Sliding mode control for a wind turbine in finite frequency

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Abstract: This paper investigates a sliding mode control for a wind turbine in finite frequency (FFSMC). The sliding mode control (SMC) method can be designed for wind energy conversion systems. However, the fluctuations of wind speed perhaps reduce the robustness of the SMC. A dynamic compensator is introduced to design the sliding surface in order to overcome the difficulty of dealing with a fluctuation of wind speed, but the fast change of this fluctuation in certain finite ranges can lead to degrade the compensator performance. In order to solve this problem, a finite frequency approach based on the generalised Kalman-Yakubovich-Popov (KYP) lemma is proposed, and the compensator parameters are obtained in terms of linear matrix inequality (LMI) which can be solved efficiently using existing numerical tools. On the other hand, the reaching law is used to reduce the chattering that is produced by the traditional approach of sliding mode. Finally, the simulation results illustrate the effectiveness of the proposed control strategy compared to a non-singular terminal sliding mode control.

Keywords: finite frequency; sliding mode control; reaching law; wind turbine; systems modelling.


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1 Introduction

Global interest for clean and renewable energy sources has been growing intensively during the last years, due to high cost of fossil fuels as the limited sources of energy, and various worldwide agreements with the aim of reduction in carbon dioxide emissions. Wind turbine is one of the important renewable energy, it has received a strong impulse, reflected in great technology advances regarding reliability, cost-efficiency (Kousksou et al., 2015; Ydersbond and Korsnes, 2016; Kaivo-oja et al., 2016). Most works done in this field aim to maximise the energy captured from the wind and minimise the stress on the drive train shafts. In this way, the control strategy remains a key factor to optimise the extracted energy from the wind, and a number of controller strategies have been developed and applied to the wind energy systems such as: adaptive neuro-fuzzy control (Hafiz and Abdenour, 2016), maximum power point tracking control (Daili, 2015), intelligent maximum power point tracking control based on reinforcement-learning (Wei et al., 2015), Optimal control based on adaptive linear prediction (Narayana et al., 2017) and robust control (Aouani et al. 2017; Ghasemi et al., 2014). All those approaches can upgrade the robustness of the system to capture the maximal wind energy. However,
its performance may be degraded due to disturbances and defects in short domains because the stochastic nature of the wind flow, furthermore, note that when system disturbances occur within a finite frequency range which be known beforehand, it is not favourable to control the system in the full frequency band, because this may introduce some conservatism and poor performance of system. Recently, the control synthesis in a finite frequency has received increasing attention because of the advantages that it offers to the concept of performance in the finite ranges of frequency (Iwasaki and Hara 2005; Li and Yang 2015; Wang et al. 2015; Zhang et al. 2014). Also the sliding mode control has been developed increasingly due to its advantages of better performance against uncertainty effects and easy realisation (Moré et al., 2015; Sefriti et al., 2013; Qian et al., 2016; Berrada and Boumhidi, 2017).

In this manuscript, the finite frequency control is proposed to improve the SMC for a wind turbine against a sudden fast change of wind speed. The main step of the designing process of SMC is to design a sliding surface and ensure that the sliding mode equation is asymptotically stable. A dynamic compensator is used to overcome the difficulty of dealing with fluctuation of wind speed. So the fast fluctuations in certain finite ranges can be degrade the compensator performance, thus, the sliding mode equation becomes unstable. The finite frequency approach based on $H\infty$ performance is considered in a middle frequency by using the generalised KYP lemma (Iwasaki, 2015; Iwasaki and Hara, 2005) to give the compensator gains according to wind speed fluctuations, and keep the system on the sliding surface. In the second step of designing process, a reaching law (Wang et al., 2009) is used to reduce the chattering that is produced by traditional approach of sliding mode. The control system acts on generator in order to apply the reference electromagnetic torque calculated from the measurements of the rotational speed of the shaft at the generator side.

This paper is organised as follows. In Section 2, we will describe the model of the wind turbine system. In Section 3, we will design the proposed sliding mode control in finite frequency. We will show the performances of the proposed control strategy by simulation tests compared with the non-singular terminal sliding mode control (Rajendran and Jena, 2015) in Section 5. Section 6 concludes paper.

