Conditional dependence between oil and exchange rate returns in a developing oil-exporting economy: an investigation with copula-based TGARCH models

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Abstract: We study the interdependence, the conditional tail dependences and the volatilities of the oil and the exchange-rate returns for the Mexican economy. We develop the analysis with four copula-based TGARCH models. The main findings show that: (1) the Clayton-TGARCH distribution seems to characterise the co-movements between the series; (2) leverage effects of the exchange rate returns are bigger than the ones of the oil returns; (3) the series show lower tail dependence; and (4) extreme downfalls in oil returns may reduce exchange-rate ones with a probability of less than 10%. The study relies on series of weekly returns for the period between 2 January 1998 and 30 September 2016.

Keywords: copulas; TGARCH models; conditional dependence; oil returns; exchange-rate returns; Mexican economy.

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1 Introduction

The understanding of the relationships and dynamics of oil prices and exchange rates is important in the global economy. Oil price shocks can induce economic recessions, inflationary pressures, trade deficits, investment uncertainty and stock market fragility (Reboredo, 2012). Exchange rate variations can induce changes in prices, in output and in trade volumes and even financial crises (Venables, 1990; Krugman, 2009). Furthermore, since long, it has been recognised that the interdependence, the volatilities and the tail dependence between the variables can explain the performance of many oil-dependent economies (Krugman, 1983; Golub, 1983; Amano and van Norden, 1995).

Paradoxically, the empirical literature on the relationships and dynamics between oil prices and exchange rates is relatively scarce (Brayek et al., 2015). This scarcity has been explained in terms of the difficulties to describe and analyse both types of series.[1] The existence of non-competitive markets, market interventions, market overreactions, informational asymmetries and diverse policy goals partially explains such difficulties. These difficulties justify the use of copula-based GARCH models to study both types of series. However, due to the lack of data, these studies usually do not focus on oil-dependent economies, but on developed ones.

Here, we study the interdependence, the tail dependences and the volatilities of the oil and the exchange-rate returns for the Mexican economy (a developing oil-exporting economy with a flexible exchange rate regime). We use estimations of four copula-based TGARCH models to develop the study. We build these models with the Clayton, the Gumbel, the Symmetrised Joe-Clayton and the *t*-Student copulas and the AR (1)-TGARCH (1, 1) model with a Student's *t*-distribution. Moreover, we analyse the behaviour of the exchange rate returns when the oil returns experience extreme rises and downfalls. The study relies on series of weekly returns for the period between January 2nd, 1998 and September 30th, 2016.

We use the copula-based TGARCH models to characterise the relationships and dynamics between the series of oil and exchange rate returns. We use such models to estimate four bivariate density distributions that characterise the behaviour of both under different assumptions of the dependence structure, the tail dependences and the volatilities of the variables. Subsequently, we analyse the tail dependences and the co-movements between the series for each estimated distribution. Moreover, we quantify the probabilities of how exchange rate returns could behave when oil returns experience extreme rises and downfalls (i.e., oil price shocks).

Methodologically, this study follows the ones of Reboredo (2012), Jondeau and Rockinger (2006) and Lorenzo-Valdés et al. (2016). Specifically, we use the copula-based GARCH approach to study the oil and the exchange rate series following the proposal by Reboredo (2012). We describe and analyse the relationships and dynamics of both series following the methodologies of Jondeau and Rockinger (2006) and Lorenzo-Valdés et al. (2016). However, the research purposes, the series analysed and the models estimated in such studies have some differences with the ones of this study.[2]

The contributions of this study focus on the analysis of the conditional dependence and tail dependence structures between the series of oil and exchange rate returns. The main contributions are the following: 1) it expands the literature on the relationships and dynamics between the analysed series through the analysis of a developing oil-dependent economy; 2) it characterises the interdependence, the tail dependences and the volatilities of the series; 3) it assesses whether tail dependences are time-varying or time-invariant processes; and 4) it analyses how exchange rate returns could behave when oil returns experience extreme rises and downfalls.

The analyses proposed may be useful for some oil-dependent developing economies. Specifically, they may be useful to reduce their vulnerabilities regarding their: 1) fiscal policies, given that their public budgets usually depend on the price of oil; 2) monetary policies, given that their central bank goals may rely on the existence of stable markets and stable capital flows; 3) international trade operations, given that both variables have impact on the terms and gains of trade of such economies; and 4) financial management practices, given that the valuation and hedging of their financial assets usually depend on the dynamics of both variables.

The study is organised in six sections. Section 2 reviews the literature regarding the relationships and dynamics between oil prices and exchange rates. Section 3 focuses on the methodological issues. Section 4 shows the statistical analyses of the series of the oil and exchange rate returns. Section 5 includes the analysis of the co-movements between both series. This section focuses on the estimations of the four copula-based TGARCH models and the assessments of their assumptions. It also includes the analysis on how exchange rate returns may behave when oil returns experience extreme rises and downfalls. Section 6 summarises and discusses the main findings.

2 Literature review

The theoretical literature on the relationships and dynamics between oil prices and exchange rates considers two main channels of transmission of the effects of oil price shocks on exchange rates.[3] Particularly, the literature that studies the *wealth-transmission channel* focuses on how such shocks change investors' international portfolio decisions and the trade balances of the economies. This branch of the literature has its origins in the models of Krugman (1983) and Golub (1983). The literature that studies the *terms-of-trade channel* focuses on how oil price shocks may change the relative prices of an economy. This branch of the literature has its origins in the two-sector model of Amano and van Norden (1995).

The theoretical literature shows that the effects of oil price shocks on a flexible exchange rate regime depend on the features of the economy. Particularly, Krugman (1983) and Golub (1983) show that a rise in oil prices can induce an exchange rate appreciation (depreciation) for oil-exporting (oil-importing) economies at least in the short-run.[4] Amano and van Norden (1995) show that a rise in oil prices can cause an exchange rate appreciation (depreciation) when the economy depends in the production of non-tradable (tradable) goods.5 Thus, the theory suggests that the relationships between oil prices and exchange rates may not be univocal nor straightforward.

The empirical literature supports these conclusions. The effects of oil prices on the exchange rates depend on the economies and the periods analysed. 6 However, we should point out that these studies are "much less extensive than those on the effects of oil prices on (other) economic activities" (Brayek et al. 2015, p.174). Particularly, regarding the literature, it has been indicated that "little is known about oil price-exchange rate co-movements" (Reboredo, 2012, p.420). Thus, further research seems necessary to understand the relationships and the dynamics between oil prices and exchange rates in economies with flexible exchange rates.

The existing literature on the relationships and dynamics between oil prices and exchanges rates relies on copula-based GARCH models. These models aim at characterising the bivariate density distributions that describe the co-movements between two variables. These models are estimated with specific copula density functions and time-series models that describe the marginal distributions of the series. Particularly, the latter distributions are estimated with models of the ARCH/GARCH family.[7] Hence, the copula-based GARCH models allow to describe and analyse the interdependence, the tail dependence and the volatilities of the series.

Copula-based GARCH models have some features that facilitate the study of the relationships and the dynamics between the oil and the exchange rate series. Particularly, we take advantage of such features to: 1) build four flexible bivariate density distributions that describe the relationships and the dynamics of the series of returns; 2) measure the interdependence and the tail dependence between the series without using linear correlation assumptions; 3) analyse the conditional dependence structure and the volatilities of the series; 4) examine how these series are linked during different periods of time; and, 5) characterise the behaviour of the series of returns.

