in constraints (2) to (6).  $CAPA_{iat}$ ,  $CAPB_{pbt}$ ,  $CAPD_{pdt}$ ,  $CAPE_{pet}$  and  $CAPF_{pft}$  are stochastic variables and these constraints are satisfied with probabilities of at least  $\alpha$ ,  $\beta$ ,  $\delta$ ,  $\kappa$  and  $\psi$ , respectively.

$$\Pr\{ZA_{iat} \leq CAPA_{iat}\} \geq \alpha \quad \forall i, a, t \tag{2}$$

$$\Pr\{ZB_{pbt} \leq CAPB_{pbt}\} \geq \beta \quad \forall p, b, t \tag{3}$$

$$\Pr \left\{ \sum_c ZCD_{pcdt} \leq CAPD_{pdt} \right\} \geq \delta \quad \forall p, d, t \quad (4)$$

$$\Pr \left\{ \sum_d ZDE_{pdet} \leq CAPE_{pet} \right\} \geq \kappa \quad \forall p, e, t \quad (5)$$

$$\Pr \left\{ \sum_d ZDF_{pdfi} \leq CAPF_{pfi} \right\} \geq \psi \quad \forall p, f, t \quad (6)$$

Inventory balance between levels, amount of storage and shortage are calculated using constraint (7) to (16):

$$\sum_a ZA_{iat} \geq \sum_{p,b} WM_{ip} \times ZB_{pbt} \quad \forall i, t \quad (7)$$

$$ZB_{pbt} \geq \sum_c ZBC_{pbct} \quad \forall p, b, t = 1 \quad (8)$$

$$ZB_{pbt} + \sum_2 ZEB_{peb(t-1)} \geq \sum_c ZBC_{pbct} \quad \forall p, b, t > 1 \quad (9)$$

$$STR_{pct} = STR_{pc(t-1)} + \sum_b ZBC_{pbct} - DMND_{pct} \quad \forall p, c, t > 1 \quad (10)$$

$$STR_{pct} = \sum_b ZBC_{pbct} - DMND_{pct} \quad \forall p, c, t = 1 \quad (11)$$

$$STR_{pct} = STRP_{pct} - STRN_{pct} \quad \forall p, c, t \quad (12)$$

$$\sum_d ZCD_{pcdt} = \sum_b WC_{pct} \times ZBC_{pbct} \quad \forall p, c, t \quad (13)$$

$$\sum_c ZCD_{pcdt} = \sum_e ZDE_{pdet} + \sum_f ZDF_{pdfi} \quad \forall p, d, t \quad (14)$$

$$\sum_f ZCF_{pdfi} = \sum_c WD_{pdt} \times ZCD_{pcdt} \quad \forall p, d, t \quad (15)$$

$$\sum_d ZDE_{pdet} = \sum_b ZEB_{pebt} \quad \forall p, e, t \quad (16)$$

Based on constraint (17), the total of demands should be responded to at the end of the last period. In other words, it is possible that demands will encounter a shortage over a period of time, but this shortage should be compensated for by the end of the last time period:

$$\sum_{b,t} ZBC_{pbct} = \sum_t DMND_{pct} \quad \forall p, c \quad (17)$$

The next category of constraints pertains to fulfilling the assumption of raw materials/products transfer among the levels. In order for the products to be transferred among the levels, centres should be established. In other words, if the centres are not set



up, no products should enter into or exit from it. These conditions are applied by the following constraints:

$$ZA_{iat} \leq XA_{iat} \times M \quad \forall i, a, t \quad (18)$$

$$ZB_{pbt} \leq XB_b \times M \quad \forall p, b, t \quad (19)$$

$$ZBC_{pbct} \leq XB_b \times M \quad \forall p, b, c, t \quad (20)$$

$$ZCD_{pcdt} \leq XD_d \times M \quad \forall p, c, d, t \quad (21)$$

$$ZDE_{pdet} \leq XE_e \times M \quad \forall p, d, e, t \quad (22)$$

$$ZDF_{pdfi} \leq XF_f \times M \quad \forall p, d, f, t \quad (23)$$

### 3 Deterministic equivalent formulation

According to chance constrained theory the stochastic model could be converted to a deterministic model (Shaw et al., 2016). This conversion will be applied to constraints 2 to 6 and the objective function and rest of the constraints will remained unchanged. Below we present the deterministic counterpart.

