Application of interactive fuzzy goal programming for multi-objective integrated production and distribution planning

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Abstract: Integrating two interconnected processes and controlling conflicting goals of a production and distribution problem is necessary for better planning. The aim of this integration is to allocate the limited available resources to produce the products over the time periods and ensure economical dispatching to distribution centres. In a responsive and competitive market, production and distribution planning modules is considered here within the framework of satisfaction level of decision maker. Triangular fuzzy numbers and the concept of minimum accepted level are employed to formulate the problem. Minimum operator and weighted average operator method is used to aggregate all fuzzy set and solved by three algorithms and goal programming method. The proposed approach aims to formulate a fuzzy multi-objective linear programming model to simultaneously minimise the total production cost, distribution cost and delivery time for multi-product and multi-time period under multiple uncertainty. AHP is used to rank multiple objectives and solved with MATLAB.

Keywords: supply chains; production and distribution planning decision; PDPD; fuzzy multi-objective linear programming; FMOLP; goal programming; GP.


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1 Introduction

Production and distribution planning problem in a supply chain system involves the allocation of production volumes among the different production lines in the manufacturing plants, and the delivery of the products to the distribution centres. That is it is a process of allocating resources and operations for manufacturing of products. This integration enables the most effective use of production resources without creating extra inventory at various stage of supply chain through considering production planning and distribution as one optimisation problem. As a result the conflicting goals of minimising cost, inventory, delivery time and the number of rejected items, and optimising customer service and flexibility are commonly seen (Chen and Vairaktarakis, 2005).

However, most existing studies have focused on a single component of the overall supply chain system, such as procurement, manufacturing, distribution, warehouses or scheduling, despite limited progress having been made towards integrating these components into a single supply chain. From the research work of Mula et al. (2009), Li and Ierapetritou (2010) and Karabuk (2008), it is seen that production planning, distribution planning, transportation planning and scheduling is done separately. Hence, integrating production and distribution planning decisions (PDPDs) in supply chains is a major issue in ensuring the effectiveness of supply chain for competitive markets market. To better satisfy market needs and to maintain their higher level of productivity many researcher emphasis on integrated production and distribution. Moattar Husseini et al. (2015) considered a bi-objective optimisation for integrated production and distribution planning in the presence of manufacturing partners (IPDP-MP). Here they showed the way to minimise total cost and ensure product quality. Sarrafha et al. (2015) integrated procurement, production, and distribution problem of a multi-echelon supply chain network design. They propose bi-objective mixed-integer nonlinear programming (MINLP) model to minimising the total SC costs as well as minimising the average tardiness of product to DCs. Tsiakis (2008) presented a paper to determine the optimal configuration of a production and distribution network subject to operational and financial constraints. Operational constraints include quality, production and supply restrictions, and are related to the allocation of the production and the work-load balance by mixed-integer linear programming. Sabri and Beamon (2000) developed a comprehensive multi-objective SC to implementing simultaneous strategic and operational SC planning. The adaptive multi-objective decision analysis allows to use a performance measurement system, which includes cost, customer service levels (fill rates), and flexibility.

Additionally, in practical PDPD problems, the decision maker (DM) must simultaneously handle multiple conflicting goals in term of the use of organisational resources, and achieve these conflicting objectives in a framework of fuzzy aspiration.
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level (Liang, 2006, 2008). When any of the conventional mathematical programming methods are used to solve PDPD problems, however, the goals and relevant inputs are generally assumed to be deterministic/crisp (Lee et al., 2002; Park, 2005). In real-world PDPD problems in supply chains, environmental coefficients and/or parameters, involving available supply, resources, capacities, forecast demand, and related operating costs, are often imprecise/fuzzy owing to information is incomplete and unavailable over the planning horizon. Clearly, conventional deterministic mathematical programming methods cannot solve all imprecise/fuzzy PDPD programming problems in uncertain environments. Park (2005) published a paper that deal with the optimisation of integrated production and distribution planning but did not consider uncertainty.

Bilgen (2010) and Rizk et al. (2009) address the production and distribution planning problem in a supply chain system that involves the allocation of production volumes among the different production lines in the manufacturing plants, and the delivery of the products to the distribution centres. In this paper, fuzzy models taking into account the fuzziness in the capacity constraints and the aspiration level of costs using different aggregation operators.

Again, Mula et al. (2010a) examined the effectiveness of a fuzzy mathematical programming system for supply chain production planning with fuzzy demand. The work incorporated a method of possibilistic programming, which makes it possible to model the epistemic uncertainty in demand, which could exist in the supply chain production planning problems as triangular fuzzy numbers. Amorim et al. (2012) published a paper to show the integrated production and distribution planning of perishable products. In this paper flexibility of decision maker is ignored and optimises integrated production and distribution planning but not interactive way. Additionally, researchers have developed several FGP methods to solve multi-objective PDPDs problems (da Silva and Marins, 2014). Some researchers proposed an additive fuzzy programming model that considered weights and priorities for all non-equivalent objectives for the PDPDs problem. El-Wahed and Lee (2006) applied interactive fuzzy goal programming for multi-objective transportation problems. Again, Arikan and Güngör (2001) and Liang (2010) applied fuzzy goal programming to a multi-objective project network problem. Moreover, Sadeghi et al. (2013) adopted fuzzy grey goal programming approach.

Liang (2008) presented a fuzzy multi-objective PDPDs with multiproduct and multi-time period in a supply chain. Liang extended a fuzzy multi-objective linear programming (FMOLP) system with piecewise linear membership function to handle integrated multi-product and multi-time period PDPDs problems where the objectives are formulated in fuzzy form. At the same time the fuzzy variables are treated by weighted average method. Torabi and Moghaddam (2012) considered trans-shipment for multi site integrated production-distribution problem. Liang (2006) proposed i-FMOLP method aims to simultaneously minimise the total distribution costs and the total delivery time with reference to fuzzy available supply and total budget at each source, and fuzzy forecast demand and maximum warehouse space at each destination.

Liang (2008) presented a fuzzy multi-objective production/distribution planning decisions with multiproduct and multi-time period in a supply chain. Liang extended a fuzzy multi-objective linear programming (FMOLP) system with piecewise linear membership function to handle integrated multi-product and multi-time period production/distribution planning decisions (PDPD) problems where the objectives are formulated in fuzzy form. At the same time the fuzzy variables are treated by weighted average method and starting point considered arbitrarily. D’Apuzzo et al. (2010)
described starting-point strategies for an infeasible potential reduction method. Torabi and Moghaddam (2012) considered trans-shipment for multi site integrated production-distribution problem. Liang (2006) proposed i-FMOLP method aims to simultaneously minimise the total distribution costs and the total delivery time with reference to fuzzy available supply and total budget at each source, and fuzzy forecast demand and maximum warehouse space at each destination. Liang (2012) published a paper where fuzzy weighted average method is used to deal with uncertainty which make the mathematical statement more complex.

From the literature review, it has been seen that the different optimisation approaches on production and distribution planning problems is not so recent. Traditional production and distribution planning problems are solved by considering deterministic parameters and ignoring flexibility. But in real life it is observed that these parameters are uncertain in nature. At the same time the most of the cases PDPDs of various operational levels taken without coordination. As a result full benefit from the analysis throughout the organisation cannot achieve. Due to changing environment or dynamic market it is difficult to optimise all variables in a production and distribution problems decision maker need some flexibility to deal with the dynamic situation. In this paper, integrated production and distribution planning problem for the multiproduct, multi-period multi-customer zone is considered under uncertainties and simultaneous coordination of production and distribution operations of the entire planning horizon.

