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## Element damage assessment in semi rigid connected structures using modal domain data

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**Abstract:** Current vibration-based damage assessment techniques assumes that the joints in a structure behave either perfectly rigid or perfectly flexible. However, joints in the bolted structures are semi-rigid, which means the elements may show rotations to a certain degree on the basis of a fixity factor. In this regard, the present work proposes an alternate solution to conventional inverse damage identification problems by providing due weightage to the joint rigidity. The beam-column joints for this study are made semi-rigid by reducing their end fixity factor. An inverse problem is formulated here using changes in the natural frequencies and mode shape vectors caused due to damages, which is then solved by a unified particle swarm optimisation algorithm. The efficacy of this procedure is demonstrated by conducting a few numerical and experimental studies. The outcomes suggest that the developed procedure is able to locate and quantify damages in a semi-rigidly connected structure with significant accuracy.

**Keywords:** element damage assessment; semi-rigid connection; modal data; inverse problem; stiffness reduction factor; SRF; unified particle swarm optimisation; UPSO.

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## 1 Introduction

Since the past three decades, there has been significant progress on damage identification methods based on vibration data, which are based on the fact that the presence of damage in a structure causes local stiffness loss and hence results in changes in its vibration characteristics. The primary benefit of such damage identification methods lies in the global nature of vibration due to which it is not necessary to conduct at-the-point measurements. Thus, these methods are quite attractive for rapid and automated damage identification of large structures. Extensive reviews on such methodologies have been reported in Doebling et al. (1998), Carden and Fanning (2004), Fan and Qiao (2011) and Das et al. (2016).

In vibration-based damage identification problems, an objective function, defined in terms of discrepancies between the vibration data identified by modal testing and those

computed from the analytical model, is optimised to derive the damage conditions (Hao and Xia, 2002). Most of the early researchers such as, Nandwana and Maiti (1997), Contursi et al. (1998), Morassi (2001), Chinchalkar (2001), Lee et al. (2004), Khiem and Lien (2004) and Patil and Maiti (2005) have preferably used natural frequencies for damage identification as it is fast and easy to measure natural frequency than mode shape. However, Aktan et al. (1994) observed that natural frequencies are less sensitive to local damages, particularly in the low-stress regions. Moreover, in structures with symmetrical conditions of geometry, material, and boundary conditions, two different states of damage in two symmetrical elements can produce the same change in natural frequencies (Dado and Shpli, 2003). To avoid such shortfalls, several authors have used a combination of natural frequencies and mode shapes for damage identification (Hao and Xia, 2002; Dado and Shpli, 2003; Teughels et al., 2002; Yeung and Smith, 2005; Wang et al., 2011; Mehrian et al., 2016). Yet there exist a few challenges in the implementation of vibration-based damage identification methods. Firstly, the vibration property of the structure is less sensitive to small and local damages. Secondly, the presence of noise in measurement data makes damage identification process very complex and unstable. Thus, damage identification in such conditions needs a powerful optimisation tool. However, the ill-conditioning of these optimisation problems creates difficulties like instability and non-assured convergence in adapting derivative-based optimisation techniques for solving such problems (Meruane and Heylen, 2011). In comparison, modern derivative-free, nature-inspired heuristic algorithms have the ability to solve such problems effectively.

Maity and Tripathy (2005) proposed a damage identification technique using the genetic algorithm (GA) and natural frequencies for the cantilever beam and plane frame. Baghmisheh et al. (2008) used binary and continuous genetic algorithms for crack identifications in numerical beams. Moreover, Gomes and Silva (2008) made a comparison among damage identification techniques based on genetic algorithm and modal sensitivity methods. Although they observed the performances of both methods are similar for simple structures, the model sensitivity method requires a lot of simplifications and assumptions, which may affect its performance in damage identification of complex structures. Perera et al. (2009) proposed a multi-criteria-based objective function, which consists of natural frequency, mode shape, and flexibility matrix, and the resulting problem is solved using a Pareto-genetic algorithm. They evaluated the performance of this algorithm for beam-like structures. Na et al. (2011) evaluated damages in an experimental shear building using structural flexibility matrix solved by the genetic algorithm. Although it is observed in the literature that many researchers have used GA in solving such problems; however, it takes a comparatively long time for convergence, and false damages are observed in many cases. Pal et al. (2013) compared performances of modal curvature, frequency response function curvature, curve fitting technique, and kurtosis-based algorithms for joint damage assessment in frame structures. It is observed that the FRF curvature-based damage indices performed better and could able to localise damage in a stable and consistent manner, especially using noisy input data. Navabian et al. (2018) developed a damage assessment algorithm based on firefly optimisation and mode shape derivatives for damage identification in plate like structures.