$$ZA_{iat} \leq \overline{CAPA_{iat}} + \Phi_{(1-\alpha)}^{-1} \times \zeta \quad \forall i, a, t \quad (24)$$

$\overline{CAPA_{iat}}$  represents the mean capacity of suppliers.  $\Phi_{(1-\alpha)}^{-1}$  is the inverse of cumulative standard normal distribution.  $\zeta$  is standard deviation of the supplier capacity. The constraint have to be satisfied with a probability of at least  $\alpha$ , where  $0 < \alpha < 1$ .

$$ZB_{pbt} \leq \overline{CAPB_{pbt}} + \Phi_{(1-\beta)}^{-1} \times \vartheta \quad \forall p, b, t \quad (25)$$

$$\sum_c ZCD_{pcdt} \leq \overline{CAPD_{pdt}} + \Phi_{(1-\delta)}^{-1} \times \zeta \quad \forall p, d, t \quad (26)$$

$$\sum_d ZDE_{pdet} \leq \overline{CAPE_{pet}} + \Phi_{(1-\kappa)}^{-1} \times \mu \quad \forall p, e, t \quad (27)$$

$$\sum_d ZDF_{pdfi} \leq \overline{CAPF_{pft}} + \Phi_{(1-\psi)}^{-1} \times \varpi \quad \forall p, f, t \quad (28)$$

$\overline{CAPA_{iat}}$ ,  $\overline{CAPB_{pbt}}$ ,  $\overline{CAPD_{pdt}}$ ,  $\overline{CAPE_{pet}}$  and  $\sum_d ZDF_{pdfi}$  represent the mean capacities of suppliers, central warehouses, collection centres, recycling centres and disposal centres, respectively.  $\Phi_{(1-\alpha)}^{-1}$ ,  $\Phi_{(1-\beta)}^{-1}$ ,  $\Phi_{(1-\delta)}^{-1}$ ,  $\Phi_{(1-\kappa)}^{-1}$  and  $\Phi_{(1-\psi)}^{-1}$  are the inverse of cumulative standard normal distributions.  $\zeta$ ,  $\vartheta$ ,  $\zeta$ ,  $\mu$  and  $\varpi$  are standard deviations of capacities of suppliers, central warehouses, collection centres, recycling centres and disposal centres, respectively. The constrains (25) to (28) are satisfied with probabilities of at least  $\beta$ ,  $\delta$ ,  $\kappa$  and  $\psi$  respectively, where  $0 < \beta, \delta, \kappa, \psi < 1$ .

#### 4 Case study

In this section, the proposed model will be implemented in the automotive lights production industry in Iran in order to evaluate its performance. This company produces all kinds of headlights, traffic lights, rear warning lights, and fog lamps for such automobiles as Pride 131, Pride 111, Peugeot 206, Peugeot 207, and Peugeot Pars. In this paper, the proposed model is implemented in the real world by using the data pertaining to two raw materials, three products, five suppliers, four potential central warehouses, seven retailers, four potential collection centres, four potential recycling centres, four potential disposal centres, and six time periods. In Table 1, the demand values for each of the retailers have been shown.

**Table 1** Retailers' demand for each product per time period

$DMND_{pct}$		1	2	3	4	5	6
1	1	2,800	3,000	2,900	3,300	3,200	2,900
1	2	3,000	3,200	3,100	3,100	2,900	3,000
1	3	2,900	2,900	3,200	3,000	3,100	3,100
1	4	2,800	3,200	3,300	2,900	3,000	2,900
1	5	3,200	2,900	2,800	2,900	3,200	3,200
1	6	3,000	3,000	3,000	3,000	3,200	3,000
1	7	3,200	3,300	2,900	3,100	3,300	3,100
2	1	3,000	3,100	3,000	3,300	3,200	2,900
2	2	3,100	2,900	3,100	2,900	3,000	3,000
2	3	3,200	3,200	3,000	3,000	2,800	2,900
2	4	3,100	3,000	3,200	3,100	3,200	2,900
2	5	3,100	2,800	3,300	2,900	2,900	2,800
2	6	3,100	3,000	2,900	3,000	2,800	3,000
2	7	3,100	3,100	3,100	2,900	3,200	3,100
3	1	2,900	2,900	3,000	3,100	3,000	2,900
3	2	3,200	3,300	3,000	3,000	2,900	2,800
3	3	3,100	3,200	2,900	2,800	2,900	3,100
3	4	3,100	3,200	3,000	3,100	3,200	3,100
3	5	2,900	3,200	2,900	3,100	2,800	2,800
3	6	3,200	3,100	2,900	2,900	3,300	2,900
3	7	2,900	2,800	3,000	2,900	3,100	2,800

