Integrated inventory system with freight costs and two types of quantity discounts

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Abstract: In this paper, we investigate an integrated inventory model for single supplier-buyer considering stochastic demand. We consider two quantity discounts with freight environment. The objective is to minimise the integrated total cost incurred by the supplier and buyer. Freight costs are the function of shipping weight and distance with two transportation modes: truckload (TL) and less-than-truckload (LTL) shipments. We develop two optimal models considering all-units quantity discount and incremental quantity discount. The study shows how freight costs and the two discount policies affect the total cost of the supplier and the buyer. Numerical examples are used to illustrate of two models.

Keywords: integrated inventory; TL and LTL; stochastic demand; all-units quantity discount; incremental quantity discount.

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1 Introduction

A more efficient management of inventories across the entire supply chain can be achieved through better coordination between the supplier and the buyer. In many practical conditions, the freight cost is often the major component of the logistic cost. Swenseth and Godfrey (2002) showed that about 50% of the total logistic cost is attributed to transportation activity.

It is a common practice for supplier to offer lower unit prices on orders for larger quantities as an economic incentive to buyer to purchase in larger lot size. The supplier benefits from sales of larger quantities by reducing per unit order processing and setup costs and by increasing volume. The buyer benefits both by the reduced per unit ordering costs and the lower unit price, at the cost of having to hold more inventory. There are two quantity discount plans are offered by supplier: the all-units discount model and the incremental discount model.

Previous research on the joint supplier-buyer problem focused on the production shipment schedule in terms of the number and size of batches transferred between both parties. Most of these integrated models assume that demand as deterministic without incorporating freight cost and quantity discount. In this paper, we assume the stochastic demand and incorporating freight costs and two type quantity discounts.

In this study, we investigate the effect of the quantity discount plans on the supplier and the buyer considering stochastic demand and freight costs. We develop coordinated supplier-buyer models to determine simultaneously the lot size, reorder point, number of deliveries and discount level. An algorithm is developed to determine the optimal solutions and numerical examples are provided to illustrate the benefits of the proposed models.

This paper is organised as follows. Section 2 reviews the related literature. Section 3 provides a problem description. The proposed algorithm is presented in Section 4. The numerical examples along with sensitivity analysis are provided in Section 5. Finally, the conclusion of the paper and future research directions are described in Section 6.

2 Literature review

One of the first articles considering an integrated supplier-buyer problem is by Goyal (1977). Banerjee (1986) assumed finite rate manufacturing and a lot-for-lot shipment. Goyal (1988) developed Banerjee's (1986) model by relaxing lot-for-lot assumption. Lu (1995) relaxed the assumption of Goyal (1988) and specified the optimal production and shipment policies when the shipment sizes are equal. Goyal (1995) then developed a model where successive shipment sizes increase by a ratio equal to the production rate divided by the demand rate. Later, Hill (1997) considered the geometric growth factor as a decision variable and he suggested a solution method based on an exhaustive search for both the growth factor and the number of shipments. Based on previous researchers, Hill (1999) developed a general optimal policy model. Goyal (2003) suggested a simple procedure for determining the optimal operating policy. Except for Liao and Shyu (1991), Ben-Daya and Hariga (2004), Ouyang et al. (2004) and Jauhari et al. (2011), most of these models assumed deterministic demand.

Baumol and Vinod (1970) were the first authors introduced the inventory-theoretic models which involving of transportation and inventory costs. Langley (1980) considered actual motor carrier freight rates function into lot sizing decision by using enumeration technique. Mendoza and Ventura (2008) presented an algorithm based on a grossly simplified freight rate structure (using either a constant charge per TL or a constant cost per unit for LTL shipments). He et al. (2010) explained an algorithm for finding the optimal purchase quantity using actual freight rates. Darwish (2008) extended the model by considering freight rate discount. Abad and Aggarwal (2005) developed a model for determining the buyer's lot sizing and pricing that there are freight and all-unit quantity discount.

Modelling freight rates is a model that estimates the freight rates based on the value of some parameter in a continuous function. Examples of these parameters include:

a the TL charge in an inverse function (Swenseth and Godfrey, 2002; Yildirmaz et al., 2009)

b distance in a proportional function (Ballou, 1991)

c the constant used as an exponent in an exponential function (Buffa, 1987, 1988)

d the smoothing constant in an adjusted inverse function (Swenseth and Buffa, 1990, 1991; Swenseth and Godfrey, 1996, 2002)

e load density, shipment weight, and shipment distance in a nonlinear model (Kay and Warsing, 2009).

Ballou (1991) argued that time, effort, and cost considerations often dictate that logistics decisions. Nie et al. (2006), Ertogral et al. (2007), and Leuveano et al. (2014a, 2014b) presented an integrated inventory model with incorporating transportation cost, however their model did not incorporate stochastic demand. Gurtu et al. (2015) modified EOQ models to include transport cost and emission cost. Wangsa and Wee (2017a, 2017b) extended the model by considering stochastic demand. Bazan et al. (2015a, 2015b, 2017) developed an integrated inventory model with greenhouse gases (GHG) emissions and different coordination mechanism. Wangsa (2017) considered stochastic demand, GHG emissions, and penalty and incentive polices for a joint economic lot size (JELS).

Researchers have studied quantity discount for a long time. However, few researchers have studied its influence on the integrated supplier-buyer model with stochastic demand. Lee (1986) is one of the earliest studies that explicitly incorporated freight rate discount into EOQ model. He assumed that the transportation cost is fixed and increases in a step function format depending on the lot size. Tersine et al. (1989) formulated an economic inventory transport model with freight discounts. Tersine and Barman (1991) incorporated transportation cost in inventory policy when quantity discounts are offered and deterministic demand. Burwell et al. (1997) extended the model of Tersine and Barman's (1991) model. They considered the demand to be dependent on price. Russell and Krajewski (1991) presented a simple analytical approach for finding the order quantity that minimises total purchase costs. They allowed for over-declaring shipments in transportation and quantity discounts where over-declaring a shipment refers to artificially inflating the shipping weight to a higher weight-break point and a lower marginal tariff. Carter et al. (1995) suggested a correction procedure for irregularities in freight schedules. Carter and Ferrin (1996) formulated a lot-sizing model that consider actual freight rate discount using enumerative techniques. Darwish (2008) developed the EOQ model with considering quantity and freight discounts under stochastic environment.

In the existing literature, several researchers have only studied single echelon inventory model involving quantity discount and deterministic demand and without actual freight cost. Tsao and Sheen (2012) studied joint multi-item replenishment policy by considering weight freight discount, credit period and deterministic demand. The freight carrier provides freight discount that depends on the weight of the cargo transported. They propose a model to determine ideal supplier credit period, retailer replenishment policy and pricing decision to maximise the total profit. Geetha and Uthayakumar (2013) developed Tsao and Sheen's (2012) model by considering the supplier's quality improvement. Jauhari (2014) investigated the quantity discount and partial backorder in stochastic vendor-buyer inventory model under variable lead time and partial backorder. Jauhari et al. (2016) extended the model by considering freight cost in the supply chain system with two discount schemes, namely: all-units quantity discount and incremental quantity discount.

In this study, we consider freight rate costs that are usually computed based on shipping weight and distance. In addition, the mentioned papers mostly focus quantity discount models on the single-echelon system or two-echelon system with deterministic demand. We identify the research gap and we focus on investigating the quantity discounts, freight cost and stochastic demand. We assume that the demand is normally distributed. Two types of discounts procedure, namely: all-units quantity discount and incremental quantity discount. The purpose of this article is to determine order quantity, number of delivery and safety factor simultaneously.

3 Problem description

The mathematical model in this paper is developed based on the following assumptions and notation.

