Approaches to strongly local phenomena in dry masonry structures

Fernando Magdalena Layos and Julián García Muñoz*

Departamento de Construcciones Arquitectónicas y su Control,
Escuela Técnica Superior de Edificación –
Universidad Politécnica de Madrid,
Av Juan de Herrera, 6. 28040, Madrid, Spain
Email: fernando.magdalena@upm.es
Email: julian.garciam@upm.es
*Corresponding author

Abstract: Sliding collapse tests performed on dry-stack masonry structures show a statistical dispersion in the results that cannot be explained by the dispersion in the properties of its constituent materials, but may correspond to an uneven distribution of stresses at the contact surfaces or within the constituent elements. In order to study these local phenomena, a method of numerical analysis of rigid blocks in unilateral contact is implemented and confronted with two types of benchmark tests: load tests and photoelastic tests. Comparing the results will allow stating that the random irregularities in contact surfaces play an important role in local behaviour. The results are very encouraging and suggest that this type of approach, common in the field of granular media but not in orthotropic media, can provide interesting insights for the analysis of local phenomena in historical masonry structures.

Keywords: masonry structures; numerical analysis; discontinuous media photoelasticity; rigid blocks; unilateral contact; friction.


Biographical notes: Fernando Magdalena Layos is a multi-skilled architect with comprehensive experience in structural design. He obtained his PhD in the Department of Structures at the School of Architecture, Technical University of Madrid (Universidad Politécnica de Madrid – UPM), Spain. He is an Associate Professor at the School of Building Engineering of UPM (Escuela Técnica Superior de Edificación de Madrid – ETSEM) since 2012, he is a Member of the Research Group Analysis and Intervention in Architectural Heritage (Análisis e Intervención en Patrimonio Arquitectónico – AIPA). His academic career revolves around research on the analysis of masonry structures using rigid solid models, a subject he studied for his PhD and on which he has published numerous papers.

Julián García Muñoz is a Building Engineer with a degree in Art History. He obtained his PhD in the Department of Construction at the School of Building Engineering, Technical University of Madrid (Universidad Politécnica de Madrid – UPM), Spain. He is a Professor at IE School of Architecture (2000–2009) and the School of Building Engineering of UPM (Escuela Técnica
1 Introduction

Several methods are available to study the behaviour of historical masonry structures. A comprehensive summary of them is beyond the scope of this work but can be found in excellent reviews (Roca et al., 2010; Tralli et al., 2014; Baraldi et al., 2015). This variety allows studying very different events with adequate tools in almost every occasion. However, many of these methods allow only the analysis of the global behaviour. The local behaviour, on the other hand, is quite elusive, especially when the properties of the material are uncertain, the characteristics of the joint materials (and even the existence of the latter) are unknown and the resistance of the bond depends on friction (Magdalena, 2013). The proposed approach falls within the micromechanical methods and will be limited to dry or quasi-dry masonry structures. Other approaches in which the joint material plays an important role are excluded.

Different load tests carried out by sliding collapse on dry-stack walls (Magdalena et al., 2015a) (Figure 1(A) and (B) show a range of results with random distribution and wide dispersion (Figure 1(C)) that cannot be explained by the dispersion in the properties of the constituent materials. Since this type of behaviour has theoretical interest and can be a practical problem in certain cases (for example in anchorages of braces on masonry structures) alternative models that allow better approximations to these local phenomena must be studied.

The comparative use of numerical and experimental methods may be interesting for understanding problems of local behaviour. Similar approaches in recent literature seek, for example, to compare the results obtained in load tests of dry-stack walls (Magdalena et al., 2015a) with numerical models of unilateral contact between rigid blocks (Magdalena et al., 2015b). This type of approach allows a good comparison of the global numerical results, but does not allow understanding the internal behaviour of the structure and therefore the causes of such behaviour. To avoid this problem, the use of photoelastic models as experimental complementary benchmark tests has recently been proposed by some authors (Bigoni and Noselli, 2010a). Such approach is controversial, as photoelastic materials and models imply serious difficulties for the study of collapse in masonry structures.

