Performance analysis of batch receiving wireless sensor networks with interference signals

Jiaqi Fan, Zhanyou Ma*, Xiangran Yu and Yang Zhang

School of Science,
Yanshan University,
Qinhuangdao, 066004, China
Email: fanjiaqi@stumail.ysu.edu.cn
Email: mzhy55@ysu.edu.cn
Email: 15231986544@163.com
Email: zhangyang@stumail.ysu.edu.cn
*Corresponding author

Abstract: In order to improve the energy efficiency, the operating situation of batch receiving wireless sensor network is discussed in this paper. Combining with environmental interference signals, repairable faults of the sensor node and other actual situation, process of receiving and transmitting data packets in wireless sensor networks is simulated. An $M^X/G/1$ vacation queueing model with customers arriving in batches, negative customers, feedback customers, wake-up periods, optional vacations and repairable faults is established. The steady state distribution of the system is solved by using the supplementary variable method, and expressions of the energy saving rate and other performance indexes are given. The equilibrium of the system is discussed by establishing revenue function of the system. Then, using MATLAB software for numerical analysis, the influence of system parameters on performance indexes of wireless sensor networks systems is analysed, and equilibrium numerical results are obtained.

Keywords: wireless sensor networks; batch receiving; repairable fault; supplementary variable method; equilibrium strategy.

Reference to this paper should be made as follows: Fan, J., Ma, Z., Yu, X. and Zhang, Y. (2021) 'Performance analysis of batch receiving wireless sensor networks with interference signals', *Int. J. Communication Networks and Distributed Systems*, Vol. 27, No. 1, pp.51–76.

Biographical notes: Jiaqi Fan received his Bachelor's in Mathematics and Applied Mathematics from the Changchun University of Science and Technology, Changchun, China. Currently, he is a Postgraduate at the School of Science, Yanshan University, Qinhuangdao, China. His research interests include cognitive radio networks, wireless sensor networks, queueing systems with vacations and performance evaluation models in communication networks.

Zhanyou Ma received his BSc in Mathematics from the Jilin Normal University, Siping, China, MSc in Operations Research and PhD in Management Science and Engineering from the Yanshan University, Qinhuangdao, China. Currently, he is a Professor at the School of Sciences, Yanshan University, Qinhuangdao, China. His research interests include cognitive radio networks, wireless sensor networks, queueing systems with vacations and performance evaluation models in communication networks.

Xiangran Yu received her Bachelor's in Mathematics and Applied Mathematics from the Hebei Agricultural University, Baoding, China. Currently, she is a Postgraduate at the School of Science, Yanshan University, Qinhuangdao, China. Her research interests include cognitive radio networks, wireless sensor networks, queueing systems with vacations and performance evaluation models in communication networks.

Yang Zhang received her Bachelor's in Mathematics and Applied Mathematics from the Ankang University, Ankang, China. Currently, she is a Postgraduate at the School of Science, Yanshan University, Qinhuangdao, China. Her research interests include cognitive radio networks, wireless sensor networks, queueing systems with vacations and performance evaluation models in communication networks.

1 Introduction

Wireless sensor networks (WSNs) have a large number of miniature wireless sensor nodes and transmit data packets through wireless communication (Majumdar and Sarkar, 2015; Chikh and Lehsaini, 2018). WSNs could detect, collect, transmit and process data inexpensively through sensor nodes, and have the characteristics of rapid extension and strong resistance (Nam et al., 2017). With the continuous development of technology, WSNs have a wide range of application values in communications, military, production, industry and other aspects, and especially play a more critical role in the internet of things (Abujubbeh et al., 2019; Shi et al., 2018; Feng, 2019). Rajput and Kumaravelu (2019) used a kind of low-cost fuzzy c-means algorithm to obtain efficient and sustainable WSNs, which are used as the infrastructure of physical monitoring and applied to the agricultural detection IoT system. Alarifi and Tolba (2019) optimised the cloud-assisted internet of things by using the adaptive neural network classification method in WSNs to reduce network latency and errors and extend the service life of the networks. Combining the mobile elements in the internet of things, Abdulsalam et al. (2018) used the cluster-based data aggregation algorithm to extend the service life of WSNs. Li et al. (2018) proposed a three-factor anonymous authentication scheme for WSNs in the internet of things, which improved the computational efficiency and enhanced the security. To obtain more accurate position information of sensor nodes and improve the intelligence of the indoor internet of things, Sotenga et al. (2017) applied WSNs in internet of things to indoor positioning.

Because WSNs are powered by batteries, more and more scholars have paid attention to the energy saving of WSNs with the expansion of the network and the multiplication of sensor nodes. Gherbi et al. (2018) proposed a hierarchical distributed adaptive clustering method to reduce the overall energy consumption, balance the energy loss between sensors and extend the service life of the system. Bouyer et al. (2015) used the fuzzy c-means algorithm which determined the optimal number and locations of sensors to improve the energy utilisation of the system. Russo et al. (2018) applied a predictive analysis method based on unsupervised learning algorithms in WSNs to reduce the transmission times of date packets in the network for energy saving. Gao et al. (2011) considered sensor nodes that can enter the sleep mode at the same time to achieve synchronous dormancy and wake up of the entire network, and the optimal transmission power based on the distance between adjacent nodes is calculated. Chauhan and Awasthi (2014) designed an energy-saving circular stealing algorithm to reduce the transmission of data packets and energy consumption of the network. In order to reduce system energy consumption and extend network life, Vahabi et al. (2019) proposed a method of mobile convergence. Alromih and Kurdi (2019) used Chebyshev distance to select the best next sensor node to improve energy utilisation of the system and reduce costs of the networks. Mazinani et al. (2018) proposed a clustering routing algorithm based on fuzzy clustering, which improved the energy saving rate and extended the service life of WSNs by introducing a fixed threshold. Goswami et al. (2019) proposed an efficient and dynamic clustering technology which selected nodes based on the nearest value of the data source to reduce the operation costs of networks. Chiasserini and Garetto (2004) studied wireless sensor networks based on the Markov model whose nodes would switch to sleep mode, and performance, capacity and data transmission delay of WSNs from the aspect of energy consumption were analysed. Sharma et al. (2019), Tamene and Nageswara (2018), Chaturvedi et al. (2016) and other scholars established different energy saving models to optimise WSNs.

At present, many scholars have applied the queueing theory method to the theoretical research of WSNs. Mitici et al. (2017) collected observation data from different sensors in WSNs for data fusion, a tandem queue with batch service was established, and the optimal scheduling of sensors was obtained through improving service rate. Niyato et al. (2006) proposed a discrete time queueing model to analyse the performance of sensor nodes in solar wireless sensor network under different strategies, and the optimal parameters for sensor nodes to dormancy and wake up were obtained by used game theory. Liu et al. (2010) established a queueing model to study the performance of WSNs and the effect of random dormancy of sensor nodes on network performance. Considering the finiteness of network resources, the frequency of resource requests and the duration of resource queries, Mann et al. (2008) proposed a queueing model for analysing the resource replication strategy of WSNs, and the optimal replication strategy for network resources was determined. Lee and Yang (2008) analysed a discrete-time Geo/G/1 queueing model with N-policy and disaster arrival, which was applied to the study of energy-saving strategies for unreliable WSNs.

Normally, there is not only one data packet arriving at the same time, data packets may arrive in batches, so it is necessary to study batch receiving wireless sensor networks. The fatal impact of interference factors on the system is considered in this paper, and a sleep mechanism is proposed to improve the energy efficiency.

The paper is organised as follows. In Section 2, based on the operation mechanism of batch receiving WSNs, an $M^X/G/1$ vacation queueing model with customers arriving in batches, negative customers, feedback customers, wake-up periods, optional vacations and repairable faults is established. In Section 3, the necessary and sufficient conditions for the steady state of the system are solved. In Section 4, the steady-state distribution of the system is obtained by using the supplementary variable method, and the performance indexes of the system are obtained. In Section 5, the revenue function of the system is constructed and the equilibrium state of the system is discussed. In Section 6, the numerical experiment is carried out to analyse the relationship between parameters and performance indexes by using MATLAB software, and equilibrium numerical results of the system are obtained. Conclusions are introduced in Section 7.

2 Model description

After received in batches by the WSNs system, data packets are temporarily stored in the memory and wait for the sensor node to transmit. The process of transmitting data packets from the sensor node is abstracted as a service process, and a WSNs system based on a vacation queueing model is established. The parameters of the system are assumed as follows:

1 In this system, the number of data packets (batches) arriving at the same time is a positive integer random variable X, whose distribution is $P(X = r) = c_r$ $(r = 1, 2, \cdots)$, $E(X^2) < \infty$. The arrival of each batch of data packets obeys the Poisson process whose arrival rate is λ^+, and arrival data packets are independent of each other. If the sensor node is in working or wake-up state, arrival data packets will be received by the system and queue for transmission. If the sensor node is in standby state, arrival data packets will be transmitted immediately after being received by the system. If the sensor node is in vacation or fault state, the system refuses to receive data packets with probability θ $(0 \le \theta \le 1)$ and receives data packets with probability $\bar{\theta}$ $(\bar{\theta} = 1 - \theta)$. Environmental factors such as network viruses and network interference will generate interference signals which affect the system. The arrival of interference signals obeys the Poisson process whose arrival rate is λ^-. If the sensor node is in working state, the arrival interference signals will lead to abnormal fault of the sensor node, and the data packet which is transmitting will be offset. The sensor node is repaired immediately after the fault, and then data packets could be transmitted again. It is assumed that the repair time H_1 after an abnormal fault obeys a general distribution, the distribution function is $\eta_1(x)$, and the fault rate function is $h_1(x)$. If the sensor node is in wake-up, standby, vacation or fault (normal or abnormal) state, the interference signal will disappear automatically.

2 In the system, due to physical factors such as the defect of the embedded system, the limitation of memory, and the fault of the wireless receiver, the sensor node would have a normal fault in the process of transmitting data packets. The sensor node is repaired immediately after the fault, and data packet which is interrupted by the fault will be retransmitted immediately after the sensor node is repaired. It is assumed that the life of the sensor nodes obeys the negative exponential distribution with the parameter of τ. The repair time H_2 after a normal fault follows a general distribution whose distribution function is $\eta_2(x)$ and failure rate function is $h_2(x)$. The system transmits data packets according to the service order of first come first served. The transmission time S of each packet follows the general distribution whose distribution function is $\varphi(x)$ and failure rate function is $\mu(x)$.