We should emphasise that the studies that use copula-based GARCH models are scarce and mainly focus on developed economies. Some representative studies are the ones of Reboredo (2012), Wu et al. (2012), Aloui et al. (2013), Chen et al. (2013) and Brayek et al. (2015). These studies mostly analyse the co-movements between the WTI oil prices and the exchange rates of different developed economies. Moreover, their findings confirm that the relationships between oil prices and exchange rates may not be univocal nor straightforward. Indeed, their findings depend on the economies and the periods analysed.[8]

Finally, we should emphasise that the contributions of this study may be particularly useful for developing oil-exporting economies with flexible exchange rates. In those economies, the relationships and the dynamics between oil prices and exchanges rates determine their overall economic and financial performance. In this context, further studies regarding the analysed variables could be helpful to reduce the vulnerability of these economies. Particularly, we believe that the copula-based GARCH approach may be useful to achieve these goals. We emphasise these considerations because they motivate this study.

3 Methodological issues

In this section, we focus on some methodological issues related to the study of the relationships and the dynamics between the Mexican oil and the Mexican exchange rate returns. We organise this section in four sub-sections. The first sub-section provides an introduction to copula functions and associated dependence measures. The second one shows the TGARCH model used to estimate the marginal functions associated to the oil and the exchange rate returns. The third one describes the four copula functions used to build the bivariate density distributions used in this investigation. Finally, the fourth sub-section focuses on some estimation and analytical issues.

3.1 Copula functions and dependence measures

Traditionally, copulas are described as functions that join or couple multivariate distribution functions with their one-dimensional marginal distribution functions (Nelsen, 2006). Thus, we can describe them as functions that allow us to build bivariate probability distributions. Formally, we can describe a bivariate copula function, $C(u_1,u_2)$, as a cumulative distribution function for a vector with support in $[0, 1]^2$ and uniform marginal distribution functions. Particularly, if we denote (U_1,U_2), as the corresponding bivariate vector, a copula function can be defined as:

$$C(u_1,u_2) = P(U_1 \le u_1, U_2 \le u_2) \tag{1}$$

This investigation relies on the results of the Sklar's representation theorem (Sklar, 1959). This theorem states that the bivariate cumulative distribution function $F(x_1,x_2)$ of any pair (X_1,X_2) of continuous random variables can be written as:

$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2)) \; x_1,x_2 \in R \tag{2}$$

where $F_1(x_1)$ and $F_2(x_2)$ are two continuous marginal distribution functions and $C:[0,1]^2 \to [0,1]$. Moreover, the theorem states that, if C is a copula cumulative distribution function and F_1 and F_2 are cumulative distribution functions, then F is a bivariate cumulative distribution function with margins F_1 and F_2.

The Sklar theorem allows us to build the copula-based GARCH models, i.e. the bivariate distributions, used to study the co-movements between the series of returns. We build the models under the consideration that the Sklar theorem does not impose any restrictions on the copula nor the marginal distributions (Fan and Patton, 2014). Particularly, the theorem allows us to describe a bivariate density distribution f as:

$$f(x_1,x_2) = c(F_1(x_1),F_2(x_2)) f_1(x_1) f_2(x_2) \tag{3}$$

where c, f_1 and f_2 are, respectively, the copula density and the marginal density functions.

Here, we build and estimate four copula-based GARCH models to study the interdependence, the conditional tail dependence and the volatilities of the oil and the exchange rate returns. We estimate each model under certain assumptions regarding the copula and the marginal density functions. Specifically, we use the Clayton, the Gumbel, the Symmetrised Joe-Clayton (SJC) and the Student-t copula functions for modelling purposes. Furthermore, we use the AR(1)-TGARCH model with a Student's t distribution to estimate the two marginal density distributions of the innovations associated to the analysed variables, i.e. f_1 and f_2.

Statistically, we use the Kendall's tau rank-order correlation to measure the degree of dependence, i.e. the interdependence, between the oil and exchange rate returns. Kendall's tau, $\tau(X_1,X_2)$, is a measure of concordance between variables.[9] Hence, it is not a measure of linear correlation. It is defined as follows:

$$\tau(X_1,X_2) = P[(X_1 - Y_1)(X_2 - Y_2) > 0] - P[(X_1 - Y_1)(X_2 - Y_2) < 0] \tag{4}$$

where the values of $\tau(X_1, X_2)$ belong to the interval $[-1, 1]$. In this context, it is interesting to point out that $\tau(X_1, X_2)$ can be defined in terms of a copula function as:

$$\tau(X_1, X_2) = 4\iint C(u_1, u_2)\, dC(u_1, u_2) - 1 \tag{5}$$

Copula functions allow to characterise the degree of dependence in the tails of the density distribution function $f(x_1, x_2)$.[10] Here, we use two complementary probabilistic measures to study how X_2 behaves when X_1 experiences extreme rises and downfalls. These measures are the lower τ^L and upper τ^U asymptotic tail dependence coefficients. Nelsen (2006) defines these measures as:

$$\tau^L = \lim_{\beta \to 0^+} P\left(X_2 \le F_2^{-1}(\beta) \mid X_1 \le F_1^{-1}(\beta)\right) = \lim_{\beta \to 0^+} \frac{C(\beta, \beta)}{\beta} \tag{6}$$

$$\tau^U = \lim_{\beta \to 1^-} P\left(X_2 > F_2^{-1}(\beta) \mid X_1 > F_1^{-1}(\beta)\right) = \lim_{\beta \to 1^-} \frac{1 - 2\beta + C(\beta, \beta)}{1 - \beta} \tag{7}$$

where β refers to the 100β-th percentile of F_1 and F_2.

We estimate the above measures of tail dependence to study the co-movements between X_2 and X_1, when X_1 experiences extreme rises and downfalls. Specifically, we estimate τ^L to quantify the probability of observing a lower X_2 when X_1 is itself lower. Conversely, we estimate τ^U to quantify the probability of observing an upper X_2 when X_1 is itself upper. In addition, we estimate the degree of tail dependence between the variables. When the values of τ^L and τ^U are zero (unitary), the variables are independent (perfectly dependent) in the tail. Particularly, in this investigation, we assume that X_1 refers to the oil returns and X_2 to the exchange rate ones.

We should point out that the four copula functions have different tail dependence structures. Particularly, the Clayton, the Gumbel and the SJC copulas are characterised by asymmetric tail dependence structures; while the Student-t copula is characterised by a symmetric structure.[11] Furthermore, the Clayton and the Gumbel copulas exhibit mainly one type of tail dependence; while the SJC and the Student-t copulas exhibit two types.[12] Thus, the estimations of the measures of tail dependence must have to take into account that the copulas have different tail dependence structures.

Methodologically, we estimate the measures of tail dependence following Patton (2006). Hence, we allow the values of τ^L and τ^U to depend on time-varying processes. This assumption contrasts with the traditional one that assumes that such measures are constant over time.[13] The equations that describe the dynamics of both measures, i.e. the conditional tail dependence structure, are the following:

$$\tau_t^L = \Lambda\left(\lambda_{0L} + \lambda_{1L}\tau_{t-1}^L + \lambda_{2L}\left|u_{1,\,t-1} - u_{2,\,t-1}\right|\right) \tag{8}$$

$$\tau_t^U = \Lambda\left(\lambda_{0U} + \lambda_{1U}\tau_{t-1}^U + \lambda_{2U}\left|u_{1,\,t-1} - u_{2,\,t-1}\right|\right), \tag{9}$$

where Λ is the logistic transformation used to keep the estimated probability values in the interval $(0,1)$.[14]

We should emphasise that the study relies on the analysis of four copula-based GARCH models that describe the co-movements between the oil and the exchange rate returns. We denominate these models as the Clayton-TGARCH, the Gumbel-TGARCH, the SJC-TGARCH and the Student-t-TGARCH bivariate density distributions. We use these models to analyse the interdependence, the tail dependence and the volatilities of both series of returns under different assumptions. Furthermore, given the assumption of conditional tail dependence, we estimate the probabilities of how the exchange rate returns could behave when the oil returns experience extreme rises and downfalls.