### 1.1 Nomenclature

- $\Omega_r$: Rotor speed $[\text{rad/s}]$
- $J_r$: Rotor inertia $[\text{kg.m}^2]$
- $K_r$: Rotor friction coefficient $[\text{N.m/rad/s}]$
- $\theta_{ls}$: Gearbox side angular deviation $[\text{rad}]$
- $T_{ls}$: Shaft torque $[\text{N.m}]$
- $B_s$: Shaft stiffness coefficient $[\text{N.m/rad}]$
- $K_s$: Shaft damping coefficient $[\text{N.m/rad/s}]$
- $\theta_{gs}$: Generator side angular deviation $[\text{rad}]$
- $T_{em}$: Generator electromagnetic torque $[\text{N.m}]$
- $J_g$: Generator inertia $[\text{kg.m}^2]$
- $\Omega_g$: Generator speed $[\text{rad/s}]$
- $\Theta_g$: Generator friction coefficient $[\text{N.m/rad/s}]$
- $\omega$: Rotation frequency $[\text{Hz}]$
- $v$: Wind speed $[\text{m/s}]$
- $\rho$: Air density $[\text{kg.m}^{-3}]$
- $n_g$: Transmission ratio

### 2 Wind turbine model

From a modelling standpoint, the wind turbine is established by combining a model of mechanical structure represent the drive trains and a nonlinear model represent the aerodynamics properties of the rotor (see Figure 1).

#### 2.1 Aerodynamics model

Applying the actuator disk theory (Liu and Yoshida, 2015), the rotor model provides the aerodynamics torque extracted from the wind by the following equation

$$P_a = \frac{1}{2} \rho \pi R^2 C_p(\lambda, \beta) v^3$$

where $\rho$ ($\text{Kg/m}^3$) is the air density, $R$ ($\text{m}$) is the rotor radius, $v$ ($\text{m/s}$) is the wind speed and $C_p(\lambda, \beta)$ (3) is the power coefficient which is a function of the tip speed ratio $\lambda$ (4) and the blade pitch angle $\beta$.
Sliding mode control for a wind turbine in finite frequency

\[ C_p(\lambda, \beta) = a_1 \left( a_2 \left( \frac{1}{\lambda + 0.08\beta} - \frac{0.035}{\beta^3 + 1} \right) - a_3\beta - a_4 \right) \times \exp \left( -a_5 \left( \frac{1}{\lambda + 0.08\beta} - \frac{0.035}{\beta^3 + 1} \right) + a_6\lambda \right) \]  

(3)

With the aerodynamic coefficients \( a_i \in \{i = 1, \ldots, 6\} \) are given in Table 1.

\[ \lambda = \frac{\Omega R}{v} \]  

(4)

According to (4), at low to medium wind speeds optimal \( \lambda \) can be achieved. This means that for the low to medium wind speeds, \( C_p \) is always at its maximum, since it is desirable to capture maximum power from the wind turbine.

2.2 Drive trains model

The mechanical model represented by a two-mass model with flexible structure, (see Figure 2).

By application of Newton’s second law of motion, the rotor inertia \( J_r \) is driven at a speed \( \Omega_r \) by the aerodynamic torque \( T_a \). Its dynamic is described by the following equation

\[ J_r \frac{d\Omega_r}{dt} = T_a - T_{hi} - K_r\Omega_r \]  

(5)

The low-speed shaft torque \( T_{hi} \) acts as a braking torque on the rotor. Its dynamic can be derived by using stiffness and damping factor of the low-speed shaft given in (6).

\[ T_{hi} = B_{hi} (\theta_i - \theta_h) + K_{hi} (\Omega_r - \Omega_{hi}) \]  

(6)

The generator inertia \( J_g \) is driven by the high-speed shaft torque and broken by the electromagnetic torque \( T_{em} \), which is developed, so that

\[ J_g \frac{d\Omega_g}{dt} = T_{gs} - K_g\Omega_g - T_{em} \]  

(7)

The main objective of gearbox is to adapt the rotation speed between the low-speed shaft and the high-speed shaft. For an ideal gearbox, the transmission ratio \( n_g \) is defined as

\[ n_g = \frac{T_{hi}}{T_{gs}} = \frac{\Omega_g}{\Omega_{hi}} = \frac{\theta_g}{\theta_{hi}} \]  

(8)