3.1 Assumptions and notation

The following assumptions are used in our model:

1. There is a single supplier and a single buyer for a single product.
2. The supplier offers all-units quantity discount or incremental quantity discount.
3. The supplier produces the product with a finite production rate P, and $P > D$.
4. The buyer orders a lot of size Q and the supplier manufactures mQ with a finite production rate in one setup, but ship quantity Q to the buyer over m times. The supplier incurs a setup cost S for each production run and the buyer incurs an ordering cost A for each order of quantity Q.
5. The demand X during lead time L follows a normal distribution with mean D_L and standard deviation $\sigma\sqrt{L}$.

6 Inventory is continuously reviewed. The buyer places the order when the on hand inventory reaches the reorder point, $R = D_L + k\sigma\sqrt{L}$, k is safety factor.

7 All items are purchased free-on-board (F.O.B). The buyer incurs freight cost.

8 Shortages are allowed with partial backorders and lost sales.

To develop the mathematical model we use the following notations:

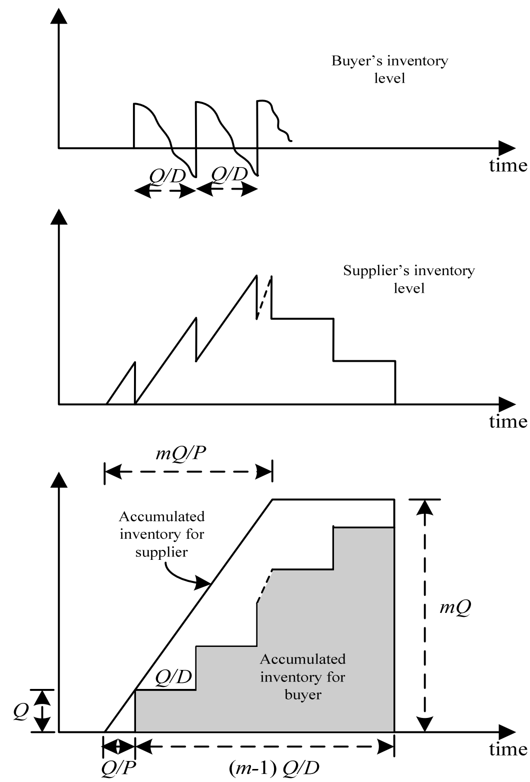
D	demand rate in units per unit time on the buyer
P	production rate in units per unit time of the supplier
A	fixed ordering cost incurred by the buyer
S	fixed setup cost incurred by supplier
C_s	supplier's production cost
C_{bj}	buyer's purchasing cost at discount level j $C_{b0} > C_{b1} > \ldots > C_{bJ} > C_s$
q_j	a sequence of integer quantities with breakpoint j, where $j = 0$ to J, q_0 is the minimum order quantity and $q_0 < q_1 < q_2 < q_3 < \ldots q_J$
Z_j	extra cost of buyer's purchasing cost in an incremental quantity discount
r	annual inventory holding cost rate per unit time
L	length of the lead times in weeks
Q	order quantity (decision variable)
R	reorder point (decision variable)
k	safety factor (decision variable)
F_0	supplier's transportation cost per trip
w	weight of a unit part in lb per unit
d	transportation distance in miles
α	discount factor for LTL shipments, $0 \leq \alpha \leq 1$
F_x	the freight rate in dollar per pound for a given per mile for full truckload (FTL)
F_y	the freight rate in dollar per pound for a given per mile for partial load
W_x	FTL shipping weight in lbs
W_y	actual shipping weight in lbs
m	number of deliveries from the supplier to the buyer (decision variable)
π_x	backorder cost per unit of the buyer
π_0	gross marginal profit per unit of the buyer
β	the backorder ratio, $0 \leq \beta \leq 1$
$B(R)$	expected demand shortage at the end of cycle

X the lead time demand, which follows a normal distribution with finite mean D_L and standard deviation $\sigma\sqrt{L}$, $X \sim N(D_L, \sigma\sqrt{L})$

$E(\cdot)$ mathematical expectation

x^+ maximum value of x and 0, i.e., $x^+ = \max\{x, 0\}$.

Figure 1 The buyer and supplier's inventory pattern



3.2 System description

Based on assumption (5), the buyer sells items to the end customers with a mean of D and standard deviation of σ. The buyer orders the item at constant lot of size Q (frequency of D / Q) from the supplier. Each time an order is placed, a fixed ordering cost A incurs. The supplier produces the product in a batch size of mQ with a finite production rate P ($P > D$) with a fixed setup cost S. During the production period, when the first Q units have been produced, the supplier will deliver to the third party (freight forwarding services/logistics services) with fixed transportation cost, F_0, after the third party received, they will make consolidation and delivers to the buyer. The buyer will receive the lot size of Q, with average every D / Q unit of time the inventory level until to zero. The inventory profile for supplier and buyer is depicted in Figure 1.

In this problem the supplier offers quantity discount to the buyer to increase the buyer's lot size of product. The purpose of this policy is the buyer has an opportunity to buy the large size of product with the lower purchase cost (C_{bj}) hence, the purchase order and the supplier setup will be decreased so the impact can be minimised the integrated total cost. The supplier's price schedule with discount is in Table 1.

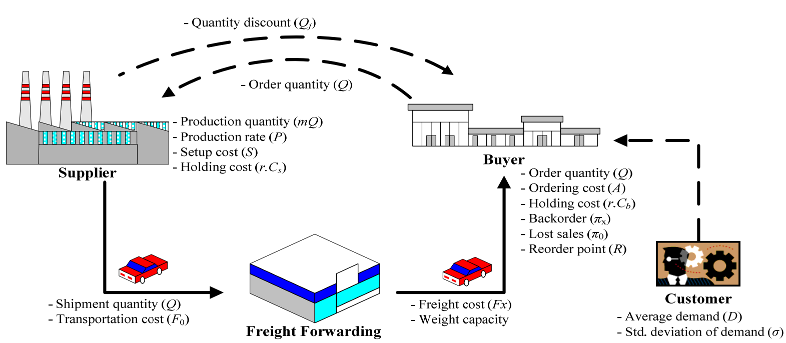
Table 1 The supplier's quantity discount schedule

j	Q *(order quantity)*	C_{bj} *(purchase cost)*
0	$q_0 \leq Q < q_1$	C_{b0}
1	$q_1 \leq Q < q_2$	C_{b1}
2	$q_2 \leq Q < q_3$	C_{b2}
…	…	…
J	$Q \geq q_J$	C_{bJ}

In Table 1, $q_0 < q_1 < q_2 < q_3 < \dots q_J$ is a sequence of integer quantities when purchase breakpoint occurs. If a quantity Q, where $q_j \leq Q < q_{j+1}$, is purchased based on all-units quantity discount model, then the unit purchase cost of each of the Q item is C_{bj} where $C_{b0} > C_{b1} > C_{b2} > \dots > C_{bJ}$. In the incremental quantity discount, as the quantity per order increases, the purchasing cost declining incrementally on additional units purchased.

The freight cost, F_x is charged to the buyer. The buyer pays freight cost to the freight forwarding services for each shipment weight, W_x which is scheduled by freight forwarder. The shipment of product from supplier to the buyer can be conducted by using some transportation modes such as motor carrier transport and railroad. In the proposed models, partial backorder π_x and lost sales π_0 are permitted. The system description of the supply chain system is illustrated in Figure 2.

Figure 2 The system description in the proposed models (see online version for colours)



3.3 Model formulation

In this section, we briefly discuss the all-units quantity discount and incremental quantity discount models.