3.2 The TGARCH model with a Student's t-distribution

Here, we describe the TGARCH model used to estimate the marginal density distributions of the innovations associated to the oil and the exchange rate returns. This model is an extension, proposed by Zakoian (1994), of the traditional GARCH one.[15] We use it because the TGARCH model has been recognised among the best ones to describe the returns of the assets of developing economies. Particularly, the TGARCH model can capture some features that characterise many financial and economic series of these economies. Among them are the existence non-constant volatilities, volatility clustering, skewed and leptokurtic distributions and leverage effects.

From a modelling perspective, the main feature of the TGARCH model is that it allows the volatility of the series of returns on period *t*, r_t, to depend on the "news" arriving to the market (i.e. the lagged innovation u_{t-1}). Such volatility is described with the following specification of the conditional variance of the innovations, σ_t^2:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \gamma u_{t-1}^2 I\left(u_{t-1} < 0\right) + \beta \sigma_{t-1}^2 \tag{10}$$

where the innovations μ_t are, by assumption, distributed as a standardised Student's t-distribution with ν degrees of freedom. The parameters α_0, α_1, β; and γ are assumed as non-negative; and I is defined as an indicator function $I = \begin{cases} 1 & \text{if } u_{t-1} < 0 \\ 0 & \text{if } u_{t-1} \geq 0 \end{cases}$.

The specification of the conditional variance given by the expression (10) allows us to analyse the effects of qualitative news on the current volatility of the series of returns. Good news, $u_{t-1} > 0$, have an effect equal to α_1 on σ_t^2. Bad news, $u_{t-1} < 0$, have an effect equal to $\alpha_1 + \gamma$. Thus, when $\gamma \neq 0$, bad news have measurable effects on the volatility of the series. Particularly, when bad news occur and $\gamma > 0$, the series exhibit the "leverage effect" (i.e. the volatility caused by bad news is bigger than the one caused by good news). Thus, γ could be considered as a measure of the sensitivity to bad news prevailing in the market.

Finally, we should emphasise that we use the AR(1)-TGARCH(1, 1) model with a Student's t-distribution to estimate the marginal distributions of the oil and the exchange rate returns. This model has a three-equations system structure. The first expression is the conditional mean of the series of returns, r_t, during the period *t*. The second one is the condition that defines an ARCH process. The third one is the specification of the conditional variance. Thus, the structure that defines the TGARCH models estimated is:

$$\begin{aligned} r_t &= \varphi_0 + u_t \\ u_t &= \sigma_t \varepsilon_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 u_{t-1}^2 + \gamma u_{t-1}^2 I\left(u_{t-1} < 0\right) + \beta \sigma_{t-1}^2 \end{aligned} \tag{11}$$

3.3 *The four copula functions*

Here, we describe the four copula functions used in this investigation. These copula functions, as we have indicated, are the Clayton, the Gumbel, the SCJ and the Student-t ones. We use them to model the bivariate density distributions used to study the relationships and the dynamics between the series of oil and exchange rate returns. Particularly, we use the Clayton and the Gumbel copulas to build two bivariate distributions characterised mainly by one type of tail dependence (lower or upper); while we use the SJC and the Student-t copula to build two bivariate distributions characterised by two types of tail dependence (lower and upper).

The four bivariate density distributions also allow us to describe, under different assumptions, how exchange rate returns might behave when oil returns experience extreme rises and downfalls. Specifically, the distributions based on the Clayton, the Gumbel and the SJC copulas assume that tail dependences are asymmetric (i.e. $\tau^L \neq \tau^U$). The distribution based on the Student-t copula assumes that tail dependences are symmetric (i.e. $\tau^L = \tau^U$). We have included the latter distribution as a reference case given that the Student-t copula has been used in several financial and economic studies.[16]

Mathematically, the Clayton copula assumes the existence of bigger dependence in the negative tail than in the positive one (Clayton, 1978). This is why it is considered as an asymmetric copula $\left(\tau^L > \tau^U\right)$. We use it to characterise the interdependence between the oil and the exchange rate returns under the assumption that both variables mainly show lower tail dependence, τ^L. The bivariate Clayton copula is defined by the following function:

$$C_\theta^{CL}\left(u_1, u_2\right) = \left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-\frac{1}{\theta}} \tag{12}$$

where $\theta \in [-1, \infty] \backslash \{0\}$ and $\tau^L = 2^{-1/\theta}$.[17]

The Gumbel (1960) copula assumes the existence of bigger dependence in the positive tail than in the negative one. Like the Clayton one, it is also considered an asymmetric copula $\left(\tau^U > \tau^L\right)$. We use it to characterise the interdependence between the oil and the exchange-rate returns under the assumption that both variables mainly show upper tail dependence, τ^U. The bivariate Gumbel copula is defined by the following function:

$$C_\theta^{GU}\left(u_1, u_2\right) = \exp\left[-\left(\left(-\ln\left(u_1\right)^\theta\right) + (-\ln\left(u_2\right)^\theta\right)^{\frac{1}{\theta}}\right] \tag{13}$$

where $\theta \in [-1, \infty]$ and $\tau^U = 2 - 2^{-1/\theta}$.[18]

The SJC copula assumes the existence of dependence in both tails (Patton, 2006).[19] We use it to characterise the interdependence between the analysed variables under the assumption that they show lower and upper tail dependences. The bivariate SJC copula is defined by the following function:

$$C^{SJC}\left(u_1,u_2 \mid \tau^U,\tau^L\right)=0.5\left[C^{JC}\left(u_1,u_2 \mid \tau^U,\tau^L\right)+C^{JC}\left(1-u_1,1-u_2 \mid \tau^L,\tau^U\right)\right] \tag{14}$$

where $C^{JC}\left(u_1,u_2 \mid \tau^U,\tau^L\right)$ is the Joe-Clayton copula defined as:

$$C^{JC}\left(u_1,u_2 \mid \tau^U,\tau^L\right)=1-\left(1-\left\{\left[1-\left(1-u_1\right)^k\right]^{-r}+\left[1-\left(1-u_2\right)^k\right]^{-r}-1\right\}^{-1/r}\right)^{1/k}$$

with $k=\dfrac{\ln(2)}{\ln\left(2-\tau^U\right)}$ and $r=\dfrac{-\ln(2)}{\ln\left(\tau^L\right)}$.

