# Computerised symbolic planetary transmission modelling for automotive design

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**Abstract:** A method and its computerised implementation are proposed for the modelling of planetary transmission systems. Using a computer symbolic scripting environment, a number of idea class objects and appropriate methods to manipulate them are defined, implementing all the relationships that govern the system kinematics, mechanical efficiency and geometric compatibility. The outcome is a system of equations involving the most pertinent design and performance parameters, which allow a systematic interrogation of the relationships between system design and performance. This method is particularly useful for the synthesis and design of novel automotive planetary transmissions, where very early in the design process an analytical solution to several inverse problems is required, making the proposed computerised symbolic modelling indispensable.

**Keywords:** planetary transmission; symbolic modelling; dynamics; efficiency.

**Reference** to this paper should be made as follows: Spitas, C., Amrin, A., Amani, A., Vasileiou, G. and Spitas, V. (2019) 'Computerised symbolic planetary transmission modelling for automotive design', *Int. J. Powertrains*, Vol. 8, No. 1, pp.58–73.

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This paper is a revised and expanded version of a paper entitled 'Computerised symbolic planetary transmission modelling for automotive design' presented at 2017 International Conference on Advanced Vehicle Powertrains in Hangzhou, China 25–26 September 2017.

## 1 Introduction

Due to the high speed and torque ratios and compact and effective structure planetary gear train systems are widely used in many fields, like robotics, vehicle transmission systems, etc. Due to the variety and complexity of structures of planetary systems, the kinematic and dynamic analysis of such systems is generally a complex and tedious process. For the kinematics and efficiency of simple planetary gearboxes, a host of solutions has been proposed over the years by Radzimovsky (1959) and Pennestri and Freudenstein (1993). The automation of physical modelling of planetary gear trains attracted many scientists in engineering domain. For the analysis of a planetary gearbox, researchers developed some methods using the traditional methodologies: tabular methodology, vector analysis technique, graph theory method, matrix theory technique, train value technique, etc. Network-based methods are one of the most efficient, computerised and flexible ones. Freudenstein (1971) and Hsu and Lam (1998) and introduced a graph theory-based method for kinematic analysis of the planetary gear trains. Tian and Li-qiao (1997) developed a matrix-based methodology for kinematic and dynamic analysis. Del Castillo (2002) also proposed a matrix-based computerised approach, towards calculating kinematics and efficiency of generalised planetary gear topologies.

Graph representation introduced by Hsu and Lam (1998) provides a general representation of gearbox topologies in competitively automated level. However, according to the methodology, the gearbox topology can be represented in different ways which can lead to different results. Tian and Li-qiao's (1997) method also provides the same methodology, but the level of automation is quite low. As the technique requires a human to create matrixes, systems can be misinterpreted and it can lead to errors. Circuits are also identified manually. Laus et al.'s (2012) method applied the graph and screw theory for gear train analysis.

It follows from the review of the thus far proposed methods that all of them are reliant on user intervention, not only in order to identify the gearbox topology, but also to translate it using prescribed guidelines into appropriate coefficient matrices, which act as inputs to the employed numerical solvers. This process is not only tedious, but also error-prone, as there is some redundancy in the human input and the coefficient matrices quickly become unwieldy and unintelligible as the number of system links increases.

In this paper, an automated method for kinematic and dynamic analysis of the planetary gear systems is introduced based on synaptic network (SN) theory (Mavrikas et al., 2015; Amrin et al., 2018) and Del Castillo's (2002) basic models, albeit stripped of their formulaic matrix-algorithmic form. The method requires the user to only input the topology and system parameters and handles both the creation and solution of the mathematical model automatically by using a symbolic scripting environment. A computer program is developed for planetary gear train analysis that can be used for planetary gear trains with any number of links. And finally, the program is applied for the analysis of three main types of planetary gear systems, showing the direct applicability of the proposed method for automating parametric design studies at the early design stage.

## 2 Planetary gearbox topologies

In the planetary gear trains, there are two types of movement that characterise links: a rotation around the fixed shafts and planetary type of movement around the same shaft. Considering the type of movement, the links are classified into two types: central links and planets. The kinematic pairs in planetary gear trains are classified into two types: rotating (turning) pairs and gearwheel pairs (Del Castillo, 2002). According to Buchsbaum and Freudenstein (1970), for a single degree-of-freedom planetary gearbox system, the number of gearwheel pairs $J$ and links $L$ are related by

$$J = L - 2 \tag{1}$$

For each gearwheel interaction, a circuit $k$ is identified, where the gearwheel pairs are denoted as gearwheel $i_k$ and gearwheel $j_k$ which are joined together by the carrier $r_k$. Classification of a gearwheel as $i_k$ or $j_k$ is an arbitrary choice, but if the planet is not an input or an output and is not fixed, it is better to define it as planet gear $j_k$, because it will give us the sum of power transmitted to the planet equal to zero.