By implementing the proposed model in GAMS win 64 24.1.2/Cplex software, the optimal values of the objective function and decision variables were determined, which have been presented as follows:

- the value of the objective function was obtained equal to 60,337,662,370 monetary units
- all five suppliers will be contacted

- central warehouses numbered 1, 2, 3, and 4 are established
- collection centres numbered 1, 2, and 4 are established
- recycling centres numbered 1, 2, 3 and 4 are established
- disposal centre numbered 1 is established
- the amount of raw material purchased from each supplier is presented in Table 2.

**Table 2** The amount of raw materials purchased from each supplier per time period

$ZA_{iat}$		1	2	3	4	5	6
1	1	12,400	2,516	14,500	0	14,000	0
1	2	6,325	14,100	14,100	11,900	0	0
1	3	12,900	13,200	0	0	8,237	10,400
1	4	13,200	0	0	13,600	10,800	0
1	5	14,500	0	0	14,800	0	0
2	1	10,800	0	0	0	0	0
2	2	12,100	0	11,700	0	0	0
2	3	14,200	11,500	10,995	10,500	11,700	0
2	4	0	0	0	12,600	14,600	0
2	5	10,600	11,816	0	8,541	0	8,602

- The amount of products transferred from the plant to the central warehouses is shown in Table 3.

**Table 3** The amount of products transferred from the plants to the central warehouses at each time period

$Zb_{pbt}$		1	2	3	4	5	6
1	1	8,800	6,988	8,800	9,900	0	9,900
1	2	9,900	3,458	1,196	9,900	2,851	1,861
1	3	9,900	0	8,800	0	8,800	0
1	4	9,900	0	0	0	1,603	0
2	1	8,800	0	0	8,800	9,900	0
2	2	7,661	3,403	329	4,425	9,900	0
2	3	5,189	9,900	8,800	8,132	8,800	0
2	4	9,900	8,800	988	0	0	0
3	1	2,068	0	5,967	0	0	0
3	2	8,800	9,420	2,890	8,671	3,311	407
3	3	9,900	0	9,900	9,900	0	0
3	4	8,800	7,208	0	6,961	9,900	5,642

- The amount of products transferred from central warehouses to retailers is shown in Table 4.

**Table 4** The amount of products transferred from central warehouses to retailers at each time period

$ZBC_{pbc}$			1	2	3	4	5	6
1	1	1	3,960	11,188	0	0	0	1,368
1	1	2	2,990	0	2,820	3,548	0	0
1	1	4	0	0	0	6,352	0	0
1	1	5	0	0	0	0	0	3,189
1	1	6	0	0	0	0	0	2,990
1	1	7	1,850	0	5,980	0	0	2,353
1	2	1	0	0	0	0	0	1,522
1	2	2	0	3,458	0	5,421	0	0
1	2	3	0	0	0	0	6,179	0
1	2	4	5,272	0	3,289	0	0	339
1	2	5	0	0	0	1,489	0	0
1	2	6	0	0	0	2,990	0	0
1	2	7	4,628	0	0	0	0	0
1	3	3	7,549	0	1,906	2,505	0	0
1	3	5	0	0	4,192	0	3,189	0
1	3	6	2,351	0	2,702	0	1,586	0
1	3	7	0	0	0	0	4,025	0
1	4	4	0	708	0	0	0	2,080
1	4	5	6,079	0	0	0	0	0
1	4	6	3,821	0	95	0	1,603	0
2	1	1	0	0	0	3,289	6,079	0
2	1	2	0	0	0	2,422	0	464
2	1	4	2,621	0	0	3,089	0	0
2	1	5	0	1,568	0	0	0	0
2	1	6	0	0	0	0	5,734	0
2	1	7	6,179	0	0	0	0	0
2	2	1	7,661	0	329	0	0	0
2	2	4	0	3,403	0	0	6,079	0
2	2	5	0	0	0	4,750	3,821	0
2	3	1	0	0	1,079	0	0	0
2	3	2	0	0	0	2,395	3,590	0
2	3	3	1,631	0	4,946	0	5,681	0
2	3	4	468	0	2,776	0	0	0
2	3	5	0	984	0	0	0	0
2	3	6	3,089	8,916	0	0	0	0
2	3	7	0	0	0	5,738	0	0
2	4	2	5,252	3,817	0	0	0	0
2	4	3	1,558	3,189	0	1,034	0	0

