Valuation of a risk-averse investor under incomplete information

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Abstract: In this study, we provide a firm valuation rule under incomplete information. Incomplete information here means that investors have not been informed the true expected return of business cash flows. We describe incomplete information using the filtering theory. We evaluate the firm value under incomplete information with a utility-based valuation rule. The utility-based rule reflects the risk aversion of investors in firm value. We also verify the relation between the quality of information and firm value using sensitivity analysis. This examination indirectly relates the quality of information and cost of capital for the firm. Furthermore, we examine the firm value using the discount cash flow (DCF) method as an example of risk-neutral valuation approaches. By comparing the results of DCF valuation, we describe how a risk-averse investor evaluates the firm under the incomplete information environment.

Keywords: valuation; utility indifference pricing; incomplete information.

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1 Introduction

1.1 Motivations

We consider the firm valuation problem in an incomplete information model. The incomplete information model in this study means that the investor who funds the firm has no full information about the cash flow growth of the firm. Of course, the firm in need of money has full information about itself. Hence, we see a sort of asymmetry of information in financial markets.

Asymmetry of information makes it difficult for firms to obtain finance from investors. Financial economics literature explains that asymmetry of information between a firm and investors leads to adverse selection in the money market and makes it difficult for the firm to obtain money. From the viewpoint of investment decision making in corporate finance theory, this means that the firm value is less than the investment cost due to asymmetry of information and hence investors cannot decide to provide money. This study shows this by using the valuation rule with utility-based pricing.

There are several studies on the relation between information and finance. Earlier studies demonstrated that the quantity or quality of information affects the cost of capital. Muramiya (2005) conducted a good survey on this and found that active disclosure (i.e., releasing more information) reduces the cost of capital (Diamond and Verrecchia, 1981; Botosan, 1997; Easley and O'Hara, 2004), and information quality (or accuracy of earnings forecast) decreases the cost of capital (Botosan and Plumlee, 2013; Francis et al., 2004; Muramiya, 2005). Also, Leuz and Verrecchia (2005) and Lambert et al. (2007) incorporate the quality of information released by firms into the cash flow model and derive valuation formulae of firm stock price and cost of capital via a general equilibrium approach. They demonstrated the influence of information quality on cost of capital and showed that improvement (deterioration) of information quality decreases (increases) the cost of capital. Furthermore, in recent, the contents of the disclosed information have been focused. Cao et al. (2017) found that management forecast reduces the cost of capital. García-Sánchez and Noguera-Gámez (2017) revealed that the integrated report improves the cost of capital. This result implies that improvement (deterioration) of information quality increases (decreases) firm value through the relation between asset price and its returns. In fact, D'Mello and Ferris (2000) showed that stock prices drop when the analysts' earnings forecasts significantly vary.

Our research verifies the relation between quality of information and firm value, and shows that incomplete information makes financing the firm difficult. We further show the need of using the valuation method that considers the risk-averse investor.

1.2 Our model

Our model assumes an investor and a firm, with the firm obtaining finance from the investor. Recall that we consider an incomplete information model in which the investor does not have full information about the growth of the firm. In this work, we represent the cash flow by stochastic differential equations (especially geometric Brownian motions) and the drift term in the process indicates growth of cash flow. We regard the drift term as the growth of the firm. While the drift term in the cash flow process is constant, the complete information model supposes that the investor knows the drift term in advance but the incomplete information model assumes that the term is unknown to the investor. Since the investor does not know the true value of the drift term in advance, she/he should estimate the drift term by the realised cash flow in the course of time. The dynamics of the cash flow incorporated with estimation of the drift term is based on the filtering theory for a given prior distribution (Monoyios, 2007). Estimation of the drift term depends on the prior distribution of the return of cash flow. A variance of distribution implies estimation error or difficulty of estimation. Following a previous research, we refer to estimation error as information quality. Leuz and Verrecchia (2005) exogenously considered the cash flow process to include incomplete information. Using filtering theory, we can naturally incorporate incompleteness of information into the cash flow process as estimated expected return.

1.3 Our approach

Our valuation formula uses a utility-based pricing method. The literature presents some motivations to use such a pricing approach. Christensen and Feltham (2009) used a general equilibrium approach to reflect a macroeconomic factor (i.e., inflation rate) into the pricing rule. Leuz and Verrecchia (2005) used a general equilibrium approach and described the information quality of an individual firm as left to the market with many investors participating.

Utility-based pricing approaches have been used in the context of mathematical finance (Henderson and Hobson, 2002; Hodges and Neuberger, 1989, Monoyios, 2007; Musiela and Zariphopoulou, 2004; Takino, 2011; Young and Zariphopoulou, 2002). The risk-neutral formula is used to price contingent claims when the payoff of financial products is completely replicated with traded assets. This financial market model is called complete (financial) market, and the Black-Scholes model is a typical complete market model. However, a product not traded in the financial market does not allow us to use the risk-neutral formula because the formula does not provide a unique price. Such a market model is called incomplete market model; the utility-based pricing approach has been applied in the incomplete market models. Henderson and Hobson (2002) regarded the cash flow of a firm as a contingent claim not traded in the market; they proposed an asymptotic utility-based formula to evaluate the present value of cash flow. At this point, in the complete market model, the expected return (i.e., drift term) of cash flow equals the risk-free rate and is removed from the pricing formula. For the incomplete market model, in general, the expected return remains in the pricing formula if one uses the utility-based pricing approach. This is why we consider the incomplete market model and use the utility-based approach.