Despite these difficulties, photoelastic models can provide interesting information if the results obtained are considered at qualitative level and if the numerical model with
which they are compared is adequate. The use of photoelastic tests as a benchmark method is here proposed not as a unique method but in combination with load tests. The benchmarking of those two approaches with a numerical model of rigid blocks will allow comparing qualitatively and quantitatively the hypothesis that the discontinuity of the medium and the inevitable irregularities in the contact between parts play a relevant role in local behaviour.

Experimental load tests, photoelastic tests and the method of numerical analysis are described below. Qualitative and quantitative comparisons between them will be discussed in order to study the relevance of this comparative methodology and its scope.

2 Experimental methods

Two experimental methods are proposed. The first method will allow studying, through a set of usual loading tests with conventional ceramic bricks, the problem of the random distribution and the wide degree of dispersion of the loads that lead to collapse. The second method will enable observing, through photoelastic tests, the internal behaviour of a discontinuous orthotropic deformable medium with some similarities with historical masonry structures.

In order to compare the results of these tests with those of the numerical analysis using rigid blocks in unilateral contact, the models proposed below are different in geometry, materials and type of stress. The photoelastic model is based on those previously used by several authors (Bigoni and Noselli, 2010a; Baig et al., 2015) and is suitable for showing the random distribution of stresses, while the brick model is designed to study the local collapse of the structure by sliding.

These differences will be essential to allow us studying them in a comparative way, and therefore understanding if the numerical analysis using rigid blocks is a correct analytical approximation for the simulation of local behaviour.

2.1 Load tests

Load tests of similar structures have been published (Oliveira, 2000; Restrepo et al., 2014) including the application of load on the wall and collapsing the structure by crushing, rocking, twisting, bending or mixed modes that include sliding. The type of load tests that seemed to fit better the proposed approach is that of pure sliding collapse. Since the late nineteenth century (Painlevé, 1895) it is a well-known fact that problems of rigid-body dynamics including slide are inconsistent (Painlevé paradox). Drucker (1953) showed that, in the presence of sliding, theorems of Standard Limit Analysis (SLA) are not applicable and therefore there is no guarantee that the solution is unique. Finally, numerical results obtained using probabilistic methods (Magdalena and Hernando, 2014) suggest that among all possible solutions, not all are equally likely -therefore some experimental tests were necessary.

As a part of a larger set, 33 pure sliding collapse tests were carried out at the Building Structures Department Laboratory (ETSAM, Universidad Politécnica de Madrid) on a dry-stack masonry wall composed of the same 98 pieces randomly replaced arranged on a wooden plank fully supported by a reinforced concrete foundation. (Magdalena et al., 2016). The tests were designed to achieve an in-plane collapse by pure sliding and in quasi-static mode. The physical model reproduced that of the numerical models that had
given a greater dispersion in the results. The aim was to magnify the effect of randomness to facilitate its comparison with previously obtained numerical results that gave a value of 45 N for the overall minimum of the Non-Standard Limit Analysis and 589 N for the maximum obtained by Standard Limit Analysis, as well as a value minimum of 178 N and a maximum of 540 N in 1000 numerical simulations, being 5% percentile 298 N.

The objective was to verify to which extent the random contact conditions influence the collapse load of physical models that are equal in all their other characteristics. The test consisted of applying a horizontal point load at the same point of the structure and increasing it until the sliding rupture began, which was measured by a dynamometer. The test scheme is shown in Figure 1(A); the results, in Figures 1(B) and (C). A full description can be found in the related paper.

