Dynamic modelling of the turbocharged gasoline direct injection air-path using mean value and linear parameter varying models

Mohammadjavad Ghomashi* and Byron Mason
Aeronautical and Automotive Engineering,
Loughborough University, UK
Email: m.ghomashi@lboro.ac.uk
Email: b.mason2@lboro.ac.uk
*Corresponding author

Mark Cary
Ford Motor Company Limited,
Dunton Engineering Centre, UK
Email: mcary@ford.com

Kambiz Ebrahimi and Aitshaam Shahzad
Aeronautical and Automotive Engineering,
Loughborough University, UK
Email: K.Ebrahimi@lboro.ac.uk
Email: a.shahzad@lboro.ac.uk

Abstract: Engine behaviour can be simply defined using mean value engine models (MVEMs) as an average over one engine cycle but there are concerns about the validity of MVEM models during transient operation. In this study, the performance of MVEM is evaluated by comparing air-path dynamics during transient and steady-state operation for a turbocharged gasoline engine. The comparison is performed experimentally by the measurement of port and manifold fast pressures and calculated air mass flow during speed and torque transient tests. In addition, engine models can be linearised to facilitate rapid estimation. In this regard, a nonlinear intake manifold model is linearised and several linear parameter varying (LPV) models are formulated. Investigation of MVEM shows the transient cycles are within the steady-state range, whilst the effects of turbocharger performance are significant. The LPV modelling approach showed approximately 90% conformity between the linear and nonlinear models for estimating manifold pressure.

Keywords: air mass flow; intake manifold model; linearisation; linear parameter varying; LPV; mean value engine model; MVEM; transient operation.


1 Introduction

Engine behaviour can be described by an average value over a cycle with cycle behaviour summarised by this single average value. Mean value engine models (MVEMs) are used to predict engine behaviour (Hendricks and Luther, 2001) resulting in a compact engine model; it is therefore an efficient approach to the simulation and control design of a nonlinear engine model (Karlsson and Fredriksson, 1999).

During steady-state operation, engine dynamics are stabilised and a mean value engine model can be developed without any concern about the neglected dynamics. On the other hand, dynamic variations can arise during engine transient operation. So, a transient mean value model inherently contains events and dynamic variations that
happen over the cycle. For evaluating transient behaviour based on steady-state, the validity of the mean value model must be investigated to establish whether the dynamic variations are effective during model operation.

In sweep based engine testing the engine control inputs are ramped continuously. This is the main idea behind the engine rapid measurement. It can be done by eliminating the stabilisation time and recording the measurement over shorter periods of time (Röpke and Knaak, 2007). The main obstacle in the transient measurement method is the slow dynamic response; this causes the differences in measurements between transient and steady-state and results in a loss of accuracy. As a consequence of this, it is necessary to compensate for the effects of the engine and sensor dynamics (Sugita et al., 2009). The instrument response time is one of the factors that can cause inaccuracy in transient measurement. Due to this reason, the maximum rate of the transient operation is limited by the response time of the instruments (Ward et al., 2002).

Transient experiments can be applied as throttle steps at constant speed, throttle steps at constant torque, torque ramps and load steps with fixed throttle angle (Hendricks and Sorenson, 1990). Fast air mass flow dynamics are negligible but there remains some doubt about the effect of transient operation on volumetric efficiency. Inertia effects can resist against the start of any changes to the intake air demands and decrease induction during tip-in and increase it during tip-out. The wave effects are produced by moving parts of the engine and cause the reflection of the pressure waves inside the intake manifold pipes. A standing wave can develop during steady state operation; this phenomena is not replicated during transient operation due to insufficient time for the settling down of the waves (Chevalier et al., 2000). It was proven using simulation results that the engine volumetric efficiency in steady-state and transient engine operation had a similar response. If the air flow adjusts very quickly to the variations of the engine, it can be treated as steady-state operation. In this case the pressure waves propagate rapidly along the intake manifold, so can be neglected during engine transient operation (Smith, 1999).