Our valuation formula is an extension of the famous discount cash flow (DCF) method as used in Takino (2011). We use the utility-based pricing approach that prices the present values of future cash flows instead of the approach of discounting the cash flow using a discount rate similar to the weighted average cost of capital (WACC). Following the result of Monoyios (2007), we determine the present values of future cash flows using the incomplete information model. We finally provide the firm value by summing up those values.

1.4 Results

All our results are obtained through numerical simulations. We verify the influences of the incomplete information model on valuation through sensitivity analyses and comparisons with the complete information model and a standard valuation rule. As shown in previous researches, a fall in information quality decreases the firm value, and this value becomes large when the investor is more risk averse. An increase in expected return of cash flow increases the firm value. The effect of information quality on firm value is more significant than the effect of expected business return. We also observe that the investor can be more sensitive to changes information quality. The incomplete information environment decreases the firm value compared to the complete information case. Furthermore, we find that the investor underestimates the firm value in the incomplete information environment when she/he is risk averse.

1.5 Remaining contents

The rest of paper is organised as follows: in Section 2, we introduce a complete information model and an incomplete information model. In Section 3, we provide valuation formulae with utility-based pricing for both complete and incomplete information models. In Section 4, we implement our valuation rules. We first compare the results with the complete information model. Next, we observe how the incomplete information environment affects firm value through sensitivity analyses. Finally, we compare our results with those of the DCF approach to find the characteristic of the valuation rule for the risk-averse investor. In Section 5, we conclude our study.

2 Model

We first define the market model for assets traded in our financial market. We consider a filtered probability space $(\Omega_W, \{\mathcal{F}_{W,t}; t \geq 0\}, \phi)$ and denote the expectation under measure ϕ by E^W. The stock price and cash flow are modelled using the two-dimensional Brownian motion $W = (W_1, W_2)$, where the correlation coefficient between W_1 and W_2 is ρ (const.), and filtration $\mathcal{F}_{W,t}$ is generated by Brownian motion; that is, $\mathcal{F}_{W,t} = \sigma(W_s : 0 \leq s \leq t)$.

We suppose that a risk-free asset B with zero interest rate and stock S are traded in the financial market. The price processes for these are represented as

$$\begin{aligned} \mathrm{d}B_t &= 0 \\ \frac{\mathrm{d}S_t}{S_t} &= \mu \mathrm{d}t + \sigma \mathrm{d}W_{1t} \end{aligned} \tag{2.1}$$

for $t \geq 0$, where μ, σ are constant and $\mu > r$ without loss of generality.

2.1 Complete information model

We introduce the cash flow process under complete information as a counter example of the incomplete information model. We consider the filtered probability space

$(\Omega_W, \{\mathcal{F}_{W,t}; t \geq 0\}, \phi)$ again. The cash flow process under complete information is driven by

$$\frac{\mathrm{d}Y_t}{Y_t} = \alpha \mathrm{d}t + \beta \mathrm{d}W_{2t} \tag{2.2}$$

for $t \geq 0$, where α, β are constants. Since S is the stock price of the firm to be evaluated, it is natural to assume that $\rho > 0$.

2.2 *Incomplete information model*

Next, we model the cash flow process under incomplete information, which is represented by

$$\frac{\mathrm{d}Y_t}{Y_t} = \alpha \mathrm{d}t + \beta \mathrm{d}W_{2t}, \tag{2.3}$$

where β is a constant; α is also a constant parameter, but the investor did not fully know its true value when she/he invested money in the firm. This is the incomplete information model in our study; it is defined precisely in the following assumption:

Assumption 2.1 (Incomplete information model): The investor providing money to the firm has no information about the true expected return of cash flow at time zero, but knows about a prior distribution for the return of cash flow. The distribution is assumed to be normal with mean m_0 and variance Γ_0.

We mathematically define our incomplete information model based on Ishijima (1999). α in (2.3) is a random variable on $(\Omega_\alpha, \mathcal{F}_\alpha, \nu)$ and satisfies $E\left[\alpha^4\right] < \infty$. E denotes the expectation under probability measure P, which is defined below. $\nu \circ \alpha^{-1}$ follows the normal distribution with mean m_0 and variance Γ_0; that is,

$$m_0 = E[\alpha],$$
$$\Gamma_0 = E[(\alpha - m_0)^2].$$

Then, filtered probability space $(\Omega, \{\mathcal{F}_t; t \geq 0\}, P)$ is defined to satisfy $\Omega = \Omega_\alpha \times \Omega_W$, $\mathcal{F}_t = \mathcal{F}_\alpha \vee \mathcal{F}_{W,t}$ and $P = \nu \otimes \phi$.