3.3.1 All-units quantity discount

First, we develop the all-units quantity discount model considering freight cost and stochastic demand. Based on the above notations and assumptions, the integrated total cost per unit time is the sum of the following elements:

$$\text{Ordering cost per unit time} = \frac{AD}{Q} \tag{1}$$

The inventory system is continuously reviewed by the buyer, when the inventory level drops to the reorder point R, an order of quantity Q is made. The expected net inventory level before receipt of an order is $R - D_L$, and the expected net inventory level immediately after the successive order is $Q + R - D_L$. Hence, the average inventory over the cycle can be approximated by $Q / 2 + R - D_L$. Hence, the buyer's expected holding cost per unit time is rC_b $(Q / 2 + R - D_L)$. Using the same approach as in Montgomery et al. (1973), the expected net inventory level just before receipt of a delivery is $R - D_L + (1 - \beta)E(X - R)^+$. The expected shortage quantity at the end of the cycle is given by $B(R) = \sigma\sqrt{L}\psi(k) = \int_R^\infty (x - R)dF(x),$ where $\psi(k) = \phi(k) - k[1 - \Phi(k)]$. The values of ϕ, Φ denotes the standard normal density function, there are: probability density function (p.d.f) and cumulative density function (c.d.f), respectively. The expected inventory immediately after the successive delivery is $[Q + R - D_L + (1 - \beta)B(R)]$.

$$\text{The holding cost per unit time} = rC_{bj}\left[\frac{Q}{2} + R - D_L + (1-\beta)B(R)\right] \tag{2}$$

As mentioned earlier, the lead time demand X has a c.d.f. with finite mean D_L and standard deviation $\sigma\sqrt{L}$; the reorder point is $R = D_L + k\sigma\sqrt{L}$. Shortage occurs when $X > R$; the expected shortage quantity at the end of cycle is given by $B(R)$. Thus, the expected backorders and lost sales per order is $\beta B(R)$ and $(1 - \beta)B(R)$, respectively.

$$\text{The shortage cost per unit time} = \frac{D}{Q}\left[\pi_x\beta + \pi_0(1-\beta)\right]B(R) \tag{3}$$

The freight rates function that able to emulate LTL shipment is adjusted inverse function (Swenseth and Godfrey, 1996, 2002). By using the transportation cost based upon Swenseth and Godfrey (2002) and Wangsa and Wee (2017a, 2017b), the freight rate for partial load F_y based on adjusted inverse function given as $F_y = F_x + \alpha F_x[(W_x - W_y) / W_y)]$, where α has is constant from 0 to 1.

$$\text{The freight cost per unit time} = \frac{D}{Q}\alpha F_x W_x d + Ddw(1-\alpha)F_x \tag{4}$$

The total cost for the buyer per unit time for each j can be formulated by considering equations (1)–(4). The total cost for the buyer (TC_{bj}) is given by:

$$TC_{bj}(Q,R)=\frac{AD}{Q}+rC_{bj}\left[\frac{Q}{2}+R-D_L+(1-\beta)B(R)\right]$$
$$+\frac{D}{Q}\left[\pi_x\beta+\pi_0(1-\beta)\right]B(R)+\frac{D}{Q}\alpha F_xW_xd \tag{5}$$
$$+Ddw(1-\alpha)F_x$$

The total cost for the supplier consists of setup cost, transportation cost and holding cost for the supplier. Supplier will produce a lot size mQ, the setup frequency of $D\,/\,(mQ)$ and delivery interval to the buyer of $D\,/\,Q$. Since S is the setup cost per setup and F_0 is the transportation cost per trip, hence the setup cost and transportation cost respectively:

$$\text{The setup cost per unit time}=\frac{SD}{mQ} \tag{6}$$

$$\text{The transportation cost per unit time}=\frac{F_0D}{Q} \tag{7}$$

The supplier's inventory level is shown in Figure 1. The supplier its average inventory can be evaluated as follows:

$$\bar{I}_s=\frac{\left\{mQ\left[\frac{Q}{P}+(n-1)\frac{Q}{D}\right]-\frac{m^2Q^2}{2P}\right\}-\frac{Q}{D}\left[1+2+\cdots+(m-1)\right]Q}{\frac{mQ}{D}}$$

$$\bar{I}_s=\frac{Q}{2}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]$$

$$\text{The supplier's holding cost per unit time}=rC_s\frac{Q}{2}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right] \tag{8}$$

The total cost for the supplier per unit time (TC_s) can be formulated by equations (6)–(8):

$$TC_s(m)=\frac{SD}{mQ}+\frac{F_0D}{Q}+rC_s\frac{Q}{2}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right] \tag{9}$$

The integrated total cost per unit time is given by equation (10) which is the total cost incurred to the buyer [equation (5)] and the supplier [equation (9)]:

$$\begin{aligned}ITC_j(Q,R,m)=&\frac{AD}{Q}+rC_{bj}\left[\frac{Q}{2}+R-D_L+(1-\beta)B(R)\right]\\&+\frac{D}{Q}\left[\pi_x\beta+\pi_0(1-\beta)\right]B(R)+\frac{D}{Q}\alpha F_xW_xd\\&+Ddw(1-\alpha)F_x+\frac{SD}{mQ}+\frac{F_0D}{Q}\\&+rC_s\frac{Q}{2}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]\end{aligned} \tag{10}$$

Substituting $R - D_L = k\sigma\sqrt{L}$ and $B(R) = \sigma\sqrt{L}\psi(k)$ into equation (10), one has:

$$\begin{aligned} ITC_j(Q,k,m) &= \frac{D}{Q}\left\{A + \frac{S}{m} + F_0 + \alpha F_x W_x d + \left[\pi_x\beta + \pi_0(1-\beta)\right]\sigma\sqrt{L}\psi(k)\right\} \\ &+ rC_{bj}\left[k\sigma\sqrt{L} + (1-\beta)\sigma\sqrt{L}\psi(k)\right] + Ddw(1-\alpha)F_x \\ &+ r\frac{Q}{2}\left\{C_{bj} + C_s\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right]\right\} \end{aligned} \tag{11}$$

To solve this nonlinear programming problem for minimise integrated total cost we used partial derivatives, $ITC_j(Q, k, m)$ with respect to Q and k. The first partial derivatives of $ITC_j(Q, k, m)$ with respect to Q and equating it to zero, one has:

$$\begin{aligned} \frac{\partial ITC_j(Q,k,m)}{\partial Q} &= -\frac{D}{Q^2}\left\{A + \frac{S}{m} + F_0 + \alpha F_x W_x d + \left[\pi_x\beta + \pi_0(1-\beta)\right]\sigma\sqrt{L}\psi(k)\right\} \\ &+ \frac{r}{2}\left\{C_{bj} + C_s\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right]\right\} = 0 \end{aligned}$$

The equation of optimal order quantity is given by:

$$Q^* = \sqrt{\frac{2D\left\{A + \frac{S}{m} + F_0 + \alpha F_x W_x d + \left[\pi_x\beta + \pi_0(1-\beta)\right]\sigma\sqrt{L}\psi(k)\right\}}{r\left\{C_{bj} + C_s\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right]\right\}}} \tag{12}$$

In order to find the optimal safety factor we take the first partial derivative of $ITC_j(Q, k, m)$ with respect to k then equating it to zero.

$$\begin{aligned} \frac{\partial ITC_j(Q,k,m)}{\partial k} &= \frac{D}{Q}\left[\pi_x\beta + \pi_0(1-\beta)\right]\sigma\sqrt{L}\left[\Phi(k) - 1\right] \\ &+ rC_{bj}\left\{\sigma\sqrt{L} + (1-\beta)\sigma\sqrt{L}\left[\Phi(k) - 1\right]\right\} = 0 \end{aligned}$$

Thus, equation of optimal safety factor, one has:

$$\Phi(k^*) = 1 - \frac{rC_{bj}Q}{\left[\pi_x\beta + \pi_0(1-\beta)\right]D + rC_{bj}Q(1-\beta)} \tag{13}$$