**Figure 1** (A) Scheme of the load test performed at the ETSAM-UPM; (B) three examples of collapse mechanisms and (C) table of values of the force that causes slide and its graphical representation: in black the histogram of the results, in red the Gaussian kernel density and in numbers the values of maximum, minimum and confidence interval for the 5% percentile characteristic value (see online version for colours)
2.2 Photoelastic method

2.2.1 Method and background

The principle on which photoelastic tests are based is simple: under the polarised light, the phenomenon of double refraction in birefringent deformable materials enables observing in photoelastic models signs (in a linguistic manner) of the presence of otherwise non-detectable stress phenomena through direct light. Photoelastic results can help to establish stress patterns, to determine areas of stress concentration or to understand the overall behaviour of a model. The quantification of these stresses, however, is difficult if high accuracy is sought, so these tests are often used to obtain only qualitative approximations.

Photoelasticity has been used since the beginning of the 20th century as an experimental method to determine the internal stresses in structural systems (Frocht, 1965) especially in the study of continuous media. Its use in the analysis of discontinuous media is far more recent, although some authors (Drescher and Josseling de Jong, 1972) used photoelastic tests on granular media with the intention of evaluating the random nature of the internal distributions of stresses. However, with regard to discontinuous orthotropic media, such as the masonry structures studied hereafter, the use of photoelasticity has been rare. McNicholas (1970) used a birefringent coating for tensile analysis in a 1 : 1 scale model; Heinrich (1977) studied a bended arc with strong friction in the beds and associated internal stresses in the material with thrust lines and collapsing...
mechanisms; Rajchenbach (2001) proposed a study on a rectangular block model similar to the one used here, in which he performed an analysis of vertical distribution through statistics; Bigoni and Noselli (2010a, 2010b) recently proposed a model of rectangular blocks, which was tested with different photoelastic materials and compared with mathematical models of random distribution; and finally Baig et al. (2015), using a model of a similar geometry to that used by Bigoni, designed an algorithm of infographic retouching to transform the photoelastic information into lines or stress streams.

2.2.2 Implementation

In order to study the irregularity of the stress distribution in the contacts between faces and to facilitate the comparison of results with the reference literature, the model proposed for photoelastic tests and its corresponding numerical simulation is similar to the one proposed by Bigoni. It represents a wall of dry-stack masonry composed by 187 cuboid pieces distributed in 22 rows. 165 have a proportion ratio of 2×1 and 22 of 3×1. They are arranged in a conventional bond, forming a 180 mm wide and 220 high set. A vertical point load is applied to the centre of the upper course using an auxiliary piece. The model can be seen inside the confinement box in Figure 2(A), and is identical to the one represented in Figure 3(A).

The photoelastic model was tested under a 100 N load. In the comparative numerical analysis, however, a load of 250 N was simulated in order to obtain the same relation between the applied load and the model’s self-weight, and therefore a similar dispersion in the contact stresses. The relationship between both loads is proportional to the thickness and specific weight in the different models.

The material used in the blocks manufacturing is a two-component epoxy resin, polymerised at room temperature. A complete description of the material used and the test conditions can be found in Magdalena et al. (2016b). Once cut, the resin blocks have been placed in dry and direct contact, assuming the irregularities of the cut.

The resulting set was vertically installed in a PMMA confinement box (Figure 2(A)) designed to limit the deformation of the model inside a vertical plane. The two confinement main surfaces, which frontally limit the bending tendency of the model, are 6 mm apart from each other. This will avoid friction between them and the 5 mm thick model, allowing only punctual contacts that would not distort the test results. In the lower area, the perimeter sash acts as a base or foundation of the model, while on the sides, it is an element that can contain thrust when necessary. Similar strategies have been used in other recent studies (Rajchenbach, 2001).

The room temperature during the tests can be estimated at 22.5°C, slightly over the 21°C of the model’s surface. The load was applied vertically to the model using a compression screw attached to a fixed axis and to several interposed PMMA blocks, being the last contact element a resin block of 20×10×5 mm. The point load applied on that last element was 100 N, which lead to stresses of 0.5 N/mm² maximum and to a mean stress in the base of 0.1 N/mm². Sixteen of such tests were performed, randomly rearranging blocks in every occasion.

