Coordinating replenishment and marketing policies for non-instantaneous deteriorating items with imprecise deterioration free time and general deterioration and holding cost rates

Amirhosein Patoghi* and Mostafa Setak

Department of Industrial Engineering,
K.N. Toosi University of Technology,
Tehran, Iran
Email: amirhpa.ah@gmail.com
Email: Setak@kntu.ac.ir
*Corresponding author

Abstract: In this paper an inventory system for non-instantaneous deteriorating items with imprecise deterioration free time is developed. We adopt a price and advertisement frequency dependent demand function, and in order to reach a general framework, arbitrary functions of deterioration and holding cost rates are hired. The major objective is to determine the optimal selling price, the optimal replenishment cycle and the optimal frequency of advertisement such that, the total profit is maximised. In order to determine the optimal solution several theoretical results are derived which indicate existence and uniqueness of the optimal solution. Thereafter, based on these theoretical results an iterative solution is developed. Finally, numerical examples are provided to demonstrate solution procedure, then sensitivity analysis is performed, it is shown that optimal policy under uncertain environment and crisp environment are identical and finally some managerial insights are proposed.

Keywords: inventory; imprecise deterioration free time; arbitrary deterioration and holding cost rates; price and advertisement dependent demand.

Reference to this paper should be made as follows: Patoghi, A. and Setak, M. (2018) 'Coordinating replenishment and marketing policies for non-instantaneous deteriorating items with imprecise deterioration free time and general deterioration and holding cost rates', *Int. J. Inventory Research*, Vol. 5, No. 1, pp.38–59.

Biographical notes: Amirhosein Patoghi graduated with Master of Industrial Engineering from the K.N. Toosi University of Technology, Tehran. His research interests are in the fields of supply chain management, inventory management and routing problems.

Mostafa Setak is an Associate Professor in the Department of Industrial Engineering, K.N. Toosi University of Technology, Tehran. He has several years of research experience in inventory management, supply chain management and vehicle routing problems.

1 Introduction

In modern economical environment, marketing activities play an important role in ensuring customer satisfaction, increasing the level of demand and the revenue of business sectors. As well as the importance of determining marketing policies such as frequency of advertisement and specially determining optimal selling price, inventory control policy is also a crucial element in improving the performance and profitability of business sectors. When the items are deteriorating the importance of these factors is increased. In literature, deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, loss of utility, or loss of marginal value of commodity, which decreases usefulness. Electronic goods, blood banks, medicine, fashionable items are instances for deteriorating items. For the first time a model with exponentially decaying inventory was presented by Ghare and Scharder (1963). Covert and Philip (1973) extended that model hiring two parameter Weibull's distributed deterioration rate. Philip (1974) stated the cases in which two parameter Weibull's distributed deterioration rate is insufficient and introduced three parameter Weibull's distributed deterioration rate. A comprehensive review of the literature on deteriorating items is provided by Goyal and Giri (2001).

It is observed that majority of commodities have a time span for maintaining quality or freshness, during which no deterioration occurs. This phenomenon is known as "non-instantaneous deterioration". In the real world almost goods, such as first hand vegetables and fruits, fashionable items and electronic products have a time span in which no deterioration occurs. Therefore, in order to reach appropriate decision for replenishment policies, the inventory problem for non-instantaneous deteriorating items must be considered. A model with non-instantaneous deteriorating items was initially proposed by Wu et al. (2006). They considered a stock dependent demand and partial backlogging in their modelling. Afterwards several researches like Ouyang et al. (2008), Chang et al. (2010), Maihami and Kamalabadi (2012), and Musa and Sani (2012) have investigated non-instantaneous deteriorating items under different conditions.

As aforementioned, price plays an important role in success of a business. Wee (1997) formulated an optimal replenishment policy for deteriorating items with varying deterioration rate in which demand is a linear function of price. Abad (2003) developed a joint pricing and lot-sizing with consideration of perishability, finite production rate and partial backlogging. Dye et al. (2007) presented a joint pricing and ordering policy for deteriorating items with partial backlogging in which backlogging rate depends on the total number of customers in waiting line. A joint pricing and replenishment policy for non-instantaneous deteriorating items with price sensitive demand was developed by Wu et al. (2009). Sana (2010) established an order quantity model for deteriorating items with price dependent demand and partial backlogging. Begum et al. (2012) developed an inventory control policy for deteriorating items with Wiebull's distributed deterioration rate and price dependent demand. Zhang et al. (2014) formulated and solved a joint pricing and replenishment model for deteriorating items with preservation technology investment. Their model enables the retailer to determine its selling price, the length of replenishment cycle and the technology investment. Maihami and Karimi (2014) developed a joint pricing and replenishment policy for non-instantaneous deteriorating items. They considered promotional efforts and price dependent stochastic demand function in their modelling. Soni and Patel (2013) studied a joint pricing and replenishment policy for non-instantaneous deteriorating items with imprecise deterioration free time and they applied triangular fuzzy number to characterise the length of deterioration free time. Soni (2013) extended the work of Chang et al. (2010) from to aspects, he discussed a model in which delay in payment is permissible, and moreover he considered price and stock sensitive demand. Afterwards, Wu et al. (2014) developed the study of Soni (2013) by adding ending inventory as salvages and considering all possible replenishment cycle time. Khurana and Chaudhary (2016) presented an optimal pricing and ordering policy for deteriorating items with partial backlogging. In their study demand rate is a function of stock level and selling price. Taleizadeh et al. (2015) developed pricing and ordering decisions in a supply chain with inspection under buyback of defective items and imperfect quality items. Shah et al. (2016) formulated and solved a joint dynamic pricing and ordering policy for deteriorating items. They considered trapezoidal demand rate and assumed controllable deterioration rate in their study. Taleizadeh et al. (2017) studied the price effect by developing a model when announced price increase occurs. They also considered probabilistic replenishment intervals and partial backordering. In another study, Tavakoli and Taleizadeh (2017) developed a model for deteriorating items in which full payment scheme is allowed.

Another marketing parameter which has become prevalent in business world is advertisement. Advertisement significantly affects the demand rate and the profit of the inventory system. Urban (1992) developed an economic production quantity (EPQ) in which demand is a function of selling price and advertisement. Urban and Baker (1997) studied a single period environment, where demand is a function of price, time and level of inventory. In that model unsold items at the end of season are sold at the lower price. Tan et al. (2003) developed a deterministic inventory model, in which demand is a function of price, advertisement expenditure and on hand inventory. Shah et al. (2013) considered an inventory system for non-instantaneous deteriorating items for optimising inventory and marketing policy. In that paper demand is a function of selling price and advertisement. In addition, they considered generalised deterioration and holding cost rates. Taleizadeh et al. (2013) developed an inventory control model to obtain the optimal order and shortage quantities for a perishable item when the supplier proffers special sale. Geetha and Udayakumar (2016) discussed an inventory model for non-instantaneous deteriorating items in which demand is a function of selling price and advertisement. They considered salvage value for deteriorating items and in their model shortages are partially backlogged. A joint dynamic pricing and replenishment policies for non-instantaneous deteriorating items was developed by Rabbani et al. (2016). They discussed simultaneous physical and quantity deterioration and time-dependent inventory holding cost in their study.

An appropriate marketing and inventory policy for non-instantaneous deteriorating items with imprecise deterioration free time and generalised type deterioration and holding cost rates is presented in this paper. The main objective is to simultaneously determine the optimal selling price, the optimal length of replenishment cycle, the optimal advertisement frequency and order quantity. To the best of our knowledge this study is the first analysis to jointly consider optimising marketing and inventory policy for non-instantaneous deteriorating items with imprecise deterioration free time and generalised type deterioration and holding cost rates.