The planetary gear train topologies can be classified into three types:

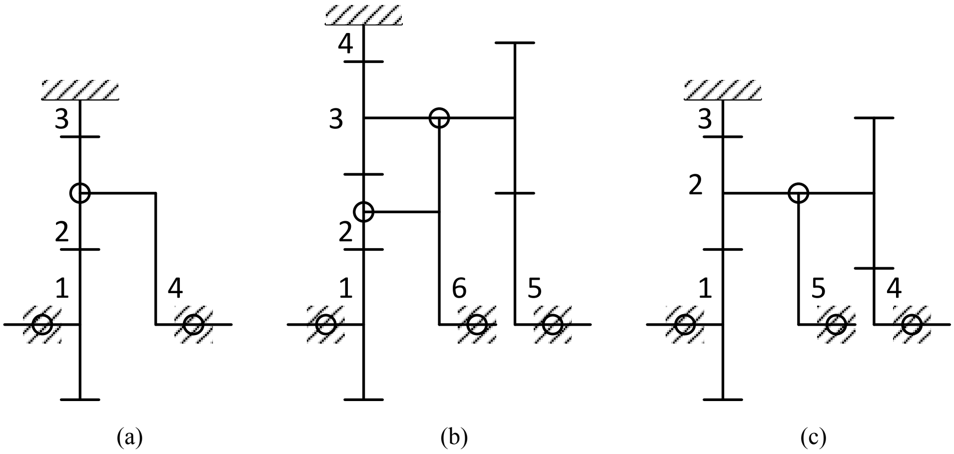
a simple planetary gear train

b meshed planetary gear train

c stepped planetary gear train.

For the case study, we analyse typical configurations for each type of the planetary systems: a simple planetary system which contains an input sun 1, a planet 2, a fixed ring 3 and an output carrier 4 [Figure 1(a)], a meshed planetary system which contains an input sun 1, a planet 2, a double planet 3, a fixed ring 4, an output sun 5 and a carrier 6 [Figure 1(b)] and a stepped planetary system which contains an input sun 1, a double planet 2, a fixed ring 3, an output sun 4 and a carrier 5.

**Figure 1** Planetary gear train topologies, (a) simple planetary (b) meshed planetary (c) stepped planetary




## 3 SN representation

An SN is a network of ideas connected through connections called synapses. Thus, any idea algebra can be visualised graphically by means of a SN. Any object or parameter of a system can be represented via an idea. For the planetary gearbox, all elements *e* and interfaces *i* of the system represented via ideas. The interfaces, which are essentially different from elements only insofar as they accept other ideas as arguments, hence written *i*(*e*1, *e*2, …), are classified into following relationships between the elements:

1 meshing interfaces (*m*)

2 bearing interfaces (*b*)

3 fixed interfaces (*f*).

No sub-classification for the elements is needed, as their function (e.g., sun, planet, carrier, etc.) is defined only by the interfaces that they are part of (arguments to). The next step is to identify types of elements which can be classified by analysing the types of interfaces. After identifying the elements, the algorithm searches the paths in the network to find flows within the network which can be used for identification of circuits. Finally, the Del Castillo's (2002) gear train analysis can be applied.

## 4 Kinematic and dynamic analysis

The kinematic and dynamic analysis of the gear trains applied based on Del Castillo's (2002) methods. For formula derivations, firstly, the table of circuits must be constructed, where the first column indicates an *i* gearwheel link, the second column indicates a *j* gearwheel link and the third column shows the carriers and each row indicates each circuit.

The gear teeth ratio can be calculated by formula (2):

$$Z = Z_{i_k j_k} = \pm \frac{z_{i_k}}{z_{j_k}} \tag{2}$$

where $z_{i_k}$ is the number of teeth of gearwheel *i* of circuit *k* and $z_{j_k}$ is the number of teeth of gearwheel *j* of circuit *k*. The positive or negative sign defines whether the gearwheels engaged externally or internally, respectively.

A system of N unknown torques can be created by the formula $[C^T\ e_{REAC}\ e_{OUT}]^T$, where $C^T$ is a torque on the *j* gears, $e_{REAC}$ is a reaction torque which is a column vector of all zero values except the fixed link element row and $e_{OUT}$ is the output torque which is a column vector of all zero values except the output row.

$$\tilde{C}_{kn} = \begin{cases} Z_k & \\ 1 & \\ -1 - Z_k & \\ 0 & \end{cases} \text{if} \begin{array}{l} n = i_k \\ n = j_k \\ n = r_k \\ \text{otherwise} \end{array} \tag{3}$$

The torque ratio can be defined as

$$R = \frac{T_{out}}{1} = \frac{\det\left(\tilde{C}_{out}\right)}{\det\left(\tilde{C}\right)} = \frac{\Delta_{out}}{\Delta} \tag{4}$$