The Student-t copula also assumes the existence of dependence in both tails (Durante and Sempi, 2016). We use it to characterise the interdependence between the variables under the assumption that the lower and upper tail dependence coefficients are identical. The bivariate Student-t copula is defined by the following function:

$$C_\theta^T\left(u_1,u_2\right)=\int_{-\infty}^{t_k^{-1}(u_1)}\int_{-\infty}^{t_k^{-1}(u_2)}\left\{\frac{1}{2\pi\sqrt{1-\theta^2}}\left(-\frac{x_1^2-2\theta x_1x_2+x_2^2}{2\left(1-\theta^2\right)}\right)-\frac{k+2}{2}\right\}dx_1\,dx_2 \tag{15}$$

where $\theta\in[-1,\ 1]$ is the linear correlation coefficient, k is the number of degrees of freedom, t_k^{-1} denotes the inverse of the Student-t density function with zero mean, and $\tau^L=\tau^U=2^{-1/\theta}$.[20]

We should emphasise that copula functions, like the Clayton, the Gumbel, the SJC and the Student-t ones, have some features that allow us to investigate the interdependence and the tail dependence between the series of returns. These features are the following: 1) Copulas are invariant to monotonic transformations of the random variables; 2) there is a direct relationship between the parameters of the copulas and the measures of concordance; 3) copulas can describe different types of tail dependence; and 4) they allow to estimate multivariate density distributions when the measures of tail dependence are time-varying or time-invariant processes.

3.4 Estimation and analytical issues

Methodologically, there are several methods to estimate the parameters of the copula-based GARCH models.[21] Here, we estimate these parameters with the traditional Maximum Likelihood (ML) method. Specifically, we estimate the parameters of each copula-based TGARCH model $\left(\theta_0,\theta_1,\theta_2\right)$ by maximising the following log-likelihood function:22

$$\ln f\left(\theta_0,\theta_1,\theta_2\right)=\sum_{t=1}^{T}\left(\ln c\left(F_1\left(r_{1t};\theta_1\right),F_2\left(r_{2t};\theta_2\right);\theta_0\right)+\ln f_1\left(r_{1t};\theta_1\right)+\ln f_2\left(r_{2t};\theta_2\right)\right) \tag{16}$$

where $c\left(F_1\left(r_{1t};\theta_1\right),F_2\left(r_{2t};\theta_2\right);\theta_0\right)$ is the density of the copula function; and f_1 and f_2 are the marginal density distributions related, respectively, to the oil and the exchange rate returns.

The estimation procedure of each copula-based TGARCH model includes three stages. Specifically, in the first stage, we use the series of returns and the TGARCH model given by (11) to estimate the marginal density distributions of the standardised innovations f_1 and f_2. In the second stage, we use these parameters as initial "guesses" to estimate the ML parameters that maximise the log-likelihood function given by (16). The ML parameters include the ones of the copula, of the marginal density distributions and of the equations that describe the dynamics of the measures of tail dependence. Finally, we estimate the values of the measures of tail dependence.

Statistically, we study the co-movements between the series with the estimations of the four copula-based TGARCH models. We analyse the estimations with goodness-of-fit statistics and significance tests. Particularly, we estimate the Akaike information criteria (*Akaike*) and the log likelihood ratio (*Logl*) to assess the goodness-of-fit of the bivariate distributions. In addition, we use conventional tests of individual significance to assess whether the parameters $\left(\theta_1,\theta_2,\lambda_{0L},\lambda_{1L},\lambda_{2L},\lambda_{0U},\lambda_{1U},\lambda_{2U}\right)$ are significant or not. Furthermore, we use the measures of tail dependence to analyse how the exchange rate returns may behave when the oil returns experience extreme rises and downfalls.

Finally, we should point out that we use some complementary tests to validate the estimation procedure. Specifically, we use ADF and KPSS tests to assess the order of integration of the series of the logs of the oil prices and of the logs of the exchange rates. We use both tests due to their complementarity and to avoid spurious estimations.[23] We use ARCH-LM tests of the type proposed by Engle (1982) to examine the convenience of using models of the ARCH/GARCH family for modelling and analysing the series of returns. Furthermore, we use Ljung-Box tests of the type proposed by Ljung and Box (1978) to assess potential misspecification problems.[24]

4 Statistical analyses of the series

We use the Economatica and the Mexican Central Bank databases to build a sample of weekly reference prices for the Mexican oil barrel and the Mexican currency (peso).[25] These reference prices are expressed in nominal dollars to facilitate the analysis in terms of a single currency. We use these prices to build the series of the oil and the exchange rate returns. These series are built under the assumption that weekly returns follow a compound interest process.[26] Each series of returns uses closing data for each Friday day between January 2nd, 1998 and September 30th, 2016. Thus, each series of returns includes 978 weekly observations.[27]

Table 1 summarises the descriptive statistics, the Jarque-Bera normality tests and the correlation estimations for the series of the oil and the exchange rate returns. The table shows that the density distributions of the series are leptokurtic and left skewed and, by consequence, that the series are not normally distributed. Furthermore, both correlation estimations show that the series of returns are correlated. Particularly, the Pearson's estimation, 0.2124, shows that that there is some positive linear correlation between the series. Moreover, the Kendall's tau estimation, 0.1341, validates that the series can be characterised by a low degree of dependence and by concordant relationships.

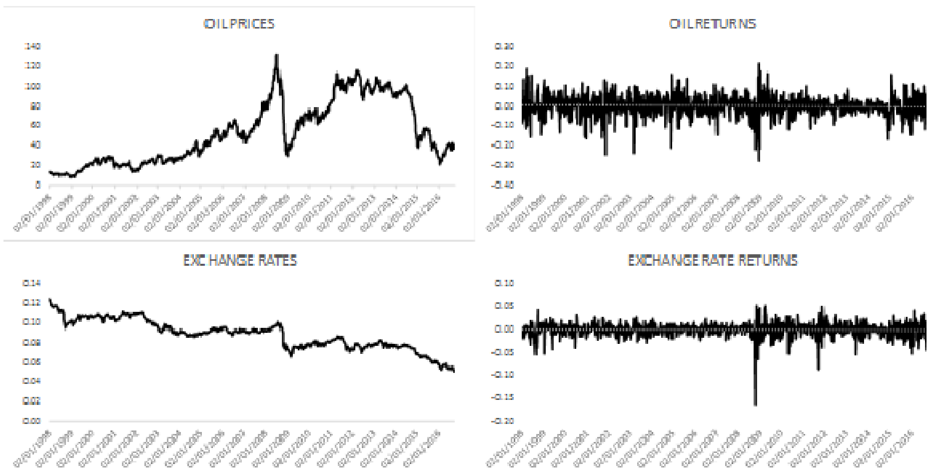
Table 1 Descriptive statistics, normality tests and correlation estimations for the series of oil and exchange rate returns

	Oil	*Exchange rate*
Mean	0.0012	–0.0009
Median	0.0034	0.0000
Maximum	0.2152	0.0527
Minimum	–0.2775	–0.1630
Standard deviation	0.0557	0.0147
Skewness	–0.5203	–1.7674
Kurtosis	5.3573	20.3581
Jarque-Bera	270.56	12787.35
P-value	0.0000	0.0000
Pearson's correlation	0.2124	
Kendall's tau correlation	0.1341	

Source: Authors' estimations with data obtained from the Economatica and the Mexican central bank databases