The most suitable engine model for control purposes is one that is the most parsimonious. Linearisation of nonlinear dynamic models can provide a surrogate model with a much simpler structure. More so, if linear models are specified locally at a number of operating points, then the combination of these results in a global linear parameter varying (LPV) model.

Inherently, systems exhibit nonlinear behaviour in the real world, but system dynamics can be represented by a linear model locally, considered as a linear-operating region (Klee and Allen, 2007). For this reason, techniques are applied to simplify these models to reduce the order of differential equations of the model into first order equations (UMASS LOWELL, 2005; Xue and Chen, 2007). Linear models are defined around an operating point where it can predict system behaviour over a certain region about that point. The equilibrium points are the most suitable operating points for the model linearisation (MathWorks, 2014). So, the operating points of a nonlinear system are calculated by setting the time-varying portion of the input and state derivatives equal to zero (Esfandiari and Lu, 2014).

Many model predictive control efforts for automotive and engine applications are based on linear models (Von Wissel et al., 2014). Chen et al. (2013) linearise an intake manifold model by considering the state equation of intake manifold absolute pressure, and estimated manifold pressure using a Kalman Filter. Engine air path model
linearisation around steady state operating points was investigated by Nilsson et al. (2006). The results show that the nonlinear, linear and truncated models can predict the required air charge properly.

LPV systems are usually represented in state-space form but estimation of its parameters is complicated (Wei et al., 2008). An alternative form is the input-output LPV system. This method is presented by Bamieh and Giarré (2002). Identification of the LPV systems can be categorised as LPV-IO or LPV-SS based methods (Tóth, 2010). Wei et al. (2008) employed the LPV-IO approach to model the air path system of a diesel engine. For a better physical understanding, the air path system was divided into three subsystems of boost pressure, exhaust manifold pressure and exhaust gas recirculation (EGR). Ngo et al. (2014) applied LPV-SS to present a reduced air path engine model. The results showed that the prediction error of the model increases when the throttle reached its opening limit, which is related to the error in pressure calculation. Pressure is not a proper scheduling parameter since it cannot be calculated accurately and depends on other variables in the model.

Currently, LPV systems are employed in a wide range of applications. Linear models with rapid estimation are advantageous for engine control purposes, so developing LPV system for a nonlinear engine model can be studied to assess the operation of these systems in engine applications.

In this study, different approaches are investigated to check the assumptions and possibilities for simplifying an engine model. Initially the validity of MVEM is evaluated during transient operation. Experimental results are employed to calculate the cycle average of manifold and port pressures to compare with steady-state and transient operations. Furthermore, an Intake manifold model is considered for studying engine model linearisation. Linear models are presented as a LPV system, which combines identified linear models in every operating point and two implementation techniques are presented based on the type of scheduling spaces.

2 Background information

The speed density equation, based on volumetric efficiency, is a recognised method for calculating the air mass flow. It shows the ability of the engine during the induction process to fill the cylinder by air flow. It can be defined by the air pressure and temperature measurements. The air mass flow can be calculated using equation (1) (Heywood, 1988).

\[ m_a = \mu_v N V_d \frac{P_{man}}{RT_{man}} \]  

(1)

where \( m_a \) is air mass flow that can be calculated using volumetric efficiency \( \mu_v \), engine speed \( N \) in rpm, displacement volume of the engine \( V_d \), manifold pressure \( P_{man} \), manifold temperature \( T_{man} \) and \( R \) as gas constant for air.

In addition, it is feasible to apply this equation as an instantaneous relation, if the air mass flow through the inlet port are supposed fast enough to neglect flow dynamics
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So the mean value of intake manifold pressure, intake manifold temperature and engine speed must be measured and used for applying this method to estimate air mass flow into the cylinder. In this case, the temperature variations in the intake manifold are slow enough and can be used in this equation. Also, the pressure sensor is fast enough to capture pressure variations in the intake manifold. So the speed density or volumetric efficiency equation is a suitable approach for the estimation of intake air mass flow by setting up pressure and temperature sensors inside the intake manifold (Andersson, 2002).