The efficiency of the gear train system can be defined as

$$\eta = \frac{R^*}{R} \tag{5}$$

where $R^*$ is the torque ratio reduced by friction of gears which is defined by gear ratios $Z_k$ and efficiency $\eta_k$ of the circuit *k*.

$$Z_k^* = Z_{i_k j_k}^* = \begin{cases} \dfrac{Z_k}{\eta_k} & \text{if } \varepsilon_k > 0 \text{ (gearwheel } i \text{ drives } j \text{ in the circuit } k\text{)} \\ Z_k \eta_k & \text{if } \varepsilon_k < 0 \text{ (gearwheel } j \text{ drives } i \text{ in the circuit } k\text{)} \end{cases} \tag{6}$$

where the $\varepsilon_k$ is the sensitivity of the circuit which can be defined as

$$\varepsilon_k = -\frac{Z_k}{R} \frac{\partial R}{\partial Z_k} \tag{7}$$

or if the output speed ration is known, it can be calculated from the algebraic equation:

$$\varepsilon_k = \frac{\Delta_{out,k}}{\Delta_{out}} - \frac{\Delta_k}{\Delta}. \tag{8}$$

## 5 Gearbox computer modelling

The SN representation of the simple gear train (Figure 2), the meshed planetary gear train (Figure 3) and the stepped gear train (Figure 4) were created based on the case study topology representations (Figure 1). For purposes of relationship identification, the fixed ring and a housing were separated into two different types of elements, where 0 is a housing element and fixed rings and other elements were named based on the case study id numbers. Ideas *i*(*e*1, *e*2) represents interfaced between element *e*1 and element *e*2.

**Figure 2** SN representation of the simple planetary gearbox system (see online version for colours)

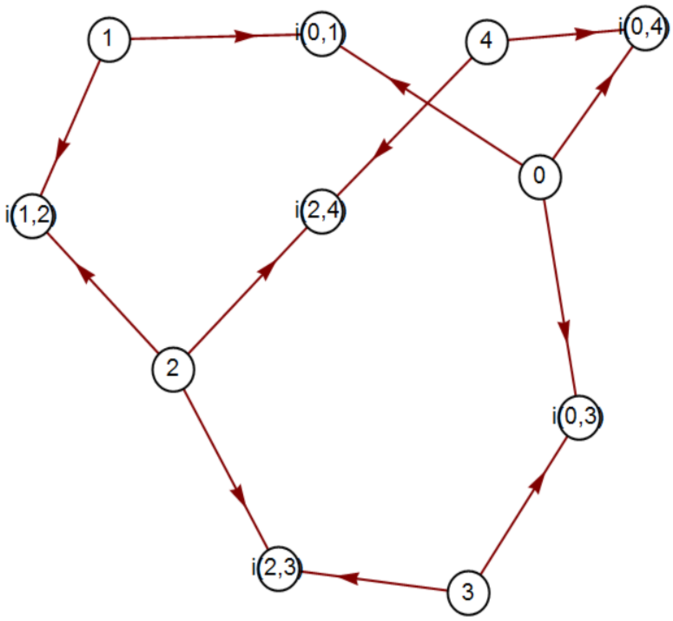



**Figure 3** SN representation of the meshed planetary gearbox system (see online version for colours)

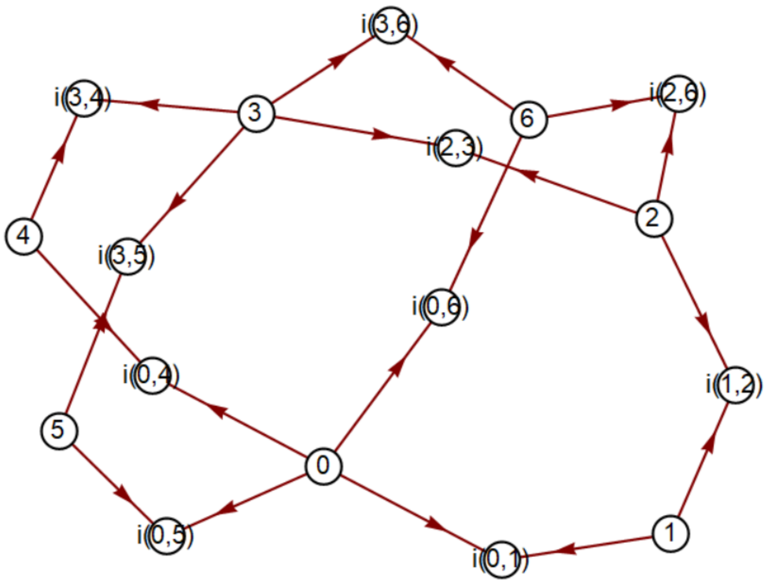
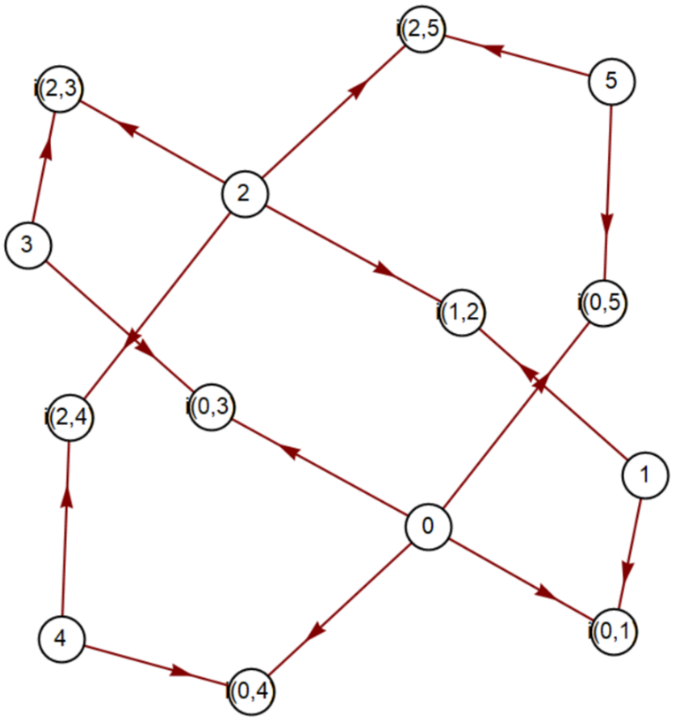