3.3.2 Incremental quantity discount

In this case, the overall cost function in incremental quantity is continuous over all values of Q. To develop the integrated total cost function, we use the same cost function in the all-units quantity discount [equation (11)]. Therefore, we modified the buyer's holding cost that follows incremental quantity discount. First, we modified the total buyer's purchasing cost on discount interval C_{bj} and C_{bj+1} can be expressed as:

$$C_{bj}(Q) = C_{b0}(q_1 - q_0) + C_{b1}(q_2 - q_1) + \cdots + C_{bj-1}(q_j - q_{j-1}) + C_{bj}(Q - q_j) \tag{14}$$

To simply equation (14), the extra purchasing cost incurred by the buyer can be expressed as:

$$Z_j = C_{b0}\left(q_1 - q_0\right) + C_{b1}\left(q_2 - q_1\right) + \cdots + C_{bj-1}\left(q_j - q_{j-1}\right) \tag{15}$$

With $Z_0 = 0$, therefore:

$$C_{bj}(Q) = Z_j + C_{bj}\left(Q - q_j\right) \tag{16}$$

The average unit purchasing cost for Q is equal to $C_{bj}(Q) / Q$, which is equal to:

$$\frac{C_{bj}(Q)}{Q} = \frac{Z_j + C_{bj}\left(Q - q_j\right)}{Q}$$

$$C_{bj} = \frac{Z_j}{Q} + \frac{C_{bj}(Q)}{Q} - \frac{C_{bj} q_j}{Q}$$

$$C_{bj} = \frac{Z_j - C_{bj} q_j}{Q} + C_{bj} \tag{17}$$

Substituting equation (17) into the buyer's holding cost [equation (2)] and we obtain:

$$\text{The holding cost per unit time} = r\left(\frac{Z_j - C_{bj} q_j}{Q} + C_{bj}\right) \left[\frac{Q}{2} + R - D_L(1-\beta)B(R)\right] \tag{18}$$

Finally, the integrated total cost per unit time (*ITC*) for incremental quantity discount, one has:

$$\begin{aligned} ITC_j(Q, k, m) = {} & \frac{D}{Q}\left\{A + \frac{S}{m} + F_0 + \alpha F_x W_x d + \left[\pi_x \beta + \pi_0(1-\beta)\right]\sigma\sqrt{L}\psi(k)\right\} \\ & + r\left(\frac{Z_j - C_{bj} q_j}{Q} + C_{bj}\right)\left[k\sigma\sqrt{L} + (1-\beta)\sigma\sqrt{L}\psi(k)\right] \\ & + \frac{r}{2}\left\{Z_j - C_{bj} q_j + C_{bj} Q + Q C_s\left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right]\right\} \\ & + Ddw(1-\alpha)F_x \end{aligned} \tag{19}$$

The optimal solution for the incremental quantity discount can be found by taking the first partial derivatives, $ITC_j(Q, k, m)$ in equation (19) with respect to Q and k then equating the results to zero, respectively. One has:

$$Q^* = \sqrt{\frac{\begin{aligned} & 2D\left\{A + \frac{S}{m} + F_0 + \alpha F_x W_x d + \left[\pi_x \beta + \pi_0(1-\beta)\right]\sigma\sqrt{L}\psi(k)\right\} \\ & + 2r\left(Z_j - C_{bj} q_j\right)\left[k\sigma\sqrt{L} + (1-\beta)\sigma\sqrt{L}\psi(k)\right] \end{aligned}}{r\left\{C_{bj} + C_s\left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right]\right\}}} \tag{20}$$

And

$$\Phi(k^*) = 1 - \frac{r\left(Z_j - C_{bj}q_j + C_{bj}Q\right)}{\left[\pi_x\beta + \pi_0(1-\beta)\right]D + r\left(Z_j - C_{bj}q_j + C_{bj}Q\right)(1-\beta)} \tag{21}$$

4 Proposed algorithm

In this section, we proposed two algorithms to minimise the integrated total cost. The proposed algorithms are modified from the basic algorithm by Leuveano et al. (2014b) and Jauhari et al. (2016). The algorithms are presented as follows:

4.1 Proposed algorithm for all-units quantity discount

Step 0 Set $m = 1$ and $ITC_j(Q^*_{m-1}, k^*_{m-1}, m-1) = \infty$.

Step 1 For discount level, $j = 0, 1, 2, \ldots, J$.

- 1.1 Use purchasing cost at discount level j, C_{bj} based on the supplier's quantity discount schedule.
- 1.2 Set $k_j = 0$ [implies $\psi(k_j) = 0.39894$ and $\Phi(k_j) = 0.50$].
- 1.3 Substitute $\psi(k_j)$ into equation (12) to evaluate Q_j.
- 1.4 Calculate actual shipping weight, $W_{yj} = Q_j.w$. If $W_{yj} \le W_x$ is satisfied go to Step (1.6). Otherwise, go to Step (1.5) if truckload (TL) constraint is not satisfied or $W_{yj} > W_x$.
- 1.5 Revised delivery lot size, $Q_j = \dfrac{W_x}{w}$ and go to Step (1.6).
- 1.6 Use Q_j to determine $\Phi(k_j)$ from equation (13) then find k_j from $\Phi(k_j)$ by checking the normal table.
- 1.7 Repeat (1.3)–(1.6) until no change occurs in the values of Q_j and k_j.

Step 2 Check the validation for the solution that obtained from Step (1).

- 2.1 For $j = 0, 1, 2, \ldots, J-1$.
  - a If $q_j \le Q_j < q_{j+1}$, set $Q^*_j = Q_j$ and $k^*_j = k_j$ as the optimal valid solutions.
  - b If $Q_j \ge q_{j+1}$, the solutions is invalid and set $ITC_j\left(Q^*_m, k^*_m, m\right) = \infty$.
  - c If $Q_j < q_j$, set $Q^*_j = q_j$ as the optimal valid solutions and compute k^*_j using equation (13).
- 2.2 For $j = J$.
  - a If $Q_j \ge q_j$, set $Q^*_j = Q_j$ and $k^*_j = k_j$ as the optimal valid solutions.
  - b If $Q_j < q_j$, set $Q^*_j = q_j$ as the optimal valid solutions and compute k^*_j using equation (13).

Step 3 Set the valid of Q_j^* and k_j^* and compute $ITC_j(Q_m^*, k_m^*, m)$ using equation (11).

Step 4 If $ITC_j(Q_m^*, k_m^*, m) \le ITC_j(Q_{m-1}^*, k_{m-1}^*, m-1)$, then go to Step (5), otherwise go to Step (6).

Step 5 Set $m = m + 1$, repeat Step 1.

Step 6 $(Q_j^*, k_j^*, m_j^*) = \left(Q_{j,m-1}^*, k_{j,m-1}^*, m_j - 1\right)$, then (Q_j^*, k_j^*, m_j^*) and $R_j^* = D_L + k_j^* \sigma \sqrt{L}$ are the optimal solution for each discount level, j.

Step 7 Set $ITC_j(Q^*, k^*, m^*) = \text{Min}_{j=1,2,\ldots,J}, \ ITC_j(Q_j^*, k_j^*, m_j^*)$ then (Q^*, k^*, m^*) as optimal minimum. Hence, the optimal reorder point is $R^* = D_L + k^* \sigma \sqrt{L}$.

4.2 Proposed algorithm for incremental quantity discount

Step 0 Set $m = 1$ and $ITC_j(Q_{m-1}^*, k_{m-1}^*, m-1) = \infty$.

Step 1 For discount level, $j = 0, 1, 2, \ldots, J$.