Air charge estimation can be more complicated for a turbocharged engine. The turbocharger influences the compression and expansion ratios and the compressor and turbine downstream temperatures (Moulin and Chauvin, 2011). In turbocharged engines, the wastegate is used for controlling the amount of exhaust gas available to the turbine. So the air mass flow to the cylinder is affected by the exhaust manifold pressure and variations of wastegate influence the turbocharger performance and introduces some unexpected transients in air flow (Andersson, 2005).

One of the most common applications of MVEMs is intake manifold airflow modelling. The intake manifold is taken to comprise from the throttle plate to inlet ports of the cylinder. The rate of mass flow inside the intake manifold volume depends on the input mass flow rate from the throttle body and output mass flow rate to the cylinder. By considering the ideal gas equation for the manifold the relation of the manifold mass flow can be determined using intake manifold temperature, volume and pressure. The rate of manifold pressure can be obtained using equation (2).

\[
P_{\text{man}} = RT_{\text{man}} \left( \dot{m}_{\text{ai}} - \dot{m}_{\text{ao}} \right) / V_{\text{man}}
\]  

Therefore, the manifold pressure ratio is defined by the rate of throttle mass flow to inlet port mass flow. The air mass flow rate is calculated applying the speed density equation (1). Stabilisation of the manifold pressure was found to occur more quickly than crankshaft speed. In addition, the manifold filling time constant depends on crankshaft speed. A fast opening of throttle plate induces air into the intake manifold instantly and generates peaks in manifold pressure responses during tip-in. On the other hand, the throttle is closed very quickly during the tip-out same as tip-in, but the air mass flow decreases gradually. So intake air mass flow shows nonlinear behaviour depending on throttle position and crankshaft speed (Hendricks and Vesterholm, 1992).

By considering the matrices of states \( x \), inputs \( u \) and outputs \( y \), the linear system can be presented in state-space form, depicted by equation (3) (Katebi et al., 2002).

\[
\Delta x(t) = Ax(t) + Bu(t) \\
\Delta y(t) = Cx(t) + Du(t)
\]  

In the above equations, it must be noted that the matrices of \( A, B, C, D \) can be defined by the equation (4).

\[
A = \frac{\partial x}{\partial x} \bigg|_{\text{wp,wp}}, \quad B = \frac{\partial x}{\partial u} \bigg|_{\text{wp,wp}}, \quad C = \frac{\partial y}{\partial x} \bigg|_{\text{wp,wp}}, \quad D = \frac{\partial y}{\partial u} \bigg|_{\text{wp,wp}}
\]

(Chevalier et al., 2000).
In this study, an intake manifold model was considered for linearisation. The main purpose of linearisation is presenting a reduced model for the intake manifold which is much faster than the more complex model. The governing differential equations in this model can be described by equations (5) and (6) (Schaal et al., 2014).

\[
\frac{dP}{dt} = \frac{RC_p}{C_V} \left( \dot{m}_{\text{in}} T_{\text{in}} - \dot{m}_{\text{out}} T_{\text{amb}} \right) \quad (5)
\]

\[
\frac{dT}{dt} = \frac{RT^2}{PV} \left( \dot{m}_{\text{out}} - \dot{m}_{\text{in}} + \frac{V}{RT} \frac{dP}{dt} \right) \quad (6)
\]

According to the above equations, an intake manifold linear model can be obtained by considering one input, inlet air mass flow and two outputs and the states of pressure and temperature.

### 3 Experimental setup

For evaluating MVEMs, an experimental engine test was performed. The test bed consists of a Ford engine 1.6 SGDI C520 that is coupled to a transient dynamometer. The test is controlled and data are collected using the sensors by AVL PUMA and IndiSmart systems. Engine instrumentation includes slow and fast pressure measurement and temperature measurement at various locations. The instrumentation of particular relevance to this study is the fast pressure measurements. These include cylinder, port and manifold pressures. Reference is also made to the thermocouple located in the intake manifold.