**Figure 4** SN representation of the stepped planetary gearbox system (see online version for colours)




The interfaces between the given elements were classified into three types: meshed (*m*), bearing (*b*) and fixed (*f*) interfaces. The automated sequence starts by parsing the network from the input element through intermediate elements till the output element. For circuits identification the meshed interfaces were analysed, where the interface elements were classified into $i_k$ or $j_k$ circuit elements. After identification of the circuits, computerised Del Castillo's (2002) algorithms were applied.

By identifying the meshing interfaces and types of the elements, the circuits were defined automatically as shown in Table 1.

**Table 1** Tabulated form of system topology, created automatically from the network description

| *Simple planetary system* | | | | *Meshed planetary system* | | | | *Stepped planetary system* | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $k$ | $i_k$ | $j_k$ | $r_k$ | $k$ | $i_k$ | $j_k$ | $r_k$ | $k$ | $i_k$ | $j_k$ | $r_k$ |
| 1 | 1 | 2 | 4 | 1 | 1 | 2 | 6 | 1 | 1 | 2 | 5 |
| 2 | 3 | 2 | 4 | 2 | 2 | 3 | 6 | 2 | 3 | 2 | 5 |
| | | | | 3 | 4 | 3 | 6 | 3 | 2 | 4 | 5 |
| | | | | 4 | 3 | 5 | 6 | | | | |

Gear ratios were defined by the program based on equation (2), as per Table 2.

**Table 2** Definition of gear ratios

| *Simple planetary system* | *Meshed planetary system* | *Stepped planetary system* |
|---|---|---|
| $Z_1 = z_1 / z_2$ | $Z_1 = z_1 / z_2$ | $Z_1 = z_1 / z_2$ |
| $Z_2 = -z_3 / z_2$ | $Z_2 = z_2 / z_3$ | $Z_2 = -z_3 / z_2$ |
| | $Z_3 = -z_4 / z_3$ | $Z_3 = z_2 / z_4$ |
| | $Z_4 = z_3 / z_5$ | |

The torque system matrix $\tilde{C}$ generated by the algorithm based on rule (3), as shown in Table 3.

**Table 3** Automatically generated torque system matrix

| *Simple planetary system* | *Meshed planetary system* | *Stepped planetary system* |
|---|---|---|
| $\begin{pmatrix} Z_1 & 1 & 0 & -1-Z_1 \\ 0 & 1 & Z_2 & -1-Z_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ | $\begin{pmatrix} Z_1 & 1 & 0 & 0 & 0 & -1-Z_1 \\ 0 & Z_2 & 1 & 0 & 0 & -1-Z_2 \\ 0 & 0 & 1 & Z_3 & 0 & -1-Z_3 \\ 0 & 0 & Z_4 & 0 & 1 & -1-Z_4 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$ | $\begin{pmatrix} Z_1 & 1 & 0 & 0 & -1-Z_1 \\ 0 & 1 & Z_2 & 0 & -1-Z_2 \\ 0 & Z_3 & 0 & 1 & -1-Z_3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$ |

The output torque system matrix $\tilde{C}_{out}$ is a result of the matrix $\tilde{C}$ where the input column was replaced by the column containing zeroes except the last row which is equal to 1, as shown in Table 4.