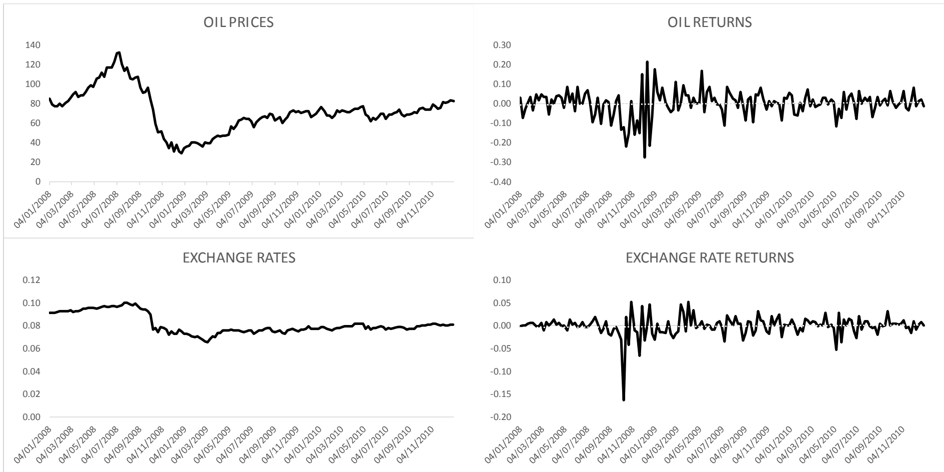
The behaviour of the series can be analysed graphically. This analysis complements our previous findings. Particularly, Figure 1 shows that, on average, oil prices gained value and that the exchange rates were depreciated between 1998 and 2016. Thus, the figure suggests that there is a negative correlation between oil prices and exchange rates. Figure 2, by contrast, shows that these variables moved together between 2008 and 2010.[28] Thus, the figures suggest that Pearson's correlation estimations are time-varying. Moreover, they suggest that the relationships and dynamics between the series may have been strengthened during extreme events (like the Global Financial crisis).[29]

Figure 1 Prices and returns for the Mexican oil barrel and for the Mexican exchange rate (January 2nd, 1998 to September 30th, 2016)



Source: Author's estimations with data obtained from the economatica and the Mexican central bank databases

Figure 2 Prices and returns for the Mexican oil barrel and for the Mexican exchange rate (January 4th, 2008 to December 31st, 2010)



Source: Authors' estimations with data obtained from the Economatica and the Mexican central bank databases

Figures 1 and 2 show that both series of returns shared similarities during the analysed period. Specifically, the figures show that the variables frequently experienced simultaneous rises, downfalls and episodes of volatility. They also show that the series have non-constant variances, volatility clusters and leverage effects. Moreover, Figure 2 suggests that such similarities seem to be highlighted between 2008 and 2010. These findings partially justify the use of time-series models of the ARCH/GARCH family for modelling and estimation purposes. The complementary justification relies on the results of the ADF, KPSS and ARCH-LM tests.

Tables 2, 3 and 4 show, respectively, the results of the ADF, KPSS and ARCH-LM tests. Particularly, Tables 2 and 3 show that the series of the logs of the oil prices and the exchange rates are difference stationary, i.e. I(1). These findings are robust to the different assumptions used in tests regarding the null hypotheses and the test equations. These findings imply that the series of the returns are stationary and that they may not induce spurious regression problems. Furthermore, Table 4 shows that ARCH effects are present in both series of returns. These finding implies that the series of returns may be adequately modelled with models of the ARCH/GARCH family.

We can summarise these findings by indicating that the Mexican series of returns during the analysed period: 1) exhibited a positive linear correlation equal to 0.2124; 2) can be characterised by a low degree of dependence and by concordant relationships; 3) have experimented simultaneous rises, downfalls and episodes of volatility; 4) strengthened their relationships and dynamics between 2008 and 2010; 5) have been characterised by non-constant variances, volatility clusters, leptokurtic and left skewed distributions and leverage effects; 6) may have been characterised by stationarity and ARCH effects; and 7) may be modelled with models of the ARCH/GARCH family.

Table 2 ADF unit root-tests. The table shows the statistics and the *P*-values of the ADF tests for the series of logarithms of the oil prices and the exchange rates (Levels) and for the series of the oil and the exchange rate returns (Differences). One, two and three asterisks indicate significance levels of 10%, 5% and 1%, respectively

	Equation test assumption						*Order of integration*
	Intercept			*Trend and intercept*			
	Levels	*Differences*		*Levels*	*Differences*		
	P-value	*P*-value		*P*-value	*P*-value		
Oil	0.3666	0.1777	***	0.8740	0.0397	***	1
Exchange rate	0.8958	0.0000	***	0.4182	0.0647	***	1

Source: Authors' estimations

Table 3 KPSS stationarity tests. The table shows the KPSS statistics for the series of logarithms of the oil prices and the exchange rates (Levels) and for the series of the oil and the exchange rate returns (Differences). An asterisk denotes the rejection of the null hypothesis under a significance level equal to 5%

	Equation test assumption						*Order of integration*
	Intercept			*Trend and intercept*			
	Levels		*Differences*	*Levels*		*Differences*	
	Statistic		Statistic	Statistic		Statistic	
Oil	2.8438	*	0.1777	0.5741	*	0.0397	1
Exchange rate	3.3704	*	0.0952	0.1741	*	0.0647	1

Source: Authors' estimations

Table 4 ARCH-LM tests for ARCH effects. The table shows the ARCH-LM tests for the series of the oil and the exchange rate returns. The number of lags included in each ARCH specification is 4. One, two and three asterisks indicate significance levels of 10%, 5% and 1%, respectively

	F Statistic	*P-value*	
Oil	41.4702	0.0000	***
Exchange rate	2.7469	0.0273	**

Source: Authors' estimations

5 Analysis of the co-movements between the series of returns

Here, we study the relationships and the dynamics between the oil and the exchange rate returns. We develop this analysis with the estimations of the four bivariate density distributions built with the copula-based TGARCH models. We use these estimations and the tests previously indicated to characterise the interdependence, the conditional tail dependence and the volatilities of the series of returns. Furthermore, we estimate the measures of conditional tail dependence to analyse how exchange rate returns could behave when oil returns experience extreme rises and downfalls. For simplicity, we use tables and figures to develop the analysis.

Table 5 Parameter and goodness-of-fit estimations for the Clayton-TGARCH and the Gumbel-TGARCH bivariate density distributions. The first six parameters are the ones of the TGARCH estimations used to estimate the marginal density distributions. The next three parameters are the ones that describe the dynamics of the measures of conditional tail dependence

Clayton-TGARCH bivariate distribution	*Oil*		*Exchange rate*		*Gumbel-TGARCH bivariate distribution*	*Oil*		*Exchange rate*	
	Coefficient	*P-value*	*Coefficient*	*P-value*		*Coefficient*	*P-value*	*Coefficient*	*P-value*
ϕ_0	0.0027	0.0553	–0.0001	0.8464	ϕ_0	0.0026	0.0610	0.0000	0.9311
α_0	0.0001	0.0111	0.0000	0.0002	α_0	0.0000	0.0234	0.0000	0.0003
α_1	0.0432	0.1546	–0.0171	0.4970	α_1	0.0401	0.1515	–0.0155	0.5784
γ	0.0935	0.0078	0.1578	0.0000	γ	0.0886	0.0075	0.1671	0.0000
β	0.8860	0.0000	0.8696	0.0000	β	0.9008	0.0000	0.8651	0.0000
ν	6.4390	0.0000	4.4700	0.0000	ν	6.4972	0.0000	4.2262	0.0000
λ_{0L}	–2.9408	0.0275			λ_{0L}	–0.4410	0.0578		
λ_{1L}	–20.7053	0.3723			λ_{1L}	–8.5791	0.0000		
λ_{2L}	1.4319	0.4220			λ_{2L}	–0.4862	0.0317		
Logl		4488.0406			*Logl*		4485.0948		
Akaike		–9.1567			*Akaike*		–9.1507		

Source: Authors' estimations with programs implemented in the Eviews software

Table 6 Parameter and goodness-of-fit estimations for the SJC-TGARCH and the Student-t-TGARCH bivariate density distributions. The first six parameters are the ones of the TGARCH estimations used to estimate the marginal density distributions. The following parameters are the ones that describe the dynamics of the measures of conditional tail dependence. The parameter for the degrees-of-freedom, *k*, is the one estimated for the Student-t copula function