At this point, Γ_0 is an estimation error of the expected return α of cash flow. In fact, when the value of Γ_0 is low, the realised α is not far from m_0. That is, the investor can correctly estimate the expected return α of cash flow. As mentioned in Lambert et al. (2007), if the firm is active in disclosing information or the quality of the released information is high, then Γ_0 is reduced, because better information of firms increases the estimate accuracy of cash flow. We perform sensitivity analysis of firm value with respect to Γ_0 in a numerical example.

From Assumption 2.1, by applying the approach of Monoyios (2007), the cash flow process can take into account the estimation of α represented by stochastic differential equation without uncertainty. To this end, we introduce the observation process G by

$$G(t, Y_t) = \int_0^t \frac{\mathrm{dY_s}}{Y_s}$$

and the natural filtration $\mathcal{F}_{G,t}$ $(s \geq 0)$ generated by G.

Theorem 2.1: The cash flow process Y in the incomplete information model is represented by the process in the complete information model, replacing α with its estimator m; that is,

$$\frac{\mathrm{d}Y_t}{Y_t} = m(t, Y_t)\mathrm{d}t + \beta \mathrm{d}\hat{W}_{2t}, \tag{2.4}$$

where $m(t, Y_t)$ is

$$m(t, Y_t) = \frac{\beta^2 m_0 + \Gamma_0 G(t, Y_t)}{\beta^2 + \Gamma_0 t} \tag{2.5}$$

with

$$G(t, Y_t) = \ln\left(\frac{Y_t}{Y_0}\right) + \frac{1}{2}\beta^2 t. \tag{2.6}$$

Also, $\hat{W}_2$ is an innovation process given by

$$\hat{W}_{2t} = \frac{1}{\beta}\left\{G(t, Y_t) - \int_0^t m(s, Y_s)\mathrm{d}s\right\},$$

and a $\mathcal{F}_{G,t}$-Brownian motion under P with a correlation coefficient ρ to W_1.

The proof of Theorem 2.1 follows from the proof of Theorem 1 in Monoyios (2007). Monoyios' (2007) theorem is based on the argument in Øksendal (2003). We also use the result of Øksendal (2003).

Proof: We define an observation process G by

$$G_t = \int_0^t \frac{\mathrm{dY_s}}{Y_s} \tag{2.7}$$

and the natural filtration of G by $\mathcal{F}_{G,t}$ $(t \geq 0)$. We set the optimal filter of α by

$$m_t := m(t, Y_t) = E[\alpha|\mathcal{F}_{G,t}], \quad m(0, Y_0) = m_0$$

and its mean-square-error at t by

$$\Gamma_t := \Gamma(t, Y_t) = E[(\alpha - m(t, Y_t))^2|\mathcal{F}_{G,t}], \quad \Gamma(0, Y_0) = \Gamma_0.$$

Also, we define the innovation process M by

$$M_t = G_t - \int_0^t m_s \mathrm{d}s.$$

From Lemma 6.2.6 of Øksendal (2003), $\hat{W}_2$, defined by

$$\hat{W}_{2t} = \int_0^t \frac{1}{\beta}\mathrm{d}M_s, \tag{2.8}$$

is the $\mathcal{F}_{G,t}$-Brownian motion under P correlated to W_1 with correlation coefficient ρ. By plugging (2.8) into (2.3), we have (2.4). Furthermore, (2.6) follows from (2.4).

Next, because α is a constant and (2.7), we have the following dynamics:

$$\mathrm{d}\alpha = 0,$$
$$\mathrm{d}G_t = \alpha \mathrm{d}t + \beta \mathrm{d}W_{2t}.$$

Then, from Theorem 6.2.8 of Øksendal (2003), filter m is driven by stochastic differential equation (SDE)

$$\mathrm{d}m_t = -\frac{\Gamma_t}{\beta^2} m_t \mathrm{d}t + \frac{\Gamma_t}{\beta^2} \mathrm{d}G_t \tag{2.9}$$

and Γ is the solution of ordinary differential equation (ODE)

$$\frac{\mathrm{d}\Gamma_t}{\mathrm{d}t} = -\frac{\Gamma_t}{\beta^2}. \tag{2.10}$$

The solution of (2.10) is given by

$$\Gamma_t = \frac{\beta^2 \Gamma_0}{\beta^2 + \Gamma_0 t}. \tag{2.11}$$

At this point, when we define $v_t = \Gamma_t/\beta^2$ with $v_0 = \Gamma_0/\beta^2$, (2.11) and SDE (2.9) become

$$v_t = \frac{\Gamma_0}{\beta^2 + \Gamma_0 t}, \tag{2.12}$$

$$\mathrm{d}m_t = -v_t m_t \mathrm{d}t + v_t \mathrm{d}G_t, \tag{2.13}$$

respectively. (2.13) is a linear SDE (see Section 5.6 in Karatzas and Shreve, 1991) and has a unique explicit solution. By substituting (2.12) into (2.13) and solving linear SDE (2.13), we have

$$m_t = \frac{m_0 + v_0 G_t}{1 + v_0 t}.$$

Plugging v_0 into this provides (2.5). □

2.3 Wealth process

We use utility indifference pricing to evaluate the present value of a future cash flow as utility-based pricing in this study. Also, this is a pricing approach of a risk-averse investor. The indifference price is derived by solving the investor's utility maximisation problems for her/his future wealth. The wealth includes a portfolio consisted with the risk free asset and the risky asset. The investor funding the firm invests her/his money $\pi(s)$ of wealth $X(s)$ at time s into stock and deposits the rest of the money into a bank account. We further assume that the portfolio strategy π satisfies the admissible strategy defied as follows:

Definition 2.1 (Admissible strategy): The portfolio strategy π is admissible if π satisfies

$$E\left[\int_0^{t_i} \pi_s^2 \mathrm{d}s\right] < \infty$$

for each $i \in \mathbb{N}$, and is self-financing strategy defined by

$$\begin{aligned} \mathrm{d}X_t &= \frac{\pi_t}{S_t}\mathrm{d}S_t + \frac{X_t - \pi_t}{B_t}\mathrm{d}B_t \\ &= \pi_t(\mu \mathrm{d}t + \sigma \mathrm{d}W_{1t}) \end{aligned} \tag{2.14}$$

for $t \geq 0$. Furthermore, we denote the set of admissible strategies by $\mathcal{A}$.

We suppose that the utility function U of the investor is given by an exponential utility,

$$U(x) = -\frac{1}{\gamma}e^{-\gamma x}, \tag{2.15}$$

where γ (> 0) is an absolute risk-averse coefficient that shows the investor's attitude to risk. When the value of γ increases, the investor becomes more risk averse.

3 Valuations

In this study, we use the valuation approach with the utility indifference price proposed by Takino (2011). This approach extends the standard DCF method and uses indifference pricing to calculate the present values of future cash flows instead of discounting the cash flow with an appropriate discount rate. This approach is constructed as follows. First, the indifference prices of future cash flows are solved as a present value. Next, we determine the firm value by summing up the indifference prices. That is, by denoting the utility indifference price $p(0; t_i)$ for cash flow Y_{t_i} at time t_i, the firm value V is given by

$$V = \sum_{i=0}^{N} p(0; t_i). \tag{3.1}$$

As to the decision making, the investor decides to fund the firm if the firm value V is larger than the investment cost I, and not otherwise.

At this point, utility indifference pricing is a pricing approach of contingent claims, like derivatives taking into account the incompleteness of the (financial) market model. Incompleteness of the market means that the contingent claims are completely replicated with traded assets. We regard the future cash flow as a contingent claim that cannot be completely replicated with traded assets, as in Henderson and Hobson (2002). If the cash flow is completely hedged, we can use the risk-neutral pricing approach. The work left for us is to solve the utility indifference price for the cash flow in both the complete and incomplete information models.

3.1 Valuation under complete information model

We next derive the utility indifference price $p(0;t_i)$ for cash flow Y_{t_i} at time t_i ($i = 0,1,2,\ldots,N$) in the complete information model (Section 2.1).

In order to derive the utility indifference price, we consider two distinct utility maximisation problems,

$$\begin{aligned} u(t,x) &= \sup_{\pi\in\mathcal{A}} E^W\left[U(X_{t_i})\middle|\, X_t = x\right], \\ v(t,x,y) &= \sup_{\pi\in\mathcal{A}} E^W\left[U(X_{t_i}+Y_{t_i})\middle|\, X_t = x, Y_t = y\right], \end{aligned} \tag{3.2}$$

for $t < t_i$. Then, the utility indifference price $p(0,t_i)$ for cash flow Y_{t_i} at time t_i is defined as follows:

Definition 3.1 (Utility indifference price): The utility indifference price $p(0,t_i)$ for cash flow Y_{t_i} at time t_i solves to

$$u(0,x) = v(0,x-p(0;t_i),y). \tag{3.3}$$

From Definition 3.1, we can solve two utility maximisation problems in (3.2). The first problem u in (3.2) yields a closed formula (c.f. Cvitanić and Zapatero, 2004). On the one hand, several papers have considered the second problem v in (3.2). Young and Zariphopoulou (2002) provided an explicit formula of value function v for an exponential utility case, and Henderson and Hobson (2002) gave an asymptotic formula of value function v with multiple units of Y_T for a power utility case. We can apply their results to our model. By solving u and v in (3.2) and substituting those into (3.3), we obtain the utility indifference price.

Before providing the indifference price of Y_T, we introduce an equivalent martingale measure Q^W with respect to ϕ.