**Table 4** The amount of products transferred from central warehouses to retailers at each time period (continued)

$ZBC_{pbc\tau}$			1	2	3	4	5	6
2	4	5	3,089	3,527	0	0	0	0
2	4	7	0	0	3,681	0	0	2,840
3	1	2	2,068	0	2,990	0	0	0
3	1	6	0	0	6,321	2,748	0	0
3	1	7	0	429	0	0	0	0
3	2	1	5,780	0	0	0	0	0
3	2	2	0	3,289	0	8,671	0	0
3	2	3	3,020	0	2,890	0	0	0
3	2	4	0	9,268	0	0	6,279	0
3	2	6	0	0	0	0	0	2,890
3	3	1	0	0	2,990	6,079	0	0
3	3	2	1,121	0	0	0	0	0
3	3	3	0	0	0	2,791	0	0
3	3	5	2,890	0	2,060	0	0	0
3	3	6	5,889	0	0	0	0	0
3	3	7	0	0	4,850	1,030	0	0
3	4	1	0	0	0	0	0	2,890
3	4	3	70	3,189	0	0	5,980	0
3	4	4	3,089	0	0	0	0	0
3	4	5	0	4,019	0	6,961	0	1,709
3	4	6	389	0	0	0	0	0
3	4	7	5,251	0	0	0	4,838	1,042

- The amount of products returned from retailers to collection centres is presented in Table 5.

**Table 5** The amount of products returned from retailers at each time period

$ZCD_{pdc\tau}$			1	2	3	4	5	6
1	1	1	0	0	0	0	0	441
1	1	2	0	1,709	0	0	0	0
1	1	4	605	0	0	0	0	0
1	2	2	0	678	553	0	0	0
1	2	4	586	0	0	1,757	0	0
1	3	1	0	0	358	0	0	0
1	3	4	1,419	0	0	471	1,161	0
1	4	1	867	116	0	0	0	398
1	4	2	0	0	541	0	0	0
1	4	4	0	0	0	1,045	0	0

**Table 5** The amount of products returned from retailers at each time period (continued)

$ZCD_{pct}$		$1$	$2$	$3$	$4$	$5$	$6$
1	5	1	0	0	0	0	320
1	5	2	0	0	421	0	320
1	5	4	611	0	0	150	0
1	6	1	0	0	283	0	302
1	6	2	623	0	0	0	0
1	6	4	0	0	0	302	322
1	7	1	0	0	801	0	315
1	7	2	0	0	0	0	539
1	7	4	868	0	0	0	0
2	1	1	956	0	0	410	759
2	1	2	0	0	176	0	0
2	2	1	0	0	0	604	0
2	2	2	659	479	0	0	0
2	2	4	0	0	0	0	451
2	3	1	484	484	0	157	0
2	3	4	0	0	751	0	863
2	4	1	0	239	195	217	427
2	4	4	217	0	0	0	0
2	5	1	465	0	0	716	0
2	5	4	0	916	0	0	576
2	6	1	0	854	0	0	0
2	6	2	296	0	0	0	549
2	7	1	0	0	0	0	276
2	7	2	0	0	0	557	0
2	7	4	600	0	357	0	0
3	1	1	0	0	207	0	200
3	1	2	400	0	0	0	0
3	1	4	0	0	0	420	0
3	2	1	612	631	573	0	0
3	2	4	0	0	0	1,663	0
3	3	1	0	580	526	508	1,088
3	3	2	562	0	0	0	0
3	4	1	583	1,748	0	0	1,184
3	5	1	456	634	325	0	270
3	5	4	0	0	0	1,099	0
3	6	1	909	0	0	398	418
3	6	4	0	0	915	0	0
3	7	1	0	38	425	90	424
3	7	4	460	0	0	0	0

- The amount of products transferred from collection centres to recycling centres is shown in Table 6.