Results allow both global (Figure 2(B)) and local (Figure 4(A)–(C)) interpretations. It should be noted that a greater concentration of isochromatic lines, and not a more apparently intense backlighting, implies a greater stress in the blocks (Frocht, 1965).
Results show, at a global scale, a random distribution of stresses similar to the one described by mathematical simulations. At a local level, however, irregular concentrations at certain points of contact appear (Figure 4(B) and (C)). These concentrations generate discontinuities, either by an excess of stress – causing stress streams – or by default – giving rise to unloaded islands (Figure 4(A)).

Figure 2  (A) Geometric characteristics of the model and conditions of the photoelastic tests:
1 – monochromatic light source; 2 – vertical polariser; 3 – point load; 4 – resin model;
5 – PMMA confinement box; 6 – horizontal polariser; 7 – capture and analysis.
(B) Three results of the photoelastic tests, in which different stress currents and low stress islets can be noticed (see online version for colours)
Figure 3  (A) Geometric characteristics of the model for the numerical analysis by rigid blocks in unilateral contact. (B) Three values of results of the numerical analysis, in which different stress streams and unloaded islands are manifested: in the first case, a substantially vertical distribution; in the second, an irregular one; and a regular one in the third case (see online version for colours)

Figure 4  (A) Detail of stress streams and unloaded islands in one of the photoelastic models tested; (B) the stress stream is transmitted vertically and (C) the stress stream is distributed in two blocks (see online version for colours)
3 Numerical analysis through rigid blocks in unilateral contact

3.1 Method and background

There is a long tradition in the field of numerical analysis of discontinuous media when it comes to approaching the problem of a set of blocks in contact, both in geomechanics (Cundall, 1971) and in structural analysis (Livesley, 1978; Gilbert and Melbourne, 1994). The method here proposed is classified as “Advanced Computer Developments Based on Limit Analysis: Analysis of Blocky Structures” (Roca et al., 2010). In its most elementary formulation using discrete elements, the problem can be considered as one of unilateral contact between rigid blocks (Fishwick, 1996; Ferris and Tin-Loi, 2001). More recent research (Orduña and Lourenço, 2005) and (Portioli et al., 2015) agree with the method proposed in the fundamental aspects of the formulation of the problem, but do not coincide in the approach to its resolution, the most important difference being that not a single solution but a multiple solution approach is sought. This paper proposes that this multiplicity of solutions is not only a characteristic of the mathematical formulation but also a physical fact that is corroborated by the load tests designed for this purpose.

The numerical model (Figure 3(A)) simulates a set of rigid solids in unilateral, dry and direct contact, with finite friction and a non-associative friction-slide law. The material of the model is one of high compressive strength – far superior to the compressions to which it is going to be subjected, and little or no tensile strength. In the case of quasi-dry masonry structures, not considering the contribution of the tensile strength of the joint material is on the safe side, as there is often no record of the damages suffered by historical constructions – and mortar joints are usually the area most susceptible to deterioration.

3.2 Implementation

This type of problem can be formulated by implementing a set of equations and static constraints (1) as a function of a vector of purely static positive variables \( y \geq 0 \); a set of equations and kinematic constraints (2) as a function of a vector of purely kinematic positive variables \( z \geq 0 \); and a set of equations representing unilateral contact conditions.

\[
\begin{align*}
\mathbf{f}(y) &= \mathbf{0}; \quad \mathbf{g}(y) \geq \mathbf{0} \quad (1) \\
\mathbf{f}(z) &= \mathbf{0}; \quad \mathbf{g}(z) \geq \mathbf{0}.
\end{align*}
\]

Sets (1) and (2) are formed by linear equations and constraints that define a linear (and therefore easy to solve) problem: the only difficulty lies, in this type of problem, in unilateral contact conditions. It is important to clarify that when slide does not intervene in the collapse mechanism, solving (1) and obtaining a valid static solution, represented as \( y \in \{ S_y \} \), or (2) obtaining a valid kinematic solution, represented as \( z \in \{ K_z \} \) allows solving the whole problem, leading to the well-known formulations of the static and kinematic theorems of the Standard Limit Analysis.