**Figure 1** Input variations of the torque ramps test and torque response (see online version for colours)
Two sets of experiments are undertaken; these are torque and throttle transients. Several experiments are run back to back within each set. A series of torque transients were performed with input conditions as shown in Figure 1. Dynamometer demand speed was kept constant though it should be noted that actual speed varied from target as shown in the figure (top right). Torque was controlled using pedal input. Initially, the engine was run at 90 Nm for a period of 35 seconds, torque demand was then ramped over 3 seconds to 200 Nm and then back down to 20 Nm. Six further transients were undertaken from 20 Nm to 200 Nm over 3 seconds while cam timing was fixed. As it can be seen from the figure, throttle position varies as it is controlled by the controller between approximately 8 and 70 degrees.

In another experiment, speed ramps were conducted with fixed throttle position as shown in Figure 2. The test was started at 3,250 rpm and run for a number of seconds at steady-state. The speed was then initially ramped to 4,500 rpm in 3 seconds and ramped down to 2,000 rpm in 3 seconds. After 4 ramps down and 3 ramps up the engine was returned to the same steady-state conditions as at the start of the test.

4 Implementation of LPV model

The intake manifold model was linearised considering the inlet air mass flow as an input and intake manifold pressure and temperature as outputs of the linear model. Throttle and speed variations have significant influences on the intake manifold dynamics. Therefore, operating points are defined in the scheduling space by specific throttle and speed values. In all of operating points, the wastegate position is completely open and all other test control parameters remain constant.

Linear time-invariant (LTI) models for every operating point were calculated. Besides linear input and outputs, the scheduling parameters, which define the location of each
LTI model in scheduling space, are implemented in the LPV system. Figure 3 schematically illustrated the implementation of intake manifold LPV model.

Figure 3  The considered structure for intake manifold LPV model (see online version for colours)

Calculated LTI models in this study are based on state-space LTI models. Conforming with interpolation methods as explained in De Caigny et al. (2009) and Pajmans et al. (2008), LTI models can be interpolated by calculating the zero, pole and gain of state-space models. The model fitting tools in model based calibration (MBC) toolbox may be used to establish an LPV scheduling function. Also, the initial values of inputs, outputs and states must be interpolated, which is a requirement for LPV model implementation. The order of LTI models can be simplified using Hankel singular value technique without making significant changes in the model fidelity (Adegas et al., 2013; Sootla, 2012).

5 Results

The experimental data collected during the speed and torque transient tests are applied to investigate the validity of MVEMs during transient operation. For this purpose, two separate methods are applied based on measured pressures or calculated air mass flow.

5.1 Torque transients

Figure 4 shows the cycles selected for comparison, selection is made at random for steady-state and based on cycle average of the manifold pressure. Manifold pressure is sampled under steady-state operation (cycles 1 and 9) and then the closest matching cycles are selected for comparison from transients (points 2 to 8). As the transient is undertaken the throttle is continuously moving (sampled at 20 Hz), meaning that at 3,250 rpm, $\theta_{\text{tr}}$ is sampled every 1.35 cycles. For this reason, when comparing transient operation with steady-state, it is necessary to select the cycle in transience whose throttle position corresponds most closely with that achieved in steady-state. The variation associated with this selection is shown in Figure 4. As can be seen from the plot whilst cycles are selected for comparison based on similar input conditions these are rarely identical.

Fast manifold pressure was sampled at 0.1 degree crank intervals (195 kHz at 3,250 rpm). The mean and standard deviations are calculated using data from 1,190 steady-state cycles. These cycles are taken from before and after the transient parts (region A and B on Figure 4).
Figure 4  (a) Cycle average of manifold pressure with location of selected cycles under steady and transient operation (b) Values of manifold pressure for each of the selected cycles and group mean and standard deviation (see online version for colours)

Figure 5  (a) Manifold pressure plotted over crank angle (b) Variation of turbocharger speed in the selected cycles under steady and transient operation (see online version for colours)

Figure 5 shows manifold pressure variations in the selected cycles plotted against crank position. The green line represents the mean value of steady-state cycles. The shaded area shows standard deviation σ around the mean value. Steady-state sampled cycles (1 and 9) as expected are close to the mean value. Considering the manifold pressure variations in the induction stroke of the engine (–360° < θeng < –180°), the 1st and 2nd ramps down (2 and 4) have higher value compared to the mean value and are located outside of the shaded range. The 3rd and 4th ramps down (6 and 8) have lower values compared to the mean value but they are within the range of the shaded area. All ramps up are lower than steady-state mean value.