3 The time when the sensor node is in normal state (working or standby state) is the working time G, and it is assumed that the working time follows an exponential distribution whose parameter is α. In order to better maintain the sensor node, the sensor node enter vacation state after the working time in spite of that there are data packets in the system. Firstly, the sensor node enters the essential vacation whose vacation time is V_1. After the essential vacation, the sensor node enters the wake-up state with probability ε $(0 \le \varepsilon \le 1)$ and enters the optional vacation state whose vacation time is V_2 with probability $\bar{\varepsilon}$ $(\bar{\varepsilon} = 1 - \varepsilon)$. After the optional vacation, the sensor node enters wake-up state. After the wake-up state, the sensor node enters normal state. It is assumed that the wake-up time U follows the general distribution, whose distribution function is $\beta(x)$ and failure rate function is $\sigma(x)$, and the vacation time V_j $(j = 1, 2)$ follows the general distribution, whose distribution function is $\psi_j(x)$ and the failure rate function is $\gamma_j(x)$. Data packet whose transmission is interrupted because of vacation will be retransmitted after the end of vacation.

4 If data packets are not affected by interference signals, the system would feedback some of data packets which have been transmitted to the system for inspection. After the transmission is completed, data packets will be feedbacked to the system for inspection with probability q $(0 \le q < 1)$ and leave the system with probability $\bar{q}$ $(\bar{q} = 1 - q)$. The inspection time of data packets is negligible, and another transmission service is required after inspection.

It is assumed that random variables H_1, H_2, G, U, V_1 and V_2 are independent of each other and have finite first and second moments. The Laplace-Stieltjes transformations of distribution functions $\beta(x)$, $\varphi(x)$, $\eta_i(x)$ $(i = 1, 2)$ and $\psi_j(x)$ $(j = 1, 2)$ are as follows:

$$\beta^*(s) = \int_0^\infty \mathrm{e}^{-sx}\mathrm{d}\beta(x), \quad \varphi^*(s) = \int_0^\infty \mathrm{e}^{-sx}\mathrm{d}\varphi(x),$$

$$\eta_i^*(s) = \int_0^\infty \mathrm{e}^{-sx}\mathrm{d}\eta_i(x),\ i = 1, 2, \quad \psi_j^*(s) = \int_0^\infty \mathrm{e}^{-sx}\mathrm{d}\psi_j(x),\ j = 1, 2.$$

The Laplace transforms of the functions $P_0(t)$, $P_i(t, x)$ $(i = 1, 2, \cdots, 6)$ and $P_i(t, 0)$ $(i = 1, 2, \cdots, 6)$ are as follows:

$$\tilde{P}_0(s) = \int_0^\infty \mathrm{e}^{-st}P_0(t)\mathrm{d}t, \quad \tilde{P}_i(s, x) = \int_0^\infty \mathrm{e}^{-st}P_i(t, x)\mathrm{d}t,\ i = 1, 2, \cdots, 6,$$

$$\tilde{P}_i(s, 0) = \int_0^\infty \mathrm{e}^{-st}P_i(t, 0)\mathrm{d}t,\ i = 1, 2, \cdots, 6.$$

For the convenience of writing, the symbols are defined as follows:

$$a = \lambda^- + \alpha + \tau, \quad \bar{\beta}(x) = 1 - \beta(x), \quad \bar{\varphi}(x) = 1 - \varphi(x),$$

$$\bar{\eta}_i(x) = 1 - \eta_i(x),\ i = 1, 2, \quad \bar{\psi}_j(x) = 1 - \psi_j(x),\ j = 1, 2,$$

$$\bar{\beta}^*(s) = \int_0^\infty \mathrm{e}^{-sx}\bar{\beta}(x)\mathrm{d}x = \frac{1 - \beta^*(s)}{s},$$

$$\bar{\varphi}^*(s) = \int_0^\infty \mathrm{e}^{-sx}\bar{\varphi}(x)\mathrm{d}x = \frac{1 - \varphi^*(s)}{s},$$

$$\bar{\eta}_i^*(s) = \int_0^{\infty} \mathrm{e}^{-sx}\bar{\eta}_i(x)\mathrm{d}x = \frac{1-\eta_i^*(s)}{s},\ i = 1, 2,$$

$$\bar{\psi}_j^*(s) = \int_0^{\infty} \mathrm{e}^{-sx}\bar{\psi}_j(x)\mathrm{d}x = \frac{1-\psi_j^*(s)}{s},\ j = 1, 2,$$

$$\xi_1(s) = \int_0^{\infty} x\mathrm{e}^{-sx}\bar{\varphi}(x)\mathrm{d}x = -[\bar{\varphi}^*(s)]',$$

$$\omega_1(s) = \int_0^{\infty} x\mathrm{e}^{-sx}\mathrm{d}\varphi(x) = -[\varphi^*(s)]',$$

$$\xi_2(s) = \int_0^{\infty} x^2\mathrm{e}^{-sx}\bar{\varphi}(x)\mathrm{d}x = [\bar{\varphi}^*(s)]'',$$

$$\omega_2(s) = \int_0^{\infty} x^2\mathrm{e}^{-sx}\mathrm{d}\varphi(x) = [\varphi^*(s)]''.$$

3 System steady state condition

According to the description, there are two situations that data packets leave the system, one is that they leave without feedback at the end of the transmission, and the other is that they are lost because of the influence of interference signals. At the same time, the sensor node undergoes an abnormal fault repair phase. Therefore, considering the time interval of two consecutive packets at the end of transmission or at the end of abnormal fault repair, general service time T is defined. To find the mean of general service time, a Markov chain with absorption state is constructed. Let $C(t)$ represent the state of the system at the time t, and details are as follows:

1 $C(t) = 0$ indicates that the data packet has been transmitted and left or the abnormal fault of the sensor node has been repaired at the time t

2 $C(t) = 1$ indicates that the sensor node is transmitting data packets at the time t

3 $C(t) = 2$ indicates that the sensor node is in wake-up state at the time t

4 $C(t) = 3$ indicates that the sensor node is in normal fault repair state at the time t

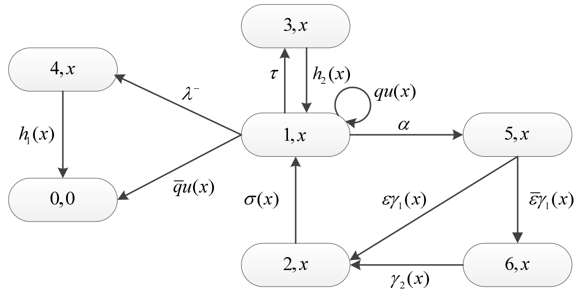
5 $C(t) = 4$ indicates that the sensor node is in abnormal fault repair state at the time t

6 $C(t) = 5$ indicates that the sensor node is in essential vacation state at the time t

7 $C(t) = 6$ indicates that the sensor node is in optional vacation state at the time t.

Let $\Delta(t)$ represent the elapsed time of the state of the sensor node at the time t. The system state transition diagram between t and $t + \mathrm{d}t$ is shown in Figure 1, where x is the value of $\Delta(t)$. Then $\{(C(t), \Delta(t)),\ t \geq 0\}$ is a Markov process with an absorption state of $(0, 0)$, and the state space is $\Omega_1 = \{(0, 0)\} \cup \{(i, x), i = 1, 2, \cdots, 6, x \geq 0\}$.

Figure 1 The system state transition diagram between t and $t + \mathrm{d}t$



Now, the probabilities are defined as follows:

$$P_0(t) = P\{C(t) = 0\},$$

$$P_i(t,x)\mathrm{d}x = P\{C(t) = i, x < \Delta(t) \le x + \mathrm{d}x\}, \quad t \ge 0, x \ge 0, i = 1, 2, \cdots, 6.$$

According to the relationship between the states, combining Kolmogorov differential equation (Zhang and Wang, 2017), the state probability equations of the system are obtained:

$$P_0(t) = \int_0^\infty P_1(t,x)\mu(x)\bar{q}\mathrm{d}x + \int_0^\infty P_4(t,x)h_1(x)\mathrm{d}x, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\right) P_1(t,x) = -(a + \mu(x))P_1(t,x), \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\right) P_2(t,x) = -\sigma(x)P_2(t,x), \tag{3}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\right) P_3(t,x) = -h_2(x)P_3(t,x), \tag{4}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\right) P_4(t,x) = -h_1(x)P_4(t,x), \tag{5}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\right) P_5(t,x) = -\gamma_1(x)P_5(t,x), \tag{6}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\right) P_6(t,x) = -\gamma_2(x)P_6(t,x). \tag{7}$$

The boundary conditions are given by

$$\begin{aligned} P_1(t,0) = &\int_0^\infty P_1(t,x)\mu(x)q\mathrm{d}x + \int_0^\infty P_2(t,x)\sigma(x)\mathrm{d}x \\ &+ \int_0^\infty P_3(t,x)h_2(x)\mathrm{d}x, \end{aligned} \tag{8}$$

$$P_2(t,0)=\int_0^\infty P_5(t,x)\gamma_1(x)\varepsilon \mathrm{d}x+\int_0^\infty P_6(t,x)\gamma_2(x)\mathrm{d}x, \tag{9}$$

$$P_3(t,0)=\int_0^\infty P_1(t,x)\tau \mathrm{d}x, \tag{10}$$

$$P_4(t,0)=\int_0^\infty P_1(t,x)\lambda^- \mathrm{d}x, \tag{11}$$

$$P_5(t,0)=\int_0^\infty P_1(t,x)\alpha \mathrm{d}x, \tag{12}$$

$$P_6(t,0)=\int_0^\infty P_5(t,x)\gamma_1(x)\bar{\varepsilon} \mathrm{d}x. \tag{13}$$

