Flexural fatigue analysis of fibre reinforced polymer concrete composites under non-reversed loading

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Abstract: Results of an investigation on the flexural fatigue performance of poly-propylene fibre reinforced polymer concrete composites (PFRPCC) are presented. Flexural fatigue lives of PFRPCC beams at different stress levels were obtained using 100 kN MTS servo-controlled actuator. The specimens incorporated three weight fractions, i.e., 0.5%, 1.0% and 2.0% of polypropylene fibres. It has been established using graphical goodness of fit procedure that the fatigue life distributions of PFRPCC at various stress levels approximately follow the two parameter Weibull distribution with correlation coefficient exceeding 0.9. The results of graphical goodness of fit procedure have been reinforced with the help of Kolmogorov-Smirnov goodness of fit test and Anderson-Darling test of goodness of fit. The fatigue strength prediction models, particularly representing S-N relationships, have been examined and the material coefficients have been obtained for PFRPCC containing different weight fractions of fibres. Furthermore, using Weibull distribution, probability of failure has been incorporated into the fatigue life data of PFRPCC to develop S-N-P_f relationships, both graphically and analytically. The two million cycle endurance limits for PFRPCC containing different amounts of fibres have also been obtained.

Keywords: polymer concrete composites; fatigue; Weibull distribution; statistical properties/methods.

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1 Introduction

Polymer concrete composites (PCC) are materials obtained by polymerisation of an aggregate resin mixture and contain no cement. Since cost of construction has become a crucial matter these days, in terms of minimising labour, maintenance and repair costs, PCC technology offers a challenging opportunity. In particular, at construction work related to infrastructure, PCC products help to reduce maintenance, repair and labour costs significantly by extending the useful life of the investment. Polymer based technology results in a service life 3.3 times longer than cement concrete (CC) based technology. For example, the service life of PCC pipes is designed for a period of time not less than 50 years, whereas CC-based pipes have a useful life of 15 years. Therefore, the high material cost of PCC should be disregarded as it offers a new choice of material which has high strength to weight ratio, lower permeability, improved chemical resistance and therefore, better durability in comparison to CC. Because of the increased product life, the per year cost of PCC products has been reported to be 40% of that of CC products (Bozkurt and Mehmet, 2013). No special equipment other than those used for producing CC, are required for the production of PCC. The only major component which controls the cost of PCC is the resin and therefore, it is desirable to use minimum resin content for achieving an acceptable strength. Since PCC does not contain cement and can employ recycled micro-fillers such as fly ash as well as resins based upon recycled waste materials like PET bottles, its environmental impact is expected to be not higher than that of CC (Rebeiz, 1996a, 1996b, 1995).

These are being utilised for a variety of mechanical engineering applications, most common being their use as a material for machine tool bases. The use of PCC in machine tool bases has been reported since 1970s in Eastern Europe and other countries (Koblischek, 1985, 1975; Salje et al., 1988; Weck and Hartel, 1985). Recent studies on machine tools having bases made of PCC and fibre reinforced polymer concrete composites (FRPCC) have concluded that components manufactured with these materials have better surface finish, tolerances when compared to those made with cast iron bases (Bruni et al., 2008, 2006; Rahman et al., 2001). The most important reason for this being the vibration damping capability of these materials which is significantly higher than conventional machine building materials like cast iron (Bedi and Singh, 2013; Cortes and Castillo, 2007; Orak, 2000).The machine elements made of PCC are undoubtedly subjected to repetitive fatigue loads, which can result in early fatigue failure. Studies on fatigue of PCC are restricted in number, wherein compression fatigue (Kobayashi et al., 1974), flexural fatigue (Vipulanandan and Mebarkia, 2001; Woelfl et al., 1981) has been characterised to a very limited extent. Very small number of specimens has been tested at different stress levels in these studies (Woelfl et al., 1981) and therefore, accurate characterisation of fatigue life distributions of PCC is not available till date. It is worthwhile to mention that due to statistical nature of fatigue phenomenon, large variability usually occurs in the fatigue life data of composites, at a given stress level, even under carefully controlled test procedures. Therefore, in investigations wherein the probabilistic analysis of the fatigue data is the prime objective, it is desirable to test relatively more number of specimens at a given stress level to obtain data which is statistically significant. Optimum design requirements these days necessitate accurate characterisation of this variability for the materials. The dispersion of fatigue life has, therefore, been a subject of statistical analysis by various researchers (Bedi and Chandra, 2009; Khashaba, 2003; Singh et al., 2006; Singh and Kaushik, 2001, 2000). Most of the models used to predict fatigue strength of composite materials provide relationship between applied stress level and number of cycles to failure (Singh and Kaushik, 2001; Vipulanandan and Mebarkia, 2001). Due to large variability in fatigue test results, it becomes essential to relate probability of failure with stress level and number of cycles to failure. An equation proposed by McCall (1958) has been used previously for fatigue strength prediction of PCC (Vipulanandan and Mebarkia, 2001; Woelfl et al., 1981).

A number of studies on FRPCC have reported improvement in mechanical properties under statically applied loads due to addition of fibres (Mebarkia and Vipulanandan, 1992; Reis, 2005; Reis and Ferreira, 2003; Sett and Vipulanandan, 2004; Vipulanandan and Mantrala, 1996). It has also been observed in case of conventional cement concrete that addition of fibres enhances the fatigue strength/life of the material (Singh et al., 2005a, 2005b; Singh et al., 2006; Singh and Kaushik, 2001, 2000). Polypropylene fibres have gained popularity for their use in conventional cement concrete mainly to enhance the shrinkage resistance and toughness of the material, rather than for enhancing the strength of conventional concrete (Alhozaimy et al., 1996). The benefits of fibre reinforcement in improving the fracture toughness, impact resistance, fatigue endurance and energy absorption capacity of concrete are also well known (Banthia and Nandakumar, 2003).Therefore, a comprehensive investigation was planned to evaluate the flexural fatigue performance of PCC and FRPCC containing different types of fibres, such as polypropylene and glass fibres, as well as differing content of fibres. The result presented here are a part of the investigation with reference to fatigue of PCC and polypropylene fibre reinforced polymer concrete composites (PFRPCC).

2 Experimental procedure

Epoxy resin, LAPOX-B47 along with hardener LAPOX-K46 supplied by Atul Ltd., Mumbai has been used in this investigation. The hardener and resin have been mixed in the ratio of 1:2 by weight. Resin dosage of 10%–14% by weight of PCC has been reported in literature when using coarse aggregates (Cortes and Castillo, 2007; Fattah and El-Hawary, 1999) whereas higher resin dosages up to 20% have been reported when using only sand as aggregate material (Ferreira, 2000). Resin dosage of 12% by weight of PCC has been used in this investigation as sufficient workability was obtained at this resin dosage. Aggregate grading plays an important role in the final properties of PCC and therefore an optimised aggregate mix suggested in literature has been used in this study (Muthukumar et al., 2003). Locally available crushed gravel has been used as aggregate in PCC. The aggregate mix had been optimised based upon the least void content criteria. A microfiller is also often added to PCC mix to reduce the void content in aggregate mixture and thereby to increase the strength. Fly ash is a by-product of the coal burning in power plants and is often used as filler because of its easy availability and usage in PCC is reported to yield better mechanical properties as well as reduced water absorption (Varughese, 1996). Addition of fly ash also improves the workability of fresh PCC mix resulting in products with excellent surface finish (Gorninski et al., 2004). F-type fly ash has been used in the ratio of 10% by total weight of PCC in this study.