Using the equations (5)–(8), the two-mass model can be written as the following state equations

\[
\begin{bmatrix}
\dot{\Omega}_r \\
\dot{\Omega}_g \\
\dot{T}_{hi}
\end{bmatrix} =
\begin{bmatrix}
\frac{-K_r}{J_r} & 0 & -\frac{1}{J_r} \\
0 & \frac{-K_g}{J_g} & \frac{1}{n_g J_g} \\
B_{hi} \frac{K_{hi}}{J_r} & \frac{1}{n_g} \left( \frac{K_{hi} K_g}{J_g} - B_{hi} \right) & -K_{hi} \left( \frac{J_r + n_g^2 J_g}{n_g^2 J_g J_r} \right)
\end{bmatrix}
\begin{bmatrix}
\Omega_r \\
\Omega_g \\
T_{hi}
\end{bmatrix} +
\begin{bmatrix}
\frac{1}{J_r} \\
0 \\
T_a
\end{bmatrix}
\]

(9)

The nonlinear term of system (9) comes from the aerodynamics torque \( T_a \) that is rewritten as

\[ T_a = \frac{1}{2} \rho \pi R^3 C_p(\lambda, \beta) v^2 \]  

(10)

This nonlinear term can be approximated around an operating point as follows

\[ T_a = T_{a0} + \Delta T_a \]  

(11)

\[ \Delta T_a = T_{a0} \Delta v + T_{a0} \Delta \Omega_r + T_{a0} \Delta \beta \]  

(12)

The coefficients \( T_{a0}, T_{a0} \Delta \) and \( T_{a0} \beta \) are given in the Appendix A.

At low wind speed range (roughly 3–12 m/s), the pitch angle \( \beta \) is fixed at the optimal value \( (\Delta \beta = 0) \). Then, the aerodynamic torque \( T_a \) (11) is rewritten as \( \Delta T_a = T_a - T_{a0} \) and

\[ \Delta T_a = T_{a0} \Delta v + T_{a0} \Delta \Omega_r \]  

(13)

Finally, the state representation of the linearised system around an operating point is given as
\[
\begin{bmatrix}
\Delta \Omega_r \\
\Delta \Omega_g \\
\Delta T_{ls}
\end{bmatrix} = \begin{bmatrix}
\frac{T_{a1} - K_r}{J_r} & 0 & -\frac{1}{J_r} \\
0 & -\frac{K_g}{J_g} & -\frac{1}{n_g J_g} \\
\frac{B_{ls} - K_r K_t}{J_r} + \frac{T_{a1} K_{ls}}{J_r} & \frac{K_t K_r}{J_g} & -\frac{K_t}{n_g J_g}
\end{bmatrix}
\]

(14)

Figure 2 Wind turbine tower mass model (see online version for colours)

The system (14) can be reduced as the following representation

\[
\begin{align*}
\Delta \dot{x} &= A_\Delta \Delta x + B_\Delta \Delta \Theta_{em} + D_\Delta \Delta v \\
\Delta y &= C_\Delta \Delta x
\end{align*}
\]

(15) (16)

Where \(\Delta x = [\Delta \Omega_r \ \Delta \Omega_g \ \Delta T_{ls}]^T\), and \(C_\Delta = [1 \ 0 \ 0]\).

The wind speed can be approximated by the output of a linear filter, so the turbulent component model of the wind speed is represented as

\[
\Delta \dot{y} = T_{sw}\left( \frac{T_{av}}{J_g} \right) + 0 \Delta v
\]

(17)

With \(p_1, \ p_2\) and \(k\) are calculated by optimising the gap between the power spectral density (PSD) of wind speed (17–18) and the nonlinear PSD (Xin, 1997).

Let \(\dot{x} = [\Delta v \ \Delta \dot{v}]^T\), the state representation (17)–(18) becomes

\[
\begin{align*}
\dot{x}_v &= A_v x_v + B_v e_v \\
\Delta v &= C_v x_v
\end{align*}
\]

(19) (20)

Where: \(e_v\) represents the wind fluctuation.