Should the onset of collapse occur including sliding, it will be necessary to formulate the problem as a whole, since a solution of onset of collapse must be a valid static solution \( y \in \{ S_y \} \), a valid kinematic solution \( z \in \{ K_z \} \) and satisfy the contact conditions.
To incorporate this last type of conditions, these problems of unilateral contact have been formulated since the 1980s as complementarity problems. The main characteristic of this type of problem is that they include, at least, a complementarity constraint, which can be expressed as a condition of orthogonality between two positive vectors (3). As dot product is null, being both vectors $y$, $z$ positive it can be stated that (4):

$$0 \leq y \perp z \geq 0$$  \hspace{1cm} (3)

$$y \cdot z = 0 ; \ y \geq 0 ; \ z \geq 0.$$  \hspace{1cm} (4)

Thus, an onset of collapse solution will be defined by a pair of positive vectors $y$, $z$ that satisfy the following conditions (5):

$$y \cdot z = 0 ; \ 0 \leq y \in \{S_y\} ; \ 0 \leq z \in \{K_z\}.$$  \hspace{1cm} (5)

Problems of this type are difficult (Cottle et al., 2009; Garey and Johnson, 1979; Hu et al., 2008) and can either have no solution, have one solution or have multiple solutions. In these problems, it is easy to verify a solution previously found, but there is no deterministic method that guarantees to find a solution, if it exists, or to demonstrate that it does not exist otherwise. Given the possible multiplicity of solutions, some lines of research have focused on trying to find the load factor of minimum global value. In this case, the main difficulty they pose, from the practical point of view, is that (Hu et al., 2008) “[many programs] are able to obtain, with no guarantee of success, some kind of solution […] but are unable to determine the quality of the solution obtained”, i.e., whether it is near or far from the absolute minimum. A method for calibrating the obtained solutions seems therefore necessary.

On the other hand, several authors have raised the objection that “finding the [absolute] minimum load factor can lead to overly conservative results” (Orduña and Lourenço, 2001) or that “only the smallest non-associative solution guarantees security […] but there is a risk of seriously underestimating the capacity of the actual structure” (Gilbert et al., 2006).

By means of these methods, local minimums can be found when they are presented as particular solutions and the obtained result depends on the starting point. The new method proposes not to take this randomness as a mathematical difficulty but as a characteristic of the problem, considering as a hypothesis that the origin of the randomness in the results can be the randomness in the contact conditions. In this way the problem can be simplified by adding the random contact conditions as constraints and then obtaining a valid static solution that satisfies $y = y'$, whereby (5) is converted to (6) and can be solved by a sequence of Linear Programs (6) and (7) according to the method proposed by Mangasarian (1995).

This method proposes to solve a Linear Complementarity Problem (or its Disjoint Bilinear Program equivalent) by means of a successive linearisation, fixing in step (6) the value of the static variables $y = y'$, solving the Linear Program to obtain the kinematic values $z'$, and by substituting in step (7) the value of the kinematic variables obtained $z'$, to solve for the Linear Program and to obtain the new value of the static $y''$. The process is repeated until $y = z = 0$ or it does not converge, in which case it would start from a different $y'$. 

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\begin{align}
    y' & \rightarrow 0 \leq z \in \{ K_z \}; \ y' \cdot z = 0 \rightarrow z = z' \\
    z' & \rightarrow 0 \leq y \in \{ S_y \}; \ y \cdot z' = 0 \rightarrow y = y^*. \tag{6}
\end{align}

For each set of contact conditions, different solutions will be obtained (Figure 3(B)). The use of this procedure as the core of a Monte Carlo simulation (Rubinstein and Kroese, 2007) allows finding a sample of solutions as large as desired (Magdalena et al., 2014). In this case, by renouncing to obtain the global minimum of the value of the load factor that causes the onset of collapse, an approximation to its value—or, more importantly, a distribution of its possible solutions—can be obtained.