Macro-monofilament type synthetic polypropylene fibres of average length 12 mm were used at 0.5%, 1.0% and 2.0% weight fractions. Details of the materials used in this study are provided in Table 1. The specimens of 40 × 40 × 160 mm size were cast on a vibratory table for flexural fatigue and static flexural strength tests. The specimen size has been chosen as per RILEM PC-2-TC113 (RILEM PCM-8 TC-113, 1995) and has been used by a number of researchers in their work on PCC (Ferreira, 2000; Ribeiro et al., 2003). Aggregate materials and fly ash were dried before preparation of specimens to reduce moisture content below 0.5% as it has been reported that moisture content of aggregates has a deleterious effect on the properties of polymer concrete (Ohama, 1973). The specimens were cured at room temperature for 7 days before testing, as per method adopted by a number of researchers (Ohama and Demura, 1982; Rebeiz, 1995; Tawfik, 2006).

Table 1 Detail of materials

Aggregate (crushed gravel)	
Particle size (mm)	*Quantity (% of total aggregate weight)*
4.76–9.52	39.6%
2.38–4.36	33.5%
0.15–0.3	26.9%
Resin and Hardener system	
Description	*Quantity (% of total weight of PCC)*
(LAPOX- B47 and K-46)	12 %
Microfiller	
Description	*Quantity (% of total weight of PCC)*
F – type fly ash	10%
Fibre reinforcement	
Description	*Quantity (% of total weight of PCC)*
Macro monofilament poly-propylene fibres	0.5%, 1.0%, 2.0%

It is essential that static flexural strength (f_r) of the test material is determined before the fatigue testing for the selection of maximum and minimum loads to be applied in particular fatigue test. The static flexural strength of the PFRPCC specimens was evaluated prior to fatigue testing. An average static flexural strength of 24.41 MPa was obtained for PCC, whereas the average static flexural strength obtained for PFRPCC containing 0.5%, 1.0% and 2.0% fibres was 26.98 MPa, 27.53 MPa and 24.7 MPa respectively. It is observed that the addition of polypropylene fibres enhances the static flexural strength of PCC to a limited extent. An increase of approximately 9% in static flexural strength is observed by addition of 0.5% fibres by weight when compared to PCC. Further increase in fibre loading to 1.0% does not have any influence on the static flexural strength. It was observed that addition of 2.0% fibres causes a decrease in the static flexural strength of PFRPCC. It can be attributed to the large volume occupied by light weight polypropylene fibres, which may have caused improper bonding, etc.

Fatigue tests were carried out on a 100 kN MTS-cyclic load testing equipment in three point bending mode. The loading span was taken as 100 mm. A stress ratio of R = 0.1 was used in fatigue testing. All the fatigue tests were carried out under constant amplitude sinusoidal loading, at a frequency of 10 Hz. The minimum fatigue stress (f_{min}) and maximum fatigue stress (f_{max}) to be applied on test specimen was selected from f_r and a particular stress level 'S' ($f_{max/fr}$). For each mix, the first test was conducted at the highest possible stress level and the number of cycles to failure was noted as fatigue life 'N'. Subsequent tests were conducted by lowering the stress levels in a systematic manner. Since fatigue testing is a time consuming and expensive process and a large number of specimens were proposed to be tested, an upper limit of number of cycles to be applied was fixed depending upon the availability of testing equipment and time constraints. A particular test was terminated as and when the failure of the specimen occurred or the upper limit was reached, which ever was earlier. Figure 1 presents the details of the loading arrangement and a view of the polypropylene fibres used in this study.

Figure 1 (a) Macro monofilament polypropylene fibres (b) Loading arrangement (see online version for colours)

(a)

(b)

3 Analysis and discussion of fatigue test results

The complete fatigue life data obtained at various stress levels for PCC is reported elsewhere (Bedi et al., 2013) and for PFRPCC with 0.5%, 1.0% and 2.0% fibres is listed in ascending order in Tables 2, 3 and 4 respectively. Chauvenet's criteria (Kennedy and Neville, 1986),as used previously (Bedi and Chandra, 2009; Singh and Kaushik, 2000) was applied to the data points at all the stress levels tested in this investigation and data points meeting this criterion for rejection were identified and excluded from further analysis.

3.1 Fatigue life distributions of PFRPCC

A number of mathematical models have been employed to study the statistical dispersion of fatigue life. One of the popular models is the logarithmic-normal (lognormal) distribution function. However, it was pointed out later that the hazard function or risk function of lognormal distribution decreases with increasing life or time (Gumble, 1963). This violates the physical phenomenon of progressive deterioration of engineering materials resulting from the fatigue process, where the damage resulting from any particular loading cycle is dependent on the damage induced by previous loading cycles. In the case of composite materials, randomness is introduced into material characteristics, during the various stages of manufacturing and testing. It has been established that fatigue failure results are influenced by the progressive damage, initial random properties and random nature of the damage accumulation process (Lee et al., 1997).

Table 2 Laboratory fatigue life data for PFRPCC-0.5% (number of cycles to failure)

S = 0.90	*S = 0.85*	*S = 0.8*	*S = 0.75*	*S = 0.7*
520#	5,088	1,806*	60,023	153,452
522#	7,021	17,612	87,903	182,345
910#	9,954	20,438	136,458	245,784
913#	11,931	39,974	165,874	392,145
1783#	12,351	40,124	170,267	524,876
–	13,214	46,299	172,367	752,318
–	16,134	47,129	196,325	812,019
–	19,232	55,101	237,737	862,546
–	19,703	65,235	325,445	985,357
–	19,841	70,124	434,540	1,102,489
–	–	80,163	–	2,000,000**

Notes: *Rejected as outlier by Chauvenet's criteria, not included in analysis.
**Run out, not included in analysis and #Used for S-N curves only.

Table 3 Laboratory fatigue life data for PFRPCC-1.0% (number of cycles to failure)

S = 0.90	*S = 0.85*	*S = 0.8*	*S = 0.75*	*S = 0.7*
984#	7,789	95*	123,565	201,326
1203#	7,852	35,382	149,862	295,623
1642#	8,526	45,325	154,781	345,625
2165#	9,347	55,367	200,356	402,351
2754#	12,451	55,559	207,461	463,589
–	14,633	67,096	260,177	736,324
–	16,836	73,529	367,834	869,247
–	17,905	77,851	498,657	1,133,565
–	21,199	118,677	656,834	1,326,546
–	28,193	158,954	698,652	1,632,581
–	–	175,301	–	2,000,000**

Notes: *Rejected as putlier by Chauvenet's criteria, not included in analysis.
**Run out, not included in analysis and #Used for S-N curves only.

Weibull distribution function has proved to be useful and versatile means of describing fatigue behaviour of cement concrete (Goel et al., 2012; Singh and Kaushik, 2000) as well as other composite materials (Bedi and Chandra, 2009; Lee et al., 1997; Talreja, 1981). Therefore, in the present investigation two parameter Weibull distribution is adopted for statistical analysis and modeling of fatigue test data of PFRPCC.

Table 4 Laboratory fatigue life data for PFRPCC-2.0% (number of cycles to failure)

S = 0.90	*S = 0.85*	*S = 0.8*	*S = 0.75*	*S = 0.7*
1562#	7,033	35,628	95,258	142,038
1589#	8,547	47,723	120,325	175,623
1865#	9,865	57,072	145,265	250,382
1952#	9,956	78,658	200,382	421,539
2569#	11,617	84,807	240,526	520,152
3457#	13,237	85,629	326,080	628,405
3685#	14,252	126,562	338,038	756,214
–	23,220	175,684	649,115	956,203
–	25,624	185,647	785,624	1,425,870
–	27,987	232,437	800,265	1,754,735

Notes: *Rejected as outlier by Chauvenet's criteria, not included in analysis.
**Run out, not included in analysis and #Used for S-N curves only.