2 Problem formulation

In general, an integrated planning problem combines at least two sub-problems from different operational functions in a single planning problem and its related optimisation model. Thus, the requirements, restrictions and optimisation goals of each sub-problem have to be combined in one integrated planning problem and considered simultaneously (Chen, 2010; Park, 2005).

Assume that \( I \) manufacturers or sources produce \( N \) types of products to satisfy the market demand of \( J \) distribution centre over a planning horizon \( H \). Based on the above characteristics of the considered integrated production and distribution problem, the fuzzy mathematical programming model is designed here with following assumptions:

1. All objective functions are fuzzy with imprecise aspiration levels.
2. All objective functions and constraints are linear equations.
3. The production costs at each source and distribution cost/time on a given route are directly proportional to the units manufactured and shipped capacity per truck, respectively.
4. The linear membership functions are specified for all of the fuzzy objectives involved in the proposed model by triangular method.
5. The minimum operator and weighted average operator are used to aggregate all of the fuzzy sets.
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Index sets

- $i$ index for source, for all $i = 1, 2, \ldots, I$
- $n$ index for product type, for all $n = 1, 2, \ldots, N$
- $h$ index for planning time period, for all $h = 1, 2, \ldots, H$
- $j$ index for destination, for all $j = 1, 2, \ldots, J$
- $g$ index for objective, for all $g = 1, 2, \ldots, K$.

Objective functions

1. $Z_1$ production costs ($\$$)
2. $Z_2$ distribution cost ($\$$)
3. $Z_3$ delivery time (hours).

Decision variables

- $Q_{inh}$ regular production volume for $n^{th}$ product by source $i$ in period $h$ (units)
- $S_{inh}$ subcontracting volume for $n^{th}$ product by source $i$ in period $h$ (units)
- $P_{inh}$ inventory level for $n^{th}$ product by source $i$ in period $h$ (units)
- $X_{inhj}$ units distributed for $n^{th}$ product from source $i$ to destination $j$ in period $h$ (units).

Parameter

- $a_{inh}$ regular production cost per unit for $n^{th}$ product by source $i$ in period $h$ ($\$/unit)
- $e_a$ escalating factor for regular production cost (%)
- $b_{inh}$ subcontracting cost per unit for $n^{th}$ product by source $i$ in period $h$ ($\$/unit)
- $e_b$ escalating factor for subcontracting cost (%)
- $c_{inh}$ inventory carrying cost per unit for $n^{th}$ product by source $i$ in period $h$ ($\$/unit)
- $e_c$ escalating factor for inventory carrying cost (%)
- $k_{inhj}$ delivery cost per unit for $n^{th}$ product from source $i$ to destination $j$ in period $h$ ($\$/unit)
- $e_k$ escalating factor for delivery cost (%)
- $u_{inhj}$ delivery time per unit for $n^{th}$ product from source $i$ to destination $j$ in period $h$ (hour/unit)
- $C_{inhj}$ capacity per truck delivered for $n^{th}$ product from source $i$ to destination $j$ in period $h$ (units)
- $D_{inhj}$ demand for $n^{th}$ product of destination $j$ in period $h$ (units)
- $l_{in}$ hour of labour per unit for $n^{th}$ product produced by source $i$ (man-hour/unit)
- $F_{ih}$ maximum labour levels available for source $i$ in period $h$ (man-hour)
2.1 Models descriptions: FMOLP model

Here multiple objectives for solving the multi-product and multi-time period integrated production and distribution problems is shown and aim is to simultaneously optimise the multiple objective with respect to the satisfaction level of decision maker. Total production cost includes regular time production cost, subcontracting cost and carrying inventory cost. Regular time production cost is considered by constant work force level over the planning horizon and utilising regular production capacity for I sources produce N types of products to satisfy the market demand of J distribution centre over a planning horizon H. Subcontracting cost is considered here because if demand exceeds the capacity of regular time production then the demand is satisfied by subcontracting. Distribution cost is considered here with respect to the unit distributed to various destinations. The distribution cost is considered only for finish goods of products N transferred from plants I to demand zone J in the planning horizon H.

- Objective 1: Minimise total production cost

\[
\text{Min } Z_1 = \sum_{i=1}^{I} \sum_{n=1}^{N} \sum_{h=1}^{H} a_{inh} Q_{inh} (1 + e_i)^h + \sum_{i=1}^{I} \sum_{n=1}^{N} \sum_{h=1}^{H} b_{inh} S_{inh} (1 + e_h)^h + \sum_{i=1}^{I} \sum_{n=1}^{N} \sum_{h=1}^{H} c_{inh} P_{inh} (1 + e_c)^h
\]

(1)

- Objective 2: Minimise total distribution cost

\[
\text{Min } Z_2 = \sum_{j=1}^{J} \sum_{n=1}^{N} \sum_{h=1}^{H} k_{inh} X_{inh} (1 + e_k)^h
\]

(2)

- Objective 3: Minimise total delivery time

\[
\text{Min } Z_3 = \sum_{j=1}^{J} \sum_{n=1}^{N} \sum_{h=1}^{H} \sum_{j=1}^{J} \left[ \frac{u_{inh}}{C_{inh}} \right] X_{inh}
\]

(3)

The symbol ‘\(\cong\)’ is used here to represent the fuzzified version of ‘=’ with respect to the aspiration level of the decision maker. Then the final objective functions are formulated as a multi-objective functions for the integrated production and distribution problem. For the proposed multi-objective linear programming, these conflicting objectives are required to be simultaneously by the decision maker in the framework fuzzy aspiration levels.
2.2 Constraints

- Constraints on carrying inventory

\[ W_{\text{inh},-1} + Q_{\text{inh}} + S_{\text{inh}} - P_{\text{inh}} = \sum_{j=1}^{J} X_{\text{inhj}} \quad \text{for } \forall i, \forall n, \forall h \]  

(4)

- Constraints on demand

\[ \sum_{j=1}^{J} X_{\text{inhj}} = D_{\text{inhj}} \quad \text{for } \forall j, \forall n, \forall h \]  

(5)

Here the symbol ‘~’ is used to show fuzziness especially for fuzzy numbers. The market demand for each destination can never be determined precisely because of the dynamic nature of market demand and supply, and the sum of regular production, inventory levels, and subcontracting levels come from various sources essentially should equal the market demand for each destination, as in equations (4) and (6).

Equation (4) stands for the constraint that the forecasted demand for each destination over a particular period can be either satisfied by regular production and inventory of previous period or by considering subcontracting by sources. Here \( W_{\text{inh},-1} \) is represent the beginning inventory level.

- Constraints on subcontracting

\[ \sum_{n=1}^{N} S_{\text{inh}} \leq \tilde{S}_{\text{ih}} \quad \text{for } \forall i, \forall h \]  

(6)

Supply capacity of each supplier is not infinity. Each has some limited supply capacity to supply of product \( n \) to the plant \( i \) in the period \( t \). At the same time there is a limit of subcontracting of the producer. Subcontracting cost is more than regular production cost and has some other limitations. As a result decision maker try to reduce subcontracting and meet the demand by the regular time production shown in equation (6).