Different options are possible for the modelling of random contact conditions. In this paper only two of all possible random distributions will be used: a uniform distribution of the point of application of the resultant of the contact forces, and a discrete distribution 0–1, the latter corresponding to the location of the contact stresses at the vertices of the blocks as proposed by Bigoni. However, this choice does not presuppose that they are the only possible or the most appropriate.

4 Discussion

The possible comparisons between the two experimental approaches and the numerical system described are of two types. On the one hand, there is the comparison between the experimental photoelastic method and the numerical analysis using rigid blocks, which can only be of a qualitative nature, given the limitations of the information obtained through the photoelastic tests. On the other hand, there is the comparison between the load tests and the numerical analysis, which allows, insofar as it provides numerical results, a quantitative discussion.

4.1 Qualitative comparison of numerical analysis and photoelastic test results

A qualitative comparison between these two approaches can be performed directly, since identical geometric characteristics and proportional loads and have been used in both. To facilitate direct visual comparison a graphical code has been implemented for the numerical representation. Contact stresses are represented by means of strokes in red in proportion to their width, as shown in Figure 5(A). Given the characteristics of the model, the numerical analysis to be implemented will only take into account vertical contact forces and stresses. The implementation is very simple and consists of finding a valid static solution $y \in \{ S_y \}$ through the following steps:

- the equations of equilibrium are considered taking into account that only compression is possible on the contact surfaces
- the points of application of the contact forces are randomly chosen
- a feasible equilibrium solution is sought for the applied external force using linear programming
- if found, this solution will be one of the possible solutions sought.
The results obtained using this type of implementation are, if an appropriate graphic representation is used, clearly similar to those obtained in the photoelastic tests (Figure 5(B)). If one of the hypotheses proposed is that the behaviour is probabilistic, finding a solution that resembles a specific photoelastic test does not demonstrate a strong correspondence between model and test, nor does it allow to analyse what type of random distribution is the most appropriate. However, it does allow questioning the general applicability of other types of deterministic models to very local phenomena, especially those that consider the material homogeneous.

4.2 Quantitative comparison of numerical analysis and load test results

Load tests described in Section 2.1 permits a quantitative comparison with the numerical analysis by rigid blocks in unilateral contact. In this model it is possible, in addition, to use the same values obtained in the test –same self-weight and same coefficient of friction, for example. The proposed implementation (Magdalena et al., 2015b) consists of obtaining multiple values of the minimum point load that will cause the onset of collapse, repeating the following steps:

- a limit equilibrium solution \( y = y' \) is obtained by randomly fixing the points of application of the contact forces
- the maximum value of the point load that causes the onset of collapse is obtained by using Linear Programming
- the solution of onset of collapse is verified
- solutions that do not meet all the conditions are rejected
- the minimum value of the point load that will cause the collapse is calculated, keeping fix the values of the conditions that have reached their limit value \( y_i = 0 \).

This method allows finding different collapse mechanisms corresponding to some of the multiple possible solutions (Figure 6).
The procedure is repeated as many times as necessary to obtain a sufficiently large sample. Steps (1)–(4) of this implementation are a particular case of the general method detailed in Section 3.1 by executing steps (6) and (7) – its meaning was explained in Section 3.2- once and rejecting results that do not meet the conditions. With all the accepted results, and since the incorporation of the Coulomb friction law indicates that the load that causes the onset of collapse is not necessarily unique, a minimisation operation is performed (step 5) incorporating to the model a principle of minimum similar to principle of minimum power proposed (Peshkin and Sanderson, 1989) for “quasistatic systems [...] essentially equivalent to Coulomb friction”. Steps 1–4 are instrumental. The aim is to obtain the minimum value of the punctual load determined by the random contact restrictions. But, as it is not possible to do this in a direct way, a solution of maximum is first obtained and, once it is proven it is an onset of collapse solution, the values of the variables that have reached their limit are fixed and a solution of the minimum is found. So each valid solution is obtained by solving three linear programs.