**Table 4** Automatically generated output torque system matrix

| *Simple planetary system* | *Meshed planetary system* | *Stepped planetary system* |
|---|---|---|
| $\begin{pmatrix} 0 & 1 & 0 & -1-Z_1 \\ 0 & 1 & Z_2 & -1-Z_2 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$ | $\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & -1-Z_1 \\ 0 & Z_2 & 1 & 0 & 0 & -1-Z_2 \\ 0 & 0 & 1 & Z_3 & 0 & -1-Z_3 \\ 0 & 0 & Z_4 & 0 & 1 & -1-Z_4 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 1 & 0 & 0 & -1-Z_1 \\ 0 & 1 & Z_2 & 0 & -1-Z_2 \\ 0 & Z_3 & 0 & 1 & -1-Z_3 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}$ |

The torque ratios of the gearbox systems are then calculated as shown in Table 5.

**Table 5** Torque ratios obtained from the solution of the matrix system

| *Simple planetary system* | *Meshed planetary system* | *Stepped planetary system* |
|---|---|---|
| $R = \dfrac{Z_1 - Z_2}{Z_1}$ | $R = \dfrac{-Z_1 Z_2 - Z_3}{-Z_1 Z_2 + Z_1 Z_2 Z_3 Z_4}$ | $R = \dfrac{-Z_1 + Z_2}{-Z_1 + Z_1 Z_2 Z_3}$ |

The sensitivity of each circuit is calculated as per Table 6.

**Table 6** Calculated circuit sensitivity

| *Simple planetary system* | *Meshed planetary system* | *Stepped planetary system* |
|---|---|---|
| $\varepsilon_1 = -\frac{Z_2}{Z_1 - Z_2}$ | $\varepsilon_1 = \varepsilon_2 = -\frac{Z_3}{-Z_1 Z_2 - Z_3}$ | $\varepsilon_1 = \frac{Z_2}{-Z_1 + Z_2}$ |
| $\varepsilon_2 = \frac{Z_1}{Z_1 - Z_2} - 1$ | $\varepsilon_3 = -\frac{Z_1 Z_2}{-Z_1 Z_2 - Z_3} + \frac{Z_1 Z_2}{-Z_1 Z_2 - Z_1 Z_2 Z_3 Z_4}$ | $\varepsilon_2 = -\frac{Z_1}{-Z_1 + Z_2} + \frac{Z_1}{-Z_1 + Z_1 Z_2 Z_3}$ |
| | $\varepsilon_4 = 1 + \frac{Z_1 Z_2}{-Z_1 Z_2 + Z_1 Z_2 Z_3 Z_4}$ | $\varepsilon_3 = 1 + \frac{Z_1}{-Z_1 + Z_1 Z_2 Z_3}$ |

The virtual gear ratios used to calculate the torque ratio reduced by the friction are shown in Table 7.

**Table 7** Calculated virtual gear ratios

| *Simple planetary system* | *Meshed planetary system* | *Stepped planetary system* |
|---|---|---|
| $Z_1^* = \frac{Z_1}{\eta_0}$ | $Z_1^* = \frac{Z_1}{\eta_0}$ | $Z_1^* = \frac{Z_1}{\eta_0}$ |
| $Z_2^* = Z_2 \eta_0$ | $Z_2^* = \frac{Z_2}{\eta_0}$ | $Z_2^* = Z_2 \eta_0$ |
| | $Z_3^* = Z_3 \eta_0$ | $Z_3^* = \frac{Z_3}{\eta_0}$ |
| | $Z_4^* = \frac{Z_4}{\eta_0}$ | |

The efficiency of each planetary gear train is finally obtained, as per Table 8.

**Table 8** Calculated efficiency

| *Simple planetary system* | *Meshed planetary system* | *Stepped planetary system* |
|---|---|---|
| $\eta = \frac{Z_1 - \eta_0^2 Z_2}{Z_1 - Z_2}$ | $\eta = \frac{-Z_1 Z_2 - \eta_0^3 Z_3}{-Z_1 Z_2 - Z_3}$ | $\eta = \frac{\eta_0^2 Z_2 - Z_1}{Z_2 - Z_1}$ |