- Constraints on labour levels

\[ \sum_{n=1}^{N} l_{\text{inh}} Q_{\text{inh}} \leq \tilde{F}_{\text{ih}} \quad \text{for } \forall i, \forall h \]  

(7)

- Constraints on machine capacity

\[ \sum_{n=1}^{N} r_{\text{inh}} Q_{\text{inh}} \leq \tilde{M}_{\text{ih}} \quad \text{for } \forall i, \forall h \]  

(8)

Equations (7) and (8) represent the limits of actual labour levels and machine capacity for each source in each period. The available resources in right hand sides of equations (7) and (8), represent the available labour levels and machine capacity for each source are often fuzzy/imprecise. Uncertainty of the demand and supply of the labour forces, worker skills, public policy, machine capacity and other factors over the planning horizon is considered here.
Constraints on warehouse space

$$\sum_{j=1}^{I} \sum_{n=1}^{N} S_{nhj} X_{nhj} \leq R_{nhj} \quad \text{for } \forall j, \forall h \quad (9)$$

Constraints on total budget

$$\sum_{j=1}^{I} \sum_{n=1}^{N} \sum_{h=1}^{H} a_{nhj} Q_{nhj} (1 + e_{a})^{h} + \sum_{j=1}^{I} \sum_{n=1}^{N} \sum_{h=1}^{H} b_{nhj} S_{nhj} (1 + e_{b})^{h}$$

$$+ \sum_{j=1}^{I} \sum_{n=1}^{N} \sum_{h=1}^{H} c_{nhj} P_{nhj} (1 + e_{c})^{h} + \sum_{j=1}^{I} \sum_{n=1}^{N} \sum_{h=1}^{H} d_{nhj} X_{nhj} (1 + e_{d})^{h} \leq B \quad (10)$$

Equation (9) represents the limits of actual warehouse capacity in each period for each destination and (10) represents the limits of total budget which means that the sum of total cost of production and distribution should be less or equal to the total budget.

Non-negativity constraints on decision variables

$$Q_{nhj}, V_{nhj}, W_{nhj}, E_{nhj} \geq 0 \quad \text{for } \forall i, \forall n, \forall h, \forall f. \quad (11)$$

2.3 Treatment of the fuzzy parameter

In this work decision maker adopt the pattern of triangular possibility distribution to represent the fuzzy variables. The primary advantages of the triangular fuzzy number are the simplicity and flexibility of the fuzzy arithmetic operations (Liou and Wang, 1992; Rommelfanger, 1996). In a real-life situation on can often estimate the maximum and minimum values, and the most likely outcome, even if he does not know the mean and standard deviation. The triangular distribution has a definite upper and lower limit, so one can avoid unwanted extreme values. That is why decision maker consider the situations in which the market demand for each destination, $D_{nhj}$, is a triangular fuzzy number with the most and least possible values and also for other fuzzy variables. Again, the most pessimistic and optimistic values which provided the boundary solutions of the fuzzy number thus should be assigned smaller weights. In the process of defuzzification, this work applies Liou and Wang’s (1992) approach for ranking fuzzy method to convert fuzzy number into a crisp number. If the minimum acceptable membership level $\alpha$, corresponding auxiliary crisp of triangular fuzzy number $\tilde{D}_{nhj}^{\alpha}$, $\tilde{D}_{nhj}^{m}$, $\tilde{D}_{nhj}^{p}$ is:

$$D_{nhj}^{\alpha} = \frac{1}{2} \left[ \alpha \tilde{D}_{nhj}^{m} + \tilde{D}_{nhj}^{p} + (1-\alpha)\tilde{D}_{nhj}^{p} \right] \quad (12)$$

For instance, Figure 1 shows the distribution of the triangular fuzzy number $D_{nhj}^{\alpha}$. In practical situations, the triangular distribution of $D_{nhj}^{\alpha}$ may:
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1. the most pessimistic value ($\hat{D}_{m,l}^n$) that has a very low likelihood
2. the most likely value ($\tilde{D}_{m,l}^n$) that definitely belongs to the set of available values
3. the most optimistic value ($\hat{D}_{m,l}^o$) that has a very high likelihood of belonging to the set of available values.

**Figure 1** Membership function of $D_{m,l}^n$

2.4 Problem formulation using aggregation operator

Here minimum operator and weighted average operator method is used to aggregate all fuzzy set. First the optimal solution yielded by the minimum operator method. But it may not be an efficient solution with respect to the satisfaction level of decision maker. The minimum operator is preferable when a decision maker wants to make values of the optimal membership functions approximately equal or when a decision maker believes that the minimum operator is an approximate representation. To overcome the disadvantage of using the minimum operator the compensatory weighted average operator is employed for to obtain overall decision maker satisfaction degree.

2.4.1 Formulation using fuzzy minimum operator and weighted average operator method

The original fuzzy multi-objective production and distribution model designed above can be solved using the fuzzy decision-making concept and fuzzy programming technique of Bellman and Zadeh (1970) and Zimmermann (1996). Here linear membership function by considering suitable upper bounds and lower bounds to the objective function is defined as follow. First, the positive ideal solution (PIS) and negative ideal solution (NIS) for each of the fuzzy objective functions can be specified for minimisation problem as follows:

$$Z_{g}^{PIS} = \text{Min } Z_{g} \text{ = lower limit and } Z_{g}^{NIS} = \text{Max } Z_{g} \text{ = upper limit}$$
And then linear membership functions are specified by requiring the DM to select the goal value interval \([Z_g^{PIS} \text{ and } Z_g^{NIS}]\). Accordingly, the corresponding, non-increasing continuous linear membership functions for the fuzzy objective functions can be expressed by

\[
 f_g (Z_g) = \begin{cases} 
 1, & Z_g \leq Z_g^{PIS} \\
 \frac{Z_g^{NIS} - Z_g}{Z_g^{NIS} - Z_g^{PIS}}, & Z_g^{PIS} \leq Z_g \leq Z_g^{NIS}, & g = 1, 2 \ldots k \\
 0, & Z_g \geq Z_g^{NIS} 
\end{cases}
\]

For this minimisation problem decision maker try to finds result as \(Z_g^{NIS} - Z_g\) in the range of \(Z_g^{PIS} \leq Z_g \leq Z_g^{NIS}\) that shown in Figure 2. While if the problem is maximisation, the decision maker will try to finds result as \(Z_g^{PIS} - Z_g\) in the range of \(Z_g^{PIS} \leq Z_g \leq Z_g^{NIS}\). In practice, the corresponding possible value interval for a fuzzy objective can be estimated based on the experience and knowledge of decision maker and/or experts, and the equivalent membership grade are normally in the interval \([0, 1]\) which indicates the satisfaction level of decision maker. If the value of \(f_g(Z_g) = 1\) in the interval of \(Z_g \leq Z_g^{PIS}\) of the equation (13) then its means that decision maker achieve highest satisfaction and the result obtained is minimum. Again if \(f_g(Z_g) = 0\) in the interval of \(Z_g \geq Z_g^{NIS}\) of the equation (13) its mean that result obtained at maximum point.

**Figure 2**  Linear membership function of \(f_g(Z_g)\) (see online version for colours)

Introducing the auxiliary variable \(L\) enables the original fuzzy MOLP problem to be converted into an equivalent ordinary LP form. This weight is given here by analytical hierarchy method with respect to the various characteristics of the case. Consequently, the complete ordinary single-objective LP form can be formulated as follows:

- **Minimum operator method**

  Maximise \(L\)  
  \[(0 \leq L \leq 1)\]

  Subject to the
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\[ L \leq \frac{Z_g^{NS} - Z_g^S}{Z_g^{NS} - Z_g^{PS}} \]  

(14)

And equations (4) to (11)

- Weighted average operator method

\[ \text{Max } L = \sum_{g=1}^{k} W_g L_g \]  

(15)

Subject to,

\[ L \leq \frac{Z_g^{NS} - Z_g^S}{Z_g^{NS} - Z_g^{PS}} \text{ for } \forall g \]  

(16)

\[ \sum_{g=1}^{k} W_g = 1 \text{ for } \forall g \]  

(17)

And equations (4) to (11)

Where \( W_g \) \((g = 1, 2, \ldots, k)\) is the corresponding weight of the \(g\)th fuzzy objective function chosen by DM.