The initial condition is given by

$$P_1(0,x)=\delta(x).$$

By taking Laplace transform on both sides of equations (1)–(13), the following results are obtained:

$$\tilde{P}_0(s)=\int_0^\infty \tilde{P}_1(s,x)\mu(x)\bar{q}\mathrm{d}x+\int_0^\infty \tilde{P}_4(s,x)h_1(x)\mathrm{d}x, \tag{14}$$

$$\frac{\partial}{\partial x}\tilde{P}_1(s,x)=-(s+a+\mu(x))\tilde{P}_1(s,x)+\delta(x), \tag{15}$$

$$\frac{\partial}{\partial x}\tilde{P}_2(s,x)=-(s+\sigma(x))\tilde{P}_2(s,x), \tag{16}$$

$$\frac{\partial}{\partial x}\tilde{P}_3(s,x)=-(s+h_2(x))\tilde{P}_3(s,x), \tag{17}$$

$$\frac{\partial}{\partial x}\tilde{P}_4(s,x)=-(s+h_1(x))\tilde{P}_4(s,x), \tag{18}$$

$$\frac{\partial}{\partial x}\tilde{P}_5(s,x)=-(s+\gamma_1(x))\tilde{P}_5(s,x), \tag{19}$$

$$\frac{\partial}{\partial x}\tilde{P}_6(s,x)=-(s+\gamma_2(x))\tilde{P}_6(s,x), \tag{20}$$

$$\begin{aligned}\tilde{P}_1(s,\ 0)=&\int_0^\infty \tilde{P}_1(s,x)\mu(x)q\mathrm{d}x+\int_0^\infty \tilde{P}_2(s,x)\sigma(x)\mathrm{d}x\\&+\int_0^\infty \tilde{P}_3(s,x)h_2(x)\mathrm{d}x,\end{aligned} \tag{21}$$

$$\tilde{P}_2(s,0)=\int_0^\infty \tilde{P}_5(s,x)\gamma_1(x)\varepsilon \mathrm{d}x+\int_0^\infty \tilde{P}_6(s,x)\gamma_2(x)\mathrm{d}x, \tag{22}$$

$$\tilde{P}_3(s,0) = \int_0^\infty \tilde{P}_1(s,x)\tau \mathrm{d}x, \tag{23}$$

$$\tilde{P}_4(s,0) = \int_0^\infty \tilde{P}_1(s,x)\lambda^- \mathrm{d}x, \tag{24}$$

$$\tilde{P}_5(s,0) = \int_0^\infty \tilde{P}_1(s,x)\alpha \mathrm{d}x, \tag{25}$$

$$\tilde{P}_6(s,0) = \int_0^\infty \tilde{P}_5(s,x)\gamma_1(x)\bar{\varepsilon} \mathrm{d}x. \tag{26}$$

The following results can be obtained by solving equations (15)–(20):

$$\tilde{P}_1(s,x) = (1+\tilde{P}_1(s,0)) \exp\{-(s+a)x\}\,\bar{\varphi}(x), \tag{27}$$

$$\tilde{P}_2(s,x) = \tilde{P}_2(s,0) \exp\{-sx\}\,\bar{\beta}(x), \tag{28}$$

$$\tilde{P}_3(s,x) = \tilde{P}_3(s,0) \exp\{-sx\}\,\bar{\eta}_2(x), \tag{29}$$

$$\tilde{P}_4(s,x) = \tilde{P}_4(s,0) \exp\{-sx\}\,\bar{\eta}_1(x), \tag{30}$$

$$\tilde{P}_5(s,x) = \tilde{P}_5(s,0) \exp\{-sx\}\,\bar{\psi}_1(x), \tag{31}$$

$$\tilde{P}_6(s,x) = \tilde{P}_6(s,0) \exp\{-sx\}\,\bar{\psi}_2(x). \tag{32}$$

Substituting equations (27)–(32) into equations (22)–(27), the results are given by

$$1+\tilde{P}_1(s,0) = \frac{1}{1-\varphi^*(s+a)(q+\alpha\beta^*(s)\psi_1^*(s)(\varepsilon+\bar{\varepsilon}\psi_2^*(s))+\tau\eta_2^*(s))}, \tag{33}$$

$$\tilde{P}_2(s,0) = \frac{\alpha\varphi^*(s+a)\psi_1^*(s)(\varepsilon+\bar{\varepsilon}\psi_2^*(s))}{1-\varphi^*(s+a)(q+\alpha\beta^*(s)\psi_1^*(s)(\varepsilon+\bar{\varepsilon}\psi_2^*(s))+\tau\eta_2^*(s))}, \tag{34}$$

$$\tilde{P}_3(s,0) = \frac{\tau\varphi^*(s+a)}{1-\varphi^*(s+a)(q+\alpha\beta^*(s)\psi_1^*(s)(\varepsilon+\bar{\varepsilon}\psi_2^*(s))+\tau\eta_2^*(s))}, \tag{35}$$

$$\tilde{P}_4(s,0) = \frac{\lambda^-\varphi^*(s+a)}{1-\varphi^*(s+a)(q+\alpha\beta^*(s)\psi_1^*(s)(\varepsilon+\bar{\varepsilon}\psi_2^*(s))+\tau\eta_2^*(s))}, \tag{36}$$

$$\tilde{P}_5(s,0) = \frac{\alpha\varphi^*(s+a)}{1-\varphi^*(s+a)(q+\alpha\beta^*(s)\psi_1^*(s)(\varepsilon+\bar{\varepsilon}\psi_2^*(s))+\tau\eta_2^*(s))}, \tag{37}$$

$$\tilde{P}_6(s,0) = \frac{\alpha\bar{\varepsilon}\varphi^*(s+a)\psi_1^*(s)}{1-\varphi^*(s+a)(q+\alpha\beta^*(s)\psi_1^*(s)(\varepsilon+\bar{\varepsilon}\psi_2^*(s))+\tau\eta_2^*(s))}. \tag{38}$$

Substituting equations (33) and (36) into equations (27) and (30) respectively, the following results can be obtained:

$$\tilde{P}_1(s,x) = \frac{\exp\{-(s+a)x\}\,\bar{\varphi}(x)}{1-\varphi^*(s+a)(q+\alpha\beta^*(s)\psi_1^*(s)(\varepsilon+\bar{\varepsilon}\psi_2^*(s))+\tau\eta_2^*(s))}, \tag{39}$$

$$\tilde{P}_4(s,x) = \frac{\lambda^- \varphi^*(s+a)\exp\{-sx\}\,\bar{\eta}_1(x)}{1-\varphi^*(s+a)(q+\alpha\beta^*(s)\psi_1^*(s)(\varepsilon+\bar{\varepsilon}\psi_2^*(s))+\tau\eta_2^*(s))}. \tag{40}$$

Substituting equations (39) and (40) into equation (14), the result is given by

$$\tilde{P}_0(s) = \frac{\varphi^*(s+a)(\bar{q}+\lambda^-\eta_1^*(s))}{1-\varphi^*(s+a)(q+\alpha\beta^*(s)\psi_1^*(s)(\varepsilon+\bar{\varepsilon}\psi_2^*(s))+\tau\eta_2^*(s))}. \tag{41}$$

Therefore, the mean of the generalised service time $E(T)$ is given by

$$\begin{aligned} E(T) &= -\tilde{P}_0'(s)\Big|_{s=0} \\ &= B_1{}^{-2}((\omega_1(a)(\bar{q}+\lambda^-)+\varphi^*(a)\lambda^- E(H_1))B_1 \\ &\quad+\varphi^*(a)(\bar{q}+\lambda^-)(\omega_1(a)(q+\alpha+\tau)-\varphi^*(a)B_2)) \end{aligned}$$

where

$$B_1 = 1-\varphi^*(a)(q+\alpha+\tau),$$

$$B_2 = \alpha(E(U)+E(V_1)+\bar{\varepsilon}E(V_2))+\tau E(H_2).$$

The WSNs system is equivalent to a classical M/G/1 queueing system, whose service time is the general service time T. In this system, when the sensor node is in working state, standby state or wake-up state, the arrival rate of data packets is λ^+; when the sensor node is in vacation state or fault repair state, the arrival rate is $\bar{\theta}\lambda^+$. Therefore, the necessary and sufficient condition for the steady state balance of the system is

$$\begin{aligned} &\lambda^+(P_I+P_B+P_Q+\bar{\theta}(P_V+P_H))E(X)E(T) \\ &= -\lambda^+(P_I+P_B+P_Q+\bar{\theta}(P_V+P_H))E(X)\tilde{P}_0'(0) < 1 \end{aligned}$$

where P_I, P_B, P_Q,P_V and P_H are the probability that the server is respectively in idle, working, wake-up, vacation and fault repair state under the condition of steady state, the specific expressions of them are shown in Section 4.3.

4 Model analysis

4.1 Steady state equations of the system

Under the condition of steady state, let $L(t)$ represent the number of data packets in the system at time t, let $C(t)$ represent the state of the system at time t, let $\Delta(t)$ represent the elapsed time of the state of the server at the time t, the specific states are defined in Section 3 and $(0,0)$ indicates that the system is idle. Thus, $\{(C(t),L(t),\Delta(t)),t\geq 0\}$ is a Markov process, and the state space is $\Omega_2=\{(0,0)\}\cup\{(i,n,x),i=1,2,3,5,6,n=1,2,\cdots,x\geq 0\}\cup\{(i,n,x),i=4,n=0,1,\cdots,x\geq 0\}$. The probabilities are as follows:

$$I(0) = \lim_{t\to\infty} P\{C(t)=0,L(t)=0\},\ t\geq 0,$$

$$B_n(x) = \lim_{t\to\infty} P\{C(t)=1, L(t)=n, x<\Delta(t)\le x+\mathrm{d}x\},$$
$$t\ge 0,\ x\ge 0,\ n\ge 1,$$
$$Q_n(x) = \lim_{t\to\infty} P\{C(t)=2, L(t)=n, x<\Delta(t)\le x+\mathrm{d}x\},$$
$$t\ge 0,\ x\ge 0,\ n\ge 1,$$
$$D_{1,n}(x) = \lim_{t\to\infty} P\{C(t)=3, L(t)=n, x<\Delta(t)\le x+\mathrm{d}x\},$$
$$t\ge 0,\ x\ge 0,\ n\ge 1,$$
$$D_{2,n}(x) = \lim_{t\to\infty} P\{C(t)=4, L(t)=n, x<\Delta(t)\le x+\mathrm{d}x\},$$
$$t\ge 0,\ x\ge 0,\ n\ge 0,$$
$$M_{j,n}(x) = \lim_{t\to\infty} P\{C(t)=4+j, L(t)=n, x<\Delta(t)\le x+\mathrm{d}x\},$$
$$t\ge 0,\ x\ge 0,\ n\ge 1,\ j=1,2.$$