The simulation is repeated 1,000 times with random contact conditions for each of the two distributions used and the results are statistically treated to compare them with those of the load test. The results of the comparison are shown in Figure 7. In this graph, as in all others used in this paper, the results have been represented by their Gaussian kernel density estimation (Wand and Jones, 1995) using the Silverman rule for bandwidth. The results of the 0–1 distribution are shown on the left side; while the right figure shows those of the uniform distribution. The continuous thin line represents the results of the test, the discontinuous thick line the maximum value (step 2) of the numerical model and the continuous thick line the minimum value (step 5).

Figure 8 shows a comparison between the distribution of the load test results and the confidence intervals (Neyman, 1937) to a level of 99% of their characteristic value –5% percentile. The left figure includes the results of several deterministic numerical methods referred to in Magdalena et al. (2016). The right figure includes the case where a better adjustment is obtained – that is, the minima using a uniform distribution- among the proposed probabilistic numerical models.
The right figure shows that the lower bound for this characteristic value is very similar and that, in any case, the difference is on the security side for the numerical model.

The results of the load tests and their comparison with the numerical results obtained by the simulation described allow to conclude that, of the two random distributions used to model the irregularities in the contact, the uniform distribution is the one that best adjusts to the results of the load tests (Figure 7(A)), the adjustment being especially good at the characteristic value. Numerical results are, in the distribution set, consistently below the experimental ones (Figure 9).
In order to check whether the use of the model reasonably coincides with the experimental results and can be considered safe in the present case, a Kolmogorov-Smirnov test was performed for two samples (Nikiforov, 1994), obtaining a non-negligible p-value of 0.447 for the coincidence $T(\lambda) = M(\lambda)$ and a very high p-value of 0.994 for the hypothesis that the distribution of the model is consistently inferior to the one of the $T(\lambda) = M(\lambda)$ test.

**Figure 9** Comparison between relative cumulative frequencies of numerical and experimental results. The thick line represents the Gaussian kernel density estimation of the numerical model; the thin line represents the test data, and the dashed line represents the empirical distribution of the test (see online version for colours)

### 4.3 The material and its limitations

For a more general case, the adjustment will be better the more the starting hypotheses are met, in particular, the low (or zero) resistance of the joint material. This would correspond, in the quoted studies of granular media (Rajchenbach, 2001), to a little or no cohesive material. The incorporation of the joint material to the model (not so much as a resistant element but as a distribution element) could be studied in future developments since including both the joint and the deformability of the blocks will undoubtedly result in more adjusted results (although not safer). If necessary, their incorporation should be carefully studied, including the appropriate safety factors depending on the degree of certainty their data allow.

This type of numerical analysis may be also valid to simulate the internal behaviour of a discontinuous material, provided that any irregularities in the contact between blocks are incorporated through the random selection of the contact points for those resulting from the contact stresses. The results obtained from the numerical analysis allow the observation of ‘stress streams’ together with other zones of ‘unloaded islands’ which are scarcely stressed, also perceptible in photoelastic results. The description of this phenomenon, under different designations (like ‘chains of forces’ or ‘networks of stresses’) can be traced in studies on granular media (Drescher and Josseling de Jong, 1972) both numerical and photoelastic. The analysis of these concentrations is of crucial importance for the study of very local behaviours, such as the action of a point load on a small number of pieces.
Regarding the information provided by photoelastic tests, some advantages and limitations of the method must be discussed—and foreseen in the analysis of case studies.

The most obvious advantages of photoelasticity applied to the study of masonry are those that have to do with the immediacy of the photoelastic tests: the irisation the model manifests is a direct consequence of its deformation, and this in turn of the stress to which they are subjected. In addition, the current possibilities of using laser-cutting systems make geometrically complex models available in an almost automatic way. The photoelastic model, moreover, does not predetermine the irregularities in the contact between elements, since they are not imposed as a restriction but produced by the manufacturing process of the model, which is as accurate as possible using the above described computer-guided laser cutting.