3 Data collection and case description

To completion of this research work secondary data has been considered and verified the model. The secondary data is taken from the previous published paper named as ‘Fuzzy multi-objective production/distribution planning decisions with multi-product and multi-time period in a supply chain’ by Liang (2008). Here two sources two products three periods and four distribution centres problem is considered. The related is given below:

Daya is the main global producer of super precision ball screw, linear bearing, and guide ways. The product model includes two types of standard ball screw, namely the external recirculation type (product 1) and the internal recirculation type (product 2). Two products are ordered to satisfy market demand from four distribution centres in Taipei, Taichung, Haulien and Kaohsiung, with production based at two plants in Touliu and Hsinchu. The planning horizon is three months from March to May.

Tables 1 to 3 summarise the related manufacture, distribution and demand data, respectively, for the coming three months. Capacity per truck from each source to various destinations is fixed to carry 100 dozen bottles. As shown in Table 2, for example, the transportation cost per dozen bottles and delivery time per truck to carry 100 dozen bottles from Touliu to Taipei are $2.8 and 5.2 h, respectively.
Table 1  Production data for two factories in the Daya case (in US dollar)

<table>
<thead>
<tr>
<th>Factory</th>
<th>Period</th>
<th>Product</th>
<th>a_{int}</th>
<th>b_{int}</th>
<th>c_{int}</th>
<th>l_{in}</th>
<th>r_{in}</th>
<th>v_{in}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Touliu</td>
<td>March</td>
<td>1</td>
<td>20</td>
<td>25</td>
<td>0.30</td>
<td>0.05</td>
<td>0.10</td>
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<tr>
<td></td>
<td></td>
<td>2</td>
<td>10</td>
<td>12</td>
<td>0.15</td>
<td>0.07</td>
<td>0.08</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>April</td>
<td>1</td>
<td>20</td>
<td>25</td>
<td>0.30</td>
<td>0.05</td>
<td>0.10</td>
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<td></td>
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<tr>
<td></td>
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<td>20</td>
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<td>0.15</td>
<td>0.07</td>
<td>0.08</td>
<td>3</td>
</tr>
<tr>
<td>Hsinchu</td>
<td>March</td>
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<td>18</td>
<td>24</td>
<td>0.28</td>
<td>0.04</td>
<td>0.09</td>
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</tr>
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<td></td>
<td></td>
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<td>9</td>
<td>12</td>
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<td>0.06</td>
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<td></td>
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<td>18</td>
<td>24</td>
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<td>0.07</td>
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Table 2  Distribution data in the Daya case (in US dollar)

<table>
<thead>
<tr>
<th>Factory</th>
<th>Product</th>
<th>Distribution centre</th>
<th>Taipei</th>
<th>Taichung</th>
<th>Haulien</th>
<th>Kaohsiung</th>
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<tbody>
<tr>
<td>Touliu</td>
<td>1</td>
<td>2.8a/5.2b</td>
<td>1.0/1.8</td>
<td>4.2/13.5</td>
<td>2.2/2.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.5/5.2</td>
<td>0.9/1.8</td>
<td>4.0/13.5</td>
<td>2.0/2.8</td>
<td></td>
</tr>
<tr>
<td>Hsinchu</td>
<td>1</td>
<td>1.2/2</td>
<td>1.5/2.5</td>
<td>5.0/15</td>
<td>3.5/6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.1/2</td>
<td>1.4/2.5</td>
<td>4.5/15</td>
<td>3.2/6</td>
<td></td>
</tr>
<tr>
<td>Available warehouse space(ft$^2$)</td>
<td>20,000</td>
<td>18,000</td>
<td>20,000</td>
<td>12,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: $^a$ denotes delivery cost per dozen bottles ($)

$^b$ denote delivery time per truck to carry 100 dozen bottles (hours)

Table 3  Demand data for the case problem (units)

<table>
<thead>
<tr>
<th>Distribution centre</th>
<th>Product</th>
<th>Period</th>
<th>March</th>
<th>April</th>
<th>May</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taipei</td>
<td>1</td>
<td>920, 1,000, 1,080</td>
<td>2,750, 3,000, 3,250</td>
<td>4,600, 5,000, 5,400</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>600, 650, 700</td>
<td>850, 910, 970</td>
<td>2,750, 3,000, 3,250</td>
<td></td>
</tr>
<tr>
<td>Taichung</td>
<td>1</td>
<td>750, 820, 890</td>
<td>2,100, 2,300, 2,500</td>
<td>3,750, 4,000, 4,250</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>450, 500, 550</td>
<td>650, 710, 790</td>
<td>2,200, 2,400, 2,600</td>
<td></td>
</tr>
<tr>
<td>Haulien</td>
<td>1</td>
<td>450, 500, 550</td>
<td>1,050, 1,200, 1,350</td>
<td>2,220, 2,400, 2,580</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>260, 300, 340</td>
<td>365, 400, 435</td>
<td>1,050, 1,150, 1,250</td>
<td></td>
</tr>
<tr>
<td>Kaohsiung</td>
<td>1</td>
<td>1,100, 1,230, 1,360</td>
<td>3,050, 3,400, 3,750</td>
<td>4,950, 5,300, 5,650</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>650, 710, 770</td>
<td>950, 1,050, 1,150</td>
<td>2,850, 3,100, 3,350</td>
<td></td>
</tr>
</tbody>
</table>
Other data

1 Initial inventory in period 1 is 400 and 200 U of product 1 and 2, respectively, for the Touliu factory, and 300 and 200 U of product 1 and 2, respectively, for the Hsinchu factory. The end inventory of each product for the two factories in period 3 is zero.

2 The available fuzzy labour levels in each period are (950, 1040, 1,130) man-hours for the Touliu factory and (850, 920, 990) man-hours for the Hsinchu factory.

3 Available fuzzy machine capacities in each period are (1,550, 1,710, 1,870) machine-hours for the Touliu factory and (1,850, 2,050, 2,250) machine-hours for the Hsinchu.

4 Available fuzzy subcontracting opportunity in each period are (350, 400, 450) units for the Touliu factory and (250, 300, 400) units for the Hsinchu factory.

5 Maximum warehouse spaces available for the distribution centres are 19,500 ft$^2$ (Taipei), 16,000 ft$^2$ (Taichung), 10,000 ft$^2$ (Haulien), and 20,000 ft$^2$ (Kaohsiung), respectively.

6 Total budget is $850,000 and the expected escalating factor for each cost categories is 2%.

7 The minimal acceptable membership level $\alpha$, for all fuzzy numbers is 0.5.

3.1 Solving procedures

Step 1 Formulate the multi-objective production/distribution planning problems according to equations (1) to (11).

Step 2 Provide the minimum acceptable membership level, $\alpha$ and then convert the fuzzy parameters into crisp ones using the fuzzy ranking number method according to equation (12).

Step 3 Solve the first objective function as a single objective production/distribution planning problem. Continue this process $K$ times for the $g^{th}$ objective functions. Then specify the lower bound ($L_k$) and the upper bound ($U_k$) by PIS and NIS.

Step 4 Define the membership function (MF) of each objective function

Step 5 Introduce the auxiliary variable $L$, thus enabling aggregation of the original fuzzy MOLP problem into an equivalent ordinary single-objective LP form using the minimum operator method.

Step 6 Solve the ordinary LP problem. If the DM is dissatisfied with the initial solutions, the model should be adjusted until a preferred satisfactory solution is obtained.