To make it easier to write equations (42)–(46), the indicator functions $I_{\{n>0\}}$ and $I_{\{n>1\}}$ are introduced as follows:

$$I_{\{n>0\}} = \begin{cases}1, n>0,\\ 0, n\le 0,\end{cases} \quad I_{\{n>1\}} = \begin{cases}1, n>1,\\ 0, n\le 1.\end{cases}$$

According to the model description and the state transition of the system, the steady state equations of the system are obtained:

$$\begin{aligned}\frac{\mathrm{d}B_n(x)}{\mathrm{d}x} &= -(\lambda^+ + \lambda^- + \tau + \alpha + \mu(x))B_n(x)\\ &\quad + I_{\{n>1\}}\sum_{r=1}^{n-1}\lambda^+ c_r B_{n-r}(x),\ n\ge 1,\end{aligned} \tag{42}$$

$$\frac{\mathrm{d}Q_n(x)}{\mathrm{d}x} = -(\lambda^+ + \sigma(x))Q_n(x) + I_{\{n>1\}}\sum_{r=1}^{n-1}\lambda^+ c_r Q_{n-r}(x),\ n\ge 1, \tag{43}$$

$$\begin{aligned}\frac{\mathrm{d}D_{1,n}(x)}{\mathrm{d}x} &= -(\lambda^+\bar\theta + h_2(x))D_{1,n}(x)\\ &\quad + I_{\{n>1\}}\sum_{r=1}^{n-1}\lambda^+\bar\theta c_r D_{1,n-r}(x),\ n\ge 1,\end{aligned} \tag{44}$$

$$\begin{aligned}\frac{\mathrm{d}D_{2,n}(x)}{\mathrm{d}x} &= -(\lambda^+\bar\theta + h_1(x))D_{2,n}(x)\\ &\quad + I_{\{n>0\}}\sum_{r=1}^{n}\lambda^+\bar\theta c_r D_{2,n-r}(x),\ n\ge 0,\end{aligned} \tag{45}$$

$$\begin{aligned}\frac{\mathrm{d}M_{j,n}(x)}{\mathrm{d}x} &= -(\lambda^+\bar\theta + \gamma_j(x))M_{j,n}(x)\\ &\quad + I_{\{n>1\}}\sum_{r=1}^{n-1}\lambda^+\bar\theta c_r M_{j,n-r}(x),\ n\ge 1,\ j=1,2,\end{aligned} \tag{46}$$

$$\lambda^{+}I(0)=\int_{0}^{\infty}B_{1}(x)\mu(x)\bar{q}\mathrm{d}x+\int_{0}^{\infty}D_{2,0}(x)h_{1}(x)\mathrm{d}x, \tag{47}$$

$$\begin{aligned}B_{n}(0)=&\ \lambda^{+}c_{n}I(0)+\int_{0}^{\infty}B_{n+1}(x)\mu(x)\bar{q}\mathrm{d}x+\int_{0}^{\infty}B_{n}(x)\mu(x)q\mathrm{d}x\\&+\int_{0}^{\infty}Q_{n}(x)\sigma(x)\mathrm{d}x+\int_{0}^{\infty}D_{1,n}(x)h_{2}(x)\mathrm{d}x\\&+\int_{0}^{\infty}D_{2,n}(x)h_{1}(x)\mathrm{d}x,\quad n\geq1,\end{aligned} \tag{48}$$

$$Q_{n}(0)=\int_{0}^{\infty}M_{1,n}(x)\gamma_{1}(x)\varepsilon\mathrm{d}x+\int_{0}^{\infty}M_{2,n}(x)\gamma_{2}(x)\mathrm{d}x,\ n\geq1, \tag{49}$$

$$D_{1,n}(0)=\int_{0}^{\infty}B_{n}(x)\tau\mathrm{d}x,\ n\geq1, \tag{50}$$

$$D_{2,n}(0)=\int_{0}^{\infty}B_{n+1}(x)\lambda^{-}\mathrm{d}x,\ n\geq0, \tag{51}$$

$$M_{1,n}(0)=\int_{0}^{\infty}B_{n}(x)\alpha\mathrm{d}x,\ n\geq1, \tag{52}$$

$$M_{2,n}(0)=\int_{0}^{\infty}M_{1,n}(x)\gamma_{1}(x)\bar{\varepsilon}\mathrm{d}x,\ n\geq1. \tag{53}$$

The normalisation condition is given by

$$\begin{aligned}&I(0)+\sum_{n=1}^{\infty}\int_{0}^{\infty}B_{n}(x)\mathrm{d}x+\sum_{n=1}^{\infty}\int_{0}^{\infty}Q_{n}(x)\mathrm{d}x+\sum_{n=1}^{\infty}\int_{0}^{\infty}D_{1,n}(x)\mathrm{d}x\\&+\sum_{n=0}^{\infty}\int_{0}^{\infty}D_{2,n}(x)\mathrm{d}x+\sum_{j=1}^{2}\sum_{n=1}^{\infty}\int_{0}^{\infty}M_{j,n}(x)\mathrm{d}x=1.\end{aligned} \tag{54}$$

4.2 Solution of steady state equations of the system

In order to solve the above equations, the functions are defined as follows:

$$B(x,z)=\sum_{n=1}^{\infty}z^{n}B_{n}(x),\ Q(x,z)=\sum_{n=1}^{\infty}z^{n}Q_{n}(x),\ D_{1}(x,z)=\sum_{n=1}^{\infty}z^{n}D_{1,n}(x),$$

$$D_{2}(x,z)=\sum_{n=0}^{\infty}z^{n}D_{2,n}(x),\ M_{j}(x,z)=\sum_{n=1}^{\infty}z^{n}M_{j,n}(x),\ j=1,2,$$

$$C(z)=\sum_{n=1}^{\infty}z^{n}c_{n}$$

where

$$|z| \leq 1.$$

The following results are obtained from equations (42)–(46) respectively:

$$\frac{\partial B(x,z)}{\partial x} = (\lambda^{+}C(z) - (\lambda^{+} + \lambda^{-} + \tau + \alpha + \mu(x)))B(x,z), \tag{55}$$

$$\frac{\partial Q(x,z)}{\partial x} = (\lambda^{+}C(z) - (\lambda^{+} + \sigma(x)))Q(x,z), \tag{56}$$

$$\frac{\partial D_1(x,z)}{\partial x} = (\lambda^{+}\bar{\theta}C(z) - (\lambda^{+}\bar{\theta} + h_2(x)))D_1(x,z), \tag{57}$$

$$\frac{\partial D_2(x,z)}{\partial x} = (\lambda^{+}\bar{\theta}C(z) - (\lambda^{+}\bar{\theta} + h_1(x)))D_2(x,z), \tag{58}$$

$$\frac{\partial M_j(x,z)}{\partial x} = (\lambda^{+}\bar{\theta}C(z) - (\lambda^{+}\bar{\theta} + \gamma_j(x)))M_j(x,z),\ j = 1,2. \tag{59}$$

The following results are obtained by solving equations (55)–(59):

$$B(x,z) = B(0,z)\exp\left\{(\lambda^{+}C(z) - (\lambda^{+} + \lambda^{-} + \alpha + \tau))x\right\}\bar{\varphi}(x), \tag{60}$$

$$Q(x,z) = Q(0,z)\exp\left\{(\lambda^{+}(C(z) - 1))x\right\}\bar{\beta}(x), \tag{61}$$

$$D_1(x,z) = D_1(0,z)\exp\left\{(\lambda^{+}\bar{\theta}(C(z) - 1)x\right\}\bar{\eta}_2(x), \tag{62}$$

$$D_2(x,z) = D_2(0,z)\exp\left\{(\lambda^{+}\bar{\theta}(C(z) - 1))x\right\}\bar{\eta}_1(x), \tag{63}$$

$$M_j(x,z) = M_j(0,z)\exp\left\{(\lambda^{+}\bar{\theta}(C(z) - 1))x\right\}\bar{\psi}_j(x),\ j = 1,2. \tag{64}$$

According to equations (47) and (48), the following result is obtained:

$$\begin{aligned} & B(0,z) + \lambda^{+}I(0)(1 - C(z)) \\ & = \int_0^\infty \frac{B(x,z)}{z}\mu(x)\bar{q}\mathrm{d}x + \int_0^\infty B(x,z)\mu(x)q\mathrm{d}x + \int_0^\infty Q(x,z)\sigma(x)\mathrm{d}x \\ & + \int_0^\infty D_1(x,z)h_2(x)\mathrm{d}x + \int_0^\infty D_2(x,z)h_1(x)\mathrm{d}x. \end{aligned} \tag{65}$$

Substituting equations (60)–(64) into equation (65), the result is given by

$$\begin{aligned} B(0,z) + \lambda^{+}I(0)(1 - C(z)) = {} & B(0,z)C_1(z)\left(\frac{\bar{q}}{z} + q\right) + C_2(z)Q(0,z) \\ & + C_3(z)D_1(0,z) + C_4(z)D_2(0,z) \end{aligned} \tag{66}$$

where

$$C_1(z) = \varphi^{*}(\lambda^{+} + \lambda^{-} + \alpha + \tau - \lambda^{+}C(z)), \quad C_2(z) = \beta^{*}(\lambda^{+}(1 - C(z))),$$

$$C_3(z) = \eta_2^*(\lambda^+\bar{\theta}(1 - C(z))), \quad C_4(z) = \eta_1^*(\lambda^+\bar{\theta}(1 - C(z))).$$

According to equations (49)–(53), the following results are obtained:

$$Q(0, z) = \int_0^\infty M_1(x, z)\gamma_1(x)\varepsilon \mathrm{d}x + \int_0^\infty M_2(x, z)\gamma_2(x)\mathrm{d}x, \tag{67}$$

$$D_1(0, z) = \tau \int_0^\infty B(x, z)\mathrm{d}x, \tag{68}$$

$$D_2(0, z) = \frac{\lambda^-}{z} \int_0^\infty B(x, z)\mathrm{d}x, \tag{69}$$

$$M_1(0, z) = \alpha \int_0^\infty B(x, z)\mathrm{d}x, \tag{70}$$

$$M_2(0, z) = \bar{\varepsilon} \int_0^\infty M_1(x, z)\gamma_1(x)dx. \tag{71}$$

Substituting equation (61) into equations (68)–(70), which lead to

$$D_1(0, z) = \tau B(0, z)C_5(z), \tag{72}$$

$$D_2(0, z) = \frac{\lambda^-}{z} B(0, z)C_5(z), \tag{73}$$

$$M_1(0, z) = \alpha B(0, z)C_5(z) \tag{74}$$

where

$$C_5(z) = \bar{\varphi}^*(\lambda^+ + \lambda^- + \alpha + \tau - \lambda^+ C(z)).$$