Definition 3.2: Set the function $\mathcal{E}^W$ as

$$\mathcal{E}^W(t) = \exp\left\{-\int_t^{t_i}\theta \mathrm{d}W_{1s} - \frac{1}{2}\int_t^{t_i}\theta^2 \mathrm{d}s\right\}$$

for each $i \in \mathbb{N}$, where $\theta = \mu/\sigma$. Then, we define an equivalent martingale measure Q^W with respect to ϕ by

$$\left.\frac{\mathrm{d}Q^W}{\mathrm{d}\phi}\right|_{\mathcal{F}_{W,t}} = \mathcal{E}^W(t).$$

Also, we denote the expectation under the measure Q^W by E^{Q_W}.

Theorem 3.1: Under the complete information model, the utility indifference price $p(0,t_i)$ for cash flow Y_{t_i} at time t_i is given by

$$p(0,t_i) = -\frac{1}{\gamma(1-\rho^2)}\ln E^{Q_W}\left[e^{-\gamma(1-\rho^2)Y_{t_i}}\right], \tag{3.4}$$

where Y is driven by

$$\frac{\mathrm{d}Y_t}{Y_t} = (\alpha - \rho\beta\theta)\mathrm{d}t + \beta \mathrm{d}W_{2t}^{Q_W} \tag{3.5}$$

with the standard Brownian motion $W_2^{Q_W}$ under Q^W.

We note that α in (3.5) is constant and known to the investor in the complete market model.

3.1.1 Valuation rule

From Theorem 3.1, the valuation rule with utility-based pricing is given by

$$V = \sum_{i=0}^{N} \left\{ -\frac{1}{\gamma(1-\rho^2)} \ln E^{Q_W} \left[e^{-\gamma(1-\rho^2)Y_{t_i}} \right] \right\}, \tag{3.6}$$

where Y is (3.5).

3.2 Valuation under incomplete information model

3.2.1 Utility indifference price under incomplete information

We derive the utility indifference price $p(0;t_i)$ for cash flow Y_{t_i} at time t_i $(i = 0, 1, 2, \ldots, N)$. Although we derive the indifference price under the incomplete information environment where the drift term α of the process Y in (2.3) is an unknown parameter, we can utilise the result of the Kalman filter approach in Section 4.1 of Monoyios (2007) for our settings in Section 2 and Assumption 2.1.

Now, we consider two distinct utility maximisation problems,

$$\begin{aligned} u(t,x) &= \sup_{\pi \in \mathcal{A}} E\left[U(X_{t_i}) \middle| X_t = x \right], \\ v(t,x,y) &= \sup_{\pi \in \mathcal{A}} E\left[U(X_{t_i} + Y_{t_i}) \middle| X_t = x, Y_t = y \right], \end{aligned} \tag{3.7}$$

where $t < t_i$. Then, the utility indifference price $p(0,t_i)$ for the cash flow Y_{t_i} at time t_i is defined as follows:

Definition 3.3 (Utility indifference price): The utility indifference price $p(0,t_i)$ for the cash flow Y_{t_i} at time t_i solves to

$$u(0,x) = v(0, x - p(0;t_i), y). \tag{3.8}$$

Monoyios (2007) solved the utility maximisation problems (3.7) under the incomplete information model and provides the utility indifference price for a claim that is bounded below. By applying the result of Monoyios (2007), we obtain an explicit formula of utility indifference price for the cash flow Y. Before providing the indifference price of Y_T, we introduce an equivalent martingale measure Q with respect to P.

Definition 3.4: Set the function $\mathcal{E}$ as

$$\mathcal{E}(t) = \exp\left\{-\int_t^{t_i} \theta \mathrm{d}W_{1s} - \frac{1}{2}\int_t^{t_i} \theta^2 \mathrm{d}s\right\}$$

for each $i \in \mathbb{N}$, where $\theta = \mu/\sigma$. Then, we define an equivalent martingale measure Q with respect to $\tilde{P}$ by

$$\left.\frac{\mathrm{d}Q}{\mathrm{d}P}\right|_{\mathcal{F}_t} = \mathcal{E}(t).$$

Also, we denote the expectation under the measure Q by E^Q.

Theorem 3.2: Under the incomplete information model provided in Definition 2.1, the utility indifference price $p(0, t_i)$ for the cash flow Y_{t_i} at time t_i is given by

$$p(0, t_i) = -\frac{1}{\gamma(1-\rho^2)} \ln E^Q\left[e^{-\gamma(1-\rho^2)Y_{t_i}}\right], \tag{3.9}$$

where Y is driven by

$$\frac{\mathrm{d}Y_t}{Y_t} = (m(t, Y_t) - \rho\beta\theta)\mathrm{d}t + \beta \mathrm{d}\hat{W}_{2t}^Q, \quad Y_0 \geq 0, \tag{3.10}$$

and $m(\cdot, Y_\cdot)$, $G(\cdot, Y_\cdot)$ are as given in Theorem 2.1.

Proof: We consider the function g of Y_T, $g(Y_T) = Y_T$ for convenience. Y_T in (3.10) is non-negative under the assumption of $Y_0 \geq 0$. Therefore, function $g(Y_T)$ is bounded below. Then, using the result of Monoyios (2007), we complete the proof. □

We need to change the measure from P to Q to solve the utility maximisation problem v. For a more precise solution, see Monoyios (2007).