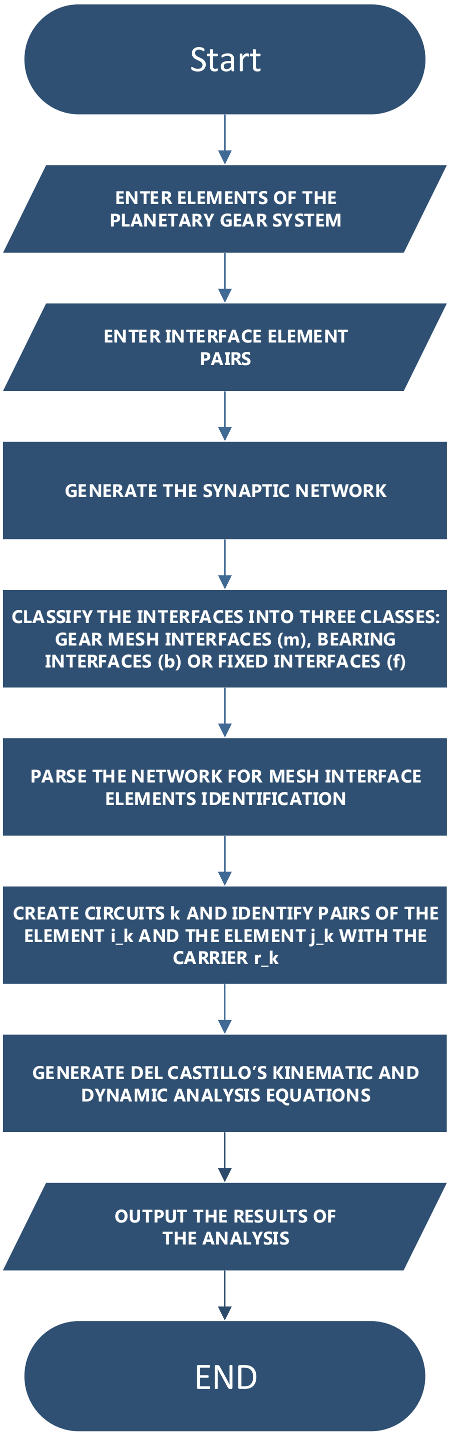
where the efficiency $\eta_k$ was replaced by a constant $\eta_0$.

Final recalculation of the torque ratio and efficiency by using gear ratio equations (2) yields the result of Table 9.

**Table 9** Calculated efficiency as function of gear tooth numbers

| *Simple planetary system* | *Meshed planetary system* | *Stepped planetary system* |
|---|---|---|
| $R = \frac{z_1 + z_3}{z_1}$ | $R = \frac{z_3 z_4 z_5 - z_1 z_3 z_5}{-z_1 z_3 z_5 - z_1 z_3 z_4}$ | $R = \frac{-z_1 z_2 z_4 - z_2 z_3 z_4}{-z_1 z_2 z_4 - z_1 z_2 z_3}$ |
| $\eta = \frac{z_1 + \eta_0^2 z_3}{z_1 + z_3}$ | $\eta = \frac{\eta_0^3 z_4 - z_1}{z_4 - z_1}$ | $\eta = \frac{-z_1 - \eta_0^2 z_3}{-z_1 - z_3}$ |

**Figure 5** The program flowchart (see online version for colours)



## 6 Computer program algorithm

Based on the above methodology, a computer program for kinematic and dynamic analysis of planetary gear train systems with any number of links was developed using Wolfram Mathematica platform. The general flowchart which describes the algorithm of the program is shown in Figure 5.

Step 1 Enter all elements of the system.

Step 2 Identify (input) element interfaces (*i*).

Step 3 Computer generates a SN.

Step 4 The program algorithm classifies said interfaces as either gear mesh interfaces (*m*), bearing interfaces (*b*) or fused/ fixed interfaces (*f*) based on types of elements that contains the interface idea.

Step 5 The program algorithm parses the gear train system SN to identify the circuits and *i* and *j* gearwheels and the carrier.

Step 6 The program algorithm generates kinematic and dynamic analysis equations based on Del Castillo's (2002) methods.

Step 7 The program outputs the results.

## 7 Validation of the model and parametric simulation

The proposed model was validated by comparing known theoretical values for speed and efficiency of configuration (a). Indeed, the corresponding formulas for transmission ratio and efficiency agree with equations provided in Pennestri and Freudenstein (1993).

In terms of time needed for the application of the proposed method, it has been possible to demonstrate significant savings in effort spent to setup the computational problem and parametric study, by means of the developed code (in Wolfram Mathematica). The calculations themselves have been nearly instantaneous in all cases, so there is no significant difference in that case. Specifically, based on the comparative study of the workflows presented in Del Castillo (2002) and in the present work, indicative time estimates have been derived as per Table 10. While the underlying fundamental models of both methods are the same, it can be seen that the automatic transition from the topology description to the definition of the mathematical system model can sped up the effort by a factor of 5–7 [expert users of Del Castillo's (2002) method may be able to perform the system model (matrix) derivation more efficiently, but with more complex system topologies the time expenditure and risk of human error can be expected to increase significantly].

**Figure 6** Parametric study results for the (a) speed ratio and (b) efficiency achieved with configuration (a) for different values of non-dimensional gear tooth numbers (normalised against $z_1$)