The wind model included as a noise model. For this, the state representation (15) increased of the state vector \(x_v\) is adopted as

\[
\Delta \dot{x} = \begin{bmatrix} A \Delta x \\ D \Delta \Theta_{em} \end{bmatrix} x_v + \begin{bmatrix} B \Delta \Theta_{em} \\ 0 \end{bmatrix} e_v
\]

(21)

Let \(x = [\Delta x \ x_v]^T\), \(u = T_{em}\) and \(w = e_v\), (21) can be reformulated as follows

\[
\begin{align*}
\dot{x} &= Ax + Bu + Dw \\
y &= Cx
\end{align*}
\]

(22) (23)

where \(C = [C_\Delta \ 0 \ 0]\).
3 Control strategy design

Two operating areas of a variable speed wind turbine can be distinguished: below and above nominal power. Below the nominal power, the main objective of the controller is to maximise the energy captured from the wind and to minimise the stress of the wind turbine devices.

The power capture \( C_p(\lambda, \beta) \) curve has a unique maximum \( C_{p\text{max}} = C_p(\lambda_{\text{opt}}, \beta_{\text{opt}}) \) which is the optimal capture of the wind power. Consequently, for a partial load operating regime, and in order to maximise the wind power capture, the blade pitch angle \( \beta \) is fixed to its optimal value \( \beta_{\text{opt}} \) and in order to maintain \( \lambda \) at its optimal value, the rotor speed must be adjusted to track the optimal reference \( \Omega_{\text{opt}} \) given by \( \Omega_{\text{opt}} = \frac{\lambda_{\text{opt}}}{R} v \).

One may obviously observe that this reference has the same shape as the wind speed. The aim of the controller is to track this optimal rotor speed \( \Omega_{\text{opt}} \), while trying to reduce the control stress and dynamic loads.

The time responses of the wind turbine electrical system are much faster than those of the other components of the wind turbine. This makes it possible to dissociate the generator and the aero-turbine (mechanical and aerodynamic parts) control designs, and thus, define a cascaded control structure through two control loops.

1. The inner control loop concerns the electric generator via the power converters.
2. The outer control loop concerns the aero-turbine that provides the reference input of the inner loop.

Many works address the electrical part control without considering the aero-turbine control as Moreira et al. (2017) and Azzaoui and Mahmoudi (2017). Making the assumption that the inner loop is well controlled, this paper focuses on the aero-turbine control. The electric generator control is not considered, since the aim of the paper is to design high-level controller considering the electromagnetic torque \( T_{\text{em}} \) as control input of the system.

Our aim in this section is to design an appropriate finite frequency sliding mode control that guarantee the asymptotic stability of the closed-loop system in order to track a reference signal and attenuate the wind turbine vibration. This section consists of two parts. Firstly, we establish a sliding mode controller. Next, we adopt a reaching law and design the finite frequency controller to establish a sliding mode controller. Next, we adopt a vibration. This section consists of two parts. Firstly, we track a reference signal and attenuate the wind turbine as asymptotic stability of the closed-loop system in order to guarantee the control input of the system.

3.1 Sliding surface design

The objective of this subsection is to design an appropriate sliding surface for system (22)-(23). The wind fluctuation \( w \) makes the control design difficult. Thus, the following dynamic compensator is designed to overcome this fluctuation

\[
\dot{z} = Kx - z
\]

where \( z \in \mathbb{R} \) is the compensator state, \( K \in \mathbb{R}^{1 \times 5} \) is a vector to be designed later.

The sliding surface is defined by

\[
S = Hx + z
\]

where \( H \in \mathbb{R}^{1 \times 5} \). Differentiating the sliding surface (26) along the solution of the system (22)-(23) we obtain

\[
\dot{S} = H \dot{x} + \dot{z} = HAx + HDw + HBu + Kx - z
\]

Let \( \dot{S} = 0 \) and \( HB = 1 \), then the equivalent control is gotten as follows

\[
u_{\text{eq}} = -HAx - HDw - Kx + z
\]

Substitute (28) into (22) and it yields the following

\[
\dot{x} = Ax + B((-HAx - HDw - Kx + z) + Dw = (A - BHA - BK)x + Bz + Dw
\]

On the sliding surface \( S = 0 \), we have from (28) and (29) the following state representation

\[
\dot{x} = A_c x + D_c w
\]

\[
y = Cx
\]

where \( A_c = A - BHA - BK - BH \) and \( D_c = D - BHD \).

3.2 Finite frequency design

Firstly, we introduce the basic lemmas. All the proofs of the lemmas are given in the associated references.