- 1.1 Compute the additional cost for the incremental discount, Z_j use equation (15) and purchasing cost at discount level j, C_{bj} based on the supplier's quantity discount schedule.
- 1.2 Set $k_j = 0$ [implies $\psi(k_j) = 0.39894$ and $\Phi(k_j) = 0.50$].
- 1.3 Substitute $\psi(k_j)$ into equation (20) to evaluate Q_j.
- 1.4 Calculate actual shipping weight, $W_{yj} = Q_j.w$. If $W_{yj} \le W_x$ is satisfied go to Step (1.6). Otherwise, go to Step (1.5) if TL constraint is not satisfied or $W_{yj} > W_x$.
- 1.5 Revised delivery lot size, $Q_j = \dfrac{W_x}{w}$ and go to Step (1.6).
- 1.6 Use Q_j to determine $\Phi(k_j)$ from equation (21) then find k_j from $\Phi(k_j)$ by checking the normal table.
- 1.7 Repeat (1.3)–(1.6) until no change occurs in the values of Q_j and k_j.

Step 2 Check the validation for the solution that obtained from Step (1).

- 2.1 For $j = 0, 1, 2, \ldots, J - 1$.
  - a If $q_j \le Q_j < q_{j+1}$, set $Q_j^* = Q_j$ and $k_j^* = k_j$ as the optimal valid solutions.
  - b If $Q_j \ge q_{j+1}$, the solutions is invalid and set $ITC_j(Q_m^*, k_m^*, m) = \infty$.
  - c If $Q_j < q_j$, set $Q_j^* = q_j$ as the optimal valid solutions and compute k_j^* using equation (21).
- 2.2 For $j = J$.
  - a If $Q_j \ge q_j$, set $Q_j^* = Q_j$ and $k_j^* = k_j$ as the optimal valid solutions.
  - b If $Q_j < q_j$, set $Q_j^* = q_j$ as the optimal valid solutions and compute k_j^* using equation (21).

Step 3 Set the valid of Q_j^* and k_j^* and compute $ITC_j(Q_m^*, k_m^*, m)$ using equation (19).

Step 4 If $ITC_j(Q_m^*, k_m^*, m) \le ITC_j(Q_{m-1}^*, k_{m-1}^*, m-1)$, then go to Step (5), otherwise go to Step (6).

Step 5 Set $m = m + 1$, repeat Step 1.

Step 6 $(Q_j^*, k_j^*, m_j^*) = (Q_{j,m-1}^*, k_{j,m-1}^*, m_j - 1)$, then (Q_j^*, k_j^*, m_j^*) and $R_j^* = D_L + k_j^* \sigma \sqrt{L}$ are the optimal solution for each discount level, j.

Step 7 Set $ITC_j(Q^*, k^*, m^*) = Min_{j=1,2,\ldots,J}$, $ITC_j(Q_j^*, k_j^*, m_j^*)$ then (Q^*, k^*, m^*) as optimal minimum. Hence, the optimal reorder point is $R^* = D_L + k^* \sigma \sqrt{L}$.

5 Numerical example and sensitivity analysis

In this study, we consider shipping distance parameter. In previous study, the freight rates are only the function of shipping weight. The numerical example here includes actual freight rate schedule and parameter data. To illustrate the above solution procedure, the parameter data (Table 2) is adopted from Leuveano et al. (2014b). The quantity schedule from supplier is shown in Table 3.

Table 2 Parameter data

Parameter	*Value*	*Unit*	*Parameter*	*Value*	*Unit*
D	10,000	units/year	π_0	300	\$/unit
P	40,000	units/year	σ	7	units/week
A	30	\$/order	β	0.25	-
S	3,600	\$/setup	α	0.11246	-
r	0.2	-	w	22	lbs/unit
C_v	190	\$/unit	d	600	miles
L	56	days	F_x	0.000040217	\$/lb/mile
F_0	50	\$/trip	W_x	46,000	lbs

Table 3 Supplier's discount schedule

j	*Q (unit)*	C_{bj}*(\$/unit)*
0	$0 \le Q < 220$	350
1	$220 \le Q < 250$	300
2	$250 \le Q < 650$	250
3	$650 \le Q < 900$	230
4	$900 \le Q < 1{,}000$	210
5	$Q \ge 1{,}000$	200

Table 4 The verification for the proposed models

		*Proposed model**	*Leuveano et al. (2014b)*	*Proposed model***		*Jauhari et al. (2016)*	
				All-units quantity discount	*Incremental quantity discount*	*All-units quantity discount*	*Incremental quantity discount*
Buyer	Q^*(units)	399.36	399.36	261.03	172.84	261.03	172.84
	R^*(units)	2,288.51	2,288.51	2.290.73	2.291.25	2.290.73	2.291.25
	K^*	2.45	2.45	2.56	2.59	2.56	2.59
	W_y^*(lb)	8,785.89	8,785.89	5,742.60	3,802.41	5,742.60	3,802.41
	j^*	-	-	2	0	2	0
	C_{bj}^*($/unit)	-	-	250	350	250	350
	Buyer total cost	20,048.27	20,048.27	10,525.57	11,808.92	10,525.57	11,808.92
Supplier	M^*	4	4	6	9	6	9
	$Q_v^* = m^*Q^*$(units)	1,597.44	1,597.44	1,566.16	1,555.53	1,566.16	1,555.53
	Supplier total cost	42,757.68	42,757.68	44,739.68	46,560.50	44,739.68	46,560.50
System	System total cost	62,805.95	62,805.95	55,265.25	58,369.42	55,265.25	58,369.42

Notes: *With C_b = $225/unit, **freight cost is zero.

In order to test our proposed algorithm is the same algorithm with Leuveano et al.'s (2014b) algorithm and Jauhari et al.'s (2016) algorithm, we assume freight cost is zero and buyer's purchasing cost is equal to \$225/unit. The verification for our proposed model with Leuveano et al. (2014b) and Jauhari et al. (2016) is shown in Table 4. The results show that when quantity discount is not an option, our model conforms to Leuveano et al.'s (2014b) result. When the freight cost is zero, our model conforms to Jauhari et al.'s (2016) result.

Table 5 presents the actual freight rate schedule by considering shipping weight and distance. The data is adopted from Swenseth and Godfrey (2002). The freight rates are redefined from the freight rate per pound to freight rate per pound per mile. For instance, we assume FTL can be delivered 600 miles with weight equal and more than 18,257 lbs and freight cost is constant charge (\$1,110/shipment). Therefore, the freight rate per pound per mile is obtained by dividing freight rate per shipment with the highest break point and distance. For example, the freight rate per pound per mile for FTL (assuming capacity: 46,000) so we obtain \$1,110 / 46,000 lb / 600 mile = \$0.000040217/lb/mile.

Unlike the case of 10,000 – 18,000 lb, the freight rate per pound is a variable rate based on the load transported by the LTL. To find freight rate per pound per mile can be obtained by dividing freight rate per pond with distance. For example, the freight rate per pound per mile for LTL we obtain \$0.0608/lb / 600 mile = \$0.000101333/lb/mile.

Table 5 Freight rate schedule as the function of shipping weight and distance

Weight break	F_x */ pound*	F_x */ pound/miles*
1–227 lb*	\$40	\$0.000293685
228–420 lb	\$0.176/lb	\$0.000293333
421–499 lb*	\$74	\$0.000247161
500–932 lb	\$0.148/lb	\$0.000246666
933–999 lb*	\$138	\$0.000230230
1,000–1,855 lb	\$0.138/lb	\$0.000230000
1,856–1,999 lb*	\$256	\$0.000213440
2,000–4,749 lb	\$0.128/lb	\$0.000213333
4,750–9,999 lb*	\$608	\$0.000101343
10,000–18,256 lb	\$0.0608/lb	\$0.000101333
18,257–more*	\$1,110	\$0.000040217

Note: *Constant charge.