The remainder of the paper is structured as follows. In Section 2, assumptions and notations are provided. In Section 3, crisp and fuzzy inventory models are developed. Section 4 explains solution procedure. Some numerical examples and sensitivity analysis are presented to illustrate the model in Section 5. Finally, Section 6 finishes the paper with concluding remarks.

2 Assumptions and notations

The mathematical model in this work is developed on the basis of following assumptions and notations.

2.1 Assumptions

1. The inventory system under consideration deals with single non-instantaneous deteriorating items.
2. The time horizon is infinite and a typical planning schedule of cycle of length T is considered.
3. Replenishment is instantaneous and the lead time is taken negligible.
4. Demand rate $D(A, p)$ is a function of marketing parameters which are frequency of advertisement (A) and the selling price (p). In this paper the power form of the selling price and the frequency of advertisement are assumed for the demand function, i.e., $D(A, p) = A^{\eta} ap^{-b}$ where $a(> 0)$ is the scaling factor, $b(> 1)$ is the index of price elasticity, and η is the shape parameter, where $0 \leq \eta < 1$. (Shah et al., 2013)
5. The length of deterioration free time t_d, is imprecise per se and defined by triangular fuzzy number $\widetilde{td} = (td - \tau_1, td, td + \tau_2)$, where $0 < \tau_1 < td$ and $0 < \tau_2$. τ_1 and τ_2 are decided by decision maker intrinsically. td is characterised through fuzzy number, since triangular fuzzy numbers are easy to handle when analytical solutions are preferred. After td the on hand inventory deteriorates with variable rate $\theta(t)$, where $0 < \theta(t) < 1$.
6. Deteriorated units are not repaired or replaced during the period under consideration.
7. Shortages are not allowed to avoid the lost sales.
8. The system operates for an infinite planning horizon.

2.2 Notations

k	The ordering cost per order
td	The length of deterioration free time

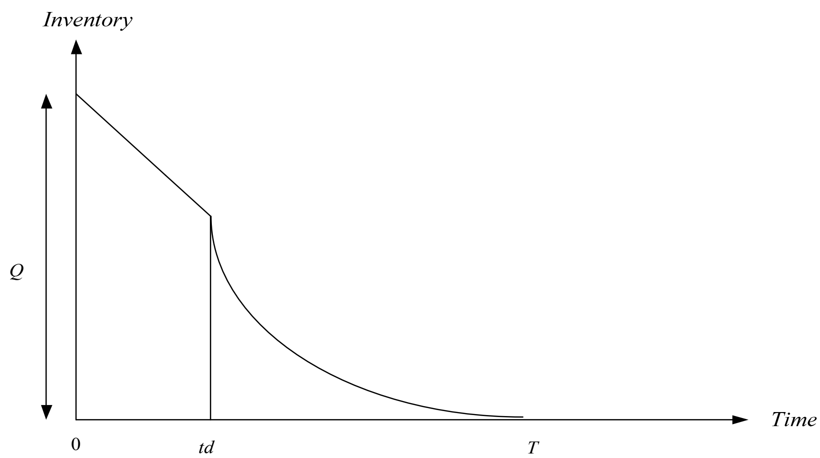
A	Frequency of advertisement per cycle
G	Cost for each advertisement
C	The purchasing cost per unit
p	The selling price per unit
$h(t)$	Unit holding cost per unit at time t
Q	The order quantity
T	Length of replenishment cycle ($T \geq td$)
$\theta(t)$	The deterioration rate of the on-hand inventory over $[td, T]$
$I_1(t)$	The inventory level at time t ($0 \leq t \leq td$) when the product has no deterioration
$I_2(t)$	The inventory level at time t ($0 \leq t \leq td$) when the product has deterioration
$\Pi(A, p, T)$	The total profit per unit time of inventory system.

3 Model formulation

3.1 Crisp inventory model

At the start of the cycle, the inventory level reaches its maximum Q units of item at time $t = 0$. During the time interval $[0, td]$, the inventory depletes due to demand. After td the inventory level declines to zero owning to demand and deterioration until the end of the current order cycle (see Figure 1).

Figure 1 Graphical representation of the inventory system



The differential equation that represents the inventory status at any instant of time $t \in [0, td]$, is given by

$$\frac{dI_1(t)}{dt} = -D(A, p), \quad 0 \le t \le td \tag{1}$$

In the second interval [*td*, *T*], the inventory level decreases due to demand and deterioration.

Thus, the following differential equation represents the inventory status.

$$\frac{dI_2(t)}{dt} = -\theta(t)I_2(t) - D(A, p), \quad td \le t \le T \tag{2}$$

With boundary conditions $I_1(0) = Q$, $I_2(T) = 0$ and using the assumptions and equations (1) and (2), we have

$$I_1(t) = Q - D(A, p)t, \quad 0 \le t \le td \tag{3}$$

$$I_2(t) = D(A, p) f(t) \int_t^T \frac{1}{f(y)} dy, \quad td \le t \le T \tag{4}$$

where $f(y) = \exp(\int_y^T \theta(x)dx)$.

From the continuity of $I(t)$ at $t = td$ and applying the condition $I_1(td) = I_2(td)$ in equations (3) and (4), we get

$$Q - D(A, p)td = D(A, p) f(td) \int_{td}^T \frac{1}{f(y)} dy$$

Which yields that the order quantity per cycle is

$$Q = D(A, p)\left[td + f(td) \int_{td}^T \frac{1}{f(y)} dy \right] \tag{5}$$

Replacing equations (5) in (3), we have

$$I_1(t) = D(A, p)\left[f(td) \int_{td}^T \frac{1}{f(y)} dy \right] + D(A, p)(td - t), \quad 0 \le t \le td \tag{6}$$

Based on the obtained inventory levels, we can obtain the inventory costs and the sale revenue per cycle, which consist of the following elements:

OC the ordering cost

$$OC = k$$

AC the advertisement cost

$$AC = G \times A$$

HC the inventory holding cost

$$HC = \left(\int_0^{td} h(t) I_1(t) dt + \int_{td}^{T} h(t) I_2(t) dt \right)$$

$$= D(A,p) \left\{ \int_0^{td} (td - t) h(t) dt + \int_{td}^{T} f(t) h(t) \int_t^T 1\!\big/\!f(y)\, dy dt + f(td) \int_0^{td} h(t) dt \int_{td}^{T} 1\!\big/\!f(y)\, dy \right\}$$

PC the purchase cost

$$PC = c \times Q$$

SR the sale revenue

$$SR = pD(A,\, p)T$$

Therefore, the total profit per unit time [denoted by $\Pi(A, p, T)$] is given by

$$\begin{aligned} \Pi(A,\, p, T) &= \frac{1}{T} \{ SR - OC - HC - PC \} \\ &= pD(A,\, p) - (K + AG) \\ &\quad - \frac{D(A,\, p)}{T} \left\{ \begin{aligned} &\int_{td}^{T} f(t) h(t) \int_t^T 1\!\big/\!f(y)\, dy dt - \int_0^{td} t h(t) dt \\ &+ \left(c + \int_0^{td} h(t) dt \right) \left(td + f(td) \int_{td}^{T} 1 / f(y) dy \right) \end{aligned} \right\} \end{aligned} \tag{7}$$