**Lemma 1.** [Generalised KYP Lemma (Iwasaki, 2015; Iwasaki and Hara, 2005)]. Consider the system (30)-(31), a symmetric matrix \( F \) and a positive scalar \( w \) be given, then the following statements are equivalents

1. The finite frequency inequality

\[
\begin{bmatrix} G(jw) & 1 \\ I & I \end{bmatrix} \Phi \begin{bmatrix} G(jw) \\ I \end{bmatrix} < 0
\]

2. There exist symmetric matrices \( P \) and \( Q \) satisfying

\[
\begin{bmatrix} A_c & D_c \\ I & 0 \end{bmatrix} \Theta \begin{bmatrix} A_c & D_c \\ I & 0 \end{bmatrix} + \Phi < 0
\]

where for the middle frequency range \( \omega_1 \leq \omega \leq \omega_2 \).
The system state will reach the sliding surface in a finite time \( \tau \) determined such that the null space bases calculations yields

\[
\begin{bmatrix}
\mathbf{0} & \mathbf{1} & \mathbf{1}
\end{bmatrix}
\begin{bmatrix}
\mathbf{A}^c \\
\mathbf{D}^c
\end{bmatrix} < 0
\]

By using some algebraic operations, we can found the inequality (38).
Theorem 2. Consider the sliding surface defined in (26), and the equivalent control in (28). If the following feedback control used for the dynamic system (22)

$$u = \dot{H}x + H\dot{w} - Kx + z - \left( \mu + \eta \|S\|^{-1} \right) S \tag{50}$$

For any $0 < \alpha < 1$ and $\mu, \eta > 0$, the sliding mode always exists, i.e., the dynamics equation (27) with the fluctuations $w$ is asymptotically stable. And the state reaches the sliding surface $S = 0$ in finite time $\tau$ (lemma 3) from any initial point.

Proof. We choose the Lyapunov function

$$V = \frac{1}{2} \dot{S}^T S \tag{51}$$

$$\dot{V} = S^T \dot{S} \tag{52}$$

$$\dot{V} = S^T (\dot{H}x + H\dot{w} + HBu + Kx - z) \tag{53}$$

$$V = S^T (\dot{H}x + H\dot{w} + Kx - z) - \left( \mu + \eta \|S\|^{-1} \right) S \tag{54}$$

$$\dot{V} = -S^T (\mu + \eta \|S\|^{-1}) S \tag{55}$$

$$\dot{V} = -\mu \|S\|^2 - \eta \|S\|^2 \tag{56}$$

From lemma 3, the state $x$ will move toward the sliding surface $S = 0$ in a finite time. When the state reaches the sliding surface $S = 0$ it will keep on this surface, and the behaviour of the state system (22) can be represented by the dynamic system (30). Combined with Theorem 1, the dynamic system (30)–(31) is asymptotically stable in a certain range of frequency when the system suddenly disturbs, and in the remainder ranges with sliding mode. So the state system (22)–(23) with the control law (50) is asymptotically stable.

Figure 3  Simulation results under fixed wind speed: (a) wind speed profile, (b) rotor speed, (c) electromagnetic torque (control law) (d) tracking error (see online version for colours)
4 Simulation results

In this paper the wind turbine described in Section 2 with the numerical values given in Table 1 is considered to evaluate the performances of the proposed controller. The simulation tests are carried out by using MATLAB software under the following operating conditions:

- The constant/variable wind speed profile with mean value 10 m/s [see Figure 3(a) and Figure 4(a)].
- The sudden change of the wind speed is applied in the time interval [20s 30s].
- The operating frequency range of the aerodynamics rotor (from 0.05 Hz to 0.7 Hz).

By solving the linear matrix inequality (38) with the optimised parameter and the obtained control gain is

$$K = \begin{bmatrix} -8.457 & -0.004 & 0.0001 & -2.5090 & 1.1395 \end{bmatrix}^T.$$

The two tests carried out by the comparison between the proposed sliding mode control in finite frequency (FFSMC), classical sliding mode control (CSMC) and terminal sliding mode control (TSMC).

From the simulation results, Figure 3(b) for constant wind speed and Figure 4(b) for variable wind speed represent the time response of the rotor speed equipped with controller FFSMC, controller TSMC and controller CSMC in dashed red line, dashed blue line and dotted blue line, respectively, and the solid black line represents the optimal signal of rotor speed. We can observe that the proposed
control strategy yields the best tracking over the interval of time [20s 30s] compared to TSMC and CSMC.