Utilising the proposed algorithms, we derived the optimal values of the both discounts policies in Table 6 and 7. Table 6 shows that all-units quantity discount policy where each level is found valid and satisfied weight. For instance, on level $j = 0$ generating the optimal order quantity is 218.26 units (within the range: $0 \leq Q < 220$ units). The optimal shipping weight (W_y^*) is 4,801.67 lbs; it satisfied the weight requirement since $W_y \leq W_x$. Besides that, the optimal number of delivery that we obtain is eight times, the optimal reorder point (R^*) is 2,289.64 units and safety factor (k^*) is 2.51. The integrated total cost on 0th level is \$69,082.47.

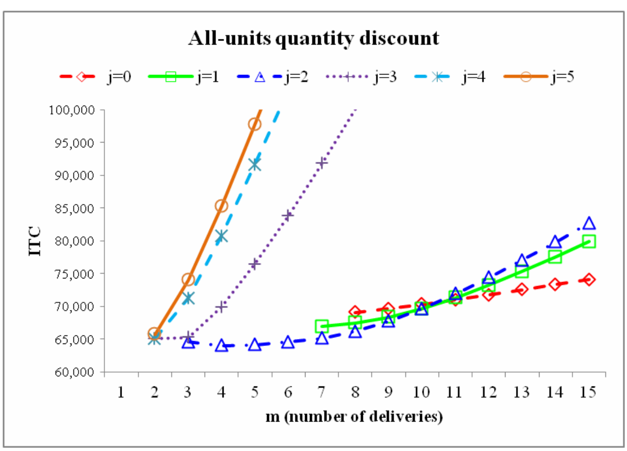
Table 6 The results of all-units quantity discount for W_x = 46,000 lbs

j	Q^*	$R^*(k^*)$	W_y^*	m^*	$ITC(Q, k, m)$	*Discounted*	*Actual weight*
0	218.26	2,289.64 (2.51)	4,801.67	8	69,082.47	Valid	Satisfied
1	246.13	2,289.88 (2.52)	5,414.81	7	66,899.69	Valid	Satisfied
2	392.62	2,287.87 (2.42)	8,637.63	4	64,036.92	Valid	Satisfied[a)]
3	694.64	2,284.28 (2.24)	15,282.15	2	65,101.55	Valid	Satisfied
4	900.00	2,282.99 (2.17)	19,800.00	2	65,087.92	Valid	Satisfied
5	1,000.00	2,282.55 (2.15)	22,000.00	2	65,744.56	Valid	Satisfied

Note: [a)]The optimal solution.

Figure 3 shows that at level 0 (price $350), the optimal total cost occurs when m equal to 8. If delivery less than eight times, the optimal order quantity will be invalid. Another example is if the price becomes $300 (first level) and the supplier conducts their delivery from 1 through 6, then the results show the discounts for these deliveries are invalid. Otherwise, if the delivery is conducted on the 7th delivery, then the discount of the optimal order quantity will be valid with 246.13 units. On the price $250 (second level), the cost function will occurs at 2nd delivery and the optimal of delivery is 4. It is similar when the price is decreased gradually of $230, $210 and $200, respectively. The impact is cost function increase on delivery equal to 2 of those prices. We find that the optimal of those prices are m^* = 2. Overall, the integrated total cost minimum of the all-units quantity discount policy is $64,036.92 per unit time when discount level, j^* = 2 (price $250) and the optimal of delivery, m^* = 4.

Figure 3 Total cost with respect m for each discount level, j on all-weight quantity discount for W_x = 46,000 (see online version for colours)



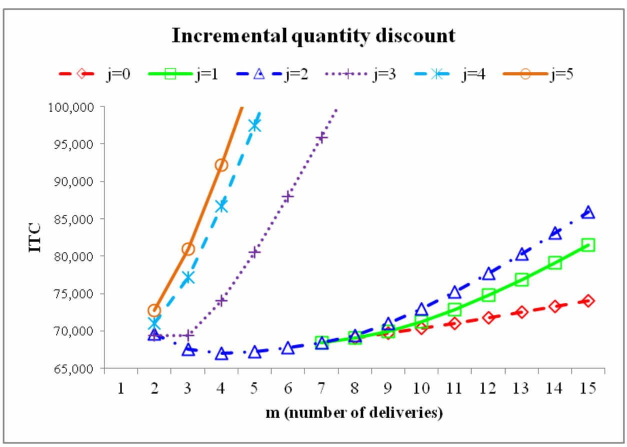
By using the incremental algorithm, the optimal of integrated total cost is $67,073.65 per unit time at level, j = 2 and delivery, m^* = 4 times. Summary and enumeration for the incremental quantity discount with respect to the number of delivery for each discount level are presented in Table 7 and Figure 4.

Table 7 The results of incremental quantity discount for W_x = 46,000 lbs

j	Q^*	$R^*(k^*)$	W_y^*	m^*	$ITC(Q, k, m)$	*Discounted*	*Actual weight*
0	211.17	2,289.87 (2.52)	4,645.65	8	69,115.43	Valid	Satisfied
1	238.06	2,289.11 (2.48)	5,237.38	7	68,490.94	Valid	Satisfied
2	371.53	2,286.64 (2.36)	8,173.61	4	67,073.65	Valid	Satisfied[a)]
3	650.00	2,283.12 (2.18)	14,300.00	2	69,373.60	Valid	Satisfied
4	900.00	2,281.00 (2.07)	19,800.00	2	71,047.17	Valid	Satisfied
5	1,000.00	2,280.33 (2.04)	22,000.00	2	72,729.84	Valid	Satisfied

Note: [a)]The optimal solution.

Figure 4 Total cost with respect m for each discount level, j on incremental quantity discount for W_x = 46,000 (see online version for colours)



We compare and analyse the results of integrated and non-integrated model for both of discount policies. Table 8 shows that the optimal discount level for integrated and non-integrated of all-units quantity discount policy is same, priced at $250 (second level). In the integrated model, we find that the optimal order quantity is 392.62 units, the number of delivery is 4 times and the reorder point is 2,287.87 units. Hence, the integrated total cost is $64,036.92 per unit time. In the non-integrated model, the total cost is $64,882.05 per unit time and the optimal order quantity smaller than the integrated model (255.18 units), the number of delivery is 6 times and the reorder point is 2,290.88 units. The integrated model results of all-units quantity discount policy give a cost saving of \$845.12 (1.30%). Similarly, the integrated model in the incremental discount policy has a cost saving of \$817.94 (1.20%).

Table 8 The comparisons between integrated and non-integrated for all-units and incremental discounts

		All-units quantity discount		*Incremental quantity discount*	
		Non-integrated	*Integrated*	*Non-integrated*	*Integrated*
Buyer	Q^*(units)	255.18	392.62	266.73	371.53
	R^*(units)	2,290.88	2,287.87	2,288.47	2,286.64
	k^*	2.57	2.42	2.45	2.36
	W_y^*(lb)	5,614.05	8,637.63	5,867.98	8,173.61
	j^*	2	2	2	2
	C_{bj}^* (\$/unit)	250.00	250.00	250.00	250.00
	Buyer total cost	\$20,016.25	\$21,191.04	\$23,250.84	\$23,855.99
Supplier	m^*	6	4	6	4
	$Q_v^* = m^*Q^*$(units)	1,531.10	1,570.48	1,600.36	1,486.11
	Supplier total cost	\$44,865.80	\$42,845.88	\$44,640.74	\$43,217.66
System	System total cost	\$64,882.05	\$64,036.92	\$67,891.59	\$67,073.65
	Saving		1.30%[a)]		1.20%[b)]

Notes: [a)]$(TC_{all\text{-}units} - ITC_{all\text{-}units}) / TC_{all\text{-}units}$ x 100%.
[b)]$(TC_{jncremental} - ITC_{jncremental}) / TC_{jncremental}$ x 100%.