3.1.1 Establishing the fatigue life distributions by graphical goodness of fit

A two parameter Weibull distribution function which is characterised by a probability density function (PDF), $f(n)$ and the cumulative distribution function (CDF), $F(n)$ as follows:

$$f(n) = \frac{\alpha}{u}\left(\frac{n}{u}\right)\exp\left[-\left(\frac{n}{u}\right)\right] \tag{1}$$

$$F(n) = 1 - \exp\left[-\left(\frac{n}{u}\right)^{\alpha}\right] \tag{2}$$

in which n = specific value of the random variable N; α = shape parameter or Weibull slope at stress level S and u = scale parameter or characteristic life at stress level S.

The probability of survival or survivorship function or reliability function, $L_R(n)$, may be defined as $L_R(n) = 1 - F(n)$ and substituting this value of $F(n)$ in equation (2) it is modified to:

$$L_R(n) = \exp\left[-\left(\frac{n}{u}\right)^{\alpha}\right] \tag{3}$$

Taking the logarithm twice of both sides of equation (3), it can be rewritten as:

$$\ln\left[\ln\left(\frac{1}{L_R}\right)\right] = \alpha\ln(n) - \alpha\ln(u) \tag{4}$$

Equation (4) represents a linear relationship between $\ln[\ln(1 / L_R)]$ and $\ln(n)$. In order to obtain a graph from equation (4), the fatigue-life data corresponding to a particular stress level are first arranged in ascending order of cycles to failure and the empirical survivorship function L_R for each fatigue-life data at a given stress level is obtained from the following relation (Kennedy and Neville, 1986):

$$L_R = 1 - \frac{i}{k+1} \tag{5}$$

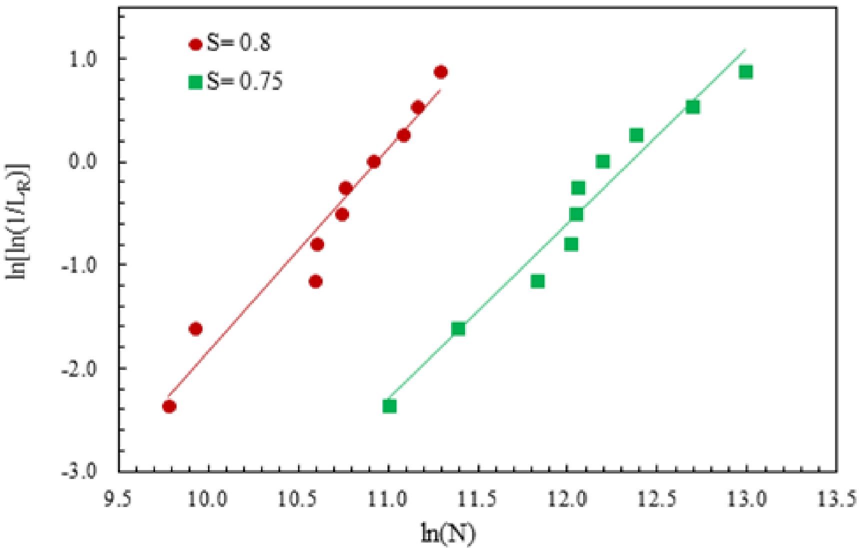
where *i* denotes the failure order number and k represents the number of data points in a data sample under consideration at a particular stress level *S*. The empirical survivorship function in the form of ln [ln(1 / L_R)] for each fatigue-life data is then plotted on a graph with the corresponding fatigue lives ln(*N*). If a linear trend is established for the data points, the best fit line is drawn using method of least squares. It can then be assumed that fatigue-life data for that particular stress level follows the two parameter Weibull distribution. The slope of the line provides an estimate of shape parameter α and the characteristic life *u* can be obtained as that value of n which corresponds to $L_R = 0.368$.

Table 5 Parameters of Weibull distribution for PFRPCC

PFRPCC-0.5%						
Stress level, S	*Graphical method*		*Method of moments*		*Average values*	
	α	*u*	α	*u*	α	*u*
0.85	2.152	15,560	2.764	15,108	2.458	15,334
0.80	1.956	56,315	2.554	54,307	2.255	55,311
0.75	1.698	230,791	1.872	223,880	1.785	227,335
0.70	1.342	709,720	1.808	676,415	1.575	693,067
PFRPCC-1.0%						
Stress level, S	*Graphical method*		*Method of moments*		*Average values*	
	α	*u*	α	*u*	α	*u*
0.85	2.110	16,685	2.294	16,335	2.202	16,510
0.80	1.808	99,653	1.870	97,233	1.839	98,443
0.75	1.500	384,798	1.599	370,043	1.550	377,421
0.70	1.396	855,413	1.568	824,716	1.482	840,065
PFRPCC-2.0%						
Stress level, S	*Graphical method*		*Method of moments*		*Average values*	
	α	*u*	α	*u*	α	*u*
0.85	1.966	17,485	2.101	17,087	2.033	17,286
0.80	1.578	128,478	1.740	124,604	1.659	126,541
0.75	1.254	424,143	1.387	405,576	1.320	414,859
0.70	1.165	801,989	1.333	766,089	1.249	784,039

Figure 2 presents the fatigue life data for a few selected stress levels plotted as described above for PFRPCC-0.5%. The approximate straight line plots in this figure with statistical correlation coefficients ‘r’ exceeding 0.9, indicate that the two-parameter Weibull distribution is a reasonable assumption for the statistical distribution of fatigue-life for PFRPCC. Similar results have been obtained for all the mixes at all the stress levels in this investigation. The estimated parameters thus obtained are listed in Table 5.

Figure 2 Graphical analysis of fatigue life data for PFRPCC-0.5% (see online version for colours)



3.1.2 Parameter estimation by method of moments

Estimating parameters by method of moments requires estimating the appropriate sample moments, such as sample mean and sample variance. A simple expression for finding the value of shape parameter α is provided in literature as follows (Wirsching and Yao, 1970):

$$\alpha = (COV)^{-1.08} \tag{6}$$

The characteristic life u can be estimated from equation (6) by substituting μ for $E(n)$ as follows:

$$u = \frac{\mu}{\Gamma\left(\frac{1}{\alpha} + 1\right)} \tag{7}$$

The parameters obtained by this method for PFRPCC containing 0.5%, 1.0% and 2.0% fibres at different stress levels are also listed in Table 5.

3.1.3 Goodness-of-fit tests

In the previous section, the graphical method also known as graphical goodness of fit procedure, has been used to establish that the statistical distribution of fatigue-life of PFRPCC, at various stress levels, can approximately be described by the two-parameter Weibull distribution with correlation coefficient values greater than 0.90. The values of the Weibull parameters for the distribution of fatigue life of PFRPCC have been estimated using different methods of analysis, i.e., graphical method and method of moments. The parameter estimation is in fact a prerequisite to proceed further for conducting any type of goodness of fit test. Goodness of fit tests are formal, statistical procedures for assessing the underlying distribution of a dataset and their results are quantifiable and more reliable. Therefore, it would be more convincing to perform goodness of fit tests to reinforce the results obtained in the preceding section which show that the Weibull distribution is a valid model for statistical distribution of fatigue life of PFRPCC. Two types of goodness of fit tests, Kolmogorov-Smirnov test and Anderson-Darling test, have been used in this investigation. The Kolmogorov-Smirnov test (Kennedy and Neville, 1986) has been applied to validate the results, which can be carried out by using the following equation:

$$D_i = \max_{i=1}^{k}\left[\left|F^*(x_i) - F(x_i)\right|\right] \tag{8}$$

where $F^*(x_i) = i / k$ = observed cumulative histogram, i = order number of the data point, k = total number of data points in the sample under consideration at a given stress level and $F(x_i)$ = hypothesised cumulative distribution given by equation (2).