Figure 3(c) and Figure 4(c) show the torque variation Tem with FFSMC in dashed red line, TSMC in dashed blue line and CSMC in dotted blue line we can notice that the control law of the proposed FFSMC is more stable without any oscillatory behaviour compared to the others (CSMC, TSMC).

Figure 3(d) and Figure 4(d) represent the tracking error of the aerodynamics rotor speed.

Table 1  Tow-mass model parameters

<table>
<thead>
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<th>Parameters</th>
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<td>$a_6$</td>
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5 Conclusions

In this paper, the sliding mode based on the reaching law lemma combined with the finite frequency control is designed for a wind turbine system, in order to reach the system at the sliding surface in finite time, and keep the system on this surface in a certain range of frequency. The designing of finite frequency approach is made using the generalised KYP lemma and the controller conditions have been derived in terms of LMI, which guarantee the asymptotic stability of the closed-loop system. The effectiveness of the proposed control strategy compared with the terminal sliding mode control has been realised and simulation results have shown an interesting performance of the proposed method to track the optimal reference faster without any oscillatory behaviour.

References


Appendix A

The aerodynamic torque $T_a$ in (10) is a nonlinear function depending on wind speed $v$, rotor speed $\Omega$, and the pitch angle $\beta$. At an operating point, the aerodynamic torque $T_a$ is developed as

$$T_a = T_{a0} + \Delta T_a$$  

(57)

$$\Delta T_a = T_a v + T_{\Omega \beta} \Delta \Omega + T_{\Omega v} \Delta \Omega$$  

(58)

where

$$\Delta \Omega = \Omega - \Omega_0, \Delta \beta = \beta - \beta_0, \Delta v = v - v_0$$  

(59)

and

$$T_{av} = \frac{\partial T_a}{\partial v}, \ T_{a\Omega} = \frac{\partial T_a}{\partial \Omega}, \ T_{a\beta} = \frac{\partial T_a}{\partial \beta}$$  

(60)

The partial derivatives of the aerodynamic torque (60) at the operating point can be calculated from the partial derivatives of the coefficient $C_q$ with respect to $\lambda$, $\beta$, and $(\partial C_q / \partial \lambda)$, where

$$C_q(\lambda, \beta) = \frac{C_p(\lambda, \beta)}{\lambda}$$  

(61)

From the expression of $T_a$ (4), we have

$$\frac{\partial T_a}{\partial \Omega} \bigg|_{pf} = \frac{R}{v_0}$$  

(62)

$$\frac{\partial T_a}{\partial v} \bigg|_{pf} = -\frac{\Omega_0 R}{v_0^2}$$  

(63)

Then it leads to the following expressions

$$T_{av} = \frac{\partial T_a}{\partial v} \bigg|_{pf} = \frac{1}{2} \rho \pi R^3 v_0 \left[ 2C_q(\lambda_0, \beta_0) - \lambda_0 \frac{\partial C_q}{\partial \lambda} \bigg|_{pf} \right]$$  

(64)

$$T_{a\Omega} = \frac{\partial T_a}{\partial \Omega} \bigg|_{pf} = \frac{1}{2} \rho \pi R^3 v_0 \frac{\partial C_q}{\partial \lambda} \bigg|_{pf}$$  

(65)

$$T_{a\beta} = \frac{\partial T_a}{\partial \beta} \bigg|_{pf} = \frac{1}{2} \rho \pi R^3 v_0^2 \frac{\partial C_q}{\partial \beta} \bigg|_{pf}$$  

(66)

At low wind speed, the linear model of aerodynamic torque $T_a$ is given as

$$T_a = T_{a0} + T_{av} \Delta v + T_{a\Omega} \Delta \Omega$$  

(67)

Where

$$T_{av} = \frac{\partial T_a}{\partial v} \bigg|_{pf} = \frac{1}{2} \rho \pi R^3 v_0 \left[ 2C_q(\lambda_0, \beta_0) - \lambda_0 \frac{\partial \lambda}{\partial \lambda} \right]$$  

(68)

$$T_{a\Omega} = \frac{\partial T_a}{\partial \Omega} \bigg|_{pf} = \frac{1}{2} \rho \pi R^3 v_0 \left[ 2C_q(\lambda_0, \beta_0) - \lambda_0 \frac{\partial \lambda}{\partial \lambda} \right]$$  

(69)

$C_q(\lambda_0)$ is obtained numerically from the tabulated function of $C_q$. 

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