Up-ramps always result in lower manifold pressure compared to steady-state conditions. This is expected as it will take some time after the ramp is initiated for the turbo to spin up. This fact is corroborated by turbo speed results on Figure 5. As can be
seen, calculated turbo speed on up-ramps is significantly lower. Looking at the same figure, it can be seen that results obtained for down-ramps are less conclusive with turbo-speed actually appearing lower for transients with manifold pressure as expected (marginally) higher in transience. It appears to show that cycles with similar wastegate positions are grouped most closely, there is however no clear correlation between position and pressure. This is likely to be a matter for repeatability; the grouping may be coincidental so should be investigated further. Having measured turbo speed, rather than estimating this is likely to reveal more precisely what is happening.

5.2 Speed transients

Figure 6 shows the cycles selected for comparison, these were selected based on cycle average manifold pressure. The same data processing procedure was applied as for the torque ramps as discussed in Section 4.1. The cycle mean and standard deviation of cycles selected at points 1 through 9 are shown in the right plot along with values of the engine speed, $N_{eng}$ for the individual selected cycles. As it can be seen from the plot whilst cycles are selected for comparison based on similar input conditions these are not identical.

Figure 7 shows manifold pressure variations in the selected cycles (1 to 9) plotted against crank angle. The mean and standard deviations shown are calculated using data from 1,763 steady-state cycles. These cycles are taken from the first and the end part of the test in steady-state (Region A and B). The shaded area shows standard deviation $\sigma$ around the mean value. Steady-state sampled cycles (1 and 9) are located within the steady-states standard deviation. Considering the manifold pressure variations in the induction stroke of the engine ($-360^\circ < \theta_{eng} < -180^\circ$), Speed ramps up always result in higher manifold pressure compared to steady-state conditions, while during speed ramps down, the manifold pressure is lower than the steady-state conditions.

Compared with the results from the torque transients, speed transients exhibit an opposite trend. The shape of the curve remains largely the same regardless of transient type or whether the engine is steady or in transience.
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Figure 7  (a) Manifold pressure plotted over crank angle (b) Variation of turbocharger speed in the selected cycles under steady and transient operation (see online version for colours)

5.3 Proportional mass flow

The next stage in the comparison attempts to evaluate differences in mass flow into the cylinder through the intake ports. This is done using the pressure measured across the valve (port and cylinder) and estimating flow using standard equations. Flow across the valve can be modelled as one dimensional quasi-steady, compressible flow. When flow is un-choked it is calculated using equation (7) (Ferguson and Kirkpatrick, 2001).

\[
\dot{m} = C_d A^* \frac{P_{op}}{\sqrt{RT_{op}}} \left( \frac{2\gamma}{\gamma-1} \right) \left( \frac{P_{down}}{P_{up}} \right)^{\frac{\gamma+2}{\gamma-1}}
\]  (7)

where \( P_{down} \) is the pressure ratio \( (P_{downstream}/P_{upstream}) \) and \( A^* \) is the minimum area normal to the flow. For choked flow i.e., pressure ratios more than the critical pressure as shown in equation (8), then equation (9) is used for calculating mass flow.

\[
P_{cv} = \frac{P_{down}}{P_{op}} = \left( \frac{\gamma+1}{2} \right)^{\frac{\gamma-1}{\gamma+1}}
\]  (8)

\[
\dot{m} = C_d A^* \frac{P_{op}}{\sqrt{RT_{op}}} \left( \frac{2}{\gamma+1} \right) \left( \frac{P_{down}}{P_{up}} \right)^{\frac{\gamma+1}{\gamma-1}}
\]  (9)

For this analysis, proportional air mass flow is calculated, \( \dot{m}_{prop} = \dot{m} \left( \frac{kg}{m^2 s} \right) \) using the equations above. Proportional mass flow is calculated since the flow area, and discharge coefficient, \( C_d \) are unknown. A calculation is made at increments of 0.1° crank and total proportional mass, \( M_{prop} = \int_{\text{tac}} \dot{m}_{prop} \) for each cycle is calculated by integration of \( m_{prop} \) when the inlet valve is open.
Figures 8 and 9 shows steady-state and transient means compared in torque transients and speed transients respectively. The shaded area indicates the respective standard deviations. There appears to be good similarity between the two. As can be seen from the plot the steady state and transient tests show good correlation with the standard deviation slightly higher on the transient tests.