The value of D_i thus obtained is compared with D_c values for each dataset obtained from Kolmogorov-Smirnov table. If $D_i < D_c$ the model is acceptable with 5% significance level. Kolmogorov-Smirnov goodness-of-fit test was applied to fatigue life data of PFRPCC at all the stress levels and it was found that the model was acceptable at 5% level of significance. The results are compiled in Table 6.

The Anderson-Darling statistic for Weibull distribution is defined as follows (Romeu and Grethlein, 2000):

$$AD = \sum_{i=1}^{n}\frac{1-2i}{n}\left\{\ln\left[1-\exp(-z_i)\right] - z_{(n+1-i)}\right\} - n \tag{9}$$

$$AD^* = \left(1 + \frac{0.2}{\sqrt{n}}\right)AD \tag{10}$$

where

$$z_i = \left[\frac{x_{(1)}}{u^*}\right]^{\alpha^*}$$

For $i = 1, \ldots\ldots, n$, n is the number of specimens and α^* and u^* are the estimated values of parameters of the Weibull distribution for the particular stress level, as listed in Table 5.

The observed significance level (OSL) is given by:

$$OSL = \frac{1}{\left\{1 + \exp\left[-0.1 + 1.24\ln\left(AD^*\right) + 4.48\left(AD^*\right)\right]\right\}} \tag{11}$$

The OSL measures the probability of observing an Anderson-Darling statistic at least as extreme as the value calculated, if in fact the data are from a two parameter Weibull distribution. If OSL ≤ 0.05, one may conclude (at five percent risk of being in error) that the population does not have a two parameter Weibull distribution. Otherwise, the hypothesis that the population has a two parameter Weibull distribution is not rejected. Anderson-Darling test was applied to the fatigue life data of PFRPCC at all stress levels tested in this investigation and it was observed that two parameter Weibull distribution is an acceptable model to represent the fatigue life distribution of PFRPCC. The results are compiled in Table 6.

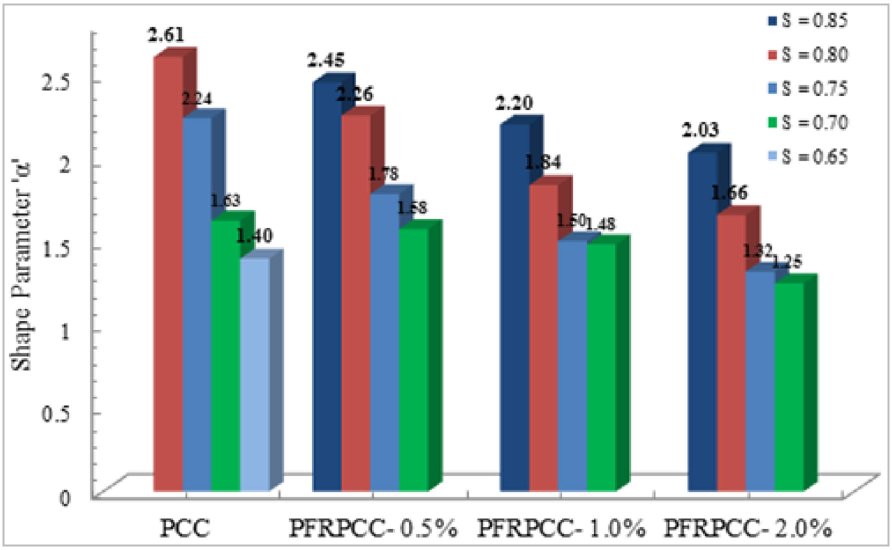
Table 6 Results of Kolmogorov-Smirnov and Anderson-Darling test for PFRPCC

			PFRPCC-0.5%		
Kolmogorov-Smirnov test				*Anderson-Darling test*	
Stress level	D_i	D_c	*Remarks*	*OSL*	*Remarks*
S = 0.85	0.1520	0.41	Accepted	0.6034	Accepted
S = 0.80	0.0996	0.41	Accepted	0.6345	Accepted
S = 0.75	0.1632	0.41	Accepted	0.5509	Accepted
S = 0.70	0.1252	0.41	Accepted	0.4910	Accepted
			PFRPCC-1.0%		
Kolmogorov-Smirnov test				*Anderson-Darling test*	
Stress level	D_i	D_c	*Remarks*	*OSL*	*Remarks*
S = 0.85	0.1515	0.41	Accepted	0.4965	Accepted
S = 0.80	0.2224	0.41	Accepted	0.2666	Accepted
S = 0.75	0.1734	0.41	Accepted	0.3038	Accepted
S = 0.70	0.1608	0.41	Accepted	0.6845	Accepted
			PFRPCC-2.0%		
Kolmogorov-Smirnov test				*Anderson-Darling test*	
Stress level	D_i	D_c	*Remarks*	*OSL*	*Remarks*
S = 0.85	0.2090	0.41	Accepted	0.1967	Accepted
S = 0.80	0.1928	0.41	Accepted	0.5998	Accepted
S = 0.75	0.1662	0.41	Accepted	0.4262	Accepted
S = 0.70	0.0864	0.41	Accepted	0.8319	Accepted

3.1.4 Variability in fatigue life

A comparison of shape parameter α of Weibull distribution for fatigue life distributions of PFRPCC with those of PCC reported elsewhere by the authors (Bedi et al., 2013) is presented in Figure 3. It is observed that there is significant decrease in the values of shape parameter with the addition of fibres to PCC, which indicates higher variability in the distribution of fatigue life data of PFRPCC compared to PCC. A maximum decrease in shape parameter, of approximately 20% at S = 0.75 has been observed with the addition 0.5% fibres to PCC. Similarly, a maximum decrease in shape parameter of the order of 32% at S = 0.75 and 41% at S = 0.75 has been observed with the addition of 1.0% and 2.0% fibres respectively. Similarly, COV values of the fatigue life of PFRPCC increase by 23%, 40% and 63% by addition of 0.5%, 1.0% and 2.0% fibres respectively, when compared to PCC. Moreover, the values of shape parameter decrease with a decrease in the fatigue stress level, which indicates higher variability in the distribution of fatigue life at lower stress levels. Similar trends have been reported for the fatigue life distributions of conventional cement concrete as well as fibre reinforced cement concrete by various authors (Singh et al., 2005a, 2005b, 2006; Singh and Kaushik, 2000).

Figure 3 Comparison of shape parameter of PFRPCC with PCC (see online version for colours)



3.2 *Fatigue strength prediction models*

3.2.1 *S-N relationships*

Fatigue life or fatigue strength estimates have traditionally been based on test data expressed in terms of the S-N curves which is a relationship between a non-dimensional term, *S* and corresponding number of cycles to failure, *N* also termed as fatigue life. The S-N relationships facilitate designers in estimating the fatigue life of a structure subjected to a particular stress history or, provide an initial estimate of fatigue life for design purpose. Several fatigue models or equations have been in use to characterise the S-N relationships for modelling of fatigue strength. Most of the researchers have used an S-N relationship represented by equation (12), as mentioned below which is also known as the Wholer equation:

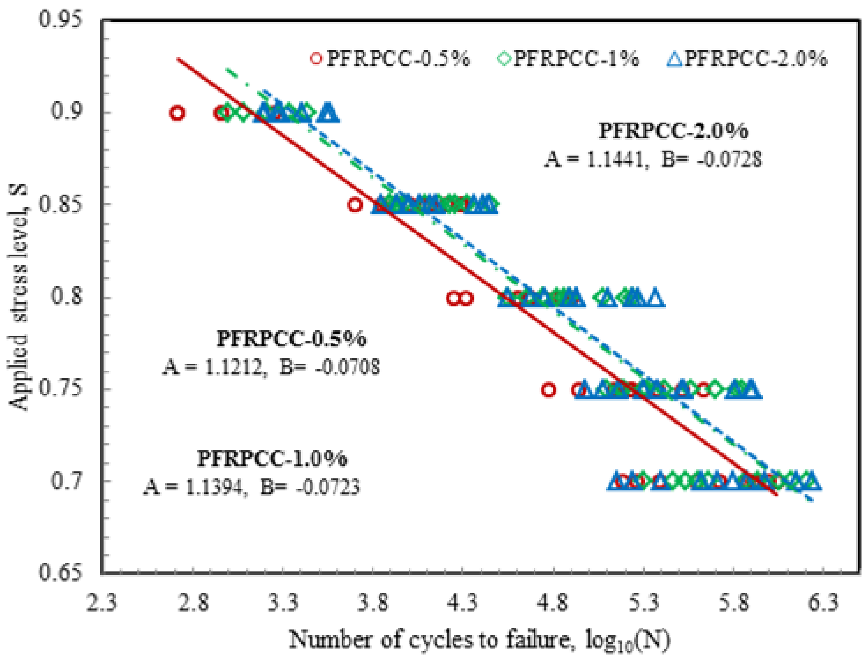
$$S = \frac{f_{\max}}{f_r} = a + b\log_{10}(N) \tag{12}$$

where *a* and *b* are material constants, which can be determined by regression from the fatigue test data. Figure 4 presents the test results in the form of S-N curves obtained in this study for PFRPCC with 0.5%, 1.0% and 2.0% fibres. Linear regression is carried out by method of least squares to determine the values of coefficients *a* and *b*. The values of coefficients obtained are listed in Table 7.

Table 7 Coefficients *a* and *b* for equation (12) and coefficient *β* for equation (13)

Material	*Coefficient a*	*Coefficient b*	*Coefficient β*
PFRPCC-0.5%	1.1212	–0.0708	0.0489
PFRPCC-1.0%	1.1394	–0.0723	0.0402
PFRPCC-2.0%	1.1441	–0.0728	0.0452
GFRPCC-0.5%	1.0773	–0.0646	0.0510
GFRPCC-1.0%	1.1184	–0.0665	0.0458
GFRPCC-2.0%	1.0963	–0.0659	0.0492

Figure 4 S-N relationships for PFRPCC from experimental data (see online version for colours)



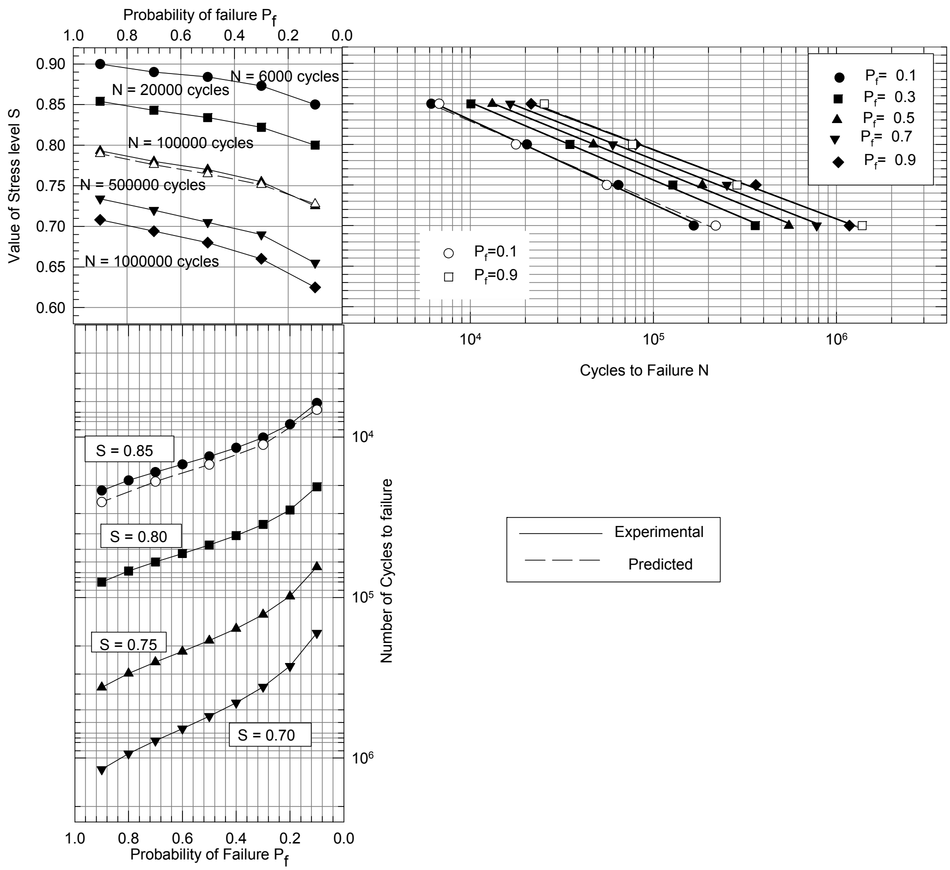
Another form of the fatigue equation used by researchers is a modification of the Wholer equation that incorporates a stress ratio R, which is the ratio of minimum fatigue stress $f_{\min}$ to the maximum fatigue stress $f_{\max}$ into the Wholer equation. The R-term is included to simulate the loading conditions in actual structures where the minimum value of the repeated stress is not zero. The modified equation takes the following form:

$$S = \frac{f_{\max}}{f_r} = 1 - \beta(1-R)\log_{10}(N) \tag{13}$$

where β is an experimental coefficient. This equation can be used for $0 \le R \le 1$ but not for stresses that vary between tension and compression. The material coefficient β in equation (13) can be obtained from the fatigue test data. Using the values of stress level S, R and $\log_{10}(N)$ the values of the coefficient β are determined for PFRPCC. The values of coefficients obtained are listed in Table 7.

As already mentioned, the results reported in this paper are from a larger investigation, to evaluate the fatigue behaviour of FRPCC containing different types of fibres. The results of fatigue behaviour of glass fibre reinforced polymer concrete composites (GFRPCC) have been reported elsewhere by the authors (Bedi et al., 2014). The coefficients a and b of equation (12) and coefficient β of equation (13) for GFRPCC are also presented in Table 7 for the reference purpose.