SJC-TGARCH bivariate distribution	*Oil*			*Exchange rate*		*Student-t-TGARCH*	*Oil*			*Exchange rate*	
	Coefficient	*P-value*		*Coefficient*	*P-value*	*bivariate distribution*	*Coefficient*	*P-value*		*Coefficient*	*P-value*
ϕ_0	0.0027	0.0555		–0.0001	0.8376	ϕ_0	0.0023	0.1854		–0.0002	0.6106
α_0	0.0001	0.0097		0.0000	0.0002	α_0	0.0001	0.0060		0.0000	0.0001
α_1	0.0434	0.1529		–0.0203	0.4247	α_1	0.0643	0.1526		–0.0403	0.2860
γ	0.0935	0.0081		0.1663	0.0000	γ	0.1628	0.0034		0.2701	0.0000
β	0.8870	0.0000		0.8679	0.0000	β	0.8759	0.0000		0.8597	0.0000
v	6.4299	0.0000		4.3358	0.0000	v	12.4442	0.0418		5.2804	0.0000
λ_{0L}	–2.2809	0.0265	λ_{0U}	–1.4898	0.0000	λ_0	–2.9544	0.0000k		211.3057	0.0000
λ_{1L}	–11.8763	0.2345	λ_{1U}	–24.3192	0.0000	λ_1	–54.8215	0.0000	*k*		
λ_{2L}	1.4837	0.2752	λ_{2U}	–1.0529	0.0064	λ_2	–0.1721	0.5271			
Logl			4490.84			*Logl*			3221.00		
Akaike			–9.1563			*Akaike*			–6.5609		

Source: Authors' estimations with programs implemented in the Eviews software

Tables 5 and 6 show the results of the ML estimations and the significance tests for the four bivariate density distributions. The results show that the signs and significance of the estimated parameters for the TGARCH models, i.e. the marginal density distributions, are consistent with the ones expected. Moreover, the estimations for leverage effects $(\gamma > 0)$ and for the degrees of freedom $(\nu > 0 \text{ and } k > 0)$ are significant. In addition, the evidence suggests that the bivariate distributions estimations may not have misspecification problems.[30] These findings suggest that such distributions may be useful to characterise the co-movements between both series.

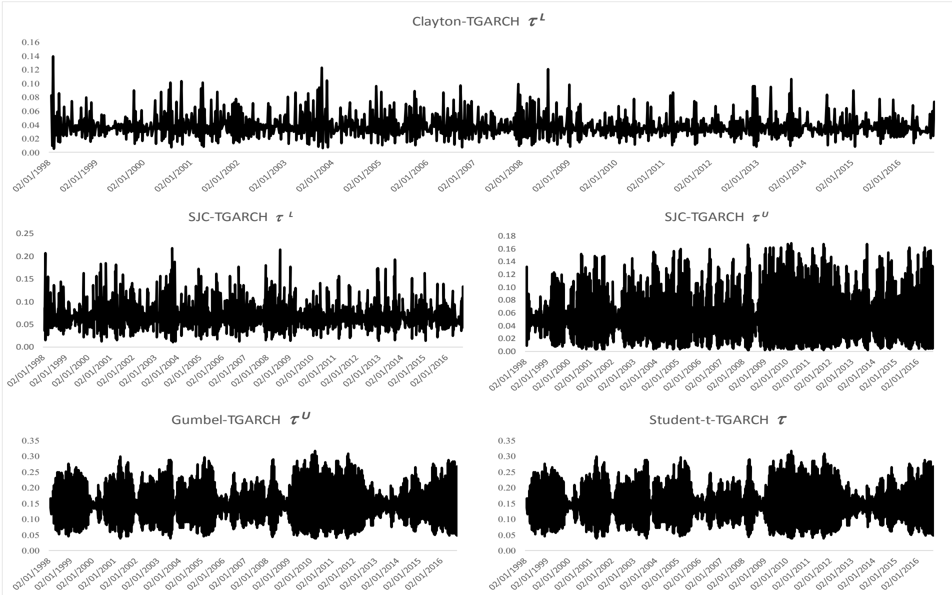
The findings of both tables have implications for the analysis of the volatilities of the series of returns. Particularly, the estimations of γ not only suggest that news have asymmetric effects on the volatilities. They also suggest that leverage effects of the exchange rate returns are bigger than the ones of the oil returns. Thus, the evidence suggests that exchange rate market may be more sensitive than the oil one. Moreover, they suggest a causal relationship from the oil market to the exchange rate one. Furthermore, the significance of the parameters ν and k suggests that the normality assumption may not be useful for taking economic nor financial decisions.

The *Akaike* and the *Logl* estimates in the last two rows of Tables 5 and 6 allow us to compare the goodness-of-fit of the four bivariate distributions. Given such estimates, it seems that the Clayton-TGARCH bivariate distribution is the best one to characterise the co-movements between the oil and the exchange rate returns.[31] The Student-t-TGARCH distribution, by contrast, is the worst one. Thus, the copula-based TGARCH models suggest that both series of returns mainly exhibit lower tail dependence $(\tau^L > \tau^U)$. In other words, the evidence suggests that an extreme exchange rate depreciation can occur due to an extreme downfall in oil returns.

Tables 5 and 6 provide further information regarding the values of the lower tail dependence, τ^L. Particularly, the estimations for the Clayton-TGARCH and the SCJ-TGARCH distributions show that only λ_{0L} is significant (see Tables 5 and 6, respectively). These findings suggest that τ^L is a constant for the analysed period. Particularly, for the Clayton-TGARCH distribution such value is approximately equal to 5.017% (the logistic distribution evaluated in the parameter value); while for the SJC-TGARCH one such value is approximately equal to 9.272%. Thus, these findings imply that value of τ^L may be less than 10%.

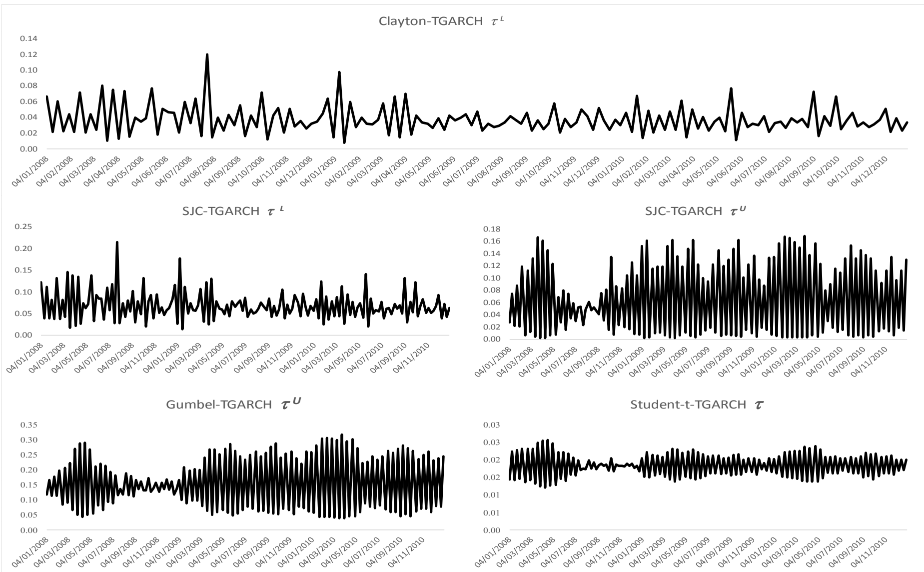
Figures 3 and 4 show the estimations of the dynamics of the measures of conditional dependence for the four bivariate density distributions. These estimations are based on equations (8) and (9). Moreover, Table 7 shows their descriptive statistics and associated normality tests. Particularly, the table shows that, for the SJC-TGARCH distribution, the mean values of τ^L and τ^U are, respectively 0.0706 and 0.0601. Thus, the table confirms that both series of returns mainly show lower tail dependence $(\tau^L > \tau^U)$. Moreover, the table confirms that, on average, extreme downfalls in the oil returns may reduce the exchange rate ones with a probability of less than 10%.