3.2.2 Valuation rule

From Theorem 3.2, the valuation rule with utility-based pricing is given by

$$V = \sum_{i=0}^{N}\left\{-\frac{1}{\gamma(1-\rho^2)} \ln E^Q\left[e^{-\gamma(1-\rho^2)Y_{t_i}}\right]\right\}, \tag{3.11}$$

where Y is (3.10).

4 Numerical result

In this section, we numerically analyse the influence of incomplete information on firm value. Through this section, we divide a year into 250 grids and set the Monte Carlo simulation time 100,000 times. We consider a project with 5 years (i.e., $N = 5$) and assume that cash flow arises once a year.

4.1 Valuation from risk-averse investor

We first compare these results with numerical results from the complete information model. Next, we observe the changes in firm value by changing the parameters in the incomplete information model. The parameters for the incomplete information model are $\mu = 0.1$, $\sigma = 0.2$, $\beta = 0.4$, $\rho = 0.75$, $\gamma = 0.1$, $m_0 = 0.02$, and $\Gamma_0 = 0.16$. The parameters for the complete information model are $\mu = 0.1$, $\sigma = 0.2$, $\alpha = 0.02$, $\beta = 0.4$, $\rho = 0.75$, and $\gamma = 0.1$. Recall that the drift parameter α of Y in the complete information model is a known and constant parameter.

4.1.1 Comparison of the incomplete information and complete information models

Table 1 gives the firm values for the incomplete and complete information models for each initial cash flow Y_0. The valuation rules for the incomplete information and complete information models are given by (3.11) and (3.6), respectively. In Table 1, for example, the firm value for incomplete information model is 54.68 and that for the complete information model is 60.81 when $Y_0 = 20$. Therefore, at $Y_0 = 20$, the firm value under incomplete information is lower than that under complete information. This property stands for all other Y_0 except $Y_0 = 10$. Hence, Table 1 shows that the firm value under incomplete information is equal to or less than that under complete information.

Table 1 Firm values for the incomplete and complete information models for each initial cash flow Y_0

Initial CF: Y_0	*10*	*20*	*30*	*40*	*50*
Incomplete information	32.13	54.68	73.65	90.52	104.85
Complete information	32.13	60.81	87.21	111.78	134.27

Note: The upper line gives the firm value under the incomplete information model, and the lower one lists the value under the complete information model.

At this point, we consider the implications for these results from viewpoint of:

a accuracy of estimation (or willingness of disclosure)

b cost of capital.

a In the simulation, we set

$$m_0 = \alpha = 0.02.$$

This implies that the firm announces information about the growth of its own business to investors. However, even if the firm is honest, the firm value is underestimated in the incomplete information model. This result is interpreted as follows: The investor should estimate the growth rate of cash flow even if the firm informs the investor about its future growth fairly. Nevertheless, the investor behaves as under incomplete information and might underestimate the firm value due to an estimation error.

This difference in model results may be due to reliability or transparency issues of the released information. As argued in Lambert et al. (2007), the investor can accurately estimate the value of the firm that provides precise and more information. Therefore, the difference in firm value between incomplete and complete information models is reduced if the investor can accurately estimate the firm value. In this study, accuracy of estimation is represented by Γ_0, the variance of prior distribution for α. Thus, the difference between the firm values of the two information models is reduced when Γ_0 becomes small. On the other hand, the firm informs higher growth of business if it wants more finance. This makes the investor overestimate α with this information to obtain a larger m_0. Hence, we need to verify how the firm is evaluated when m_0 becomes large under incomplete information. These are analysed in the next section.

b We next discuss the difference in firm value between incomplete and complete information models from the viewpoint of cost of capital. We have not estimated the cost of capital or used the DCF approach. However, the decrease in firm value implies the increase in cost of capital because the asset price and its return have a trade-off relation in general. Recall that the firm value for the incomplete information model is smaller than that for the complete market model. This is interpreted as the cost of capital increasing when the uncertainty of as 'inaccuracy of information' is added to the uncertainty of cash flow by incorporating incomplete information.

4.1.2 Sensitivity analysis

In this section, we examine how the firm value varies when each parameter is changed under the incomplete information model. We first show the firm values for each m_0 and Γ_0 (Table 2). We observe that the firm value monotonically increases when Γ_0 becomes small or m_0 becomes large.

Table 2 Firm values under the incomplete information model for each Γ_0 and m_0

m_0 \ Γ_0	*0.01*	*0.04*	*0.09*	*0.16*	*0.25*
0.000	124.62	116.41	108.55	102.30	96.52
0.005	126.04	117.52	109.19	102.81	97.11
0.010	127.59	118.40	109.94	102.80	97.78
0.015	128.59	119.93	110.81	104.37	98.58
0.020	130.17	120.71	111.67	105.10	99.47

According to the previous section, we next analyse:

1 the variation in firm value when Γ_0 is changed

2 whether the effect of Γ_0 on firm value is related to risk aversion or not

3 the change in firm value when estimator m_0 of the expected return of cash flow is changed

4 which parameter, Γ_0 or m_0, has more influence on firm value.