Substituting equations (64) and (74) into equation (71), the result is given by

$$M_2(0, z) = \alpha\bar{\varepsilon}B(0, z)C_5(z)\psi_1^*(\lambda^+\bar{\theta}(1 - C(z))). \tag{75}$$

Substituting equations (64), (74) and (75) into equation (67), the result is given by

$$Q(0, z) = \alpha B(0, z)C_5(z)\psi_1^*(\lambda^+\bar{\theta}(1 - C(z)))(\varepsilon + \bar{\varepsilon}\psi_2^*(\lambda^+\bar{\theta}(1 - C(z)))). \tag{76}$$

Substituting equations (72), (73) and (76) into equation (66), the result is given by

$$B(0, z) = \frac{\lambda^+ zI(0)(1 - C(z))}{C_1(z)(\bar{q} + qz) + C_5(z)Y_1(z) - z} \tag{77}$$

where

$$\begin{aligned} Y_1(z) = {} & C_2(z)\alpha z\psi_1^*(\lambda^+\bar{\theta}(1 - C(z)))(\varepsilon + \bar{\varepsilon}\psi_2^*(\lambda^+\bar{\theta}(1 - C(z)))) \\ & + C_3(z)z\tau + C_4(z)\lambda^-. \end{aligned}$$

Substituting equation (77) into equations (72)–(76) which lead to

$$Q(0,z) = \frac{\lambda^+ zI(0)(1-C(z))}{C_1(z)(\bar{q}+qz)+C_5(z)Y_1(z)-z} \cdot \alpha C_5(z)\psi_1^*(\lambda^+\bar{\theta}(1-C(z)))(\varepsilon+\bar{\varepsilon}\psi_2^*(\lambda^+\bar{\theta}(1-C(z)))), \tag{78}$$

$$D_1(0,z) = \frac{\lambda^+ zI(0)(1-C(z))\tau C_5(z)}{C_1(z)(\bar{q}+qz)+C_5(z)Y_1(z)-z}, \tag{79}$$

$$D_2(0,z) = \frac{\lambda^+ I(0)(1-C(z))\lambda^- C_5(z)}{C_1(z)(\bar{q}+qz)+C_5(z)Y_1(z)-z}, \tag{80}$$

$$M_1(0,z) = \frac{\lambda^+ zI(0)(1-C(z))\alpha C_5(z)}{C_1(z)(\bar{q}+qz)+C_5(z)Y_1(z)-z}, \tag{81}$$

$$M_2(0,z) = \frac{\lambda^+ zI(0)(1-C(z))\alpha\bar{\varepsilon} C_5(z)\psi_1^*(\lambda^+\bar{\theta}(1-C(z)))}{C_1(z)(\bar{q}+qz)+C_5(z)Y_1(z)-z}. \tag{82}$$

Substituting equations (77)–(82) into equations (60)–(64), and then integrating x, the following results are obtained:

$$B(z) = \int_0^\infty B(x,z)\mathrm{d}x = \frac{\lambda^+ zI(0)(1-C(z))C_5(z)}{C_1(z)(\bar{q}+qz)+C_5(z)Y_1(z)-z}, \tag{83}$$

$$B(z) = \int_0^\infty B(x,z)\mathrm{d}x = \frac{\lambda^+ zI(0)(1-C(z))C_5(z)}{C_1(z)(\bar{q}+qz)+C_5(z)Y_1(z)-z}, \tag{84}$$

$$Q(z) = \int_0^\infty Q(x,z)\mathrm{d}x = \frac{\lambda^+ zI(0)(1-C(z))\bar{\beta}^*(\lambda^+(1-C(z)))}{C_1(z)(\bar{q}+qz)+C_5(z)Y_1(z)-z} \cdot \alpha C_5(z)\psi_1^*(\lambda^+\bar{\theta}(1-C(z)))(\varepsilon+\bar{\varepsilon}\psi_2^*(\lambda^+\bar{\theta}(1-C(z)))), \tag{85}$$

$$D_1(z) = \int_0^\infty D_1(x,z)\mathrm{d}x = \frac{\lambda^+ zI(0)(1-C(z))\tau C_5(z)\bar{\eta}_2^*(\lambda^+\bar{\theta}(1-C(z)))}{C_1(z)(\bar{q}+qz)+C_5(z)Y_1(z)-z}, \tag{86}$$

$$D_2(z) = \int_0^\infty D_2(x,z)\mathrm{d}x = \frac{\lambda^+ I(0)(1-C(z))\lambda^- C_5(z)\bar{\eta}_1^*(\lambda^+\bar{\theta}(1-C(z)))}{C_1(z)(\bar{q}+qz)+C_5(z)Y_1(z)-z}, \tag{87}$$

$$M_1(z) = \int_0^\infty M_1(x,z)\mathrm{d}x = \frac{\lambda^+ zI(0)(1-C(z))\alpha C_5(z)\bar{\psi}_1^*(\lambda^+\bar{\theta}(1-C(z)))}{C_1(z)(\bar{q}+qz)+C_5(z)Y_1(z)-z}, \tag{88}$$

$$
\begin{aligned}
M_2(z) &= \int_0^\infty M_2(x,z)\mathrm{d}x \\
&= \frac{\lambda^+ z I(0)(1-C(z))\alpha\bar{\varepsilon}C_5(z)\psi_1^*(\lambda^+\bar{\theta}(1-C(z)))}{C_1(z)(\bar{q}+qz)+C_5(z)Y_1(z)-z} \\
&\cdot \bar{\psi}_2^*(\lambda^+\bar{\theta}(1-C(z))).
\end{aligned} \tag{89}
$$

Applying L'Hospital's rule in equations (83)–(89), the results are given by

$$
B(1) = \frac{\lambda^+ I(0)\bar{\varphi}^*(a)E(X)}{1-(q\varphi^*(a)+\lambda^+E(X)Y_2+\bar{\varphi}^*(a)(\alpha+\tau))}, \tag{90}
$$

$$
Q(1) = \frac{\lambda^+ \alpha I(0)\bar{\beta}^*(0)\bar{\varphi}^*(a)E(X)}{1-(q\varphi^*(a)+\lambda^+E(X)Y_2+\bar{\varphi}^*(a)(\alpha+\tau))}, \tag{91}
$$

$$
D_1(1) = \frac{\lambda^+ \tau I(0)\bar{\varphi}^*(a)\bar{\eta}_2^*(0)E(X)}{1-(q\varphi^*(a)+\lambda^+E(X)Y_2+\bar{\varphi}^*(a)(\alpha+\tau))}, \tag{92}
$$

$$
D_2(1) = \frac{\lambda^+ \lambda^- I(0)\bar{\varphi}^*(a)\bar{\eta}_1^*(0)E(X)}{1-(q\varphi^*(a)+\lambda^+E(X)Y_2+\bar{\varphi}^*(a)(\alpha+\tau))}, \tag{93}
$$

$$
M_1(1) = \frac{\lambda^+ \alpha I(0)\bar{\varphi}^*(a)\bar{\psi}_1^*(0)E(X)}{1-(q\varphi^*(a)+\lambda^+E(X)Y_2+\bar{\varphi}^*(a)(\alpha+\tau))}, \tag{94}
$$

$$
M_2(1) = \frac{\lambda^+ \alpha\bar{\varepsilon} I(0)\bar{\varphi}^*(a)\bar{\psi}_2^*(0)E(X)}{1-(q\varphi^*(a)+\lambda^+E(X)Y_2+\bar{\varphi}^*(a)(\alpha+\tau))} \tag{95}
$$

where

$$
\begin{aligned}
Y_2 = {} & \omega_1(a)+a\xi_1(a) \\
& +\bar{\varphi}^*(a)(\alpha E(U)+\bar{\theta}(\alpha E(V_1)+\alpha\bar{\varepsilon}E(V_2)+\tau E(H_2)+\lambda^- E(H_1))).
\end{aligned}
$$

Substituting equations (90)–(95) into equation (54), the result is given by

$$
I(0) = \frac{1-(q\varphi^*(a)+\lambda^+E(X)Y_2+\bar{\varphi}^*(a)(\alpha+\tau))}{1-(q\varphi^*(a)+\lambda^+E(X)Y_2+\bar{\varphi}^*(a)(\alpha+\tau))+\lambda^+\bar{\varphi}^*(a)E(X)Y_3} \tag{96}
$$

where

$$
Y_3 = 1+\alpha\bar{\beta}^*(0)+\tau\bar{\eta}_2^*(0)+\lambda^-\bar{\eta}_1^*(0)+\alpha\bar{\psi}_1^*(0)+\alpha\bar{\varepsilon}\bar{\psi}_2^*(0).
$$

Substituting equation (96) into equations (90)–(95), the expressions of $B(1)$, $Q(1)$, $D_1(1)$, $D_2(1)$, $M_1(1)$ and $M_2(1)$ can be obtained.