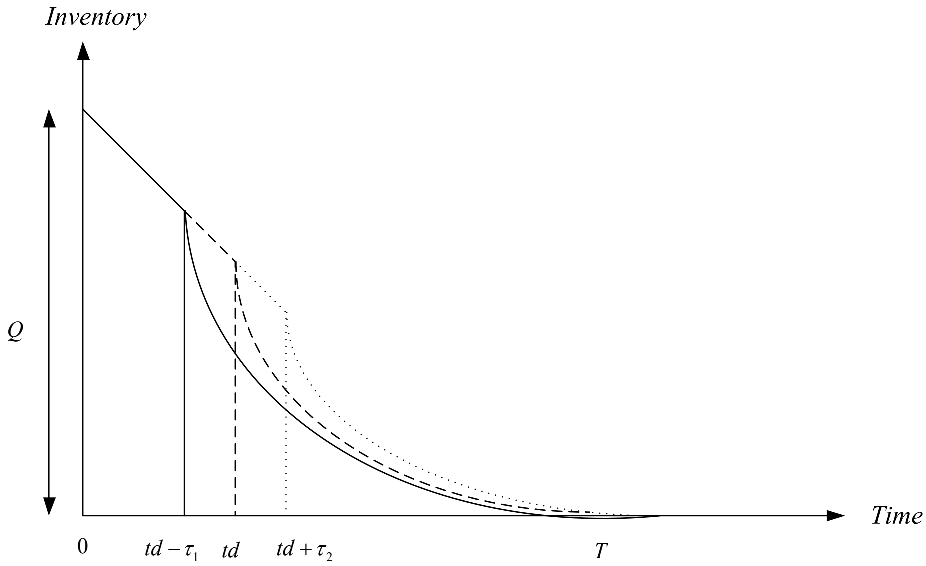
Therefore, the crisp model is $\Pi(A, p, T)$

Subject to $T \geq td$ (8)

3.2 Fuzzy inventory model

In this paper, we have defined the length of deterioration free time with a triangular fuzzy number. The behaviour of inventory system when deterioration free time is characterised as triangular fuzzy number is projected in Figure 2.

Figure 2 Graphical representation of the fuzzy inventory system



Therefore, under fuzzy framework our crisp problem described in equation (8) converts as follow

$$\text{Maximise } \widetilde{\Pi}(A, p, T)$$

$$\text{Subject to } Cr\left\{\widetilde{td} \le T\right\} \ge \alpha \tag{9}$$

Note that $\alpha \in (0, 1)$ is pre-determined confidence level of fuzzy constraint and $\widetilde{\Pi}(A, p, T)$ is defined as follow

$$\widetilde{\Pi}(A, p, T) = pD(A, p) - (K + AG) - \frac{D(A, p)}{T}\left\{\begin{array}{l}\int_{\widetilde{td}}^{T} f(t)h(t)\int_{t}^{T} {}^{1}\!/\!{}_{f(y)}\, dydt - \int_{0}^{\widetilde{td}} th(t)dt \\ + \left(c + \int_{0}^{\widetilde{td}} h(t)dt\right)\left(\widetilde{td} + f(\widetilde{td})\int_{\widetilde{td}}^{T} 1/f(y)dy\right)\end{array}\right\} \tag{10}$$

According to Soni and Potel (2013), the described problem in equation (9) converts into a multi-objective non-linear programming which is as follows

$$\text{Maximise } [\Pi_L(A, p, T), \Pi_R(A, p, T)]$$
$$\text{Subject to } T \geq td + (2\alpha - 1)\tau_2 \text{ or } T \geq td - (1 - 2\alpha)\tau_1 \tag{11}$$

where

$$\Pi_L(A, p, T) = pD(A, p) - (K + AG) - \frac{D(A, p)}{T}\left\{\begin{array}{l}\int_{td_l}^{T} f(t)h(t)\int_{t}^{T} 1/f(y)\,dydt - \int_{0}^{td_l} th(t)dt \\ +\left(c + \int_{0}^{td_r} h(t)dt\right)\left(td_r + f(td_l)\int_{td_l}^{T} 1/f(y)dy\right)\end{array}\right\} \tag{12}$$

$$\Pi_R(A, p, T) = pD(A, p) - (K + AG) - \frac{D(A, p)}{T}\left\{\begin{array}{l}\int_{td_r}^{T} f(t)h(t)\int_{t}^{T} 1/f(y)\,dydt - \int_{0}^{td_r} th(t)dt \\ +\left(c + \int_{0}^{td_l} h(t)dt\right)\left(td_l + f(td_l)\int_{td_r}^{T} 1/f(y)dy\right)\end{array}\right\} \tag{13}$$

Also the order quantity per cycle is as follow

$$\widetilde{Q} = [Q_L, Q_R] = \left[D(A, p)\left[td_l + f(td_r)\int_{td_r}^{T} 1/f(y)\,dy\right], D(A, p)\left[td_r + f(td_l)\int_{td_l}^{T} 1/f(y)\,dy\right]\right] \tag{14}$$

4 Solution procedure

First, the multi-objective problem defined in equation (11) must be transferred to a single objective optimisation problem. Herein, $\Pi_L(A, p, T)$ and $\Pi_R(A, p, T)$ can be construed as the pessimistic and optimistic returns, respectively. Hence, in order to be able to have interaction between pessimistic and optimistic decisions, we hire weighted sum method (WSM) to solve the multi-objective problem defined in equation (11). Note that, WSM transfers a multi-objective problem into single objective by applying relative weight of objective functions. In other words, WSM enables retailer to have interaction between his/her pessimistic and optimistic returns. Thus, the problem defined in equation (11) transfers into single objective problem given by

Maximise $\Pi_w(A, p, T)$

Subject to $T \geq td + (2\alpha - 1)\tau_2$ or $T \geq td - (1 - 2\alpha)\tau_1$

where w_1 and w_2 are weighting coefficients and

$$\Pi_w(A, p, T) = w_1\Pi_L(A, p, T) + w_2\Pi_R(A, p, T) \tag{15}$$

Due to the high complexity of aforementioned equations, it is not possible to hire Hessian matrix to prove concavity of the total profit. Instead, to obtain optimal solution we apply a search procedure which has been incorporated in Wu et al (2006), Shah et al (2013) and Soni and Patel (2013) as well. Accordingly, first of all for given p and A existence of a unique optimal value of T is proved. Afterwards for fixed A and known T unique optimal value of p which maximises the total profit is obtained. Then, we prove concavity of profit function with respect to A, and since frequency of advertisement is a discrete variable, the aforementioned procedure is implemented for all different value of A.