Step 7 Formulate the problem according to the weighted average operator method and solve original fuzzy MOLP problem into an equivalent ordinary single-objective LP by various techniques as shown previous.
3.2 Model implementation for the case

For the implementation of the above approach, two sources two products three periods and four distribution centres problem is considered.

First, formulate the original fuzzy MOLP model for solving the multi product and multi-time period production and distribution planning problem according to equations (1) to (11) and solve the multi objective problem using the ordinary single-objective LP problem to obtain the initial solutions for each of the objective functions to determine $Z_1^{PIS}$ and $Z_1^{NIS}$ for objective $Z_1$ and $Z_2^{PIS}$ and $Z_2^{NIS}$ for objective $Z_2$ and $Z_3^{PIS}$ and $Z_3^{NIS}$ for objective $Z_3$ shown in Table 4.

Table 4  PIS and NIS for each objective

<table>
<thead>
<tr>
<th>Objective functions</th>
<th>PIS</th>
<th>NIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_1$ ($)</td>
<td>649,350</td>
<td>763,550</td>
</tr>
<tr>
<td>$Z_2$ ($)</td>
<td>86,447</td>
<td>138,240</td>
</tr>
<tr>
<td>$Z_3$ (hrs)</td>
<td>168,190</td>
<td>275,350</td>
</tr>
</tbody>
</table>

3.3 Solving MOLP by minimum operator and weighted average operator method

The minimum operator of the fuzzy decision-making of Bellman and Zadeh (1970) is used to aggregate all fuzzy sets. Furthermore, introducing the auxiliary variable $L$ enables the original FMOLP problem to be converted into an equivalent single objective linear programming form. For the analysed case the corresponding single objective linear programming as follow:

- Minimum operator method

Max $L$

Subjected to:

$$L \leq \frac{763,550 - Z_1}{763,550 - 649,350}$$

$$L \leq \frac{138,240 - Z_2}{138,240 - 86,447}$$

$$L \leq \frac{275,350 - Z_3}{275,350 - 168,190}$$

And equations (4) to (11)

Where $L$ value ($0 \leq L \leq 1$) represents the overall DM satisfaction with the given goal values.
• Weighted average operator method

\[
\text{Max } = W_1L_1 + W_2L_2 + W_3L_3
\]

Subjected to:

\[
L_1 \leq \frac{763,550 - Z_1}{763,550 - 649,350}
\]

\[
L_2 \leq \frac{138,240 - Z_2}{138,240 - 86,447}
\]

\[
L_3 \leq \frac{275,350 - Z_3}{275,350 - 168,190}
\]

\[
W_1 + W_2 + W_3 = 1
\]

And equations (4) to (11)

Where \( L \) value (0 ≤ \( L \) ≤ 1) represents the overall DM satisfaction with the given goal values.

### 3.4 Results of MOLP

After transforming multi-objective linear programming to single objective linear programming by minimum operator method, the model is solved by three algorithms named interior-point, active set and simplex. The aim of interior-point methods is for complementary slackness while maintaining primal and dual feasibility on the other hand active set methods aim for dual feasibility while maintaining primal feasibility and complementary slackness. Again simplex and interior point methods are a mature field from an algorithmic point of view. They both work very well in practice. The good reputation of interior point methods (IPM) is due to its polynomial complexity in the worst case. That is not the case for simplex which has combinatorial complexity. Each iteration of simplex are relatively easy to compute, but it can require exponentially much iteration in the worst case (Gondzio, 2012). The mathematical representation of the method is given above where the PIS and NIS for each of the fuzzy objective functions are taken as lower limit and upper limit respectively and Matlab computer software is used to run this ordinary LP model. The proposed model provides the overall levels of DM satisfaction (\( L \) value) given the multiple fuzzy goal values (\( Z_1, Z_2 \) and \( Z_3 \)). If the solution is \( L = 1 \), then each goal is fully satisfied; if \( 0 < L < 1 \), then all of the goals are satisfied at the level of \( L \), and if \( L = 0 \), then none of the goals are satisfied. For the minimum acceptable membership level \( \alpha = 0.5 \), the overall degree of DM satisfaction \( (L) \) is 0.818 with the goal values (\( Z_1 = $670,160 \) and \( Z_2 = $95,881 \) and \( Z_3 = 1,834 \) hrs).

Again the active set algorithm gives same result of interior point algorithm that shown in Table 5. But the simplex algorithm gives deferent result. For the minimum acceptable membership level \( \alpha = 0.5 \), the overall degree of DM satisfaction \( (L) \) is 0.815 with the goal values (\( Z_1 = $670,415 \), \( Z_2 = $95,983 \), \( Z_3 = 1,832 \) hrs).
### Table 5
Result of interior-point method for minimum operator model

<table>
<thead>
<tr>
<th>Item</th>
<th>Output of the problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective, $L$</td>
<td>$Z_1 = $670,160$, $Z_2 = $95,881$, $Z_3 = 1,834$ hrs, $L = 0.818$</td>
</tr>
<tr>
<td>$Q_{oh}$ (units)</td>
<td>$Q_{11i} = 9,550$, $Q_{12i} = 6,385$, $Q_{112} = 0$, $Q_{113} = 0$, $Q_{123} = 0$, $Q_{211} = 19,899$, $Q_{221} = 2,066$, $Q_{212} = 0$, $Q_{222} = 1,213$, $Q_{213} = 0$, $Q_{223} = 0$</td>
</tr>
<tr>
<td>$S_{oh}$ (units)</td>
<td>$S_{11i} = 0$, $S_{12i} = 0$, $S_{112} = 0$, $S_{113} = 0$, $S_{123} = 0$, $S_{21i} = 0$, $S_{22i} = 0$, $S_{212} = 0$, $S_{213} = 0$, $S_{223} = 0$</td>
</tr>
<tr>
<td>$P_{oh}$ (units)</td>
<td>$P_{11i} = 8,720$, $P_{12i} = 5,875$, $P_{112} = 5,300$, $P_{122} = 4,825$, $P_{113} = 0$, $P_{123} = 0$, $P_{21i} = 17,879$, $P_{22i} = 816$, $P_{212} = 11,400$, $P_{222} = 0$, $P_{213} = 0$, $P_{223} = 0$</td>
</tr>
<tr>
<td>$X_{ohl}$ (units)</td>
<td>$X_{111i} = 0$, $X_{1112} = 0$, $X_{1113} = 0$, $X_{1114} = 1,230$, $X_{121i} = 0$, $X_{1212} = 0$, $X_{1213} = 0$, $X_{1214} = 3,400$, $X_{122i} = 0$, $X_{1222} = 0$, $X_{1223} = 0$, $X_{1224} = 1,050$, $X_{131i} = 0$, $X_{1312} = 0$, $X_{1313} = 0$, $X_{1314} = 5,300$, $X_{132i} = 0$, $X_{1322} = 1,725$, $X_{1323} = 0$, $X_{1324} = 3,100$, $X_{211i} = 1,000$, $X_{212i} = 820$, $X_{213i} = 500$, $X_{214i} = 0$, $X_{221i} = 650$, $X_{222i} = 500$, $X_{223i} = 300$, $X_{224i} = 0$, $X_{2122} = 3,000$, $X_{2123} = 2,000$, $X_{2124} = 0$, $X_{2222} = 910$, $X_{2223} = 720$, $X_{2224} = 400$, $X_{2222} = 0$, $X_{2223} = 5,000$, $X_{2224} = 4,000$, $X_{2233} = 2,400$, $X_{2234} = 0$, $X_{2231} = 3,000$, $X_{2232} = 675$, $X_{2233} = 1,150$, $X_{2234} = 0$</td>
</tr>
</tbody>
</table>