Subsequent results discuss the effect of changing shipping weight capacity (W_x) from 25,000 lb; 20,000 lb; 10,000 lb and 5,000 lb. Table 9 shows when capacity, W_x = 25,000 lb, the optimal total cost occurs at discount level, j^* = 2 and delivery is five times for both discount policies. The results show that for all discount levels and actual weight are valid and satisfied, respectively. However, in 20,000 lb capacity, Table 10 shows that the 5th of discount level is not satisfied, (the actual weight is over-capacity). Thus, the 5th of discount level can be ignored for both discounts. The minimum integrated total cost for both discounts on second level and delivery is five times. When the capacity is reduced to 10,000 lb; the actual weight on only three discount levels are satisfied. We find that the minimum integrated total cost of both discounts at level equal to 2 and optimal delivery from the supplier to the buyer is 5 (Table 11). In contrast to 5,000 lb capacity (see Table 12), the satisfied weight is only two levels (\$350 and \$300) for both discounts. Hence, the comparison is only performed on both of levels. We find that the optimal solution at price \$300 for both discounts and supplier's delivery of their product to the buyer is seven times.

From the results of Tables 9 to 12, it is shown that the shipping weight capacity gives an impact on cost and making decision. When the capacity is decreased stepwise, there is tighter freight restriction on deliver the product from the supplier to the buyer. The effects to the buyer are smaller discount level and lot size. Consequently, the effect on the buyer is lower holding costs. The impacts on the supplier are more frequent shipment and minimise the supplier's setup cost. It can be concluded that these results are consistent with the concept of win-win solution.

Table 9 The results of both quantity discounts for W_x = 25,000 lbs

Discount	*j*	Q^*	$R^*(k^*)$	W_y^*	m^*	*ITC(Q, k, m)*	*Discounted*	*Actual weight*
All-units quantity	0	208.61	2,289.96 (2.52)	4,589.40	8	66,412.45	Valid	Satisfied
	1	236.23	2,290.16 (2.53)	5,197.11	7	64,536.84	Valid	Satisfied
	2	318.13	2,289.36 (2.49)	6,998.90	5	62,377.01	Valid	Satisfied[a)]
	3	684.75	2,284.39 (2.24)	15,064.45	2	64,275.28	Valid	Satisfied
	4	900.00	2,282.99 (2.17)	19,800.00	2	64,454.73	Valid	Satisfied
	5	1,000.00	2,282.55 (2.15)	22,000.00	2	65,174.69	Valid	Satisfied
Incremental quantity	0	201.83	2,290.18 (2.53)	4,440.24	8	66,443.96	Valid	Satisfied
	1	228.59	2,289.36 (2.49)	5,028.89	7	66,147.52	Valid	Satisfied
	2	304.51	2,287.76 (2.41)	6,699.28	5	65,531.33	Valid	Satisfied[a)]
	3	650.00	2,283.12 (2.18)	14,300.00	2	68,496.87	Valid	Satisfied
	4	900.00	2,281.00 (2.07)	19,800.00	2	70,413.98	Valid	Satisfied
	5	1,000.00	2,280.33 (2.04)	22,000.00	2	72,159.97	Valid	Satisfied

Note: [a)]The optimal solution.

Table 10 The results of both quantity discounts for W_x = 20,000 lbs

Discount	*j*	Q^*	$R^*(k^*)$	W_y^*	m^*	*ITC(Q, k, m)*	*Discounted*	*Actual weight*
All-units quantity	0	206.24	2,290.03 (2.53)	4,537.39	8	65,758.32	Valid	Satisfied
	1	233.81	2,290.23 (2.54)	5,143.91	7	63,959.51	Valid	Satisfied
	2	315.65	2,289.41 (2.50)	6,944.26	5	61,948.83	Valid	Satisfied[a)]
	3	682.37	2,284.42 (2.24)	15,012.14	2	64,076.78	Valid	Satisfied
	4	900.00	2,282.99 (2.17)	19,800.00	2	64,303.97	Valid	Satisfied
	5	1,000.00	2,282.55 (2.15)	22,000.00	2	65,039.01	Valid	*Not satisfied*
Incremental quantity	0	199.54	2,290.26 (2.54)	4,389.90	8	65,789.47	Valid	Satisfied
	1	226.27	2,289.42 (2.50)	4,977.95	7	65,575.23	Valid	Satisfied
	2	302.18	2,287.81 (2.41)	6,647.86	5	65,108.98	Valid	Satisfied[a)]
	3	650.00	2,283.12 (2.18)	14,300.00	2	68,288.13	Valid	Satisfied
	4	900.00	2,281.00 (2.07)	19,800.00	2	70,263.22	Valid	Satisfied
	5	1,000.00	2,280.33 (2.04)	22,000.00	2	72,024.28	Valid	*Not satisfied*

Note: [a)]The optimal solution.

Table 11 The results of both quantity discounts for W_x = 10,000 lbs

Discount	*j*	Q^*	$R^*(k^*)$	W_y^*	m^*	*ITC(Q, k, m)*	*Discounted*	*Actual weight*
All-units quantity	0	208.70	2,289.95 (2.52)	4,591.44	8	73,598.07	Valid	Satisfied
	1	236.33	2,290.16 (2.53)	5,199.19	7	71,719.47	Valid	Satisfied
	2	318.23	2,289.36 (2.49)	7,001.04	5	69,553.83	Valid	Satisfied[a)]
	3	650.00	2,284.78 (2.26)	14,300.00	2	71,521.09	Valid	*Not satisfied*
	4	900.00	2,282.99 (2.17)	19,800.00	2	71,620.72	Valid	*Not satisfied*
	5	1,000.00	2,282.55 (2.15)	22,000.00	2	72,340.09	Valid	*Not satisfied*
Incremental quantity	0	201.92	2,290.18 (2.53)	4,442.20	8	73,629.60	Valid	Satisfied
	1	228.68	2,289.35 (2.49)	5,030.88	7	73,329.96	Valid	Satisfied
	2	304.60	2,287.76 (2.41)	6,701.30	5	72,707.93	Valid	Satisfied[a)]
	3	650.00	2,283.12 (2.18)	14,300.00	2	75,665.14	Valid	*Not satisfied*
	4	900.00	2,281.00 (2.07)	19,800.00	2	77,579.97	Valid	*Not satisfied*
	5	1,000.00	2,280.33 (2.04)	22,000.00	2	79,325.36	Valid	*Not satisfied*

Note: [a)]The optimal solution.

Table 12 The results of both quantity discounts for $W_x = 5{,}000$ lbs

Discount	*j*	Q^*	$R^*(k^*)$	W_y^*	m^*	*ITC(Q, k, m)*	*Discounted*	*Actual weight*
All-units quantity	0	202.69	2,290.15 (2.53)	4,459.28	8	71,937.38	Valid	Satisfied
	1	227.27	2,290.43 (2.55)	5,000.00	7	70,259.59	Valid	Satisfied[a]
	2	250.00	2,291.02 (2.58)	5,500.00	6	68,554.57	Valid	*Not satisfied*
	3	650.00	2,284.78 (2.26)	14,300.00	2	70,996.35	Valid	*Not satisfied*
	4	900.00	2,282.99 (2.17)	19,800.00	2	71,242.06	Valid	*Not satisfied*
	5	1,000.00	2,282.55 (2.15)	22,000.00	2	71,999.41	Valid	*Not satisfied*
Incremental quantity	0	217.66	2,289.66 (2.51)	4,788.50	7	71,931.57	Valid	Satisfied
	1	222.80	2,289.51 (2.5)	4,901.59	7	71,878.61	Valid	Satisfied[a]
	2	250.00	2,288.81 (2.47)	5,500.00	6	71,842.56	Valid	*Not satisfied*
	3	650.00	2,283.12 (2.18)	14,300.00	2	75,140.40	Valid	*Not satisfied*
	4	900.00	2,281.00 (2.07)	19,800.00	2	77,201.31	Valid	*Not satisfied*
	5	1,000.00	2,280.33 (2.04)	22,000.00	2	78,984.69	Valid	*Not satisfied*

Note: [a]The optimal solution.