The first order partial derivative of $\Pi w(A, p, T)$ with respect to T gives

$$\begin{aligned}
\frac{\partial\Pi_w(A, p, T)}{\partial T} = \frac{K + AG}{T^2} + \frac{D(A, p)}{T^2}\Bigg\{ & -Tw_1\left(f(td_l)\int_0^{td_r} h(t)dt + \int_{td_l}^{T} f(t)h(t)dt \right) \\
& +w_1\left(\int_0^{td_l} (td_l - T)h(t)dt + c\left(td_l - f(td_l)T + f(td_r)\int_{td_r}^{T} \frac{1}{f(y)}dy \right.\right. \\
& \left.\left. +f(td_r)\int_0^{td_l} h(t)dt\int_{td_r}^{T} \frac{1}{f(y)}dy + \int_{td_r}^{T} f(t)h(t)\int_t^T \frac{1}{f(y)}dydt \right)\right) \\
& -Tw_2\left(f(td_r)\int_0^{td_l} h(t)dt + \int_{td_r}^{T} f(t)h(t)dt \right) \\
& +w_2\left(\int_0^{tdr} (td_r - T)h(t)dt + c\left(td_r - f(td_r)T + f(td_l)\int_{td_l}^{T} \frac{1}{f(y)}dy \right.\right. \\
& \left.\left. +f(td_l)\int_0^{td_r} h(t)dt\int_{td_l}^{T} \frac{1}{f(y)}dy + \int_{td_l}^{T} f(t)h(t)\int_t^T \frac{1}{f(y)}dydt \right)\right)\Bigg\}
\end{aligned} \tag{16}$$

Motivated from equation (16), the auxiliary function $g(T)$, $T \in [td, \infty]$ is defined as follow

$$\begin{aligned} g(T) = \Bigg\{ & -Tw_1\left(f(td_l)\int_0^{td_r} h(t)dt + \int_{td_l}^{T} f(t)h(t)dt\right) \\ & +w_1\left(\int_0^{td_l}(td_l - T)h(t)dt + c\left(td_l - f(td_l)T + f(td_r)\int_{td_r}^{T} \frac{1}{f(y)}dy\right.\right. \\ & \left.\left. +f(td_r)\int_0^{td_l} h(t)dt\int_{td_r}^{T}\frac{1}{f(y)}dy + \int_{td_r}^{T} f(t)h(t)\int_t^T \frac{1}{f(y)}dydt\right)\right) \\ & -Tw_2\left(f(td_r)\int_0^{td_l} h(t)dt + \int_{td_r}^{T} f(t)h(t)dt\right) \\ & +w_2\left(\int_0^{tdr}(td_r - T)h(t)dt + c\left(td_r - f(td_r)T + f(td_l)\int_{td_l}^{T}\frac{1}{f(y)}dy\right.\right. \\ & \left.\left. +f(td_l)\int_0^{td_r} h(t)dt\int_{td_l}^{T}\frac{1}{f(y)}dy + \int_{td_l}^{T} f(t)h(t)\int_t^T \frac{1}{f(y)}dydt\right)\right)\Bigg\} \end{aligned} \tag{17}$$

Taking the first order derivative of $g(T)$, $T\in [td, \infty]$ with respect to $T\in [td, \infty]$, we obtain

$$\begin{aligned} \frac{dg}{dT} = -T\Bigg\{ & w_1\left(h(T) + \theta(T)\int_{td_l}^{T} f(t)h(t)dt + f(td_l)\left(c + \int_0^{td_r} h(t)dt\right)\right) \\ & +w_2\left(h(T) + \theta(T)\int_{td_l}^{T} f(t)h(t)dt + f(td_l)\left(c + \int_0^{td_r} h(t)dt\right)\right)\Bigg\} < 0 \end{aligned}$$

Therefore, $g(T)$ is a strictly decreasing function of $T\in [td, \infty]$. Furthermore we can obtain the result $\lim_{T\to\infty} g(T) = -\infty$.

For notational convenience, let

$$\Delta_1 = g\left(td - (1-2\alpha)\tau_1\right) \tag{18}$$

and

$$\Delta_2 = g\left(td + (2\alpha - 1)\tau_2\right) \tag{19}$$

Part 1. If $\Delta_1 \geq 0$ (or $\Delta_2 \geq 0$), applying intermediate value theorem, there exist a unique value $T(T_1 \in [td - (1 - 2\alpha)\tau_1, \infty]$ or $T_1 \in [td + (2\alpha - 1)\tau_2, \infty])$ such that $g(T_1) = 0$. This reveals that T_1 is the unique solution of $\partial\Pi(A, p, T)/\partial T = 0$. From equations (16) and (17) we know the following

$$\frac{\partial\Pi(A, p, T)}{\partial T} = \frac{D(A, p)g(T)}{T^2} \tag{20}$$

At point $T = T_1$

$$\left[\frac{\partial^2\Pi(A,p,T)}{\partial T^2}\right]_{T=T_1}$$
$$=\frac{-D(A,p)}{T_1}\left\{w_1\left(h(T_1)+\theta(T_1)\int_{td_l}^{T_1} f(t)h(t)dt+f(td_l)\left(c+\int_0^{td_r} h(T_1)dt\right)\right)\right.$$
$$\left.+w_2\left(h(T_1)+\theta(T_1)\int_{td_l}^{T_1} f(t)h(t)dt+f(td_l)\left(c+\int_0^{td_r} h(T_1)dt\right)\right)\right\}<0$$

Therefore, $T_1 \in [td + (2\alpha - 1)\tau_2, \infty]$ ($T_1 \in [td - (1 - 2\alpha)\tau_1, \infty]$) is the global maximum solution of $\Pi_w(A, p, T)$.

Part 2. If $\Delta_1 < 0$ (or $\Delta_2 < 0$), since $g(T)$ is a strictly decreasing function of $T \in [td + (2\alpha - 1)\tau_2, \infty]$ ($T \in [td - (1 - 2\alpha)\tau_1, \infty]$), then $g(T)$, $\forall T \in [td + (2\alpha - 1)\tau_2, \infty]$ ($T \in [td - (1 - 2\alpha)\tau_1, \infty]$). It follows that $\Pi_w(A, p, T)$ is a strictly decreasing function of $T \in [td + (2\alpha - 1)\tau_2, \infty]$ ($T \in [td - (1 - 2\alpha)\tau_1, \infty]$). As a result $\Pi_w(A, p, T)$ reaches its maximum value at $td + (2\alpha - 1)\tau_2$ (or $td - (1 - 2\alpha)\tau_1$).

Hence, for fixed A and known p, Pareto optimal solution T^* which maximises $\Pi_w(A, p, T)$ is as follow

$$T^*=\begin{cases} T_1 & \textit{if}\ \Delta_1 \ge 0 \text{ or } \Delta_2 \ge 0 \\ td-(1-2\alpha)\tau_1 & \textit{if}\ \Delta_1 < 0 \text{ and } \alpha \le 0.5 \\ td+(2\alpha-1)\tau_2 & \textit{if}\ \Delta_2 < 0 \text{ and } \alpha \ge 0.5 \end{cases} \tag{21}$$

Lemma 1. There exist a unique value p^* which maximises the profit function $\Pi_w(A, p, T)$ for fixed A and $T^* \in [td\,(1 - 2\alpha)\tau_1, \infty]$ (or $T^* \in [td + (2\alpha - 1)\tau_2, \infty]$).

Proof. The first partial derivative of $\Pi_w(A, p, T)$ with respect to is as follow

$$\begin{aligned}\frac{\partial\Pi_w(A,p,T)}{\partial p} &= pD'(A,p)+D(A,p)\\ &-\frac{D'(A,p)}{T}\left\{w_1\left(\begin{aligned}&\int_{td_l}^{T} f(t)h(t)\int_t^T \tfrac{1}{f(y)}dydt-\int_0^{td_l} th(t)dt\\ &+\left(c+\int_0^{td_r} h(t)dt\right)\left(td_r+f(td_l)\int_{td_l}^{T} 1/f(y)dy\right)\end{aligned}\right)\right.\\ &\left.+w_2\left(\begin{aligned}&\int_{tdr}^{T} f(t)h(t)\int_t^T \tfrac{1}{f(y)}dydt-\int_0^{tdr} th(t)dt\\ &+\left(c+\int_0^{tdl} h(t)dt\right)\left(td_l+f(td_l)\int_{td_r}^{T} 1/f(y)dy\right)\end{aligned}\right)\right\}\end{aligned} \tag{22}$$

where $D'(A, p)$ is the derivative of $D(A, p)$.