4.3 Performance indexes

Under the conditions of steady state, some performance indexes of the WSNs system can be obtained:

1 The probability that the sensor node is in idle state is given by

$$P_I = I(0).$$

2 The probability that the sensor node is in working state is given by

$$\begin{aligned} P_B &= B(1) \\ &= \frac{\lambda^+ \bar{\varphi}^*(a) E(X)}{1 - (q\varphi^*(a) + \lambda^+ E(X) Y_2 + \bar{\varphi}^*(a)(\alpha + \tau)) + \lambda^+ \bar{\varphi}^*(a) E(X) Y_3}. \end{aligned}$$

3 The probability that the sensor node is in wake-up state is given by

$$\begin{aligned} P_Q &= Q(1) \\ &= \frac{\lambda^+ \alpha \bar{\beta}^*(0) \bar{\varphi}^*(a) E(X)}{1 - (q\varphi^*(a) + \lambda^+ E(X) Y_2 + \bar{\varphi}^*(a)(\alpha + \tau)) + \lambda^+ \bar{\varphi}^*(a) E(X) Y_3}. \end{aligned}$$

4 The probability that the sensor node is in abnormal fault state is given by

$$\begin{aligned} P_{H_1} &= D_2(1) \\ &= \frac{\lambda^+ \lambda^- \bar{\varphi}^*(a) \bar{\eta}_1^*(0) E(X)}{1 - (q\varphi^*(a) + \lambda^+ E(X) Y_2 + \bar{\varphi}^*(a)(\alpha + \tau)) + \lambda^+ \bar{\varphi}^*(a) E(X) Y_3}. \end{aligned}$$

The probability that the sensor node is in normal fault state is given by

$$\begin{aligned} P_{H_2} &= D_1(1) \\ &= \frac{\lambda^+ \tau \bar{\varphi}^*(a) \bar{\eta}_2^*(0) E(X)}{1 - (q\varphi^*(a) + \lambda^+ E(X) Y_2 + \bar{\varphi}^*(a)(\alpha + \tau)) + \lambda^+ \bar{\varphi}^*(a) E(X) Y_3}. \end{aligned}$$

The probability that the sensor node is in fault state is given by

$$\begin{aligned} P_H &= P_{H_1} + P_{H_2} \\ &= \frac{\lambda^+ \bar{\varphi}^*(a) E(X)(\lambda^- \bar{\eta}_1^*(0) + \tau \bar{\eta}_2^*(0))}{1 - (q\varphi^*(a) + \lambda^+ E(X) Y_2 + \bar{\varphi}^*(a)(\alpha + \tau)) + \lambda^+ \bar{\varphi}^*(a) E(X) Y_3}. \end{aligned}$$

5 The probability that the sensor node is in essential vacation state is given by

$$\begin{aligned} P_{V_1} &= M_1(1) \\ &= \frac{\lambda^+ \alpha \bar{\varphi}^*(a) \bar{\psi}_1^*(0) E(X)}{1 - (q\varphi^*(a) + \lambda^+ E(X) Y_2 + \bar{\varphi}^*(a)(\alpha + \tau)) + \lambda^+ \bar{\varphi}^*(a) E(X) Y_3}. \end{aligned}$$

The probability that the sensor node is in optional vacation state is given by

$$\begin{aligned} P_{V_2} &= M_2(1) \\ &= \frac{\lambda^+ \alpha \bar{\varepsilon} \bar{\varphi}^*(a) \bar{\psi}_2^*(0) E(X)}{1 - (q\varphi^*(a) + \lambda^+ E(X) Y_2 + \bar{\varphi}^*(a)(\alpha + \tau)) + \lambda^+ \bar{\varphi}^*(a) E(X) Y_3}. \end{aligned}$$

The probability that the sensor node is in vacation state is given by

$$P_V = P_{V_1} + P_{V_2} = \frac{\lambda^+ \alpha \bar{\varphi}^*(a)E(X)(\bar{\psi}_1^*(0) + \bar{\varepsilon}\bar{\psi}_2^*(0))}{1 - (q\varphi^*(a) + \lambda^+ E(X)Y_2 + \bar{\varphi}^*(a)(\alpha+\tau)) + \lambda^+ \bar{\varphi}^*(a)E(X)Y_3}.$$

6 The energy saving rate Φ of the system represents the reduction of energy consumption per unit time. Energy saving rate is an important index to measure energy saving strategy of WSNs. When the sensor node is in vacation state, the energy saved by the system per unit time is set to A_1. When the sensor node is in abnormal fault state, the energy additionally consumed by the system per unit time is set to A_2. When the sensor node is in normal fault state, the energy additionally consumed by the system per unit time is set to A_3. The energy saving rate Φ based on the steady state solution of the system is given by

$$\Phi = A_1 P_V - A_2 P_{H_1} - A_3 P_{H_2}.$$

7 The loss rate of data packets is given by

$$W_f = \lambda^+ \theta(P_V + P_H) + \lambda^- P_B = \frac{\lambda^+ \bar{\varphi}^*(a)E(X)(\lambda^+\theta(\alpha(\bar{\psi}_1^*(0) + \bar{\varepsilon}\bar{\psi}_2^*(0)) + \lambda^- \bar{\eta}_1^*(0) + \tau\bar{\eta}_2^*(0)) + \lambda^-)}{1 - (q\varphi^*(a) + \lambda^+ E(X)Y_2 + \bar{\varphi}^*(a)(\alpha+\tau)) + \lambda^+ \bar{\varphi}^*(a)E(X)Y_3}.$$

8 The average number of data packets in the system is given by

$$E(L) = \left.\frac{\mathrm{d}B(z)}{\mathrm{d}z}\right|_{z=1} + \left.\frac{\mathrm{d}Q(z)}{\mathrm{d}z}\right|_{z=1} + \sum_{i=1}^{2} \left.\frac{\mathrm{d}D_i(z)}{\mathrm{d}z}\right|_{z=1} + \sum_{j=1}^{2} \left.\frac{\mathrm{d}M_j(z)}{\mathrm{d}z}\right|_{z=1} = \frac{K_4(\lambda^+)^4 + K_3(\lambda^+)^3 + K_2(\lambda^+)^2 + K_1\lambda^+}{2(K_5(\lambda^+)^2 + K_7\lambda^+ + K_6^2)}$$

where

$$K_1 = -\bar{\varphi}^*(a)(E(X^2) + E(X))Y_3(q\varphi^*(a) + \bar{\varphi}^*(a)(\alpha+\tau) - 1),$$

$$K_2 = E(X^2)Y_6(q\varphi^*(a) + \bar{\varphi}^*(a)(\alpha+\tau) - 1) - \bar{\varphi}^*(a)(E(X^2) + E(X))Y_3E(X)Y_2 + ((E(X^2) - E(X))(\omega_1(a) + \xi_1(a)) + 2E(X)(q\omega_1(a) + (\alpha+\tau)\xi_1(a))) \cdot E(X)\bar{\varphi}^*(a)Y_3,$$

$$K_3 = (E(X))^3(Y_6Y_2 + \omega_2(a) + a\xi_2(a) + 2\xi_1(a)Y_2) + (E(X))^2\xi_1(a)Y_5\bar{\varphi}^*(a)Y_3,$$

$$K_4 = (E(X))^4\xi_1(a)\bar{\varphi}^*(a)Y_3Y_4, \quad K_5 = -(E(X))^2(\bar{\varphi}^*(a)Y_3 - Y_2)Y_2,$$

$$K_6 = 1 - q\varphi^*(a) - \bar{\varphi}^*(a)(\alpha+\tau), \quad K_7 = E(X)K_6(\bar{\varphi}^*(a)Y_3 - 2Y_2),$$

$$Y_4 = \alpha E(U^2) + \tau\bar{\theta}^2E(H_2^2) + \lambda^-\bar{\theta}^2E(H_1^2) + 2\bar{\theta}E(U)(E(V_1) + \bar{\varepsilon}E(V_2)) + \alpha\bar{\theta}(E(V_1^2) - \bar{\varepsilon}E(V_2) + E(V_1)E(V_2) + \bar{\varepsilon}E(V_2^2)),$$

$$
\begin{aligned}
Y_5 = \; & E(X^2)(\alpha E(U) + \bar{\theta}(\tau E(H_2) + \alpha\bar{\varepsilon}E(V_2) - \alpha)) \\
& + \alpha E(X)(2E(V_1) + \bar{\theta}(2\bar{\varepsilon}E(V_2) + 1) - \alpha\bar{\varepsilon}E(V_2)) \\
& + E(X)(\bar{\theta}\lambda^- E(H_1) + \tau E(H_2)(2 - \bar{\theta})),
\end{aligned}
$$

$$
\begin{aligned}
Y_6 = \; & \bar{\theta}(\tau[\bar{\eta}_2^*(0)]' + \lambda^-[\bar{\eta}_1^*(0)]' + \alpha[\bar{\psi}_1^*(0)]' + \bar{\varepsilon}E(V_1)\bar{\psi}_2^*(0) + \alpha\bar{\varepsilon}[\bar{\psi}_2^*(0)]')) \\
& + 2\bar{\varphi}^*(a)(\alpha\bar{\beta}^*(0)(1 - \bar{\theta}(E(V_1) + \bar{\varepsilon}E(V_2))) - 2\xi_1(a)Y_3.
\end{aligned}
$$

The average length of time from the moment that a data packet is received by the system to the moment that the data packet leaves the system after transmission is completed is defined as the average delay W of the data packet. Using the Little formula of the queueing theory (Yue et al., 2009), the average delay W is given by

$$
W = \frac{E(L)}{\lambda^+(P_I + P_B + P_Q + \bar{\theta}(P_V + P_H))}.
$$

5 Equilibrium strategy

It is assumed that the state of the sensor node can not be predicted before the data packet is received by the system. Let λ ($\lambda = \lambda^+ p$) represent the effective reception rate of data packets in the system. The data packets which arrive at the system will be received with probability p and they will leave the system with probability $\bar{p}$ ($\bar{p} = 1 - p$). The equilibrium strategy of the system will be considered. At this time, the average delay of the data packets is

$$
\begin{aligned}
W(\lambda) &= \frac{E(L)}{\lambda(P_I + P_B + P_Q + \bar{\theta}(P_V + P_H))} \\
&= \frac{K_4\lambda^3 + K_3\lambda^2 + K_2\lambda + K_1}{2(K_8\lambda^2 + K_9\lambda + K_6^2)}
\end{aligned}
$$

where

$$
K_8 = (E(X))^2\bar{\varphi}^*(a)(1 + Y_7 - Y_2)Y_2, \quad K_9 = E(X)K_6(\bar{\varphi}^*(a)(1 + Y_7) - 2Y_2),
$$

$$
Y_7 = 1 + \alpha\bar{\beta}^*(0) + \tau\bar{\eta}_2^*(0) + \lambda^-\bar{\eta}_1^*(0) + \alpha\bar{\theta}(\bar{\psi}_1^*(0) + \bar{\varepsilon}\bar{\psi}_2^*(0)).
$$

The first derivative of the average delay is given by

$$
W'(\lambda) = \frac{J_4\lambda^4 + J_3\lambda^3 + J_2\lambda^2 + J_1\lambda + J_0}{4(K_8\lambda^2 + K_9\lambda + K_6^2)^2}
$$

where

$$
J_0 = 2K_2K_6^2 - K_1K_9, \quad J_1 = 4K_3K_6^2 + K_2K_7 - K_1K_8,
$$

$$
J_2 = 6K_4K_6^2 + 3K_3K_9, \quad J_3 = 5K_4K_7 + 2K_3K_8, \quad J_4 = 4K_4K_8.
$$