1 Table 3 shows the firm values under the incomplete information model with changing Γ_0 and the firm value under the complete information model for each Y_0. As shown in Tables 1 and 2, the firm value under incomplete information is equal to or less than that under complete information, and it increases with a decrease in Γ_0. Table 3 shows that the firm value under incomplete information becomes close to that under complete information when Γ_0 decreases for each Y_0. Recall that a small Γ_0 implies accuracy of estimation for α. From the results, the difference between the firm values when evaluated by the firm and when evaluated by the investor is reduced when the accuracy of estimation is improved (i.e., when Γ_0 becomes small).

Table 3 Convergence of firm values under incomplete information to the firm values under complete information when Γ_0 becomes small for each Y_0

Incomplete information							*Complete information*
Γ_0	*0.000*	*0.002*	*0.004*	*0.006*	*0.008*	*0.01*	
$Y_0 = 10$	32.13	32.06	32.04	32.04	31.84	31.79	32.13
$Y_0 = 20$	60.80	60.53	60.42	60.11	60.03	59.55	60.81
$Y_0 = 30$	86.89	86.25	86.09	85.68	85.39	84.69	87.21

Note: The second to seventh columns, list the values under the incomplete information model. The last column shows the firm value under the complete information model.

2 Table 4 shows the elasticity of firm value with respect to Γ_0 for each γ. Suppose that the elasticity is calculated by

$$\left|\frac{\Delta V/V}{\Delta\Gamma_0/\Gamma_0}\right|.$$

We have already observed that an increase in Γ_0 reduces the firm value in 1. We now examine how the deterioration of information quality decreases the firm value with the elasticity. Note that an increase in Γ_0 decreases the firm value even if all values are positive. From the table, at $\Gamma_0 = 0.01$, the elasticities are 2.31%, 3.13%, and 3.64% for $\gamma = 0.10, 0.20$, and $\gamma = 0.30$, respectively. At $\Gamma_0 = 0.16$, the elasticities are 10.49%, 14.93%, and 16.70% for $\gamma = 0.10, 0.20$, and $\gamma = 0.30$, respectively. That is, an increase in risk aversion makes the elasticity more sensitive. This characteristic stands for other Γ_0. Therefore, the investor evaluates the firm value in response to information quality sensitively when she/he becomes more risk averse.

Table 4 Elasticity of Γ_0 on firm value for each risk aversion

Γ_0	0.01	0.04	0.09	0.16
γ=0.10	2.31%	5.67%	7.79%	10.49%
γ=0.20	3.13%	8.19%	11.41%	14.93%
γ=0.30	3.64%	9.21%	14.11%	16.70%
γ=0.40	3.78%	10.24%	14.94%	18.50%
γ=0.50	4.08%	10.73%	15.20%	19.82%

Note: The elasticity is calculated by $\left|\frac{\Delta V/V}{\Delta\Gamma_0/\Gamma_0}\right|$.

3 Table 5 describes the relation between firm value and m_0. We set $\Gamma_0 = 0.0025, 0.005, 0.0075, 0.01$, that is, we consider a situation in which the estimation error is relatively small. The last line “Complete” in the table shows the firm value in the complete information model with $\alpha = 0.02$. Recall that the firm wanting finance has full information about itself. Hence, the firm value under complete information is considered to be the money amount that the firm wants to obtain. From the table, the funding that the firm wants to obtain is 134.27.

When $m_0 = 0.02$, the firm value is 133.26 for $\Gamma_0 = 0.0025$. This value is very close to the value under complete information; and the firm values for other Γ_0 fall and are lower than the value under the complete information model. The case of $m_0 = \alpha = 0.02$ means that the investor correctly estimates the growth of business. Nevertheless, the firm value under incomplete information does not reach that under complete information. The table also shows the value of m_0, where the firm value under incomplete information exceeds the firm value under the complete model, differs according to the level of Γ_0. For example, the firm value under incomplete information exceeds the value under complete information by around $m_0 = 0.025$ when $\Gamma_0 = 0.0025$. The firm value under incomplete information exceeds the value under complete information by around $m_0 = 0.04$ at $\Gamma_0 = 0.01$. From Table 5, the threshold of m_0 at which the firm value under incomplete information exceeds the firm value under the complete model becomes high when Γ_0 increases. Dechow et al. (1996) argued that the firm that needs more money tends to window-dress its financial statements. However, the figure shows that the firm cannot obtain sufficient firm value even if the investor correctly estimates the growth of business.