By solving $\dfrac{\partial \Pi_w(A,p,T)}{\partial p}=0$, p^* results

$$p^*=\frac{b}{(b-1)T}\left\{w_1\left(\begin{array}{l}\int_{td_l}^{T} f(t)h(t)\int_{t}^{T} \frac{1}{f(y)}dydt-\int_{0}^{td_l} th(t)dt\\ +\left(c+\int_{0}^{td_r} h(t)dt\right)\left(td_r+f(td_l)\int_{td_l}^{T} 1/f(y)dy\right)\end{array}\right)\right.$$
$$\left.+w_2\left(\begin{array}{l}\int_{tdr}^{T} f(t)h(t)\int_{t}^{T} \frac{1}{f(y)}dydt-\int_{0}^{tdr} th(t)dt\\ +\left(c+\int_{0}^{tdl} h(t)dt\right)\left(td_l+f(td_l)\int_{td_r}^{T} 1/f(y)dy\right)\end{array}\right)\right\} \tag{23}$$

At point $p=p^*$

$$\left[\frac{\partial^2 \Pi_w(A,\, p,T)}{\partial p^2}\right]_{p=p^*}=\frac{(1-b)D(A,\, p)}{p}<0$$

Therefore, p^* is the global Pareto optimal solution of $\Pi_w(A, p, T)$ for fixed A and $T^* \in [td\ (1-2\alpha)\tau_1, \infty]$ (or $T^* \in [td + (2\alpha-1)\tau_2, \infty]$).

Taking the second order partial derivative of $\Pi_w(A, p, T)$ with respect to A, for fixed p and T, we have

$$\frac{\partial^2 \Pi_w(A,\, p,\, T)}{\partial A^2}=\frac{D(A,p)\eta(\eta-1)(pT-X)}{A^2T}\le 0$$

where

$$X=w_1\left(\begin{array}{l}\int_{td_l}^{T} f(t)h(t)\int_{t}^{T} \frac{1}{f(y)}dydt-\int_{0}^{td_l} th(t)dt\\ +\left(c+\int_{0}^{td_r} h(t)dt\right)\left(td_r+f(td_l)\int_{td_l}^{T} 1/f(y)dy\right)\end{array}\right)$$
$$+w_2\left(\begin{array}{l}\int_{tdr}^{T} f(t)h(t)\int_{t}^{T} \frac{1}{f(y)}dydt-\int_{0}^{tdr} th(t)dt\\ +\left(c+\int_{0}^{tdl} h(t)dt\right)\left(td_l+f(td_l)\int_{td_r}^{T} 1/f(y)dy\right)\end{array}\right)$$

Thus, $\Pi_w(A, p, T)$ is concave function of A. Hence, find a local optimal solution results the optimal frequency of advertisement A^*.

Now, Due to the concavity behaviour of the objective function with respect to all decision variables, the following algorithm which is similar to ones proposed in Wu et al. (2009), Shah et al. (2013) and Soni and Potel (2013), is developed to find global Pareto optimal solution of (A, p, T). In addition, as explained in Wu et al. (2009) convergence of the proposed algorithm can easily be demonstrated by applying a similar graphical technique employed in Hadley and Whitin (1963).

Algorithm 1	
Step 1	set $A = 1$.
Step 2	set $k = 1$ and initialise the value of $p^{(k)} = c$.
Step 3	if $\alpha \leq 0.5$ go to step 4 otherwise go to step 5.
Step 4	calculate Δ_1 from equation (18) and apply any one of the following cases.
(4.1)	if $\Delta_1 \geq 0$, then obtain the value of T_1^k by solving $\frac{\partial \Pi_w(A, p, T)}{\partial T} = 0$ .Substitute the value of T_1^k into equation (23) in order to find corresponding $p^{(k)}$. Set $p^{(k+1)} = p^{(k)}$ and $T^k = T_1^k$.
(4.2)	if $\Delta_1 < 0$, set $T^{(k)} = td - (1 - 2\alpha)\tau_1$ and calculate the value of $p^{(k)}$ from equation (23). Set $p^{(k+1)} = p^{(k)}$.
Step 5	calculate Δ_2 from equation (19) and apply any one of the following cases.
(5.1)	if $\Delta_2 \geq 0$, then obtain the value of T_1^k by solving $\frac{\partial \Pi_w(A, p, T)}{\partial T} = 0$. Substitute the value of T_1^k into equation (23) in order to find corresponding $p^{(k)}$. Set $p^{(k+1)} = p^{(k)}$ and $T^k = T_1^k$. .
(5.2)	if $\Delta_2 \geq 0$, set $T^{(k)} = td - (1 - 2\alpha)\tau_1$ and calculate the value of $p^{(k)}$ from equation (23). Set $p^{(k+1)} = p^{(k)}$.
Step 6	if $\lvert p^{(k+1)} = p^{(k)} \rvert < Epsilon$, then set $(p^*, T^*) = (p^{(k+1)}, T^{(k)})$ and go to step 7. (p^*, T^*) is the Pareto optimal solution. Otherwise set $k = k + 1$ and go to step 2.
Step 7	calculate $\Pi_w(A, p^*, T^*)$. This amount is the maximum value of the objective function for fixed A.
Step 8	set $A' = A + 1$ and repeat step 2 to 7 to find $\Pi_w(A', p^*, T^*)$ and go to step 9.
Step 9	if $\Pi_w(A', p^*, T^*) \geq \Pi_w(A, p^*, T^*)$, set $A = A'$ and go to step 8, otherwise go to step 10.
Step 10	set $(A^*, p^*, T^*) = (A, p^*, T^*)$ and (A^*, p^*, T^*) is the Pareto optimal solution.
Step 11	compute $\Pi_L(A^*, p^*, T^*)$, $\Pi_R(A^*, p^*, T^*)$ and respectively from equations (12), (13) and (15). Also calculate corresponding Q_L and Q_R from equation (14).

5 Experimental results

To illustrate solution procedure and validity of proposed model, we apply the proposed algorithm to solve the following numerical examples.

Example1. Same parameters of the model are taken from Soni and Potel (2013) and Shah et al. (2013) and adopted to our model. k = \$250 per order, c = \$3 per unit, G = \$80 per advertisement, τ_1 = 3/365 year, τ_2 = 5/365 year, $w_1 = w_2$ and θ = 0.08. Holding cost function and demand function are as follows

$$h(t)=\begin{cases} 0.4, & t \le td \\ 0.4+0.2(t-td), & t \ge td \end{cases}$$

$$D(A, p) = 400{,}000 A^{0.04} p^{-25}$$

First, we obtain the results of the above example by applying the proposed algorithm. As shown in Table 1, the optimal solution is (A^*, p^*, T^*) = (3, 5.23, 0.45) and thus corresponding profit function Π_W (A^*, p^*, T^*) is 12,896.85. Figure 3 illustrates concavity behaviour of the objective function for fixed A, hence optimal solution is a global maximum solution. Moreover, we investigate the case in which deterioration free time $\widetilde{td}$, is not fuzzy. In this case, for crisp value of td, we consider the middle value of interval [td^l, td^r], i.e., $td = {td^l + td^r}/{2} = 26/365$. As Table 1 reveals these optimum values does not differ much from optimum values obtained by applying multi-objective approach with equal weighting coefficients. This result reveals that the proposed model is advantageous whenever the uncertainty exists in deterioration free time.