**Table 6** The amount of products transferred from collection centres to recycling centres at each time period

$ZDE_{pdet}$			1	2	3	4	5	6
1	1	1	0	95	0	0	0	0
1	1	2	0	0	1,177	0	0	1,450
1	1	4	708	0	0	0	0	0
1	2	1	0	0	1,328	0	0	0
1	2	2	0	0	0	0	754	0
1	2	3	547	2,093	0	0	0	0
1	4	1	0	0	0	3,328	0	0
1	4	3	3,653	0	0	0	0	0
1	4	4	0	0	0	0	1,326	0
2	1	1	0	1,434	0	0	0	0
2	1	2	1,733	0	0	0	1,078	0
2	1	3	0	0	177	0	0	0
2	1	4	0	0	0	1,913	0	304
2	2	1	0	0	0	0	464	0
2	2	2	0	405	0	470	0	0
2	2	3	0	0	148	0	0	0
2	2	4	807	0	0	0	0	0
2	4	1	762	855	0	0	0	0
2	4	2	0	0	1,034	0	1,762	0
3	1	1	0	3,345	0	0	0	0
3	1	3	0	0	1,894	0	0	902
3	1	4	2,358	0	0	917	2,484	0
3	2	1	780	0	0	0	0	0
3	4	1	0	0	854	0	0	0
3	4	3	429	0	0	2,968	0	0

- The amount of products transferred from collection centres to the disposal centre is shown in Table 7.

**Table 7** The amount of products transferred from collection centres to the disposal centre at each time period

$ZDF_{pdfi}$			1	2	3	4	5	6
1	1	1	160	21	265	0	0	327
1	2	1	77	294	186	0	106	0
1	4	1	435	0	0	396	158	0
2	1	1	173	143	18	191	108	30
2	2	1	148	74	27	87	85	0
2	4	1	55	61	74	0	126	0
3	1	1	202	287	162	79	213	77
3	2	1	182	0	0	0	0	0
3	4	1	31	0	61	214	0	0

- The amount of products transferred from recycling centres to central warehouses is shown in Table 8.

**Table 8** The amount of products transferred from recycling centres to central warehouses at each time period

$ZEB_{pebt}$			1	2	3	4	5	6
1	1	2	0	0	0	3,328	0	0
1	1	3	0	0	1,328	0	0	0
1	1	4	0	95	0	0	0	0
1	2	3	0	0	1,177	0	0	1,450
1	2	4	0	0	0	0	754	0
1	3	1	4,200	0	0	0	0	0
1	3	2	0	2,093	0	0	0	0
1	4	4	708	0	0	0	1,326	0
2	1	1	762	0	0	0	464	0
2	1	4	0	2,288	0	0	0	0
2	2	3	0	0	0	470	0	0
2	2	4	1,733	405	1,034	0	2,840	0
2	3	2	0	0	325	0	0	0
2	4	1	807	0	0	1,913	0	304
3	1	1	0	3,345	854	0	0	0
3	1	2	780	0	0	0	0	0
3	3	1	429	0	1,894	0	0	902
3	3	2	0	0	0	2,968	0	0
3	4	2	2,357	0	0	0	2,484	0
3	4	4	0	0	0	917	0	0

- The amount of products stored by each retailer is shown in Table 9.

**Table 9** The amount of products stored by each retailer at each time period

$STRP_{pct}$			1	2	3	4	5
1	1	1,170	9,368	6,478	3,189	0	
1	2	0	269	0	5,880	2,990	
1	3	4,658	1,768	485	0	3,089	
1	4	2,482	0	0	3,462	472	
1	5	2,890	0	1,401	0	0	
1	6	3,182	192	0	0	0	
1	7	3,289	0	3,089	0	736	
2	1	4,672	1,582	0	0	2,890	
2	2	2,163	3,089	0	1,926	2,526	
2	3	0	0	1,956	0	2,890	
2	4	0	413	0	0	2,890	



**Table 9** The amount of products stored by each retailer at each time period (continued)

<i>STRP<sub>pct</sub></i>		1	2	3	4	5
2	5	0	3,289	0	1,860	2,791
2	6	0	5,926	3,036	46	2,990
2	7	3,089	0	591	3,439	249
3	1	2,890	0	0	2,990	0
3	2	0	0	0	5,681	2,791
3	3	0	0	0	0	3,089
3	4	0	6,079	3,089	0	3,089
3	5	0	830	0	3,872	1,081
3	6	3,089	0	3,431	3,289	0
3	7	2,361	0	1,860	0	1,748