### 3.5 Fuzzy goal programming method

In many important real world decision making situations, it may not be feasible, or desirable to try to reduce all goals of an organisation. For example, rather than focusing only on maximising profit or minimising cost, the organisation may be simultaneously be interested in maintaining all goals. Goal programming is an extension of linear programming with an additional feature of including conflicting objectives while still yielding a solution that is optimum with respect to the decision maker’s specification of goal priorities. In goal programming, achievement of a set of goals, at some priority is always preferable to the achievement of a set of goals at a lower ranking priority. Accordingly, it is possible to include several weighted goals within each ranking. Here fuzzy goal programming method is employed and the weight of each objective value is determined by analytical hierarchy process (AHP) to find the weight of each objective various measure is taken into account and the weighting. Again TOPSIS and some other multi-criteria decision making method also used for this purpose. Sachdeva et al. (2009) use TOPSIS for multi-factor failure mode critically analysis. Then Chakraborty and Mishra (2014) develop a methodology to identify of positive deviance from the goal. Marriott et al. (2013) initiate low volume-high integrity product manufacturing methodology by priorities. Then the fuzzy goal programming approach is used to achieve the highest degree of each of the membership goals and thereby obtain the most satisfactory solution for all decision makers in a multi-objective problem.

#### 3.5.1 Weighting the objectives by AHP

Various scenario analyses is done from which AHP analysis weights each objectives

- **Scenario 1**: Removing $Z_3$ (distribution time), consider only $Z_1$ (total production costs) and $Z_2$ (distribution costs) simultaneously.
- **Scenario 2**: Removing $Z_2$ (distribution costs), consider only $Z_1$ (total production costs) and $Z_3$ (distribution time) simultaneously.
• **Scenario 3:** Removing $Z_1$ (total production costs), consider only $Z_2$ and $Z_3$ (distribution time) simultaneously. Table 6 presents the implementation results for scenarios 1 to 3.

• **Scenario 4:** The specific membership value for each of the objective functions strongly affects the overall level of satisfaction of decision maker shown in Table 7.

• **Scenario 5:** If the minimum acceptable level is increased from 0.5 to higher value that is if the decision maker wants to take risk on the basis of market demand, then the value of demand will be increased according to $D_{nj}^\alpha = \frac{1}{2}[\alpha \tilde{D}_{nj} + \tilde{D}_{nj}] + (1-\alpha)\tilde{D}_{nj}$ results shown in Table 8.

• **Scenario 6:** If the regular time production cost increased by one US dollar from the corresponding cost, then strongly affects the overall level of satisfaction of DM results shown in Table 9.

• **Scenario 7:** If the demand is increased, then strongly affects the overall level of satisfaction of decision maker and the objective value results shown in Table 10.

### Table 6  Results of implementation in Scenario 1, 2 and 3

<table>
<thead>
<tr>
<th>Item</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>L value</td>
<td>0.81</td>
<td>0.84</td>
<td>0.99</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>$670,150</td>
<td>$667,622</td>
<td>-</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>$95,871</td>
<td>-</td>
<td>$86,964</td>
</tr>
<tr>
<td>$Z_3$</td>
<td>-</td>
<td>1,853 hours</td>
<td>1,693 hours</td>
</tr>
</tbody>
</table>

### Table 7  Pair wise comparisons for the membership value to satisfaction levels

<table>
<thead>
<tr>
<th>Increase in membership value</th>
<th>$Z_1$</th>
<th>$Z_2$</th>
<th>$Z_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_1$</td>
<td>-</td>
<td>0.88</td>
<td>0.97</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>0.96</td>
<td>-</td>
<td>0.99</td>
</tr>
<tr>
<td>$Z_3$</td>
<td>0.974</td>
<td>0.98</td>
<td>-</td>
</tr>
</tbody>
</table>

### Table 8  Pair wise result of scenario 5 for $\alpha = 0.6$

<table>
<thead>
<tr>
<th>Item</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>L value</td>
<td>0.88</td>
<td>0.90</td>
<td>1</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>$672,730</td>
<td>$670,266</td>
<td>-</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>$94,451</td>
<td>-</td>
<td>$86,428</td>
</tr>
<tr>
<td>$Z_3$</td>
<td>-</td>
<td>1,823 hours</td>
<td>1,682 hours</td>
</tr>
</tbody>
</table>

### Table 9  Pair wise result of scenario 6

<table>
<thead>
<tr>
<th>Item</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>L value</td>
<td>0.824</td>
<td>0.842</td>
<td>1.0</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>$670,706</td>
<td>$671,454</td>
<td>-</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>$95,958</td>
<td>-</td>
<td>$86,428</td>
</tr>
<tr>
<td>$Z_3$</td>
<td>-</td>
<td>1,850 hours</td>
<td>1,681 hours</td>
</tr>
</tbody>
</table>
Table 10  Pair wise result of scenario 7

<table>
<thead>
<tr>
<th>Item</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>L value</td>
<td>0.87</td>
<td>0.89</td>
<td>1.0</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>$672,996$</td>
<td>$670,944$</td>
<td>-</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>$94,581$</td>
<td>-</td>
<td>$87,747$</td>
</tr>
<tr>
<td>$Z_3$</td>
<td>-</td>
<td>1,829 hours</td>
<td>1,709 hours</td>
</tr>
</tbody>
</table>

The result can be modified by expectation of decision maker that is how the weight is given by decision maker to each objective. The value of the relative weights among of multiple goals can be adjusted subjectively based on the decision maker’s experience and knowledge. Then the total average weight is determined from the above information. The criteria and the corresponding values are obtained from the relevant table. Here weighting is done by considering the relationship between each of the objective functions with respect to the satisfaction level. AHP is used for ranking each of the objective value. This process is done by weighting the pair wise objective with respect to the satisfaction value for the increase in demand case shown in Table 12. This is done by weighting each objective function between (1–9) scale, where 1 means equal relation and 9 means extreme relation that shown in Table 11. At the same time, geometric mean and normalised weight is found for the increase in demand case. In the same way for other case the normalised weight is found. After that average normalised weight is calculated for each of the selected criteria that shown in Table 13.

Then by using the above average normalised weight from Table 13 and the PIS and NIS for each of the fuzzy objective functions are taken as lower limit and upper limit respectively and Matlab computer software is used to run this ordinary LP model. Result of goal programming method of weighted average aggregation model is given in Table 14.

Table 11  The fundamental scale for preference weight

<table>
<thead>
<tr>
<th>Level of preference</th>
<th>Weights definition</th>
<th>Level of preference</th>
<th>Weights definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equally preferred</td>
<td>9</td>
<td>Extreme importance</td>
</tr>
<tr>
<td>3</td>
<td>Moderately</td>
<td>2, 4, 6, 8</td>
<td>Intermediate values</td>
</tr>
<tr>
<td>5</td>
<td>Strong importance</td>
<td>Reciprocals</td>
<td>Reciprocals for inverse comparison</td>
</tr>
<tr>
<td>7</td>
<td>Noticeable dominance</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 12  AHP analysis for with respect to the increase in demand

<table>
<thead>
<tr>
<th>Attributes</th>
<th>Production cost (C1)</th>
<th>Distribution cost (C2)</th>
<th>Delivery time (C3)</th>
<th>Geometric mean</th>
<th>Normalised weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production cost (C1)</td>
<td>1.00</td>
<td>0.17</td>
<td>5.00</td>
<td>0.94</td>
<td>0.312</td>
</tr>
<tr>
<td>Distribution cost (C2)</td>
<td>6.02</td>
<td>1.00</td>
<td>0.14</td>
<td>0.95</td>
<td>0.315</td>
</tr>
<tr>
<td>Delivery time (C3)</td>
<td>0.20</td>
<td>7.09</td>
<td>1.00</td>
<td>1.12</td>
<td>0.373</td>
</tr>
<tr>
<td>Total</td>
<td>7.22</td>
<td>8.26</td>
<td>6.14</td>
<td>3.01</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Application of interactive fuzzy goal programming