Table 1 Computational result for crisp *td* vs. fuzzy *td*

td	*A**	*p**	*T**	*Q**	*Π**
Fuzzy	3	5.23	0.456	3,085.77	12,869.9
Crisp	3	5.22	0.459	3,113.75	12,927.2

Figure 3 Profit function for fixed *A* (see online version for colours)

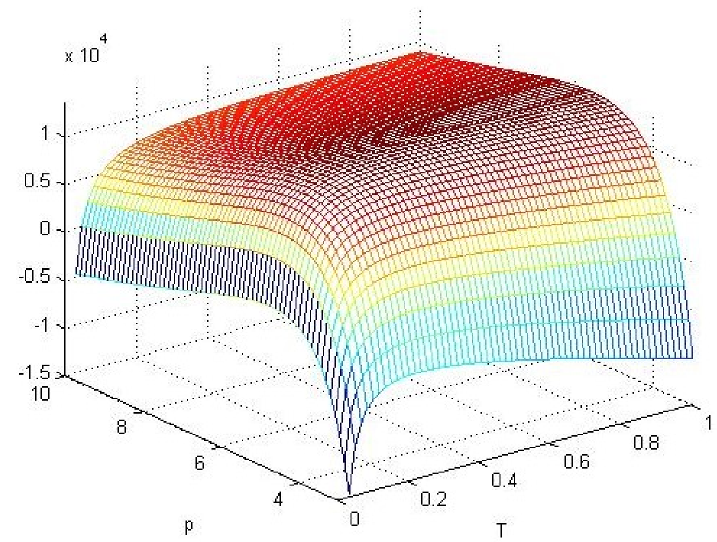


Table 2 Computational results for different value of η and td when $\theta = 0.08$

θ	η	td	δ	A^*	p^*	T^*	Q_w^*	Π_w^*
0.08	0.03	0	0	2	5.23623	0.43859	2,905.36	12,699.2
			0.2	2	5.23647	0.42101	2,786.58	12,659.2
			0.4	2	5.23703	0.40644	2,687.89	12,622.1
			0.6	2	5.23777	0.39405	2,603.68	12,587.5
			0.8	2	5.23865	0.38328	2,530.38	12,554.8
			1	2	5.23962	0.37378	2,465.61	12,523.9
		15/365	0	2	5.21994	0.43792	2,914.41	12,761.6
			0.2	2	5.21891	0.42262	2,812.25	12,731.8
			0.4	2	5.21821	0.40977	2,726.26	12,704.1
			0.6	2	5.21772	0.39872	2,652.18	12,678.3
			0.8	2	5.2174	0.38904	2,587.23	12,654
			1	2	5.21719	0.38045	2,529.49	12,631
		30/365	0	2	5.20607	0.43892	2,932.36	12,818.5
			0.2	2	5.20431	0.42583	2,845.85	12,796.8
			0.4	2	5.20287	0.41468	2,772.1	12,776.6
			0.6	2	5.20165	0.405	2,707.93	12,757.8
			0.8	2	5.20061	0.39645	2,651.25	12,740.1
			1	2	5.19971	0.38881	2,600.55	12,723.4
	0.04	0	0	3	5.25653	0.47577	3,199.15	12,842.6
			0.2	3	5.25685	0.45527	3,058.35	12,795
			0.4	3	5.25755	0.43852	2,942.85	12,751.1
			0.6	3	5.25848	0.4244	2,845.2	12,710.2
			0.8	3	5.25956	0.41222	2,760.8	12,671.9
			1	3	5.26074	0.40154	2,686.63	12,635.6
		15/365	0	3	5.24005	0.4749	3,208.32	12,906.3
			0.2	3	5.23896	0.45685	3,085.77	12,869.9
			0.4	3	5.23826	0.4419	2,983.96	12,836.3
			0.6	3	5.23782	0.42916	2,897.09	12,805.2
			0.8	3	5.23758	0.4181	2,821.49	12,775.9
			1	3	5.23747	0.40834	2,754.69	12,748.4
		30/365	0	3	5.2258	0.47559	3,225.77	12,964.8
			0.2	3	5.22385	0.45992	3,120.48	12,937.5
			0.4	3	5.22229	0.44676	3,031.91	12,912.4
			0.6	3	5.22101	0.43544	2,955.63	12,889.1
			0.8	3	5.21993	0.42554	2,888.78	12,867.2
			1	3	5.21901	0.41674	2,829.37	12,846.6

Table 3 Computational results for different value of η and td when θ = 0.01

θ	η	td	δ	A^*	p^*	T^*	Q_w^*	Π_w^*
0.1	0.03	0	0	2	5.24762	0.41956	2,773.7	12,612.4
			0.2	2	5.24784	0.40473	2,673.39	12,575.8
			0.4	2	5.24831	0.39218	2,588.32	12,541.5
			0.6	2	5.24896	0.38134	2,514.61	12,509.3
			0.8	2	5.24972	0.37181	2,449.69	12,478.7
			1	2	5.25058	0.36332	2,391.76	12,449.7
		15/365	0	2	5.22724	0.4188	2,784.79	12,690.2
			0.2	2	5.22626	0.406	2,699.21	12,663.2
			0.4	2	5.22554	0.39503	2,625.73	12,638
			0.6	2	5.22501	0.38544	2,561.47	12,614.2
			0.8	2	5.22462	0.37695	2,504.45	12,591.8
			1	2	5.22434	0.36934	2,453.27	12,570.5
		30/365	0	2	5.21005	0.42004	2,806.46	12,760.8
			0.2	2	5.2084	0.40918	2,734.59	12,741.4
			0.4	2	5.20701	0.39975	2,672.15	12,723.3
			0.6	2	5.20581	0.39143	2,617.02	12,706.2
			0.8	2	5.20477	0.384	2,567.73	12,690.1
			1	2	5.20384	0.3773	2,523.22	12,674.8
	0.04	0	0	3	5.26898	0.45514	3,053.75	12,746.8
			0.2	3	5.26926	0.43784	2,934.76	12,703.2
			0.4	3	5.26986	0.42339	2,835	12,662.6
			0.6	3	5.27067	0.411	2,749.31	12,624.6
			0.8	3	5.27162	0.40019	2,674.33	12,588.6
			1	3	5.27266	0.39062	2,607.8	12,554.5
		15/365	0	3	5.24835	0.45415	3,065.01	12,826.1
			0.2	3	5.24729	0.43903	2,962.21	12,793.1
			0.4	3	5.24656	0.42623	2,875	12,762.5
			0.6	3	5.24606	0.41515	2,799.4	12,733.8
			0.8	3	5.24573	0.40541	2,732.8	12,706.7
			1	3	5.24553	0.39672	2,673.36	12,681.1
		30/365	0	3	5.23066	0.45503	3,086.11	12,898.7
			0.2	3	5.22882	0.442	2,998.48	12,874.4
			0.4	3	5.2273	0.43083	2,923.25	12,851.7
			0.6	3	5.22602	0.42108	2,857.46	12,830.5
			0.8	3	5.22492	0.41243	2,799.08	12,810.5
			1	3	5.22397	0.40467	2,746.68	12,791.6

We now study the effect of changes in key parameters of system and based on this sensitive analysis, managerial insights which are analogous to Shah et al. (2013) are extended and the computational results are summarised in Tables 2 and 3.