Figure 3 Measures of conditional tail dependence associated to each estimated bivariate distribution (January 2nd, 1998 to September 30th, 2016)



Source: Authors' estimations with data estimated from programs implemented in the Eviews software

Figure 4 Measures of conditional tail dependence associated to each estimated bivariate distribution (January 4th, 2008 to December 31st, 2010)



Source: Authors' estimations with data estimated from programs implemented in the Eviews software

Table 7 Descriptive statistics and normality tests for the series of the measures of conditional tail dependence associated to each estimated bivariate distribution

	Clayton-TGARCH	*Gumbel-TGARCH*	*SJC-TGARCH*		*Student-t-TGARCH*
	t^L	τ^U	r^L	τ^U	τ
Mean	0.0387	0.1501	0.0706	0.0601	0.0182
Medium	0.0349	0.1375	0.0640	0.0484	0.0181
Maximum	0.1395	0.3155	0.2174	0.1689	0.0466
Minimum	0.0054	0.0372	0.0106	0.0018	0.0040
Standard deviation	0.0185	0.0741	0.0334	0.0489	0.0028
Skewness	1.2643	0.2364	1.1419	0.4624	1.3359
Kurtosis	5.4053	1.7032	4.7304	1.8435	15.9489
Jarque–Bera	495.30	77.49	333.87	89.17	7109.01
P-value	0.0000	0.0000	0.0000	0.0000	0.0000

Source: Authors' estimations with data estimated from programs implemented in the Eviews software

We summarise these findings by indicating that the analysis of co-movements between the series of returns shows that: 1) the Clayton-TGARCH distribution seems to characterise adequately the co-movements between the series; 2) leverage effects of the exchange rate returns are bigger than the ones of the oil returns; 3) the normality assumption may not be useful for taking economic and financial decisions; 4) the series of returns mainly show lower tail dependence, τ^L; 5) τ^L seems to be constant during the analysed period; and 6) extreme downfalls in the oil returns may reduce the exchange rate ones with a probability of less than 10%.

6 Conclusions and discussion

Here, we have studied the interdependence, the conditional tail dependence and the volatilities of the oil and the exchange-rate returns for the Mexican economy. We have used estimations of four copula-based TGARCH models to develop the study. We have built these models with the Clayton, the Gumbel, the Symmetrised Joe-Clayton and the t-Student copulas and the AR(1)-TGARCH(1, 1) model with a Student's t-distribution. Moreover, we have analysed the behaviour of the exchange rate returns when the oil returns may experience extreme rises and downfalls. The study has relied on series of weekly returns for the period between January 2nd, 1998 and September 30th, 2016.

The statistical analyses suggest that the Mexican series of returns during the analysed period: 1) exhibited a positive linear correlation equal to 0.2124; 2) can be characterised by a low degree of dependence and by concordant relationships; 3) have experimented simultaneous rises, downfalls and episodes of volatility; 4) strengthened their relationships and dynamics between 2008 and 2010; 5) have been characterised by non-constant variances, volatility clusters, leptokurtic and left skewed distributions and leverage effects; 6) may have been characterised by stationarity and ARCH effects; and 7) may be adequately modelled with time-series models of the ARCH/GARCH family.

The main findings of the analysis of co-movements between the series of returns suggest that: 1) the Clayton-TGARCH distribution seems to characterise adequately the co-movements between the series; 2) leverage effects of the exchange rate returns are bigger than the ones of the oil returns; 3) the normality assumption may not be useful for taking economic and financial decisions; 4) the series of returns mainly show lower tail dependence, τ^L ; 5) τ^L seems to be constant during the analysed period; and 6) extreme downfalls in the oil returns may reduce the exchange rate ones with a probability of less than 10%.

The previous findings suggest that the theories on the relationships and dynamics between oil prices and exchange rates may be complementary. Particularly, the estimations of Kendall's tau and Pearson's correlations not only show that there is dependence and linear correlation between the series. The estimations also support the prediction of the wealth-transmission channel theory that a rise in oil prices may induce an exchange rate appreciation for oil-exporting economies (Krugman, 1983; Golub, 1983). Moreover, the estimations of the measures of tail dependence provide evidence that, at least during times of turmoil, the series strength the aforementioned relationships and dynamics.

The estimations also support the prediction of the terms-of-trade channel theory that a decrease in oil prices may cause an exchange rate depreciation when the economy depends in the production of non-tradable goods (Amano and van Norden, 1995). The validity of this prediction can be supported under the consideration that during the analysed period: 1) the Mexican current account was always negative; 2) the traditional measure of the terms of trade, i.e. the ratio of exports to imports, was on average equal to 0.9680; and 3) Mexican exports were always less than 40% of the Mexican GDP, i.e., the production of the economy used to depend on non-tradable goods.

The findings also have financial and economic implications for the Mexican economy. Specifically, they imply that: 1) it is more likely that an extreme downfall in oil returns may cause a large currency depreciation than an extreme rise in oil returns may cause a large currency appreciation; 2) exchange rates may be more sensitive to bad news than oil prices; 3) trade deficits, international portfolio outflows and low terms-of-trade ratios may weaken the concordance and the positive correlations between the analysed variables; and 4) normality assumptions should be avoided to take fiscal, monetary, trading and financial management decisions.

Finally, we should emphasise that further studies seem necessary to understand the relationships and dynamics of oil prices and exchange rates in developing economies. These studies may be relevant not also for oil-exporting economies, but also for oil-importing ones. Particularly, we believe that such studies should focus on the estimation of bivariate distributions using other copula functions, like the dynamic ones and other time-series models of the ARCH/GARCH family. Hopefully, these studies will enhance our understanding about the interdependence, the tail dependence and the volatilities between the oil and the exchange rate returns.

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Notes

1 Oil prices and exchange rates are considered among the most difficult variables to forecast and analyse. See Bashiri-Behmiri and Pires-Manso (2013) and Fan and Li (2015) for reviews on the forecasting techniques used to study oil prices. See Moosa (2000) and Sarno and Taylor (2003) for reviews on the techniques used to study exchange rates.

2 The studies of Jondeau and Rockinger (2006) and Lorenzo-Valdés et al. (2016) use several copula-GARCH models to investigate the conditional dependence structure between the series of returns of various markets. Specifically, Jondeau and Rockinger (2006) analyse the interactions between four European stock markets. They use the Gaussian and the t-Student copulas and the GARCH model with a Hansen's skewed Student-t distribution to build the copula-GARCH models used in their investigation. Lorenzo-Valdés et al. (2016) analyse the tail dependence between the Brent oil market, the Mexican oil markets and the Mexican stock market. They use the Clayton and the Gumbel copulas and the TGARCH model with a Student-t distribution to build the copula-GARCH models used in their investigation.

3 See Beckmann and Czudaj (2013), Tiwari et al. (2013) and Kin and Courage (2014) for comprehensive introductions to the theories on the impacts of oil price shocks on exchange rates.