The main limitations of the photoelastic tests are due to the differences between the physical properties of the photoelastic usual materials and the materials implemented in the numerical models, which are intended to simulate a dry-stack masonry structure. The main limitation is the difficulty of simulating the self-weight of the blocks, as the weight of the resin is insignificant in relation to the stress required for a minimum irisation. It is also difficult to replicate the friction between blocks, which is almost zero in the resin model but not in real dry-stack masonry structures. But there are also some other difficulties:

- the Young’s modulus of photoelastic materials is very different from that of the usual masonry materials.
- the friction angle of photoelastic materials is very different from that of the usual masonry materials—although this problem can be partially solved by adding a sheet with a suitable coefficient of friction to the contact faces.
- the contact stresses for which collapse by sliding may occur are much smaller than those that produce appreciable photoelastic results.

However, similar limitations would arise with other small-scale models. The real problem with photoelastic tests is the deformability inherent to the photoelastic material, since the resins used in these tests lack the rigidity and fragility of the components of a standard dry-stack masonry structure. But this high deformability is essential for photoelastic tests, since the irisations that can be detected in them do not represent the internal stresses in the material but the deformations provoked in the block affected by external stresses. There is an evident contradiction: the test material must be highly deformable to manifest its internal stress state, but replaces a real material (dry-stack masonry, in this occasion) so incapable of assuming deformations that are usually considered non-deformable by many numerical approximations.

5 Conclusions

Since the constituent material of masonry structures is heterogeneous, anisotropic, has high compressive strength but little or no tensile strength and is virtually discontinuous, a numerical model capable of implementing these characteristics can be used for a simplified analysis of historic masonry buildings, especially where the damage suffered over time by this material is unknown.
The comparison between the results obtained by the photoelastic and numerical methods (Figure 5) allows noticing great similarities with regard to the randomness in the distribution of the contact stresses. Although the influence of the deformability of the material is a subject that must be carefully studied, this random behaviour is clearly observed both in the tests presented here and in the examples of the quoted authors, some made with very different materials.

The random distribution of these stresses produces similar effects in all models: they all show, in different form and degree, the appearance of stress streams and unloaded islands.

The comparison of the results obtained by the application of the numerical method with uniform distribution and those obtained through the load test (Figures 8 and 9) reinforces the idea that the consideration of both the discontinuity and the randomness in the contact conditions between blocks plays an important role in explaining their resistant behaviour.

Even assuming that the overall behaviour of a masonry structure may not differ when analysed as a continuous or as a discontinuous medium, at a local level great differences can be observed if one or another is considered, especially if the randomness in the contact conditions typical of historical masonry structures is taken into account. The appearance of unloaded islands inside masonry structures, crucial for understanding local behaviour, goes unnoticed when analysed globally.

From the above it can be inferred that the proposed approach is –within the limits clarified in the discussion- valid for the analysis of the local behaviour of structures whose material can be characterised as a discontinuous orthotropic medium. This approach is similar to that of some studies on granular media (Drescher and Josseling de Jong, 1972) but has not been used in the study of orthotropic media.

Unlike methods based on computation, whose results depend on the assumed hypotheses, photoelastic methods – with the provisos made regarding the influence of the material- allow a direct approach to the studied phenomena, including the observation of internal stress distribution. Photoelasticity can be, in this context, a useful method to deepen the study of the local behaviour of masonry structures. However, given the limitations of the photoelastic analysis procedure and the complexity of performing such essays in a number sufficient to allow reliable statistical analysis, it can only be used as support for other methods. Its use in combination with numerical analysis procedures such as the one here proposed allows the validation of the starting hypotheses on which the latter are based and makes possible a practical approach of great interest to local problems, especially in the case of historical buildings.

References


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