Nowadays, swarm-based corporative search algorithms are widely being used for solving damage identification problems. Majumdar et al. (2012, 2013) used ant colony optimisation (ACO) technique for estimating damages in the beam, and truss-like

structure whereas Majumdar et al. (2014), Nanda et al. (2014c) and Yu and Xu (2011) used real coded ACO for damage identification in beams and shear frames. Braun et al. (2015) used different versions of ACO metaheuristic solely or coupled with the Hooke-Jeeves (HJ) local search algorithm to identify structural stiffness coefficients of a damped spring-mass system. Begambre and Laier (2009) used a hybrid version of particle swarm optimisation (PSO) by combining the simplex algorithm (PSOS) to identify damages. Using frequency domain data, they validated the developed algorithm for identification of damages in a ten-bar truss, beams, and a nonlinear oscillator where they observed good identification results. Sandesh and Shankar (2010) carried out crack identification of plate structures using time-domain acceleration responses, which is solved using a hybrid version of GA and PSO. Kang et al. (2012) used an artificial immune system based mechanisms such as selection, receptor editing, and vaccination in the PSO algorithm to provide immunity enhanced PSO, which they used for damage identification in beams and trusses. Seyedpoor (2012) provided a two-stage damage identification method using a modal strain energy-based index and PSO algorithm. Mohan et al. (2013) used FRF data and PSO algorithm for damage identification in the beam and frame-like structures. Nanda et al. (2012) used natural frequency and incremental PSO algorithms for damage identification. Further, Nanda et al. (2014a) demonstrated that the location and amount of cracks could be decided in a frame structure from changes in modal parameters using a unified version of PSO. Moreover, Kaveh and Maniat (2015), Vaez and Fallah (2017) and Wei et al. (2018) have proposed the use of different versions of PSO technique for assessment of structural damages.

From all these extensive literature reviews, it is observed that most of the damage assessment techniques assume that the joints in a structure behave either perfectly rigid or perfectly flexible. However, joints in the bolted structures are semi-rigid, which means the elements may show rotations to a certain degree based on a fixity factor. This leaves a potential source of error. Sometimes, these errors are eliminated by adjusting the material properties of the elements. However, this assumption also provides another source of error. Instead, due consideration should be given to these errors by adjusting the joint rigidity. In this regard, the present work proposes an alternate solution to conventional damage identification inverse problems by providing due weightage to the joint rigidity. In this case, the unified PSO algorithm is used for solving the inverse problem relating the cause and effect. The performance of the method is demonstrated by a numerical and experimental plane frame structures.

## 2 Theory and formulation

This section discusses the finite element formulation of a structure with semi-rigid connections. Then a discussion on unified PSO is elaborated, which is followed by discussion on the development of the objective function for the present study.

### 2.1 Elemental matrixes for semi-rigid connected structure

In the present study, the semi-rigid connections are modelled as rotational springs of infinitesimal length attached at the ends of the beam, which is shown in Figure 1. The rotational stiffness ( $k'$ ) for this is given by Eurocode 3 (2005).