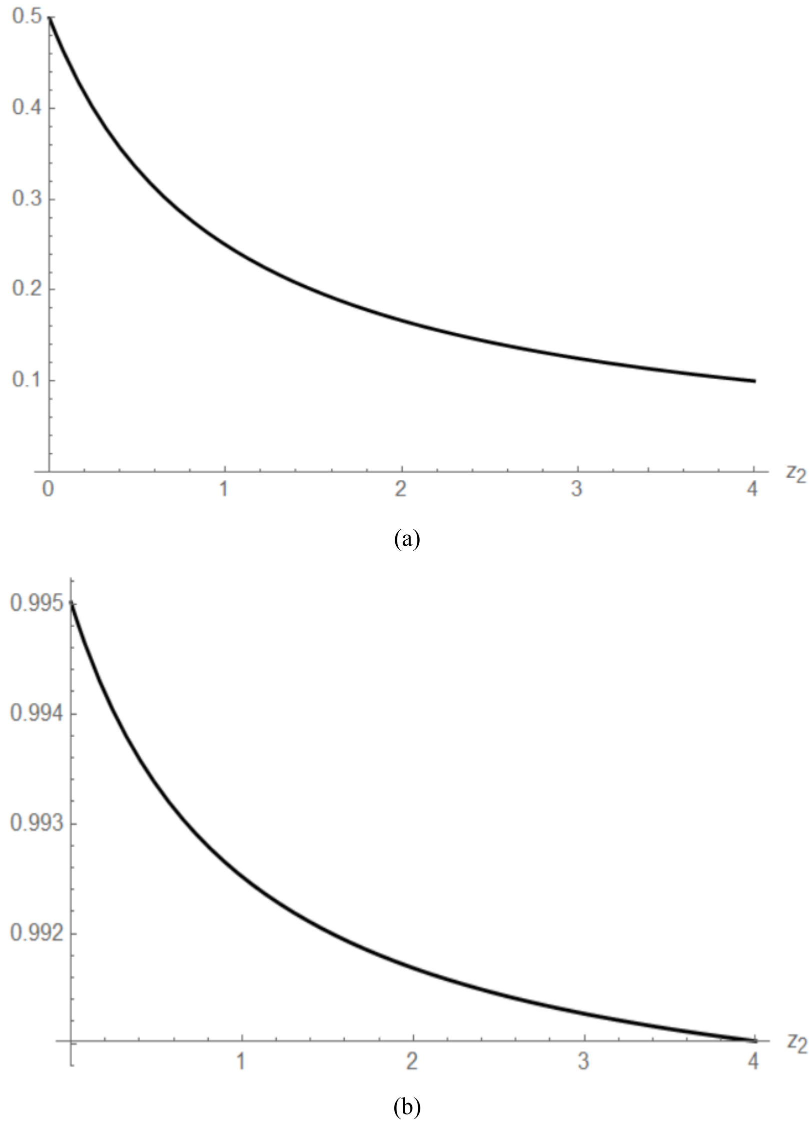
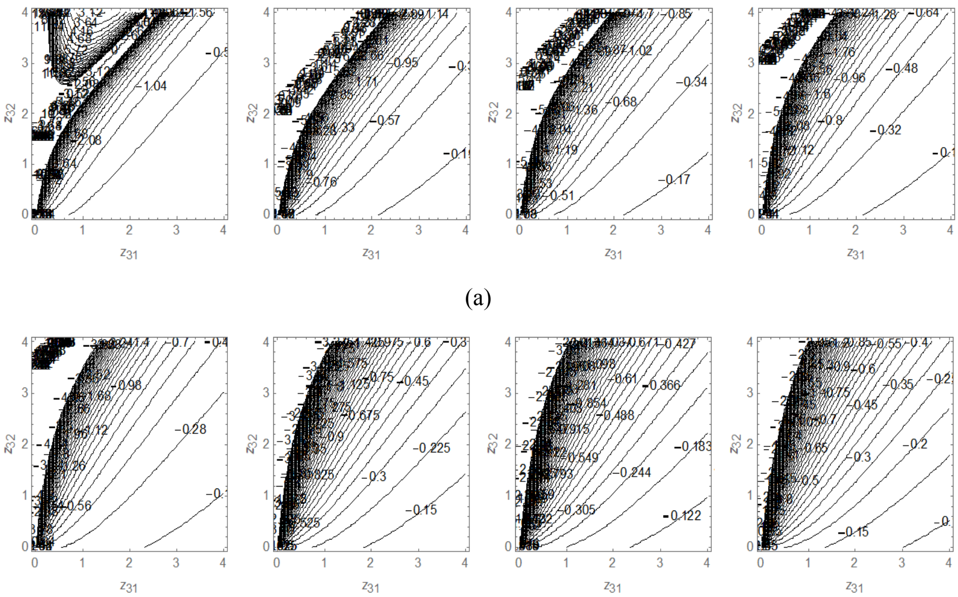


**Figure 7** Parametric study results for the (a) speed ratio and (b) efficiency achieved with configuration (b) for different values of non-dimensional gear tooth numbers (normalised against $z_1$)




(a)

(b)