Table 13  Weight determination of each objective

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Normalised weight</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>With respect to the relation of satisfaction levels with each objectives</td>
<td>0.25</td>
<td>0.37</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>With respect to the membership value to satisfaction levels with each objectives</td>
<td>0.38</td>
<td>0.35</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>With respect to the increase in minimum acceptable level ( \alpha )</td>
<td>0.27</td>
<td>0.37</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>With respect to the increase in regular time production cost</td>
<td>0.26</td>
<td>0.38</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>With respect to the increase in demand</td>
<td>0.31</td>
<td>0.32</td>
<td>0.37</td>
<td></td>
</tr>
<tr>
<td>Average normalised weight</td>
<td>0.294</td>
<td>0.358</td>
<td>0.348</td>
<td></td>
</tr>
</tbody>
</table>

Table 14  Result of fuzzy goal programming method for weighted average aggregation

<table>
<thead>
<tr>
<th>Item</th>
<th>Output of the problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective, ( L )</td>
<td>( Z_1 = $696,035, Z_2 = $87,405, Z_3 = 1,695 ) hours, ( L = 0.866 )</td>
</tr>
<tr>
<td>( Q_{inh} ) (units)</td>
<td>( Q_{111} = 16.052, Q_{121} = 1.310, Q_{112} = 4.698, Q_{122} = 6.995, Q_{113} = 0, Q_{123} = 0, Q_{211} = 8.700, Q_{221} = 1.360, Q_{212} = 0, Q_{222} = 0, Q_{213} = 0, Q_{223} = 0 )</td>
</tr>
<tr>
<td>( S_{inh} ) (units)</td>
<td>( S_{111} = 0, S_{211} = 0, S_{112} = 0, S_{212} = 0, S_{113} = 0, S_{213} = 0, S_{114} = 0, S_{214} = 0, S_{224} = 0 )</td>
</tr>
<tr>
<td>( P_{inh} ) (units)</td>
<td>( P_{111} = 13,902, P_{121} = 0, P_{112} = 11,700, P_{122} = 48.25, P_{113} = 0, P_{123} = 0, P_{211} = 8,000, P_{221} = 910, P_{212} = 5,000, P_{222} = 0, P_{213} = 0, P_{223} = 0 )</td>
</tr>
<tr>
<td>( X_{inhj} ) (units)</td>
<td>( X_{1111} = 0, X_{1112} = 820, X_{1113} = 500, X_{1114} = 1,230, X_{1211} = 0, X_{1212} = 500, X_{2111} = 300, X_{2112} = 710, X_{1213} = 0, X_{1214} = 2,300, X_{1221} = 0, X_{1222} = 720, X_{1223} = 4,000, X_{1224} = 1,050, X_{1121} = 0, X_{1122} = 4,000, X_{1123} = 2,400, X_{1124} = 5,300, X_{1231} = 0, X_{1232} = 710, X_{1233} = 0, X_{1234} = 3,400, X_{1241} = 0, X_{1242} = 1,000, X_{1243} = 3,100, X_{1244} = 5,000, X_{2121} = 0, X_{2122} = 1,000, X_{2123} = 0, X_{2124} = 0, X_{2221} = 0, X_{2222} = 1,825, X_{2223} = 0, X_{2224} = 0 )</td>
</tr>
</tbody>
</table>

For the minimum acceptable membership level \( \alpha = 0.5 \), the overall degree of DM satisfaction (\( L \)) is 0.866 with the goal values (\( Z_1 = \$696,035, Z_2 = \$87,405, Z_3 = 1,695 \) hours). However, if the DM did not accept the initial overall degree of this satisfaction value, then an adjustment is necessary for the \( L \) value to seek a set of better compromise solutions.

3.6 Interactive fuzzy goal programming

In multi-objective programming problems, the final decision is made on the basis of the value judgment of DM. Hence it is important how decision maker elicit the value judgment. In many practical cases, the vector objective function is scalarised in such a manner that the value judgment of DM can be incorporated.

The value judgment of decision maker is reflected by the weight. Although this type of scalarisation is widely used in many practical problems, there is a serious drawback in it. That’s why the solutions obtained by fuzzy goal programming for weighted average operator model can be improved by interactive goal programming.

The value of the objective function obtained by fuzzy goal programming is modified interactively and the findings are given in Tables 15 and 16.
Table 15  Optimum production and distribution plan by interactive goal programming

<table>
<thead>
<tr>
<th>Item</th>
<th>Output of the problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective, $L$</td>
<td>$Z_1 = 680,587, Z_2 = 91,139, Z_3 = 1,744$ hrs, $L = 0.94$</td>
</tr>
<tr>
<td>$Q_{inh}$ (units)</td>
<td>$Q_{111} = 13,630, Q_{121} = 4,337, Q_{112} = 0, Q_{122} = 3,967, Q_{113} = 0, Q_{123} = 0, Q_{211} = 15,820, Q_{221} = 1,360, Q_{212} = 0, Q_{222} = 0, Q_{213} = 0, Q_{223} = 0$</td>
</tr>
<tr>
<td>$S_{inh}$ (units)</td>
<td>$S_{111} = 0, S_{121} = 0, S_{112} = 0, S_{122} = 0, S_{113} = 0, S_{123} = 0, S_{211} = 0, S_{221} = 0, S_{112} = 0, S_{222} = 0, S_{213} = 0, S_{223} = 0$</td>
</tr>
<tr>
<td>$P_{inh}$ (units)</td>
<td>$P_{111} = 12,300, P_{121} = 3,027, P_{112} = 7,700, P_{122} = 4,825, P_{113} = 0, P_{123} = 0, P_{211} = 14,300, P_{221} = 910, P_{212} = 9,000, P_{222} = 0, P_{213} = 0, P_{223} = 0$</td>
</tr>
<tr>
<td>$X_{inhj}$ (units)</td>
<td>$X_{1111} = 0, X_{1112} = 0, X_{1113} = 500, X_{1114} = 1,230, X_{1211} = 0, X_{1212} = 500, X_{1213} = 1,200, X_{1214} = 3,400, X_{1221} = 720, X_{1222} = 400, X_{1223} = 1,050, X_{1224} = 0, X_{1231} = 0, X_{1232} = 575, X_{1233} = 1,150, X_{1234} = 3,100, X_{2111} = 1,000, X_{2112} = 820, X_{2113} = 0, X_{2114} = 0, X_{2121} = 650, X_{2122} = 0, X_{2123} = 0, X_{2124} = 0, X_{2131} = 3,000, X_{2132} = 2,300, X_{2133} = 0, X_{2134} = 0, X_{2211} = 910, X_{2212} = 0, X_{2213} = 0, X_{2214} = 0, X_{2221} = 0, X_{2222} = 0, X_{2223} = 0, X_{2224} = 0, X_{2231} = 0, X_{2232} = 0, X_{2233} = 0, X_{2234} = 0$</td>
</tr>
</tbody>
</table>

Table 16  Result of interactive fuzzy goal programming

<table>
<thead>
<tr>
<th>Interactive</th>
<th>Run-1</th>
<th>Run-2</th>
<th>Run-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>0.9</td>
<td>0.92</td>
<td>0.94</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>680,587</td>
<td>685,899</td>
<td>680,587</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>91,139</td>
<td>89,855</td>
<td>91,139</td>
</tr>
<tr>
<td>$Z_3$</td>
<td>1,744</td>
<td>1,727</td>
<td>1,744</td>
</tr>
</tbody>
</table>

4 Results and findings

Significant management implications for the practical application of the proposed model are as follows.