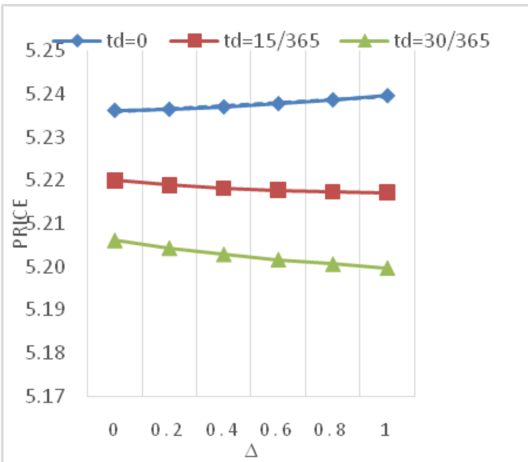
1 By increasing the value of δ when other parameters are fixed, the length of optimal replenishment cycle T^*, economic order quantity Q_w^* and the total system profit Π_w^* decrease, while optimal frequency of advertisement A^* remains unchanged. This result indicates that when holding cost increases, retailer prefers to order smaller quantity of items and storage this amount of items for a shorter period of time. When $td = 0$ (instantaneous deteriorating items) an increase in δ causes an increase in optimal selling price p^*, while when $td \neq 0$ (non-instantaneous deteriorating items) p^* decreases with an increase in the value of the δ. This behaviour of system is reflected in Figure 4.

2 By increasing the value of deterioration rate when other parameters remain unchanged, optimal length of replenishment cycle T^*, economic order quantity Q_w^* and total system profit Π_w^* decrease, while optimal frequency of advertisement A^* is not sensitive to deterioration rate.

3 When the value of deterioration free time td increases, economic order quantity Q_w^*, total system profit Π_w^* increase and optimal selling price p^* will decreases, while optimal frequency of advertisement A^* remains unchanged with this increase.

4 When the value of shape parameter η increase, optimal frequency of advertisement A^*, optimal length of replenishment cycle T^* and optimal selling price p^* will increase.

Figure 4 Variation of price with δ when $\eta = 0.03$ (see online version for colours)



Example 2. In this example we investigate the effect of demand function parameters a and b. In this regard same set of input data are considered as in Example 1. The computational results for $a \in (200{,}000, 400{,}000, 600{,}000)$ and $b \in (2, 2.5, 3)$ are summarised in Table 4.

Table 4 Computational results for different value of *a* and *b*

a	b	A^*	p^*	T^*	Q_w^*	Π_w^*
200,000	2	4	6.37796	0.50872	2,712.87	15,452.5
	2.5	2	5.34756	0.55461	1,774.49	5,911.9
	3	1	4.88188	0.66065	1,176.05	2,297.76
400,000	2	7	6.30685	0.42391	4,704.44	32,367.7
	2.5	3	5.25515	0.42311	2,851.72	12,719.9
	3	1	4.75267	0.45998	1,753.52	5,185.44
600,000	2	9	6.26814	0.37694	6,398.65	49,684.6
	2.5	4	5.21866	0.36991	3,837.31	19,738.3
	3	2	4.72245	0.41154	2,459.06	8,223.86

Based on computational results, we can derive the following managerial insights

1 By increasing the value of scaling factor a, the optimal frequency of advertisement A^*, the optimal weighted order quantity Q_w^* and the optimal weighted total profit Π_w^* increase, while the optimal replenishment cycle T^* and the optimal selling price p^* decrease with an increase in scaling factor a.

2 By increasing the value of price elasticity index b, the optimal frequency of advertisement A^*, the optimal selling price p^* and the optimal weighted total profit Π_w^* decrease.

Example 3. In this example we discuss the influence of changes in these two fuzzy parameters τ_1 and τ_2 on Pareto optimal solution of Example 1.The computational results for different values of τ_1 and τ_2 are summarised in Table 5.

Based on these computational results, we can obtain the following managerial insights

1 For fixed value of τ_2, by increasing the value of parameter τ_1, the optimal weighted quantity Q_w^*, the optimal length of replenishment cycle T^*, the optimal total weighted profit Π_w^* and the individual optimum value of objective function Π_L^* decrease whereas the optimal selling price p^* and the individual optimum value of objective function Π_R^* increase with an increase in value of parameter τ_1. It is obvious that by increasing the value of τ_1, spread on left from t_d would be wider. Therefore, the length of deterioration free time deceases and as a result reduces optimal order quantity and optimal total profit.

2 For fixed value of τ_1, by increasing the value of parameter τ_2, the optimal weighted quantity Q_w^*, the optimal length of replenishment cycle T^*, the optimal total weighted profit Π_w^* and the individual optimum value of objective function increase whereas the optimal selling price p^* and the individual optimum value of objective function Π_R^* decrease with an increase in value of parameter τ_1. By an increase in fuzzy parameter τ_2, it is clear that the average length of deterioration free time increases and thereby from view point of retailer, when the length of deterioration

free time is longer, he/she should reduce the selling price and order larger quantity per cycle to gain more profit per unit time.

It should be noted that frequency of advertisement is not sensitive to fuzzy parameters τ_1 and τ_2.

Table 5 Computational results for different value of τ_1 and τ_2

τ_2	τ_1	A^*	p^*	T^*	Q_w^*	Π_L^*	Π_R^*	Π_w^*
3/365	1/365	5	5.1871	0.367981	5,182.741	18,749.54	35,492.65	27,121.1
	3/365	5	5.1877	0.367933	5,181.133	14,560.73	39,671.39	27,116.06
	5/365	5	5.1883	0.367902	5,179.7	10,374.08	43,847.66	27,110.87
	7/365	5	5.189	0.367889	5,178.442	6,190.324	48,020.71	27,105.52
5/365	1/365	5	5.1867	0.368105	5,185.139	14,569.42	39,681.42	27,125.42
	3/365	5	5.1873	0.368071	5,183.662	10,382.35	43,858.19	27,120.27
	5/365	5	5.1879	0.368053	5,182.36	6,198.054	48,031.86	27,114.96
	7/365	5	5.1886	0.368052	5,181.233	2,017.274	52,201.7	27,109.49
7/365	1/365	5	5.1863	0.368247	5,187.712	10,391	43,868.17	27,129.59
	3/365	5	5.1869	0.368225	5,186.366	6,206.288	48,042.35	27.124.32
	5/365	5	5.1875	0.36822	5,185.195	2,024.968	52,212.81	27,118.89
	7/365	5	5.1882	0.368233	5,184.199	–2,152.22	56,378.83	27,113.3

6 Concluding remarks

In this paper we have established joint marketing and inventory policy optimisation for non- instantaneous deteriorating items. In order to be consistent with real life, the length of deterioration free time is considered imprecise and to tackle this uncertainty, the imprecise length of deterioration free time is defined as triangular fuzzy number. To achieve a general framework an arbitrary holding cost rate and arbitrary deterioration rate are incorporated. The proposed study considers a demand rate function which in addition to selling price depends on frequency of advertisement. To the best of our knowledge, this study is the first analysis to consider optimal inventory and marketing parameters (replenishment cycle, order quantity, selling price and frequency of advertisement) for non-instantaneous deteriorating items with imprecise deterioration free time and arbitrary functions of holding cost and deterioration rate. To illustrate the optimal solution, some useful theoretical results are derived based upon which an iterative solution algorithm is developed. Computational results indicate that optimal policy under uncertain environment is identical with the optimal policy under the crisp environment. As a result the developed model is capable to aid retailers to decide about optimal policy when uncertainty exists. The work presented here could have several extensions. One could extend the proposed model by considering the effect of preservation technology investment. Also consideration of trade credit and time value of money could be another extension of the proposed model. It would also be interesting to study accommodate planned shortages, stochastic demand, multi items and so forth. Another idea would be to consider supplier selection or multi-stage supply chain.

References

Abad, P.L. (2003) 'Optimal pricing and lot-sizing under conditions of perishability, finite production and partial backordering and lost sale', *European Journal of Operational Research*, Vol. 144, No. 3, pp.677–685.