Figure 5 S-N-P_f diagram for PFRPCC-0.5%

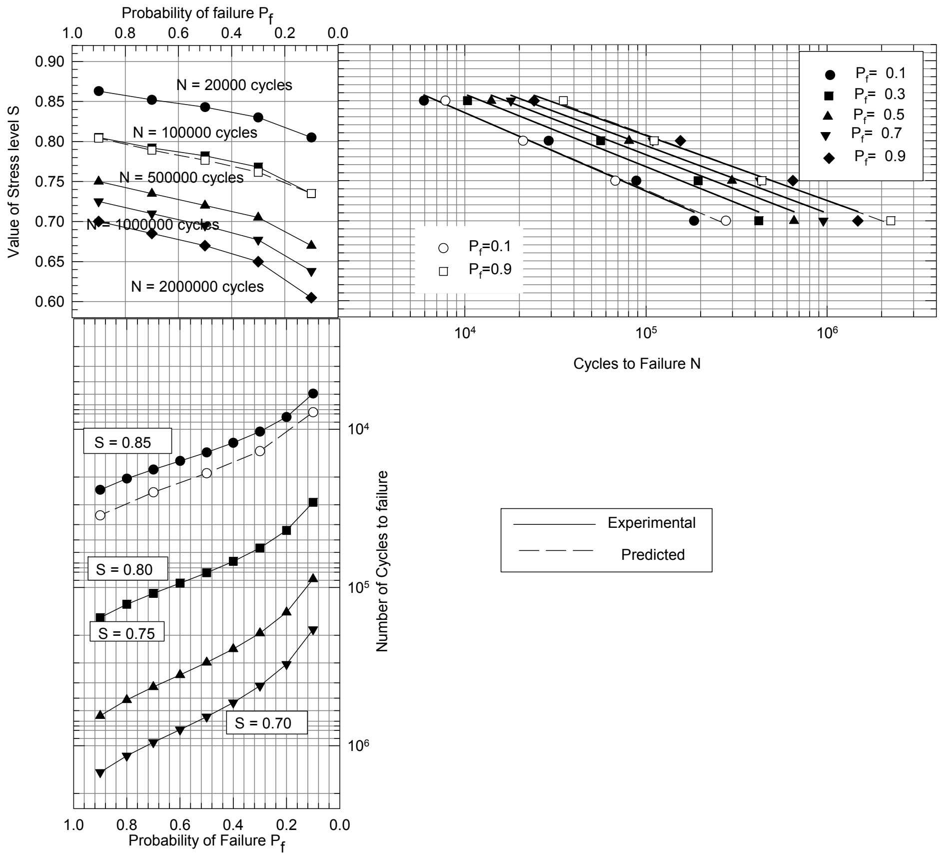


3.2.2 S-N-Pf relationships

As the fatigue test results show a lot of scatter, it is necessary to incorporate failure probability P_f, in the S-N relationships. The Weibull distribution has been employed here to incorporate the failure probability into S-N relationships for PFRPCC. Substituting $1 - P_f = L_R$ in equation (4), the following relation is obtained

$$n = \ln^{-1}\left[\frac{\ln\left\{\ln\left(\frac{1}{1-P_f}\right)\right\} + \alpha\ln(u)}{\alpha}\right] \tag{14}$$

Figure 6 S-N-Pf diagram for PFRPCC-1.0%



Thus, using the average values of the parameters of Weibull distribution for fatigue life at any stress level (as listed in Table 5), equation (14) has been used to calculate the fatigue lives at a given stress level corresponding to different failure probabilities. These calculated values of fatigue lives for failure probabilities of $P_f = 0.1$, 0.3, 0.5, 0.7 and 0.9, are plotted in Figures 5 to 7 for PFRPCC with 0.5%, 1.0% and 2.0% respectively.

To analytically represent the relationship between stress level *S*, Number of cycles to failure *N* and probability of failure P_f, called as S-N-P_f the following equation has been used by researchers:

$$L_N = (10)^{-a(S)^b (\log N)^c} \tag{15}$$

where *a*, *b* and c are experimental coefficients and L_N is the probability of survival which is $(1 - P_f)$. The experimental coefficients have been obtained by using the method described elsewhere (Singh et al., 2005a, 2005b).

The final equations obtained for PFRPCC-0.5%, PFRPCC-1.0% and PFRPCC-2.0% are provided below as equation (16) to (18) respectively:

$$L_N = (10)^{-2.83\times10^{-12}(S)^{37.79}(\log N)^{22.07}} \tag{16}$$

$$L_N = (10)^{-2.11\times10^{-11}(S)^{34.36}(\log N)^{19.92}} \tag{17}$$

$$L_N = (10)^{-2.19\times10^{-10}(S)^{28.65}(\log N)^{17.52}} \tag{18}$$

Using these material coefficients, some typical predicted curves have also been plotted alongside the curves obtained from test data in Figures 5 to 7 for PFRPCC with 0.5%, 1.0% and 2.0% fibres respectively. It can be observed that the predicted curves are quite close to the experimental curves, which proves the applicability of equation (15) for predicting the fatigue live of PFRPCC at different probabilities of failures. The coefficients *a*, *b* and *c* of equation (15) for PFRPCC are presented in Table 8 along with those obtained for PCC and GFRPCC as a part of a larger investigation.

Figure 7 S-N-Pf diagram for PFRPCC-2.0% (see online version for colours)

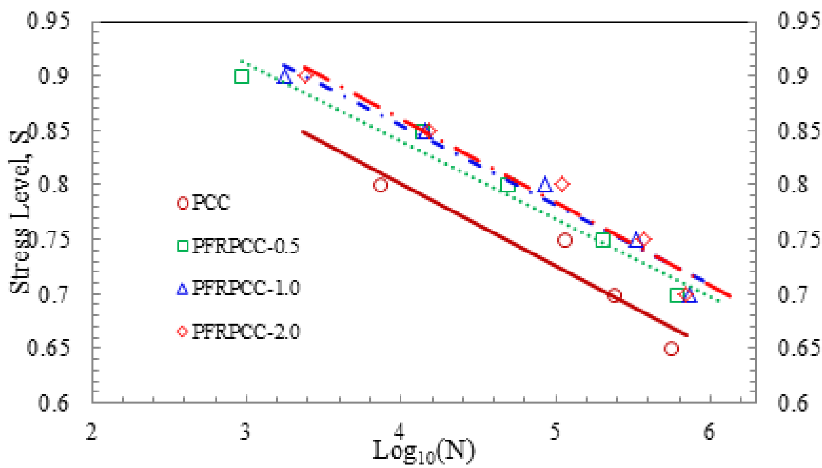


Table 8 Coefficients a, b and c for equation (15)

Material	*Fibre description*	*Weight fraction, Wf (%)*	*Coefficients of equation (14)*		
			a	*b*	*c*
PCC	–	–	1.01×10^{-10}	35.58	20.95
GFRPCC	Macro glass fibres, 12 mm average length, specific gravity – 2.68 g/cm^3	0.5	2.47×10^{-9}	46.93	20.03
		1.0	1.06×10^{-11}	28.56	19.05
		2.0	1.96×10^{-9}	28.28	16.46
PFRPCC	Macro-monofilament polypropylene fibres, 12 mm average length, specific gravity – 0.91 g/cm^3	0.5	2.83×10^{-12}	37.78	22.07
		1.0	2.11×10^{-11}	34.36	19.92
		2.0	2.19×10^{-10}	28.65	17.52

3.3 *Two million cycle endurance limits*

The endurance limit/fatigue strength has been determined for PFRPCC from the experimental data. The two million cycle fatigue strength of PFRPCC has been estimated as 67.5% of the corresponding static flexural strength for 0.5% fibre content. Further addition of fibres to the tune of 1.0% and 2.0% does not have significant influence on the fatigue strength, the fatigue strength for 1.0% and 2.0% fibre content remains almost 68%.In comparison to two million cycle fatigue strength for PCC which was obtained as 62% of the corresponding static flexural strength, there is a 9% enhancement in the same by addition of 1.0% fibres. The same is attributed to the crack arresting properties of the fibres in the matrix. Fibres resist the nucleation of cracks by acting as stress-transfer bridges and once cracks nucleate, fibres abate their propagation by providing crack tip plasticity and increased fracture toughness (Banthia and Nandakumar, 2003). Figure 8 presents the comparison of the two million cycle endurance limit for PCC with PFRPCC containing different weight fraction of fibres as obtained in this investigation.

4 Scope for further research

The objectives of the present investigation were to study the characteristics of PFRPCC under fatigue loading. It is well known that fatigue testing in itself is a time consuming and expensive process and the properties of FRPCC depend significantly upon the constituent materials such as resin type and content, aggregate type and mix proportions, microfiller type, fibre reinforcements and content, etc. To model the scatter in the fatigue life data, a large number of specimens are required to be tested for improving the reliability of widely scattering fatigue test results. Since the two-million cycles endurance limits were also proposed to be determined, it was rather difficult to widen the scope of this investigation by using different types of resins and their dosage, microfiller materials, fibres and their sizes, etc. Within these constraints, it was decided to use a fixed dosage of epoxy resin, single type of microfiller, i.e., fly ash, one type of optimised aggregate mix and specific type of polypropylene fibres. Further the fatigue test parameters, i.e., stress ratio, R was kept as 0.1 and the test frequency was fixed at 10 Hz throughout the investigation.