1 Table 6 demonstrate the interaction of trade-offs and conflicts among dependent objective functions. From Table 6, the total production costs, total distribution costs, and distribution time have diverse meanings. For instance, the combination of the total production costs and carrying and distribution costs in scenario1 of case 3 was $Z_1 = 670,150$ and $Z_2 = 89,571$ with $L$ value 0.81. Moreover, the combination of the total production costs and the total distribution time in Scenario 2 of case 3 was $Z_1 = 667,622$ and $Z_3 = 1,853$ hrs with $L$ value 0.84. Finally, the combination of the total distribution costs and total distribution time in Scenario 3 of case 3 was $Z_2 = 86,964$ and $Z_3 = 1,693$ hours with $L$ value 0.99. These solutions indicate that a fair difference and interaction exists in the trade-offs and conflicts among dependent objective functions. Different combinations of arbitrary objective function may influence the objective and L values. Accordingly, the proposed FMOLP model meets the requirements of the practical application since it can simultaneously minimise the total production costs, total distribution costs, and total distribution time.
The proposed model provides the overall levels of DM satisfaction given the multiple fuzzy goal values \((Z_1, Z_2, \text{ and } Z_3)\). If the solution is \(L = 1\), then each goal is fully satisfied; if \(0 < L < 1\), then all of the goals are satisfied at the level of \(L\), and if \(L = 0\), then none of the goals are satisfied. For example, for the minimum acceptable membership level \(\alpha = 0.5\), the overall degree of DM satisfaction \((L)\) is 0.818 with the goal values \((Z_1 = $670,160 \text{ and } Z_2 = $95,881 \text{ and } Z_3 = 1,834 \text{ hrs})\) was initially generated by interior point active set algorithm. Again by simplex algorithm, DM satisfaction \((L)\) is 0.815 with the goal values \((Z_1 = $670,415, Z_2 = $95,983, \text{ and } Z_3 = 1,832 \text{ hrs})\). At the same time if the decision maker weight the three objective functions by 0.30, 0.36 and 0.34 respectively then for the minimum acceptable membership level \(\alpha = 0.5\), the overall degree of DM satisfaction \((L)\) is 0.87 with the goal values \((Z_1 = $696,035, Z_2 = $87,405, \text{ and } Z_3 = 1,695 \text{ hours})\). Here interactive goal programming technique interactively finds a compromise solution with the overall degree of DM satisfaction \((L)\) is 0.94 with the goal values \((Z_1 = $680,716, Z_2 = $91,139, \text{ and } Z_3 = 1,774 \text{ hours})\). Figure 3 demonstrate the effect of various weights of the corresponding objectives to the satisfaction level.

Figure 3 Relation between weight and satisfaction level (see online version for colours)

4.1 Results comparison

Here minimum operator method and weighted average operator method is used to aggregate all fuzzy set and solution is obtained by various method shown in table below. The proposed model aims to simultaneously minimise the total production costs and the total delivery time with reference to the satisfaction of decision maker. In this production-distribution planning problem, the demand fluctuation of uncertain market, fuzzy labour hour and fuzzy warehouse space is considered. At the same time the proper integration of interconnected process influence on the total cost and corresponding satisfaction level.

First, the proposed method yields an efficient compromise solution. Generally, the \(L\) value may be adjusted to identify better results if the DM did not accept the initial overall degree of this satisfaction value. Additionally, the optimal solution yielded by the minimum operator method may not be an efficient solution, and the computational efficiency of the solution is not been assured. The minimum operator is preferable when a DM wants to make values of the optimal membership functions approximately equal or when a DM believes that the minimum operator is an approximate representation. To overcome the disadvantage of using the minimum operator the compensatory weighted average operator is employed for to obtain overall DM satisfaction degree.
The interactive FMOLP is used for solving the developed multi-product, multi-period integrated production and distribution planning problems with the imprecise forecasted demand, production cost, capacities and inventory holding costs. The interior-point, simplex and active-set are well known algorithm for solving linear programming problem. However, from the application and previous analysis, it is clear that there is no guarantee that simplex will work well on all possible linear programs (Klee and Minty, 1970). The interior point methods are a family of algorithms solving linear programs which come along with an efficient performance guarantee. Form the result obtained before it is interior-point and active-set algorithm give same result.

Here weighting the objectives with respect to the expectation that is describe here as goal programming. In goal programming, achievement of a set of goals, at some priority is always preferable to the achievement of a set of goals at a lower ranking priority. Accordingly, it is possible to include several weighted goals within each ranking. After that interactively modify the result is shown in Table 16 and final comparison shown in Table 17.

**Table 17** Results comparison

<table>
<thead>
<tr>
<th></th>
<th>Interior-point</th>
<th>Simplex</th>
<th>Active-set</th>
<th>Fuzzy goal programming</th>
<th>Interactive fuzzy goal programming</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>0.818</td>
<td>0.815</td>
<td>0.818</td>
<td>0.866</td>
<td>0.94</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>670,160</td>
<td>670,415</td>
<td>670,160</td>
<td>696,035</td>
<td>680,716</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>95,881</td>
<td>95,983</td>
<td>95,881</td>
<td>87,405</td>
<td>91,139</td>
</tr>
<tr>
<td>$Z_3$</td>
<td>1,834</td>
<td>1,832</td>
<td>1,834</td>
<td>1,695</td>
<td>1,744</td>
</tr>
</tbody>
</table>

### 4.2 Sensitivity analysis

1. If the minimum acceptable level is increased from 0.5 to higher value that is if the decision maker wants to take risk on the basis of market demand then the result will changed. For $\alpha = 0.6$ the corresponding result is $Z_1 = $672,730, $Z_2 = $94,451, $Z_3 = 2,021$ hours and the satisfaction value $L = 0.88$.

2. If the regular time production cost increased by one US dollar from the corresponding cost, then strongly affects the overall level of satisfaction of decision maker and the objective value. If the regular time production cost is increased, the corresponding result is $Z_1 = $670,706, $Z_2 = $95,578, $Z_3 = 1,830$ hours and the satisfaction value $L = 0.8236$.

3. At the same time if the decision maker weight the three objective functions by 0.32, 0.34 and 0.34 respectively then for the minimum acceptable membership level $\alpha = 0.5$, the overall degree of DM satisfaction ($L$) is 0.86 with the goal values ($Z_1 = $680,717, $Z_2 = $91,139, $Z_3 = 1,745$ hours).

### 5 Conclusions

This research focused on realistic and flexible integrated production plan by FMOLP method for multi-product and multi-time period in a supply chain under multiple uncertainty conditions. Here minimum operator method is used to aggregate all fuzzy set
at first and then an AHP-based weighted average method is developed with respect to the results obtained from minimum operator method. AHP is used here to determine the relative importance between the three objective functions with respect to the various characteristics in order to obtain realistic results. The problem is solved by various techniques (interior-point, active set, simplex algorithm and goal programming) with reference to fuzzy available supply and total budget at each source and fuzzy forecast demand and warehouse space constraints at each destination in the framework of satisfaction level of decision maker. Again interactively adjust the result gives better result for integrated production cost and ensures economical dispatching the products to the distribution centres. As a result, the proposed method integrates production and distribution problems under flexibility. It attempts to provide a systematic decision-making framework that facilitates a decision maker to interactively adjust the search direction until the preferred efficient solution is obtained.

References