Begum, R., Sahoo, R.R. and Sahu, S.K. (2012) 'A replenishment policy for items with price-dependent demand, time-proportional deterioration and no shortages', *International Journal of Systems Science*, Vol. 43, No. 5, pp.903–910.

Chang, C.T., Teng, J.T. and Goyal, S.K. (2010) 'Optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand', *International Journal of Production Economics*, Vol. 123, No. 1, pp.62–68.

Covert, R.P. and Philip, G.C. (1973) 'An EOQ model for items with Weibull distribution deterioration', *AIIE Transactions*, Vol. 5, No. 4, pp.323–326.

Dye, C.Y., Hsieh, T.P. and Ouyang, L.Y. (2007) 'Determining optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging', *European Journal of Operational Research*, Vol. 181, No. 2, pp.668–678.

Geetha, K.V. and Udayakumar, R. (2016) 'Optimal lot sizing policy for non-instantaneous deteriorating items with price and advertisement dependent demand under partial backlogging', *International Journal of Applied and Computational Mathematics*, Vol. 2, No. 2, pp.171–193.

Ghare, P.M. and Schrader, G.F. (1963) 'A model for exponentially decaying inventory', *Journal of industrial Engineering*, Vol. 14, No. 5, pp.238–243.

Goyal, S.K. and Giri, B.C. (2001) 'Recent trends in modeling of deteriorating inventory', *European Journal of Operational Research*, Vol. 134, No. 1, pp.1–16.

Hadley, G. and Whitin, T. (1963) *Analysis of Inventory Systems*, pp.169–172, Prentice-Hall, Englewood Cliff NJ.

Khurana, D. and Chaudhary, R. (2016) 'Optimal pricing and ordering policy for deteriorating items with price and stock dependent demand and partial backlogging', *Uncertain Supply Chain Management*, Vol. 4, No. 4, pp.307–318.

Lin, C., Tan, B. and Lee, W-C. (2000) 'An EOQ model for deteriorating items with time-varying demand and shortages', *International Journal of Systems Science*, Vol. 31, pp.391–400.

Maihami, R. and Kamalabadi, I.N. (2012) 'Joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand', *International Journal of Production Economics*, Vol. 136, No. 1, pp.116–122.

Maihami, R. and Karimi, B. (2014) 'Optimizing the pricing and replenishment policy for non-instantaneous deteriorating items with stochastic demand and promotional efforts', *Computers and Operations Research*, Vol. 51, pp.302–312 [online] http://www.sciencedirect.com/science/article/pii/S0305054814001579.

Musa, A. and Sani, B. (2012) 'Inventory ordering policies of delayed deteriorating items under permissible delay in payments', *International Journal of Production Economics*, Vol. 136, No. 1, pp.75–83.

Ouyang, L.Y., Wu, K.S. and Yang, C.T. (2008) 'Retailer's ordering policy for non-instantaneous deteriorating items with quantity discount, stock-dependent demand and stochastic backorder rate', *Journal of the Chinese Institute of Industrial Engineers*, Vol. 25, No. 1, pp.62–72.

Philip, G.C. (1974) 'A generalized EOQ model for items with Weibull distribution deterioration', *AIIE Transactions*, Vol. 6, No. 2, pp.159–162.

Rabbani, M., Zia, N.P. and Rafiei, H. (2016) 'Joint optimal dynamic pricing and replenishment policies for items with simultaneous quality and physical quantity deterioration', *Applied Mathematics and Computation*, Vol. 287, pp.149–160 [online] http://www.sciencedirect.com/science/article/pii/S0096300316302661.

Sana, S.S. (2010) 'Optimal selling price and lot size with time varying deterioration and partial backlogging', *Applied Mathematics and Computation*, Vol. 217, No. 1, pp.185–194.

Shah, N.H., Shah, D.B. and Patel, D.G. (2016) 'Optimal preservation investment, pricing and ordering policies for deteriorating inventory with trapezoidal demand', *International Journal of Operational Research*, Vol. 26, No. 3, pp.367–381.

Shah, N.H., Soni, H.N. and Patel, K.A. (2013) 'Optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates', *Omega*, Vol. 41, No. 2, pp.421–430.

Soni, H.N. (2013) 'Optimal replenishment policies for non-instantaneous deteriorating items with price and stock sensitive demand under permissible delay in payment', *International Journal of Production Economics*, Vol. 146, No. 1, pp.259–268.

Soni, H.N. and Patel, K.A. (2013) 'Joint pricing and replenishment policies for non-instantaneous deteriorating items with imprecise deterioration free time and credibility constraint', *Computers and Industrial Engineering*, Vol. 66, No. 4, pp.944–951.

Taleizadeh, A.A., Mohammadi, B., Cárdenas-Barrón, L.E. and Samimi, H. (2013) 'An EOQ model for perishable product with special sale and shortage', *International Journal of Production Economics*, Vol. 145, No. 1, pp.318–338.

Taleizadeh, A.A., Noori-daryan, M. and Tavakkoli-Moghaddam, R. (2015) 'Pricing and ordering decisions in a supply chain with imperfect quality items and inspection under buyback of defective items', *International Journal of Production Research*, Vol. 53, No. 15, pp.4553–4582.

Taleizadeh, A.A., Zarei, H.R. and Sarker, B.R. (2017) 'An optimal control of inventory under probabilistic replenishment intervals and known price increase', *European Journal of Operational Research*, Vol. 257, No. 3, pp.777–791.

Tan, B., Jung Hsieh, P. and Lin, C. (2003) 'Marketing policies on economic order quantity', *Journal of Information and Optimization Sciences*, Vol. 24, No. 1, pp.197–210.

Tavakoli, S. and Taleizadeh, A.A. (2017) 'An EOQ model for decaying item with full advanced payment and conditional discount', *Annals of Operations Research*, pp.1–22 [online] https://link.springer.com/article/10.1007/s10479-017-2510-7.

Urban, T.L. (1992) 'Deterministic inventory models incorporating marketing decisions', *Computers and Industrial Engineering*, Vol. 22, No. 1, pp.85–93.

Urban, T.L. and Baker, R.C. (1997) 'Optimal ordering and pricing policies in a single-period environment with multivariate demand and markdowns', *European Journal of Operational Research*, Vol. 103, No. 3, pp.573–583.

Wee, H.M. (1997) 'A replenishment policy for items with a price-dependent demand and a varying rate of deterioration', *Production Planning and Control*, Vol. 8, No. 5, pp.494–499.

Wu, J., Skouri, K., Teng, J.T. and Ouyang, L.Y. (2014) 'A note on 'optimal replenishment policies for non-instantaneous deteriorating items with price and stock sensitive demand under permissible delay in payment'', *International Journal of Production Economics*, Vol. 155, pp.324–329 [online] http://www.sciencedirect.com/science/article/pii/S0925527313005768.

Wu, K.S., Ouyang, L.Y. and Yang, C.T. (2006) 'An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging', *International Journal of Production Economics*, Vol. 101, No. 2, pp.369–384.

Wu, K.S., Ouyang, L.Y. and Yang, C.T. (2009) 'Coordinating replenishment and pricing policies for non-instantaneous deteriorating items with price-sensitive demand', *International Journal of Systems Science*, Vol. 40, No. 12, pp.1273–1281.

Zhang, J., Bai, Z. and Tang, W. (2014) 'Optimal pricing policy for deteriorating items with preservation technology investment', *Management*, Vol. 10, No. 4, pp.1261–1277.