Table 13 The effect of change parameters for all-units quantity discount

Parameter and changed value		Non-integrated					Integrated					Saving[a)]
		Q^*	$R^*(k^*)$	m^*	j^*	TC	Q^*	$R^*(k^*)$	m^*	j^*	ITC	
D	2,500	250.00	600.78 (2.06)	3	2	33,081.31	250.00	600.78 (2.06)	3	2	33,081.31	0.00%
	5,000	1,000.00	1,156.88 (1.86)	1	5	45,538.62	1,000.00	1,156.88 (1.86)	1	5	45,538.62	0.00%
	15,000	310.98	3,412.29 (2.64)	7	2	76,823.13	419.38	3,410.26 (2.54)	5	2	76,368.05	0.59%
	17,500	335.36	3,972.81 (2.67)	7	2	81,393.38	404.72	3,971.55 (2.6)	6	2	81,043.70	0.43%
σ	2	250.65	2,254.57 (2.58)	6	2	62,955.08	391.01	2,253.69 (2.42)	4	2	62,093.29	1.37%
	4	252.45	2,269.12 (2.57)	6	2	63,724.66	391.65	2,267.36 (2.42)	4	2	62,870.86	1.34%
	11	900.00	2,307.56 (2.17)	2	4	66,288.15	393.91	2,315.19 (2.42)	4	2	65,591.15	1.05%
	13	900.00	2,319.84 (2.17)	2	4	66,888.27	394.56	2,328.84 (2.42)	4	2	66,368.03	0.78%
w	6	255.18	2,290.88 (2.57)	6	2	61,455.41	392.62	2,287.87 (2.42)	4	2	60,610.28	1.38%
	11	255.18	2,290.88 (2.57)	6	2	62,526.23	392.62	2,287.87 (2.42)	4	2	61,681.11	1.35%
	33	255.18	2,290.88 (2.57)	6	2	67,237.86	392.62	2,287.87 (2.42)	4	2	66,392.74	1.26%
	40	255.18	2,290.88 (2.57)	6	2	68,737.02	392.62	2,287.87 (2.42)	4	2	67,891.90	1.23%
d	150	250.00	2,291.02 (2.58)	6	2	57,740.26	266.92	2,290.58 (2.55)	6	2	57,625.37	0.20%
	300	250.00	2,291.02 (2.58)	6	2	60,166.46	317.14	2,289.38 (2.49)	5	2	59,850.32	0.53%
	900	900.00	2,282.99 (2.17)	2	4	68,137.23	403.50	2,287.67 (2.41)	4	2	67,960.71	0.26%
	1,050	900.00	2,282.99 (2.17)	2	4	69,661.89	900.00	2,282.99 (2.17)	2	4	69,661.89	0.00%

Note: [a)]$(TC – ITC) / TC x$ 100%.

Table 14 The effect of change parameters for incremental quantity discount

Parameter and changed value		Non-integrated					Integrated					Saving[a)]
		Q^*	$R^*(k^*)$	m^*	j^*	*TC*	Q^*	$R^*(k^*)$	m^*	j^*	*ITC*	
D	2,500	250.00	598.16 (1.93)	3	2	36,174.70	163.65	601.48 (2.1)	4	0	36,129.19	0.13%
	5,000	250.00	1,163.72 (2.21)	4	2	49,600.35	311.67	1,162.44 (2.14)	3	2	49,360.56	0.48%
	15,000	320.70	3,410.33 (2.54)	7	2	79,867.55	409.56	3,408.98 (2.47)	5	2	79,307.15	0.70%
	17,500	344.45	3,971.01 (2.58)	7	2	84,337.61	401.41	3,970.17 (2.53)	6	2	83,991.94	0.41%
σ	2	254.01	2,253.92 (2.46)	6	2	65,481.85	368.42	2,253.34 (2.36)	4	2	64,716.76	1.17%
	4	259.12	2,267.79 (2.46)	6	2	66,438.06	369.66	2,266.67 (2.36)	4	2	65,660.45	1.17%
	11	276.77	2,315.87 (2.44)	6	2	69,860.70	374.01	2,313.23 (2.35)	4	2	68,953.61	1.30%
	13	281.75	2,329.49 (2.43)	6	2	70,856.82	375.26	2,326.51 (2.35)	4	2	69,891.77	1.36%
w	6	266.73	2,288.47 (2.45)	6	2	64,464.94	371.53	2,286.64 (2.36)	4	2	63,647.01	1.27%
	11	266.73	2,288.47 (2.45)	6	2	65,535.77	371.53	2,286.64 (2.36)	4	2	64,717.83	1.25%
	33	266.73	2,288.47 (2.45)	6	2	70,247.40	371.53	2,286.64 (2.36)	4	2	69,429.47	1.16%
	40	266.73	2,288.47 (2.45)	6	2	71,746.56	371.53	2,286.64 (2.36)	4	2	70,928.62	1.14%
d	150	250.00	2,288.81 (2.47)	6	2	61,028.25	298.16	2,287.88 (2.42)	5	2	60,849.77	0.29%
	300	250.00	2,288.81 (2.47)	6	2	63,454.45	303.58	2,287.78 (2.41)	5	2	63,006.96	0.71%
	900	310.90	2,287.65 (2.41)	5	2	71,672.29	381.69	2,286.48 (2.35)	4	2	70,982.83	0.96%
	1,050	330.75	2,287.3 (2.39)	5	2	73,540.75	475.12	2,285.15 (2.28)	3	2	72,920.96	0.84%

Note: [a)](*TC* – *ITC*) / *TCx* 100%.

We then analyse the effect of changing in average demand, standard deviation of demand, weight per unit and distance on the proposed models. The values will be changed from –75%, –50%, 50% and 75%. The results of sensitivity are given in Table 13 and Table 14. Through the analysis, it is shown that the percentage saving is not sensitive to demand, but inversely proportion to increasing standard deviation of demand. The percentage savings are also sensitive to the weight of per unit and distance. Hence, it could be concluded that the freight costs are influenced by the actual capacity, stochastic environment, weight of per unit and distance from the supplier to the buyer. Table 13 and Table 14 show that the integrated model performs better than the non-integrated models, but the cost savings are not always significant. To entice coordination, the cost savings obtained from integrated policy can be shared by using a mechanism such as credit payment from the supplier to the buyer.

6 Conclusions

This paper provides an integrated inventory with freight cost, quantity discount and stochastic demand. Two types of quantity discount are formulated based on all-units quantity and incremental quantity policies. Shortage is allowed, and reorder point is one of our decision variables. We seek to minimise the integrated total cost by simultaneously finding the ordering quantity, the reorder point and the number of delivery from the supplier to the buyer. We propose two algorithms to find the solution of the proposed models. Numerical examples are performed to see the impact of supplier's discount schedule, freight cost and shipping weight. A comprehensive analysis is carried out based on the sensitivity analysis showing a detailed comparison of two types of quantity discounts. The analysis showed that if the supplier offers a lower unit price and the logistics providers offers a lower shipping weight then the optimal lot size of the buyer become a little more.

This work can be extended in several ways. For instance, the impact of crashing cost on variable lead time and fuel price based on a real case study. In addition, deterioration of quality and GHG emissions are factors to be considered for future research.

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