Table 5 Firm values as a function of m_0 for each Γ_0 ($\Gamma_0 = 0.00251, 0.0050, 0.0075, 0.0100$)

m_0	*0.000*	*0.005*	*0.010*	*0.015*	*0.020*	*0.025*	*0.030*	*0.035*	*0.040*	*0.045*
Γ_0 = 0.0025	127.55	128.95	130.57	131.46	133.26	134.77	136.12	137.21	138.98	140.12
Γ_0 = 0.0050	126.55	127.97	129.35	130.10	131.68	133.19	134.50	135.99	137.12	139.18
Γ_0 = 0.0075	125.81	126.90	128.55	129.51	131.11	131.83	133.58	135.27	136.28	137.36
Γ_0 = 0.0100	125.12	125.88	127.06	128.59	129.84	131.29	132.51	133.85	135.32	136.03
Complete	134.27	134.27	134.27	134.27	134.27	134.27	134.27	134.27	134.27	134.27

Note: The last line labelled ‘Complete’ shows the firm value under the complete information model.

4 Finally, we demonstrate which parameter Γ_0 or m_0, has more influence on the firm value. To this end, we additionally introduce an elasticity,

$$\left|\frac{\Delta V/V}{\Delta m_0/m_0}\right|.$$

This measures the elasticity of m_0 on the firm value V. By using the results listed in Table 2, we calculate both elasticities $\left|\frac{\Delta V/V}{\Delta\Gamma_0/\Gamma_0}\right|$ and $\left|\frac{\Delta V/V}{\Delta m_0/m_0}\right|$. Tables 6 and 7 show the elasticities $\left|\frac{\Delta V/V}{\Delta\Gamma_0/\Gamma_0}\right|$ and $\left|\frac{\Delta V/V}{\Delta m_0/m_0}\right|$, respectively. Comparing both tables, at $(m_0, \Gamma_0) = (0.015, 0.01)$, the elasticity of m_0 is larger than the elasticity of Γ_0.

For the other m_0 and Γ_0, the elasticities of Γ_0 are larger than those of m_0. Therefore, the effect of quality of information on firm value is more significant than the expected return of cash flow in overview. This result implies that the investor more sensitively evaluates the firm with change in information quality.

Table 6 Elasticity of Γ_0 on the firm value for each expected return of cash flow m_0

m_0 \ Γ_0	*0.01*	*0.04*	*0.09*	*0.16*
0.005	2.25%	5.67%	7.51%	9.86%
0.010	2.40%	5.72%	8.35%	8.68%
0.015	2.25%	6.08%	7.47%	9.87%

Note: The elasticity is calculated by $\left|\frac{\Delta V/V}{\Delta\Gamma_0/\Gamma_0}\right|$.

Table 7 Elasticity of m_0 on the firm value for each information quality Γ_0

m_0 \ Γ_0	*0.01*	*0.04*	*0.09*	*0.16*
0.005	1.23%	0.75%	0.69%	0.01%
0.010	1.57%	2.57%	1.57%	3.06%
0.015	3.69%	1.95%	2.34%	2.08%

Note: The elasticity is calculated by $\left|\frac{\Delta V/V}{\Delta m_0/m_0}\right|$.

4.2 Comparison with DCF approach

In this section, we implement the DCF approach with WACC as valuation of the risk-neutral investor. We then verify that our approach with the risk-averse utility function is consistent with the results provided by previous researches.

We assume that the firm obtains all its funding by issuing shares only for convenience. Thus, the shareholders' equity to total assets is 100%, and we have

$$WACC = \mu$$

where μ is the expected return of the stock introduced in Section 2. Hence, the valuation rule by the DCF with WACC is given by

$$DCF = \sum_{i=1}^{10} E[e^{-\mu t_i} Y_{t_i}], \tag{4.1}$$

where Y is (2.4) in the incomplete information model.

Table 8 shows the firm values simulated with the DCF approach (4.1) for the incomplete information and complete information models. The table shows that the firm value under incomplete information is significantly larger than the one under the complete information for each Y_0. Thus, the cost of capital in the incomplete information model is smaller than that in the complete information model. Also, this contradicts the results of our valuation approach (i.e., risk-averse investor's valuation) as well as previous researches.

Table 8 Firm values for the incomplete information and the complete information model for each initial cash flow Y_0

Initial CF: Y_0	*10*	*20*	*30*	*40*	*50*
Incomplete information	44.91	88.32	133.76	177.78	218.34
Complete information	29.24	58.47	87.88	117.37	146.05

Notes: Firm values are computed by DCF approach (4.1) for both models. The first line shows the firm value under the incomplete information model, and the second line lists the firm value for the complete information model.

5 Summary

In this work, we proposed a firm valuation approach with utility-based pricing under an incomplete information model where the investor does not precisely know about the growth of business. We use utility-based pricing to calculate the present value of future cash flows; our valuation formula reflects the risk aversion of the investor. We also use the DCF approach, which corresponds to the valuation rule of a risk-neutral investor.

We verified the impacts of incomplete information and risk aversion on firm value. All analyses were examined with numerical simulation. Incomplete information reduces firm value compared to the complete information case. Thus, our result gives the relation between cost of capital and the information used in several previous empirical researches. While difficulty of estimation and increased risk aversion decrease firm value, the influence of improved information quality on firm value is more significant than improvement in business growth. Our study also shows why incomplete information reduces firm value. A comparison with the DCF approach shows that while the risk-averse investor decreases the value of the firm, the incomplete information model does not underestimate the firm at all.

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