4 These conclusions are normally supported by the studies on the wealth-transmission channel. These conclusions rely on the consideration that oil prices determine the balance of payments and international portfolio choices. Some wealth-transmission models are the ones of Coudert et al. (2007) and Bodenstein et al. (2011).

5 Usually, these conclusions are supported by the studies on the terms-of-trade channel. These studies rely on the consideration that oil prices determine the terms of trade. Some terms-of-trade models are the ones of Backus and Crucini (2000), Bénassy-Quéré et al. (2007) and Dauvin (2014).

6 See Aloui et al. (2013), Brayek et al. (2015), Abed et al. (2016) and Beckmann et al. (2017) for contemporary reviews on the empirical literature about the relationships between oil prices and exchange rates. See Ruiz-Porras and Anguiano-Pita (2016, 2017) for reviews on the empirical studies that have analysed, respectively, the series of the Mexican oil and the Mexican exchange rate returns.

7 The ARCH/GARCH family includes more than a hundred time-series models. Particularly, the ARCH and GARCH acronyms stand for AutoRegressive Conditional Heteroscedasticity and Generalised AutoRegressive Conditional Heteroscedasticity. These time-series models have their origins in the ones proposed by Engle (1982) and Bollerslev (1986).

8 Reboredo (2012) studies the co-movements of the WTI oil returns with eight exchange rate returns. His findings show that there is no tail dependence between oil prices and exchange rates. Wu et al. (2012) study the dependence structure between the WTI oil returns and the returns of the US dollar index futures. Their findings show that there is asymmetric tail

dependence. Aloui et al. (2013) study the conditional dependence structures of the WTI and the Brent oil returns with five exchange rate returns. Their findings show that there is symmetric tail dependence between the variables. Chen et al. (2013) study the co-movements between the returns of the WTI oil futures with the returns of the US dollar index. Their findings show that there is asymmetric volatility in the oil market. Brayek et al. (2015) study the co-movements of the returns of the WTI oil and the Brent index with seven exchange rates. Their findings show a positive dependence between the variables after the financial crisis.

9 Two points, (x_1, x_2) and (y_1, y_2) are concordant if $(x_1 - y_1)(x_2 - y_2) > 0$, and discordant if $(x_1 - y_1)(x_2 - y_2) < 0$. Two random vectors $(X_1, X_2), (Y_1, Y_2)$ are concordant if the probability $P[(X_1 - Y_1)(X_2 - Y_2) > 0]$ is bigger than $P[(X_1 - Y_1)(X_2 - Y_2) < 0]$. In other words, when X_1 and X_2 increase together, the vectors are concordant. Otherwise, they are discordant.

10 We should point out that not all copula functions exhibit tail dependences. For example, the Gaussian copula does not exhibit tail dependence.

11 Notice that, the existence of symmetric tail dependence structures implies that oil shocks of the same magnitude and different sign will have the same effects on the likelihood of occurrence of exchange rate variations. When such structures are asymmetric, oil shocks of the same magnitude and different sign, will have differentiated effects on the likelihood of occurrence of exchange rate variations. Particularly, the Clayton, the Gumbel and the SJC copulas have asymmetric tail structures. We use these three copulas because they allow to build density distributions under the assumption that investors' preferences in the markets may not be risk-neutral. See Bakshi and Madan (2006, 2007) for discussions on how the features of density distributions may be related to investors' preferences.

12 The bivariate density distribution defined by the SJC copula exhibits two different tail dependences. The Student-t copula exhibits two identical tail dependences.

13 There are several ways to define the measures of tail dependence. Particularly, any time-constant measure can be a particular case of a time-varying process. The main condition is that the estimated measures must be in the interval [0, 1].

14 We should point out that the tail dependences and their measures vary according to the equations that describe the dynamics of the tails; but the dependence structure defined by the copula-TGARCH model remains over time.

15 The TGARCH acronym stands for Threshold Generalised AutoRegressive Conditional Heteroscedasticity.

16 See Jondeau and Rockinger, (2006) and Patton (2009, 2012) for references of financial and economic studies that use Student-t copulas.

17 For the Clayton copula, the degree of dependence between the variables is given by the parameter θ (Aloui et al. 2013). They are independent when $\theta \to 0$; while they exhibit perfect positive dependence when $\theta \to \infty$.

18 Like the previous case, for the Gumbel copula, the degree of dependence between the variables is given by θ. They are independent when $\theta = 1$; while they exhibit perfect positive dependence when $\theta \to \infty$.

19 The SCJ copula allows the existence of asymmetric and symmetric tail dependences. Particularly, the copula allows for symmetric dependence tail when $\tau^U = \tau^L$.

20 Notice that the Student-t copula becomes a Gaussian copula when $k \to \infty$.

21 Some alternative methods for calibrating multivariate functions based on copulas are the Exact Maximum Likelihood (EML), the Inference Functions for Margins (IFM) and the Canonical Maximum Likelihood (CML) methods. See Patton (2012) and Ibragimov and Prokhorov (2017) for introductions to these methods.

22 These parameters are estimated with programs implemented in the Eviews software.

23 The ADF and the KPSS tests have complementary null hypotheses. The null hypothesis of the ADF test is that the data generating process contains a unit root. The null hypothesis of the KPSS test is that the data generating process is stationary. The joint use of both tests allows us to guarantee the estimation of robust results regarding the order of integration of the series.

24 We are aware that the Ljung-Box test has low power (see Godfrey and Tremayne, 2011). However, we use the test because of its extensive use in the econometric literature.

25 The Economatica database is available online at https://economatica.com/. The Mexican Central Bank database is freely available online at http://www.banxico.org.mx/sistema-financiero/indexEn.html#St.

26 We define the return for an asset during the week t, r_t, as the change in the logs of the weekly price of such asset, P_t. Therefore, $r_t = \ln P_t - \ln P_{t-1}$.

27 We are aware that the use of weekly observations is not common in the literature. However, we follow the Mexican convention of reporting financial data, like exchange rate returns, in weekly periods. In this context, it is interesting to point out how Mohammadi and Su (2010) justify the use of weekly series to study of the dynamics of crude oil prices with data of oil-dependent developing economies. Specifically, they indicate that "We have chosen weekly data for two reasons: first, it allows us to examine the robustness of previous studies to alternative data frequencies. Second, we are limited by the availability of data with other frequencies for such a large number of markets" (Mohammadi and Su, 2010, p.1001)

28 The Mexican economy experimented the effects of the Global Financial Crisis between 2008 and 2010. Particularly, during 2009, the Mexican GDP decreased 4.7%.

29 The previous conclusions derived from the graphical analysis can be confirmed with pairwise correlations. Particularly, the estimated correlation between oil prices and exchange rates is –0.4820 for the period between January 2nd, 1998 and September 30th, 2016. The estimated correlation between the same variables is 0.8876 for the period between January 4th, 2008 and December 31st, 2010. Moreover, both correlations are significant with P-values equal to 0.0000.

30 This conclusion relies on estimations of the correlograms of the residuals and the squared residuals and on assessments for serial correlation using the Ljung-Box tests.

31 We should point out that the Akaike estimates suggest that the Clayton-TGARCH distribution is the best one to describe the joint behaviour of the series. However, the Logl estimates suggest that the SJC-TGARCH distribution is the best one (see Tables 5 and 6). Burnham and Anderson (2002) argue that, for most empirical applications, the Akaike Information Criterion is a better measure of goodness-of-fit than the Log Likelihood Ratio.