In order to find the maximum value of $W(\lambda)$, the value of λ which let $W'(\lambda) = 0$ is demand. Because of $4(K_8\lambda^2 + K_9\lambda + K_6^2)^2 > 0$, the equation $J_4\lambda^4 + J_3\lambda^3 + J_2\lambda^2 +$

$J_1\lambda + J_0$=0 is solved to get the required value. The values of K_4 and K_8 are both greater than zero, then $J_4 > 0$, and the equation is transformed into $\lambda^4 + \frac{J_3}{J_4}\lambda^3 + \frac{J_2}{J_4}\lambda^2 + \frac{J_1}{J_4}\lambda + \frac{J_0}{J_4}$=0. In order to ensure that the left and right sides of the equation are perfect square trinomials after the equation deformation and the equation has roots, parameter t is introduced and it satisfies the equation $t^3 - \frac{J_2}{J_4}t^2 + \left(\frac{J_3 J_1}{J_4^2} - \frac{4J_0}{J_4}\right)t + 4\frac{J_2 J_0}{J_4^2} - \frac{J_3^2 J_0}{J_4^3} - \frac{J_1^2}{J_4^2} = 0$. Using the Ferrari's solution method, the roots of the equation can be obtained:

$$\lambda_{1,2} = \frac{1}{2}\left(\alpha - \frac{J_3}{2J_4}\right) \pm \frac{1}{2}\sqrt{\left(\frac{J_3}{2J_4} - \alpha\right)^2 - 4\left(\frac{t}{2} - \beta\right)},$$

$$\lambda_{3,4} = -\frac{1}{2}\left(\alpha + \frac{J_3}{2J_4}\right) \pm \frac{1}{2}\sqrt{\left(\frac{J_3}{2J_4} + \alpha\right)^2 - 4\left(\frac{t}{2} + \beta\right)}$$

where α and β satisfy the following relationship

$$\alpha^2 = \frac{J_3^2}{4J_4^2} - \frac{J_2}{J_4} + t, \quad \beta^2 = \frac{t^2}{4} - \frac{J_0}{J_4}, \quad \alpha\beta = \frac{J_3 t}{4J_4} - \frac{J_2}{2J_4}.$$

There are four same roots of λ under each individual value t and each group α, β. In actual solution, the four roots could be solved under a value t and a group α, β (Guo, 2011). Let λ^*=max$\{\lambda_1, \lambda_2, \lambda_3, \lambda_4, 0\}$, according to the properties of the quartic function (Izadi et al., 2018), $W'(\lambda^*) = 0$, and $W(\lambda^*)$ is the minimum value of $W(\lambda)$ when $\lambda \in (0, \lambda^+)$.

Every sensor node wants to get the most benefit from the system. It is assumed that R represents the benefit of each data packet from the moment that it is received by the system to the moment that it leaves the system after transmission is completed, m represents the cost of each data packet per unit time, and Γ represents the total revenue obtained by the system. Fixing other parameters, the revenue function about λ is constructed, and the function is as follows:

$$\Gamma(\lambda) = R - mW(\lambda).$$

The system can be discussed in the following situations:

1 When $mW(\lambda^*) \geq R$, the revenue of the system is non-positive regardless of the reception rate λ of data packets. Data packets enter the system with the equilibrium probability $p_e = 0$.

2 When $mW(\lambda^*) < R$, the situation of $\Gamma(\lambda) = R - mW(\lambda)$=0 exists. The reception rate λ at this time is delimited as the equilibrium reception rate λ_e, and $W(\lambda_e) = \frac{R}{m}$. If $\lambda_e < \lambda^+$, the revenue of the system is positive when $\lambda \in (0, \lambda_e)$, the revenue of the system is negative when $\lambda \in (\lambda_e, \lambda^+)$, and data packets enter the system with the equilibrium probability $p_e = \frac{\lambda_e}{\lambda^+}$. If $\lambda_e \geq \lambda^+$, the revenue of the system is positive when $\lambda \in (0, \lambda^+)$, and data packets enter the system with the equilibrium probability $p_e = 1$. When λ=λ^*, the revenue of the system is the most, and $\Gamma(\lambda^*) = R - mW(\lambda^*)$.

Because the expressions are more complicated, numerical experiments are used in next section to analyse the influence of the change of parameters on performance indexes.

6 Numerical experiments

Combining the obtained expressions of performance indexes, the relationship charts of performance indexes of the WSNs system with the change of parameters are obtained by using MATLAB, and the changes of performance indexes influenced by parameters are analysed through the relationship charts. It is assumed that the transmission time S, the wake-up time U, the repair time H_1 due to environmental interference, the repair time H_2 due to physical damage, the essential vacation time V_1 and the optional vacation time V_2 respectively obey negative exponential distributions with parameters of μ, σ, h_1, h_2, γ_1 and γ_2. The parameters of the WSNs system are shown in Table 1.

Table 1 The parameters of the WSNs system

Parameter	*Value*	*Parameter*	*Value*	*Parameter*	*Value*	*Parameter*	*Value*
λ^+	0.3	σ	1.0	h_1	1.0	A_1	50
λ^-	0.2	θ	0.9	h_2	0.5	A_2	1.0
μ	1.0	q	0.1	γ_1	1.0	A_3	0.1
c	0.5	ε	0.75	γ_2	1.0	R_0	30
α	0.01	τ	0.001	m	1.0		

Figure 2 describes the trend of the energy saving rate Φ with the essential vacation timer parameter γ_1 and the sensor node lifetime parameter τ. When τ remains unchanged, with the increase of γ_1, the vacation time of the sensor node becomes shorter, the sensor node becomes more active, and the system energy saving rate decreases. When γ_1 remains unchanged, with the increase of τ, the probability P_{H_2} that the sensor node is in normal fault state increases, and the energy saving rate of the system decreases.

Figure 2 The relationship of Φ with γ_1 and τ

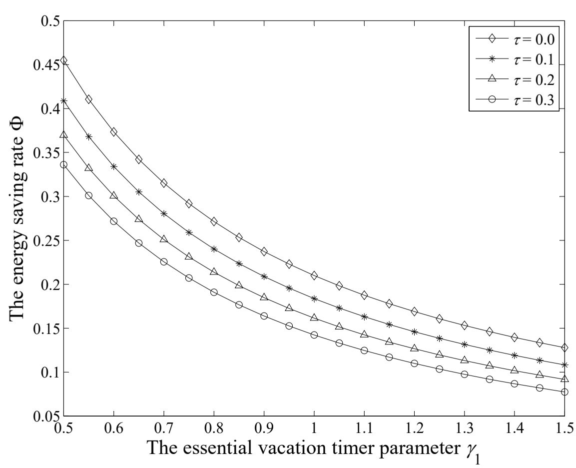


Figure 3 The relationship of W with γ_1 and ε

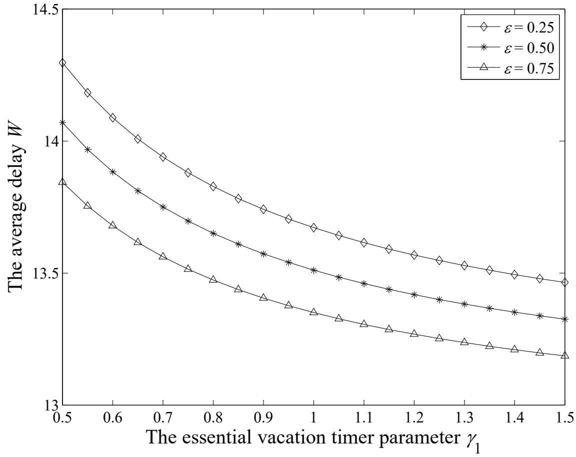


Figure 3 describes the trend of the average delay W with the essential vacation timer parameter γ_1 and the probability ε of the sensor node entering wake-up state. When ε remains unchanged, with the increase of γ_1, the vacation time of the sensor node becomes shorter, the sensor node becomes more active, which improves data packets transmission efficiency of the system. The retention time of data packets in the system becomes shorter and the average delay of data packets is reduced. When γ_1 remains unchanged, with the increase of ε, the sensor node skips the optional vacation state and directly enter into wake-up state. The retention time of data packets becomes shorter and the average delay of data packets is reduced.

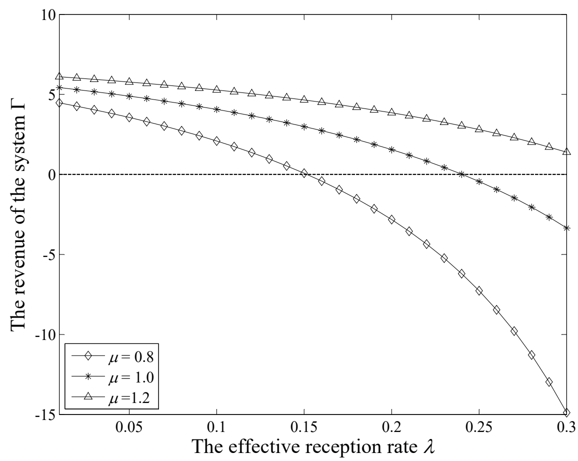
Table 2 The influence of interference signals on performance indexes

λ^-	P_B	P_Q	P_V	P_H	W_f	$E(L)$	W	Φ
0.0	0.6609	0.0066	0.0083	0.0013	0.0026	6.9777	23.4613	0.4131
0.1	0.5651	0.0057	0.0071	0.0576	0.0740	4.8246	17.0763	0.2910
0.2	0.4935	0.0049	0.0062	0.0997	0.1273	3.6236	13.3504	0.1999
0.3	0.4380	0.0044	0.0055	0.1323	0.1686	2.8751	10.9401	0.1292
0.4	0.3937	0.0039	0.0049	0.1583	0.2016	2.3720	9.2681	0.0728
0.5	0.3576	0.0036	0.0045	0.1795	0.2285	2.0145	8.0476	0.0268
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

In Table 2, the change of performance indexes of system is investigated when the arrival rate λ^- of interference signals is changed. With the increase of λ^-, the WSNs system is affected by more environmental interference, the probability P_H of the sensor node in fault state increases, and the probability P_B that the sensor node is in working state, the probability P_Q that the sensor node is in wake-up state and the probability P_V that the sensor node is in vacation state decrease. The loss of data packets is getting more and more serious, which results the decreases of the average number $E(L)$ of data packets and the average delay W in the system. The energy consumption of the WSNs system increases, the energy saving effect is weakened.