Keeping in view the above, the following studies are recommended to supplement the information obtained in this investigation:

1 Fatigue studies may be carried out to investigate the probability distributions of PCC and FRPCC mixes using the different types of resins, aggregate mix, microfillers and fibres. Similarly resin dosage, microfiller content and fibre content can also be taken as variables in future fatigue studies.

2 Since considerable interest has developed in fibre hybridisation nowadays, it will be prudent to study the influence of fibre hybridisation on the flexural fatigue characteristics of FRPCC, particularly using different types/combinations of metallic and non-metallic fibres. This will result in better understanding of the positive effects of hybridisation on the flexural fatigue performance of FRPCC.

3 Fatigue studies may also be carried out to investigate the effect of specimen size, load history, stress reversal, and rest periods, etc. on PCC as well as FRPCC.

5 Conclusions

Experimental investigation has been carried out to obtain the fatigue lives of PFRPCC specimens containing different contents of fibres at different stress levels. It has been observed that the fatigue life distributions for PFRPCC, at any stress level, approximately follow two parameter Weibull distribution. Parameters of Weibull distribution have been obtained by graphical as well as method of moments. The values of the shape parameter for PFRPCC have been found to be considerably lower than those for PCC, indicating higher variability in the fatigue life data of PFRPCC compared to PCC. The test data has been used to generate S-N curves for PFRPCC and equations have been proposed to predict the flexural fatigue strength of PFRPCC. Fatigue test data has been used to develop S-N-P_f curves for PFRPCC and a relationship amongst stress level, fatigue life and survival probability has been established. The material coefficients of the fatigue equation representing family of S-N-P_f curves have been obtained for PFRPCC. The proposed equations can be used to predict the flexural strength of PFRPCC using the appropriate coefficients obtained in this investigation. Two million cycle fatigue strength has been obtained for PFRPCC and it has been observed that addition of fibres to PCC enhances the fatigue strength of the resulting material.

References

Alhozaimy, A.M., Soroushian, P. and Mirza, F. (1996) 'Mechanical properties of polypropylene fiber reinforced concrete and the effects of pozzolanic materials', *Cem. Concr. Compos.*, Vol. 18, No. 2, pp.85–92.

Banthia, N. and Nandakumar, N. (2003) 'Crack growth resistance of hybrid fiber reinforced cement composites', *Cem. Concr. Compos.*, Vol. 25, No. 1, pp.3–9.

Bedi, R and Chandra, R. (2009) 'Fatigue-life distributions and failure probability for glass-fiber reinforced polymeric composites', *Compos. Sci. Technol.*, Vol. 69, pp.1381–1387 [online] https://doi.org/10.1016/j.compscitech.2008.09.016.

Bedi, R. and Singh, A. (2013) 'Adaptive neuro-fuzzy inference system in modelling damping performance of epoxy polymer concrete', *Int. J. Mater. Eng. Innov.*, Vol. 4, No. 1, pp.18–34.

Bedi, R., Chandra, R. and Singh, S.P. (2013) 'Probabilistic analysis of fatigue life of polymer concrete', *J. Exp. Appl. Mech.*, Vol. 4, No. 3, pp.22–28.

Bedi, R., Singh, S.P. and Chandra, R. (2014) 'Flexural fatigue-life assessment and strength prediction of glass fibre reinforced polymer concrete composites', *ISRN Mater. Sci.*, ID-928278, No. 2014, pp.1–8.

Bozkurt, O. and Mehmet, E.L. (2013) 'Comparison of cement-based and polymer-based concrete pipes for analysis of cost assessment', *Int. J. Polym. Sci.*, No. 2013, pp.1–6.

Bruni, C., Forcellese, A., Gabrielli, F. and Simoncini, M. (2006) 'Effect of the lubrication-cooling technique, insert technology and machine bed material on the workpart surface finish and tool wear in finish turning of AISI 420B', *Int. J. Mach. Tools Manuf.*, Vol. 46, pp.1547–1554 [online] https://doi.org/10.1016/j.ijmachtools.2005.09.007.

Bruni, C., Forcellese, A., Gabrielli, F. and Simoncini, M. (2008) 'Hard turning of an alloy steel on a machine tool with a polymer concrete bed', *J. Mater. Process. Technol.*, Vol. 202, pp.493–499 [online] https://doi.org/10.1016/j.jmatprotec.2007.10.031.

Cortes, F. and Castillo, G. (2007) 'Comparison between the dynamical properties of polymer concrete and grey cast iron for machine tool applications', *Mater. Des.*, Vol. 28, pp.1461–1466 [online] https://doi.org/10.1016/j.matdes.2006.03.012.

Fattah, A. and El-Hawary, M. (1999) 'Flexural behavior of polymer concrete', *Constr. Build. Mater.*, Vol. 13, No. 5, pp.253–62.

Ferreira, A.J.M. (2000) 'Flexural properties of polyester resin concretes', *J. Polym. Eng.*, Vol. 20, No. 6, pp.459–68.

Goel, S., Singh, S.P. and Singh, P. (2012) 'Fatigue analysis of plain and fiber-reinforced self-consolidating concrete', *ACI Mater. J.*, Vol. 109, No. 5, pp.573–582.

Gorninski, J.P., Dal Molin, D.C. and Kazmierczak, C.S. (2004) Study of the modulus of elasticity of polymer concrete compounds and comparative assessment of polymer concrete and portland cement concrete', *Cem. Concr. Res.*, Vol. 34, pp.2091–2095 [online] https://doi.org/10.1016/j.cemconres.2004.03.012.

Gumble, E.J. (1963) 'Parameters in the distribution of fatigue life', *J. Eng. Mech.*, ASCE October, Vol. 89, No. 1963, pp.45–63.

Kennedy, J. and Neville, A. (1986) *Basic Statistical Methods for Engineers and Scientists*, A Dun-Donnelley Publishers, New York.

Khashaba, U.A. (2003) 'Fatigue and reliability analysis of unidirectional GFRP composites under rotating bending loads', *J. Compos. Mater.*, Vol. 37, No. 4, pp.317–331.

Kobayashi, K., Ohama, Y. and Ito, T. (1974) 'Fatigue properties of resin concrete under repeated compression loads', *Seisan Kenkyu*, Vol. 26, No. 3, pp.116–18.

Koblischek, P. (1975) 'Synthetic resin bound concrete', in *1st International Congress on Polymer Concretes – Polymers in Concrete*, pp.409–419.

Koblischek, P. (1985) 'MOTEMA-acrylic concrete for machine tool frames', *Int. J. Cem. Compos. Light. Concr.*, Vol. 7, pp.55–57 [online] https://doi.org/10.1016/0262-5075(85)90027-2.

Lee, J., Harris, B., Almond, D. and Hammett, F. (1997) 'Fibre composite fatigue life determination', *Compos. Part A*, Vol. 28, No. 1, pp.5–15.

McCall, J.T. (1958) 'Probability of fatigue failure of plain concrete', *ACI J.*, Vol. 55, No. 8, pp.233–244.

Mebarkia, S. and Vipulanandan, C. (1992) 'Compressive behavior of glass-fiber reinforced polymer concrete', *J. Mater. Civ. Eng.*, Vol. 4, p.91. [online] https://doi.org/10.1061/(ASCE)0899-1561(1992)4:1(91).