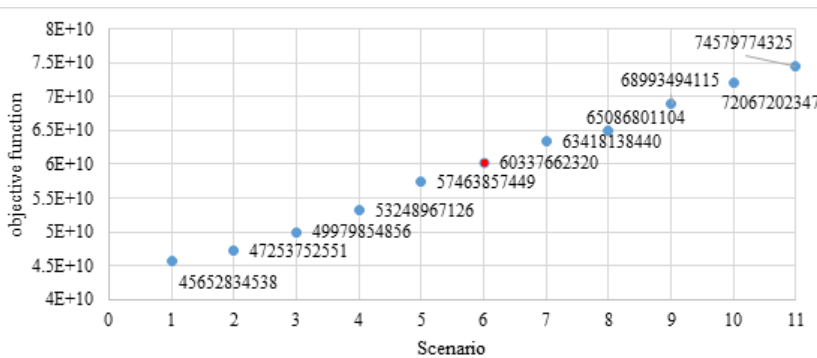
- No shortage has been observed at any time period.

Thus, following the implementation of the proposed model by means of the case study data in GAMS software, such issues as the rate of optimal purchase from suppliers, the level of product transfer among the levels, the amount of storage and shortage, and, in general, the structure of the supply chain network were determined. Then, the model sensitivity is measured using changes in demand values. Next, the accuracy and precision of the model will be examined.

### 5 Sensitivity analysis

In order to evaluate the validity of the proposed model, the sensitivity of the problem under study is analysed based on changes in demand values. Accordingly, it is expected that the value of the objective function does not improve (either stay constant or increase) with the increase of demands; and it is also expected that the value of the objective function does not deteriorate (either stay constant or decrease) with the reduction of demands. Hence, the results of the sensitivity analysis are presented as demand values in Table 10.

**Figure 1** The change process of the objective function as a result of implementing different scenarios (see online version for colours)



**Table 10** The values of the objective function for changes in demand values

<i>Scenario</i>	<i>Amount of demand</i>	<i>Lower bound of objective function</i>	<i>Upper bound of objective function</i>	<i>Objective function</i>
S01	$0.75 \times demand$	42,592,640,972	135,024,066,562	45,652,834,538
S02	$0.8 \times demand$	45,481,228,094	138,551,552,409	47,253,752,551
S03	$0.85 \times demand$	48,363,536,360	139,729,149,483	49,979,854,856
S04	$0.9 \times demand$	51,264,129,007	144,435,885,563	53,248,967,126
S05	$0.95 \times demand$	54,181,486,226	152,090,506,393	57,463,857,449
S06	$1 \times demand$	57,099,720,450	152,616,313,410	60,337,662,320
S07	$1.05 \times demand$	60,039,556,221	158,745,256,791	63,418,138,440
S08	$1.1 \times demand$	62,954,981,721	159,131,074,337	65,086,801,104
S09	$1.15 \times demand$	65,877,135,709	159,138,983,637	68,993,494,115
S10	$1.2 \times demand$	68,834,735,302	163,397,336,751	72,067,202,347
S11	$1.25 \times demand$	71,811,719,637	163,996,741,380	74,579,774,325

As shown in Table 10 and Figure 1, the value of the objective function has decreased with the reduction of demand values; and it has increased with an increase in the value of the demand values, which was in line with our logical expectation of the model implementation. Therefore, the validity of the proposed model is confirmed.

## 6 Conclusions

In this paper we develop a MILP in order to model a multi-product multi-period multi-echelon CLSCN which contains collection centres, retailers, central warehouses, plant, suppliers, recycling centres, and disposal centres. This model also consider inventory-location problem under uncertainty. The proposed model is an integrated strategic-operational model for purchasing plan, production planning, distribution planning, and inventory planning.

In this model the supplier capacity and the centres have some inherent uncertainty and we used chance constrained approach to deal with uncertainty. We then used automotive lights production data to do some sensitivity analysis on the result of the model. The results from the model show an improvement in overall operation. Below we list the future studies related to this paper:

- considering the vehicle routing problems especially pickup and delivery problem
- considering environmental and social factors to develop a sustainable modelling
- applying an exact algorithm such as benders decomposition algorithm for solving the problem in large scale
- developing a novel algorithm (heuristic/meta-heuristic) to solve the problem at large scale.

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