Figure 4 describes the trend of the revenue Γ of the system with the effective reception rate of data packets $\lambda(\lambda \in (0, \lambda^{+}))$ and the transmission rate μ of data packets. When λ remains unchanged, with the increase of μ, the number of data packets transmitted by the system per unit time increases, so the total revenue of the system increases. When μ remains unchanged, with the increase of λ, the system receives more data packets per unit time, and the average delay of data packets increases. Then the costs of system per unit time increase, which results in the decreases of the revenue of the WSNs system.

Figure 4 The relationship of Γ with λ and μ



According to the trend of the revenue of system, the appropriate ranges of the equilibrium probability p_e of data packets entering the system and the equilibrium reception rate λ_e of the system can be obtained under different the transmission rates of data packets. Table 3 shows the results of the equilibrium numerical of the revenue Γ of system at different the transmission rates μ. It is found that the equilibrium reception rate λ_e increases with the increase of the transmission rate μ.

Table 3 The equilibrium numerical results of the revenue of system

μ	λ_e		p_e	
	Minimum	*Maximum*	*Minimum*	*Maximum*
0.8	0.15	0.16	0.50	0.53
1.0	0.24	0.25	0.80	0.83
1.2	-	-	1.00	1.00

7 Conclusions

Because of the complexity of the network environment, some interference factors will have a fatal impact on WSNs. Factors of environmental disturbance are introduced at the time when the energy-saving mechanism of wireless sensor networks is being studied. Considering the fact that sensor nodes can be repaired in practice and the operating mechanism of wireless sensor networks, the batch receiving wireless sensor networks system based on an M^X/G/1 vacation queueing model with customers arriving in batches, negative customers, feedback customers, start-up periods, optional vacations and repairable faults was established. The steady state distribution of the system was obtained by using the supplementary variable method, and the expressions of the average delay of date packets and the energy saving rate of system were obtained. The changes of the performance indexes influenced by parameters were obtained through numerical analysis by using MATLAB software. It was found that reducing the influence of interference signals, replacing sensor nodes with longer service life and increasing the time of sensor nodes to adjust dormancy could increase the energy saving rate of the system and make the system operate better. Constructing revenue function of the system, equilibrium states of the system were discussed, and the numerical results of equilibrium states were solved.

Acknowledgements

This work was supported in part by the National Natural Science Foundation of China under Grant Nos. 61973261, 61872311, Natural Science Foundation of Hebei Province under Grant Nos. A2020203010, A2018203088, Key Foundation of Higher Education Science and Technology Research of Hebei Province under Grant No. ZD2017079.

References

Abdulsalam, H.M., Ali, B.A. and Alroumi, E. (2018) 'Usage of mobile elements in internet of things environment for data aggregation in wireless sensor networks', *Computers and Electrical Engineering*, Vol. 72, pp.789–807.

Abujubbeh, M., Al-Turjman, F. and Fahrioglu, M. (2019) 'Software-defined wireless sensor networks in smart grids: an overview', *Sustainable Cities and Society*, Vol. 51 [online] https://doi.org/10.1016/j.scs.2019.101754.

Alarifi, A. and Tolba, A. (2019) 'Optimizing the network energy of cloud assisted internet of things by using the adaptive neural learning approach in wireless sensor networks', *Computers in Industry*, Vol. 106, pp.133–141.

Alromih, A. and Kurdi, H. (2019) 'An energy-efficient gossiping protocol for wireless sensor networks using Chebyshev distance', *Procedia Computer Science*, Vol. 151, pp.1066–1071.

Bouyer, A., Hatamlou, A. and Masdari, M. (2015) 'A new approach for decreasing energy in wireless sensor networks with hybrid LEACH protocol and fuzzy C-means algorithm', *International Journal of Communication Networks and Distributed Systems*, Vol. 14, No. 4, pp.400–412.

Chaturvedi, A., Goswami, D.N. and Singh, S. (2016) 'Energy efficient cluster head selection for cross layer design over wireless sensor network', *International Journal of Communication Networks and Distributed Systems*, Vol. 16, No. 4, pp.335–351.

Chauhan, S. and Awasthi, L.K. (2014) 'An energy efficient cycle stealing algorithm for best effort services in wireless sensor networks', *International Journal of Communication Networks and Distributed Systems*, Vol. 12, No. 3, pp.275–298.

Chiasserini, C.F. and Garetto, M. (2004) 'Modeling the performance of wireless sensor networks', *Joint Conference of the IEEE Computer and Communications Societies*, IEEE, Hong Kong, China, pp.220–231.

Chikh, A. and Lehsaini, M. (2018) 'Multipath routing protocols for wireless multimedia sensor networks: a survey', *International Journal of Communication Networks and Distributed Systems*, Vol. 20, No. 1, pp.60–81.

Feng, Y.F. (2019) 'Analysis of wireless sensor network technology development', *Information Systems Engineering*, No. 5, p.38, in Chinese.

Gao, D., Zhang, L. and Wang, H. (2011) 'Energy saving with node sleep and power control mechanisms for wireless sensor networks', *The Journal of China Universities of Posts and Telecommunications*, Vol. 18, No. 1, pp.49–59.

Gherbi, C., Aliouat, Z. and Benmohammed, M. (2018) 'Energy efficient with time synchronised and service coverage guarantee in wireless sensor networks', *International Journal of Communication Networks and Distributed Systems*, Vol. 21, No. 1, pp.56–79.

Goswami, P., Yan, Z., Mukherjee, A., Yang, L., Routray, S. and Palai, G. (2019) 'An energy efficient clustering using firefly and HML for optical wireless sensor network', *Optik*, Vol. 182, pp.181–185.

Guo, M. (2011) 'Tracing to the source of the solution of the formula of the quadratic equation of the one variable', *Friends of Mathematics*, No. 3, pp.72–73, in Chinese.

Izadi, F., Rasool, N.F. and Amaneh, A.V. (2018) 'Fourth power diophantine equations in Gaussian integers', *Proceedings – Mathematical Sciences*, Vol. 128, No. 18 [online] https://doi.org/10.1007/s12044-018-0390-7.

Lee, D.H. and Yang, W.S. (2013) 'The N-policy of a discrete time Geo/G/1 queue with disasters and its application to wireless sensor networks', *Applied Mathematical Modelling*, Vol. 37, No. 23, pp.9722–9731.

Li, X., Niu, J.W., Kumari, S., Wu, F., Sangaiah, A.K. and Choo, K.R. (2018) 'A three-factor anonymous authentication scheme for wireless sensor networks in internet of things environments', *Journal of Network and Computer Applications*, Vol. 103, pp.194–204.

Liu, J., Jiang, X., Horiguchi, S. and Lee, T.T. (2010) 'Analysis of random sleep scheme for wireless sensor networks', *International Journal of Sensor Networks*, Vol. 7, Nos. 1–2, pp.71–84.

Majumdar, A. and Sarkar, D. (2015) 'Various types of routing protocols in wireless sensor network with vulnerabilities: a review', *Institute of Electrical and Electronics Engineers Inc* [online] https://doi.org/10.1109/iemcon.2015.7344446.

Mann, C.R., Baldwin, R.O., Kharoufeh, J.P. and Mullins, B.E. (2008) 'A queueing approach to optimal resource replication in wireless sensor networks', *Performance Evaluation*, Vol. 65, No. 10, pp.689–700.

Mazinani, A., Mazinani, S.M. and Mirzaie, M. (2018) 'FMCR-CT: an energy-efficient fuzzy multi cluster-based routing with a constant threshold in wireless sensor network', *Alexandria Engineering Journal*, Vol. 58, No. 1, pp.127–141.

Mitici, M., Goseling, J., Ommeren, J.V., Graaf, M.D. and Boucherie, R.J. (2017) 'On a tandem queue with batch service and its applications in wireless sensor networks', *Queueing Systems*, Vol. 87, Nos. 1–2, pp.81–93.

Nam, W.H., Kim, T., Hong, E.M., Choi J.Y. and Kim J.T. (2017) 'A wireless sensor network (WSN) application for irrigation facilities management based on information and communication technologies (ICTs)', *Computers and Electronics in Agriculture*, Vol. 143, pp.185–192.

Niyato, D., Hossain, E. and Fallahi, A. (2006) 'Sleep and wakeup strategies in solar powered wireless sensor/mesh networks: performance analysis and optimization', *IEEE Transactions on Mobile Computing*, Vol. 6, No. 2, pp.221–236.

Rajput, A. and Kumaravelu, V.B. (2019) 'Scalable and sustainable wireless sensor networks for agricultural application of internet of things using fuzzy c-means algorithm', *Sustainable Computing: Informatics and Systems*, Vol. 22, pp.62–74.

Russo, A., Verdier, F. and Miramond, B. (2018) 'Energy saving in a wireless sensor network by data prediction by using self-organized maps', *Procedia Computer Science*, Vol. 130, pp.1090–1095.

Sharma, P., Murthy, G.R. and Chilamkurti, N. (2017) 'Mathematical modelling of energy wastage in absence of levelling and sectoring in wireless sensor networks', *International Journal of Communication Networks and Distributed Systems*, Vol. 19, No. 1, pp.84–98.

Shi, B., Sreeram, V., Zhao, D., Duan, S. and Jiang, J. (2018) 'A wireless sensor network-based monitoring system for freshwater fishpond aquaculture', *Biosystems Engineering*, Vol. 172, pp.57–66.

Sotenga, P.Z., Djouani, K., Kurien, A.M. and Mwila, M.M. (2017) 'Indoor localisation of wireless sensor nodes towards internet of things', *Procedia Computer Science*, Vol. 109, pp.92–99.

Tamene, M. and Nageswara, K. (2018) 'Intelligent clustering protocol design and optimization in wireless sensor networks', *International Journal of Communication Networks and Distributed Systems*, Vol. 20, No. 2, pp.190–213.

Vahabi, S., Eslaminejad, M. and Dashti, S.E. (2019) 'Integration of geographic and hierarchical routing protocols for energy saving in wireless sensor networks with mobile sink', *Wireless Networks*, Vol. 25, No. 5, pp.2953–2961.

Yue, W.Y., Takahashi, Y. and Takagi, H. (2009) *Advances in Queueing Theory and Network Applications*, Springer Science+Business Media, New York.

Zhang, Y. and Wang, J.T. (2017) 'Equilibrium pricing in an M/G/1 retrial queue with reserved idle time and setup time', *Applied Mathematical Modelling*, Vol. 49, pp.514–530.