Muthukumar, M., Mohan, D. and Rajendran, M. (2003) 'Optimization of mix proportions of mineral aggregates using Box Behnken design of experiments', *Cem. Concr. Compos.*, Vol. 25, pp.751–758 [online] https://doi.org/10.1016/S0958-9465(02)00116-6.

Ohama, Y. (1973) *Mix proportions and Properties of Polyester Resin Concretes*, SP 40-13, pp.283–294, American Concrete Institute.

Ohama, Y. and Demura, K. (1982) 'Relation between curing conditions and compressive strength of polyester resin concrete', *Int. J. Cem. Compos. Light. Concr.*, Vol. 4, pp.241–244 [online] https://doi.org/10.1016/0262-5075(82)90028-8.

Orak, S. (2000) 'Investigation of vibration damping on polymer concrete with polyester resin', *Cem. Concr. Res.*, Vol. 30, pp.171–174 [online] https://doi.org/10.1016/S0008-8846(99)00225-2.

Rahman, M., Mansur, M.., Lee, L. and Lum, J. (2001) 'Development of a polymer impregnated concrete damping carriage for linear guideways for machine tools', *Int. J. Mach. Tools Manuf.*, Vol. 41, pp.431–441 [online] https://doi.org/10.1016/S0890-6955(00)00072-9.

Rebeiz, K. (1995) 'Time-temperature properties of polymer concrete using recycled PET', *Cem. Concr. Compos.*, Vol. 17, pp.119–124 [online] https://doi.org/10.1016/0958-9465(94)00004-I.

Rebeiz, K. (1996a) 'Strength and durability properties of polyester concrete using pet and fly ash wastes', *Adv. Perform. Mater.*, Vol. 3, pp.205–214 [online] https://doi.org/10.1007/BF00136746.

Rebeiz, K. (1996b) 'Precast use of polymer concrete using unsaturated polyester resin based on recycled PET waste', *Constr. Build. Mater.*, Vol. 10, pp.215–220 [online] https://doi.org/10.1016/0950-0618(95)00088-7.

Reis, J. (2005) 'Mechanical characterization of fiber reinforced polymer concrete', *Mater. Res.*, Vol. 8, pp.357–360 [online] https://doi.org/10.1590/S1516-14392005000300023.

Reis, J. and Ferreira, A.J. (2003) 'Fracture behavior of glass fiber reinforced polymer concrete', *Polym. Test.*, Vol. 22, pp.149–153 [online] https://doi.org/10.1016/S0142-9418(02)00063-6.

Ribeiro, M., Tavares, C.M.L., Figueiredo, M., Ferreira, A.J.M. and Fernandes, A.A. (2003) 'Bending characteristics of resin concretes', *Mater. Res.*, Vol. 6, pp.247–254 [online] https://doi.org/10.1590/S1516-14392003000200021.

RILEM PCM-8 TC-113 (1995) 'Method of test for flexural strength and deflection of polymer-modified mortar', in *Symposium on Properties and Test Methods for Concrete – Polymer Composites*, Oostende, Belgium, pp. 184–185.

Romeu, J.L. and Grethlein, C. (2000) *A Practical Guide to Statistical Analysis of Material Property Data*, AMPTIAC.

Salje, E., Gerloff, H. and Meyer, J. (1988) 'Comparison of machine tool elements made of polymer concrete and cast iron', *CIRP Ann. – Manuf. Technol.*, Vol. 37, pp.381–384 [online] https://doi.org/10.1016/S0007-8506(07)61659-X.

Sett, K. and Vipulanandan, C. (2004) 'Properties of polyester polymer concrete with glass and carbon fibers', *ACI Mater. J.*, Vol. 101, No. 1, pp.30–41.

Singh, B., Singh, S.P. and Kaushik, S.K. (2005a) 'Probability of fatigue failure of steel fibrous concrete', *Mag. Concr. Res.*, Vol. 57, pp.65–72 [online] https://doi.org/10.1680/macr.2005.57.2.65.

Singh, S.P., Mohammadi, Y. and Kaushik, S.K. (2005b) 'Flexural fatigue analysis of steel fibrous concrete containing mixed fibers', *ACI J.*, Vol. 102, No. 6, pp.438–444.

Singh, S.P. and Kaushik, S.K. (2000) 'Flexural fatigue life distributions and failure probability of steel fibrous concrete', *ACI Mater. J.*, Vol. 97, No. 6, pp.658–667.

Singh, S.P. and Kaushik, S.K. (2001) 'Flexural fatigue analysis of steel fiber-reinforced concrete', *ACI Mater. J.*, Vol. 98, No. 4, pp.306–312.

Singh, S.P., Mohammadi, Y. and Madan, S.K. (2006) 'Flexural fatigue strength of steel fibrous concrete containing mixed steel fibres', *J. Zhejiang Univ. Sci. A*, Vol. 7, pp.1–7 [online] https://doi.org/10.1631/jzus.2006.A0155.

Talreja, R. (1981) 'Estimation of Weibull parameters for composite material strength and fatigue life data', *Fatigue of Fibrous Composite Materials*, ASTMSTP 723, American Society for Testing and Materials, pp.291–311.

Tawfik, M.E. (2006) 'Polymer concrete from marble wastes and recycled poly (ethylene terephthalate)', *J. Elastomers Plast.*, Vol. 38, pp.65–79 [online] https://doi.org/10.1177/0095244306055569.

Varughese, K. (1996) 'Fly ash as fine aggregate in polyester based polymer concrete', *Cem. Concr. Compos.*, Vol. 18, pp.105–108 [online] https://doi.org/10.1016/0958-9465(95)00006-2.

Vipulanandan, C. and Mantrala, S.K. (1996) 'Behavior of fiber reinforced polymer concrete', in *Proceedings of Materials Engineering Conference*, pp.1160–69.

Vipulanandan, C. and Mebarkia, S. (2001) *Fatigue Crack Growth in Polyester Polymer Concrete*, American Concrete Institute, California.

Weck, M. and Hartel, R. (1985) 'Design, manufacture and testing of precision machines with essential polymer concrete components', *Precis. Eng.*, Vol. 7, pp.165–170 [online] https://doi.org/10.1016/0141-6359(85)90045-5.

Wirsching, P. and Yao, J. (1970) 'Statistical methods in structural fatigue', *J Struc Div.*, *Proc ASCE*, June, pp.1201–1219.

Woelfl, G., McNerney, M. and Chang, C. (1981) 'Flexural fatigue of polymer concrete', *Cem. Concr. Aggregates*, Vol. 3, p.84 [online] https://doi.org/10.1520/CCA10209J.

Nomenclature

COV	Coefficient of variation of the data sample under consideration.
E	Expectation.
$f(n)$	Probability distribution function
$F(n)$	Cumulative distribution function
PCC	Polymer Concrete Composite
PFRPCC-0.5%	Polypropylene fibre reinforced polymer concrete composite-0.5% fibres
PFRPCC-1.0%	Polypropylene fibre reinforced polymer concrete composite-1.0% fibres
PFRPCC-2.0%	Polypropylene fibre reinforced polymer concrete composite-2.0% fibres
i	Order number of the data point in a sample under consideration.
k	Total number of data points in a sample under consideration or sample size.
LR	Survivorship function/survival probability / reliability function
n	Number of cycles
N	Number of cycles to failure or fatigue-life.
P_f	Failure probability.
r	Statistical co-relation coefficient.
u	Characteristic life or scale parameter of Weibull distribution.
α	Shape parameter of the Weibull distribution or Weibull slope
S	Applied fatigue stress level.
σ	Standard deviation of the data sample under consideration.
$\Gamma()$	Gamma function
μ	Mean value of the data sample under consideration