The graph appears noisy though there seems initially at least some correlation between ramp type and value of the integrated proportional mass flow. The down transients and up transients appear to be well separated. This could be related to the turbo as previously discussed.
5.4 Evaluation of the LPV models

Two main validation tests as shown in Figures 10 and Figure 11 were designed to evaluate the operation of the LPV models in comparison with the nonlinear intake manifold model.

**Figure 10** Test control inputs for the throttle steps with speed ramps (see online version for colours)

Due to the interpolation characteristic in LPV model, many of the LTI models are potentially unnecessary. So, the most desirable LPV is a system that consists of the least number of LTI models and operating points whilst retaining accuracy. Normalised root mean square error (NRMSE) is calculated to compare estimation accuracy of every LPV systems with the nonlinear engine intake manifold model. NRMSE is presented for both manifold pressure and temperature. For the throttle steps with speed ramps test, the comparison results show that the LPV systems with 60, 56 and 32 LTI models have better NRMSE values for manifold pressure and temperature than the LPV system with 105 LTI models. Also, the results for 45 and 24 LTI models are acceptable, whilst they are even more accurate in manifold temperature than a LPV model with 105 LTI models.

The validation test, with the engine speed steps with the throttle angle ramps makes the model selection narrower than previous test. According to the NRMSE results, all of
LPV systems with 24, 32, 45, 56 and 60 LTI models have an acceptable value compared with LPV with 105 LTI models. For a distinctive comparison, the NRMSE result is plotted versus the number of LTI models in the LPV systems were described above.

**Figure 11** Test control inputs for the speed steps with throttle ramps (see online version for colours)

![Throttle](image1)

![Engine Speed](image2)

**Figure 12** Evaluation of LPV intake manifold models conformity based on number of LTI models in the LPV system for manifold pressure and temperature in two different validation tests of throttle steps with speed ramps (lozenge) and speed steps with throttle ramp (see online version for colours)

![NRMSE vs. No. LTI Models](image3)
A regular grid scheduling space with 77 operating points was defined using the engine speed and throttle angle as scheduling parameters. It is a non-uniform regular grid, such that the steps between the throttle angles were considered 5 degree for the lower throttle values (5 to 35 degree) and 10 degree for higher ones (45 to 75 degree). The linearised models were calculated at each operating point. LTI intake manifold models are defined by 11 states in the nonlinear engine model. In this case the LPV system is an 11th order system. All the LTI models are stable and the scheduling space as shown in Figure 13. The Hankel singular value for every state of LTI models is calculated. Figure 14 shows the effective state energy for every state of the LTI models. According to this figure, the first state in the LTI models has the most significant effect on the LTI models. Also, the second and third states are more effective compared with the other states.
Figure 15  LPV intake manifold model with the different LTI orders for throttle steps with speed ramps validation test (see online version for colours)

Figure 16  LPV intake manifold model with the different LTI orders for speed steps with throttle ramps validation test (see online version for colours)
According to the Hankel singular value analysis of the model, LTI models were truncated to the 1st and 2nd order. Two different methods were used for LPV system implementations. Initially, the LPV structure were constructed using a set of state-space models; secondly, lower order LPV models were generated with model fitting tools in MBC toolbox and implemented in Simulink. Intake manifold LPV systems with 1st, 2nd and 11th order LTI models are compared with the nonlinear engine intake manifold model in the Figures 15 and 16 for every validation test.

In both tests, all of the LPV models show an accurate fit to the nonlinear model. Test results show that the lower order linear models are similar to the higher order models. An identical operation with slight differences can be observed by comparing LPV systems with the various LTI orders in the steady-state and ramps. The only advantage of higher order LPV system is its ability to estimate some of rapid dynamics in the intake manifold. This can be seen on the manifold temperature plots in which the LPV system with 11th order LTI model shows a better rapid dynamic estimation in comparison with the other low order LPV systems.