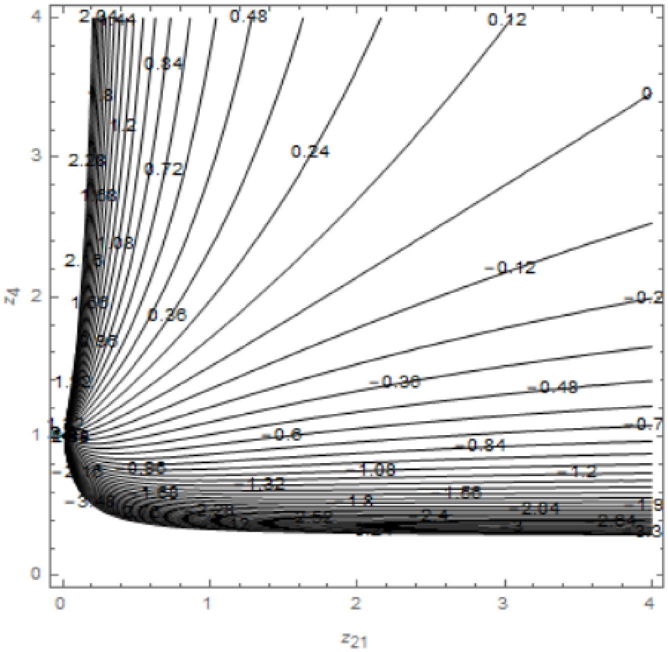
Note: From top left to bottom right, the values for $z_2$ are correspondingly 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75 and 2.00.

**Table 10** Comparison of time spent using workflows/models for configurations of the complexity of (b) and (c)

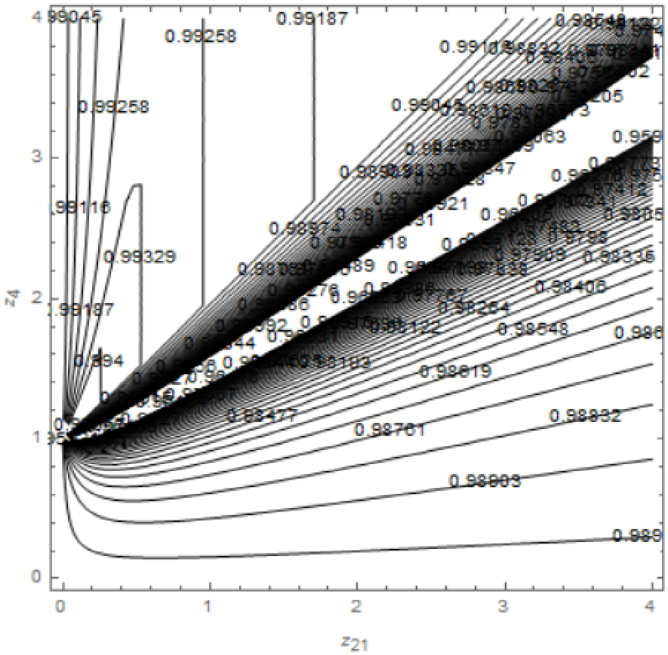
| | *Del Castillo (2002), in min.* | *This work, in min.* |
|---|---|---|
| Determine the system topology | 1 (manual) | 1 (manual) |
| Derive the system model | 5–7 (manual) | 0 (automatic) |
| Solve the system model | Nearly 0 (automatic) | Nearly 0 (automatic) |
| Derive performance metrics (ratio, efficiency) | 0 (automatic) | 0 (automatic) |

Furthermore, based on the model results, parametric studies were run for configurations (a), (b) and (c) to determine the effect and feasibility of selecting different parameter values on the transmission ratio and the efficiency of the considered systems. These are reported in Figures 6–8, respectively. As can be seen, a complete exploration of the parametric design space was achieved. Additional parametric constraints can be included in the analysis, such as to ensure minimum numbers of teeth (incl. the dependent parameters) for all gears; however, this exceeds the scope of this study and is not demonstrated here.

**Figure 8** Parametric study results for the (a) speed ratio and (b) efficiency achieved with configuration (c) for different values of non-dimensional gear tooth numbers (normalised against $z_1$)




(a)




(b)

While recognising the utility of the proposed approach, it should be noted that the method is only intended and suitable for the early phases of the design process, where more accurate data on individual gear efficiencies, errors and influence of system dynamics are not yet available. When sufficient details become available at later stages in the design process, more in-depth analysis based on physics simulations and experimental in-the-loop characterisation will typically be needed.

## 8 Conclusions

Design of planetary gear trains is a complex and tedious process. The kinematic and efficiency analysis of gear trains during the first conceptual stages of design is a mostly human-dependent process which can lead to various mistakes, like misinterpretations of the topology, wrong calculations, etc. In this paper, a new methodology for automatic kinematic and efficiency analysis of planetary gear trains is introduced. A computer program is developed based on the SN equipped with Del Castillo's (2002) methods. The program significantly decreases time spent on analysis and simplifies the process drastically.

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## Nomenclatures

$J$ Number of gearwheel pairs

$L$ Number of links

$R$ Speed ratio (or torque ratio)

$z$ Gear tooth number

$Z$ Gear tooth ratio

$\varepsilon$ Sensitivity of a circuit

$\eta$ Efficiency of tooth mesh