The conformity of LPV systems in comparison with nonlinear model assisted testing for engine characterisation (MATEC) model were evaluated as shown in Table 1. The results show that the LPV system with higher order LTI models are slightly better than lower order ones. The differences between LPV systems are more significant in manifold temperature. It is as a result of more rapid dynamics in manifold temperature and the NRMSE is much higher for LPV system with 11th order LTI models.

Figure 17 Comparison of implementation methods for 1st order LPV systems using throttle steps with speed ramps validation test (see online version for colours)
Figure 18  Comparison of implementation methods for 1st order LPV systems using speed steps with throttle ramps validation test (see online version for colours)

Figure 19  Comparison of implementation methods for 2nd order LPV systems using throttle steps with speed ramps validation test (see online version for colours)
The techniques presented for implementation of the LPV system were compared. Again, same validation tests were performed and results are depicted in Figures 17 and 18 for the 1st order models, and Figures 19 and 20 for the 2nd order models. Results clearly show an applicable operation in both approaches. Using MBC model fitting approach generates some inaccuracy in the initial values. Consequently, the LPV results show a better estimation at steady-state.

Table 1  LPV fitting accuracy based on LTI model order

<table>
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<th>Test type</th>
<th>Throttle steps with speed ramps</th>
<th>Speed steps with throttle ramps</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>LPV 11th</td>
<td>LPV 1st</td>
</tr>
<tr>
<td>Pmanifold</td>
<td>92.68%</td>
<td>92.4%</td>
</tr>
<tr>
<td>Tmanifold</td>
<td>69.93%</td>
<td>64.16%</td>
</tr>
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</table>

The NRMSE results are summarised in Table 2 for the throttle steps and speed step validation tests respectively. Comparing the NRMSE for both methods shows insignificant difference. The LPV results are slightly better in throttle steps with speed ramps tests whilst MBC models show a negligibly higher NRMSE in speed steps with throttle ramps tests.
<table>
<thead>
<tr>
<th>Test type</th>
<th>Throttle steps with speed ramps</th>
<th>Speed steps with throttle ramps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LPV 1st</td>
<td>MBC 1st</td>
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<tr>
<td>Pmanifold</td>
<td>92.4%</td>
<td>92.06%</td>
</tr>
<tr>
<td>Tmanifold</td>
<td>64.16%</td>
<td>64.03%</td>
</tr>
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</table>
6 Conclusions

MVEMs were investigated experimentally by mean value of manifold and port pressures over a cycle and calculated air mass flow in every cycle of engine during transient and steady-state operation. The observed port pressures in increasing throttle ramps are always lower than steady-state values. This is due to the variation of turbo speed, wastegate positions and inertia effect. Since opposite trends can be seen in speed transient between pressure and speed signal higher port pressures are expected than steady-state cycles for the same reasons. In similar way, the most of port pressures are higher in transient cycles compared to the steady-state during decreasing throttle ramps, and they are lower in decreasing speed ramps.

Comparing LPV systems with different LTI orders verified LPV systems with lower order can estimate nonlinear response as well as the higher one. Although higher order linear models have a better estimation in rapid variations and a minor reduction can be observed in the estimation accuracy of lower order LPV systems.

According to the estimation results, the NRMSE values are nearly equal in both implementation methods and a preferred method cannot be selected, though some disadvantage was observed for each implementation method. Regular grid LPV system applies linear interpolation and the accuracy of the model can be reduced when using a low number of operating points due to the large gaps between operating points. In this case, if there is not any linear relation between operating points, then interpolation cannot provide an estimation correctly.

As a disadvantage to the MBC method, a loss of accuracy can occur due to the calculated initial values for each operating point. Though this type of error is insignificant in most cases, it mainly influences the steady-state operation. It is suggested that explicit initial values are provided or even linear models obtained by linearisation procedure are preferred to the MBC model, which relies on interpolation.

References