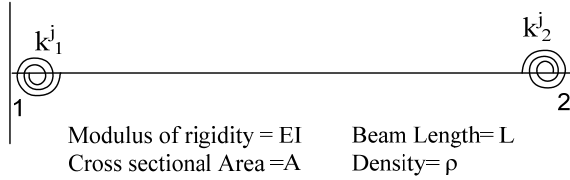
$$k^j = \alpha \frac{EI}{L} \tag{1}$$

where the terms  $L$ ,  $E$ , and  $I$  have the usual meaning as shown in the figure. The term  $\alpha$  is a multiplication factor named as stiffness index. The joint fixity factor ( $J_i$ ) is calculated from the stiffness index as given below (Eurocode 3, 2005; Yun et al., 2001).

$$J_i = \frac{\alpha}{3 + \alpha} \tag{2}$$

The value for the joint fixity factor varies from 0 to 1, where the lower limit corresponds to a perfectly pinned joint. In contrast, the upper limit corresponds to a perfectly rigid joint. Eurocode 3 (2005) specifies the range for stiffness index as [0.5–25] in an un-braced semi-rigid connection. Corresponding to this, the values of the end fixity factor ( $J_i$ ) for a semi-rigid connection varies in between [0.143–0.893].

**Figure 1** Schematic representation of semi-rigid connected beam-column joints



Considering  $J_1$  and  $J_2$  denotes the end fixity factors for the joints 1 and 2 respectively, the stiffness matrix for the semi-rigidly connected frame element is given by Yun et al. (2001) and Katkhuda et al. (2010).

$$[k] = \frac{EI}{L} \begin{bmatrix} \frac{A}{I} & 0 & 0 & -\frac{A}{I} & 0 & 0 \\ 0 & \frac{4(\alpha_1 + \alpha_2 + \alpha_3)}{L^2} & \frac{2(2\alpha_1 + \alpha_2)}{L} & 0 & -\frac{4(\alpha_1 + \alpha_2 + \alpha_3)}{L^2} & \frac{2(\alpha_2 + \alpha_3)}{L} \\ 0 & \frac{2(2\alpha_1 + \alpha_2)}{L} & 4\alpha_1 & 0 & -\frac{2(2\alpha_1 + \alpha_2)}{L} & 2\alpha_2 \\ \frac{A}{I} & 0 & 0 & \frac{A}{I} & 0 & 0 \\ 0 & -\frac{4(\alpha_1 + \alpha_2 + \alpha_3)}{L^2} & \frac{2(2\alpha_1 + \alpha_2)}{L} & 0 & \frac{4(\alpha_1 + \alpha_2 + \alpha_3)}{L^2} & -\frac{2(\alpha_2 + 2\alpha_3)}{L} \\ 0 & \frac{2(\alpha_2 + \alpha_3)}{L} & 2\alpha_2 & 0 & -\frac{2(\alpha_2 + 2\alpha_3)}{L} & 4\alpha_3 \end{bmatrix} \tag{3}$$

Sym

where the values of  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$  are given by:

$$\left. \begin{aligned} \alpha_1 &= \frac{3J_1}{4 - J_1J_2} \\ \alpha_2 &= \frac{3J_1J_2}{4 - J_1J_2} \\ \alpha_3 &= \frac{3J_2}{4 - J_1J_2} \end{aligned} \right\} \quad (4)$$

The consistent mass matrix for a frame element is also modified by the inclusion of the semi-rigid joint as given by Katkhuda et al. (2010) and Filho et al. (2004).

$$[m] = \frac{\rho AL}{420D^2} \begin{bmatrix} 140D^2 & 0 & 0 & 70D^2 & 0 & 0 \\ & 4f_1 & 2Lf_2 & 0 & 2f_3 & -Lf_4 \\ & & 4L^2f_5 & 0 & Lf_4 & -L^2f_6 \\ & & & 140D^2 & 0 & 0 \\ & & & & 4f_1 & -2Lf_2 \\ \text{Sym} & & & & & 4L^2f_5 \end{bmatrix} \quad (5)$$

where the where the  $D$  and  $f_i$  parameters are defined using joint fixity factors as:

$$\left. \begin{aligned} D &= 4 - J_1J_2 \\ f_1 &= 560 + 224J_1 + 32J_1^2 - 196J_2 - 328J_1J_2 - 55J_1^2J_2 + 32J_2^2 + 50J_1J_2^2 + 32J_1^2J_2^2 \\ f_2 &= 224J_1 + 64J_1^2 - 160J_1J_2 - 86J_1^2J_2 + 32J_1J_2^2 + 25J_1^2J_2^2 \\ f_3 &= 560 - 28J_1 - 64J_1^2 - 28J_2 - 184J_1J_2 + 5J_1^2J_2 - 64J_2^2 + 5J_1J_2^2 + 41J_1^2J_2^2 \\ f_4 &= 392J_2 - 100J_1J_2 - 64J_1^2J_2 - 128J_2^2 - 38J_1J_2^2 + 55J_1^2J_2^2 \\ f_5 &= 32J_1^2 - 31J_1^2J_2 + 8J_1^2J_2^2 \\ f_6 &= 124J_1J_2 - 64J_1^2J_2 - 64J_1J_2^2 + 31J_1^2J_2^2 \end{aligned} \right\} \quad (6)$$

## 2.2 Elemental matrixes for semi-rigid connected structure

In the present study, the damage is considered as a reduction in stiffness without any change in the mass of the structure. This is achieved by incorporating a parameter called ‘stiffness reduction factor (SRF)’ which is defined as the ratio of reduction in stiffness of the element at the damaged state to the stiffness at the undamaged state. The SRF on multiplication with undamaged stiffness matrix provides the damaged stiffness matrix for the structure. The value of SRF ranges from 0 to 1, where 0 signifies no damage in the element. In contrast, a value near to 1 means the stiffness is completely lost for the element. Mathematically for the  $i^{\text{th}}$  element, the damaged stiffness matrix is expressed as:

$$[K_d^i] = (1 - \beta_i)[K_u^i] \quad (7)$$

and

$$\beta_i = \frac{EI_u^i - EI_d^i}{EI_u^i} \quad (8)$$

where  $K_u^i$  denotes the undamaged stiffness matrix,  $E$  denotes Young's modulus of elasticity, and  $I$  denotes the moment of inertia. The subscript  $u$  and  $d$  denote the undamaged and damaged states, respectively. The parameter  $\beta$  denotes the SRF. The eigen equation for a damaged, undamped and freely vibrating system is given by:

$$[[K_d] - \omega_{id}^2 [M]] \{\phi_i^d\} = \{0\} \quad (9)$$

where  $[K]$  and  $[M]$  indicate the stiffness and mass matrix whereas  $\omega_i$  and  $\phi_i$  denote the  $i^{\text{th}}$  natural frequency and corresponding mode shape, respectively. The subscript  $d$  denotes the damaged values of corresponding parameters.

### 2.3 Unified particle swarm optimisation (UPSO)

PSO algorithm is inspired by the collective motions of birds, which involve both social interaction and intelligence (Kennedy and Eberhart, 1995). This process of social interaction is realised by the formation of neighbourhoods, which are generally two types, the global (particles share information with every member in the swarm) and the local (particles share information with their immediate neighbours according to certain topology rule). The global variant converges faster as all particles are attracted by the same best position and hence promotes exploitation but simultaneously demotes the exploration. This means the global variant is more susceptible to be trapped in some local optimum. On the contrary, the local variant has more exploration ability as the information regarding the best position of each neighbourhood is gradually communicated to the rest of the particles through their neighbours. A good optimisation technique desirably has good exploration ability at the beginning of the optimisation process while good exploitation ability at a later stage. UPSO is a scheme that harnesses both exploration and exploitation ability by combining both variants of PSO (Parsopoulos and Vrahatis, 2007, 2010).

For a swarm of  $P$ -particles in  $S$ -dimensional search space, if  $G_{ij}^{t+1}$  and  $L_{ij}^{t+1}$  denotes the velocity update for  $i^{\text{th}}$  particle in  $j^{\text{th}}$  dimension in global and local variants of PSO for  $(t+1)^{\text{th}}$  iteration then they can be represented mathematically as:

$$G_{ij}^{t+1} = \chi \left[ v_{ij}^t + c_1 r_1 (pbest_{ij} - x_{ij}^t) + c_2 r_2 (gbest_{ij} - x_{ij}^t) \right] \quad (10)$$

and

$$L_{ij}^{t+1} = \chi \left[ v_{ij}^t + c_1 r_3 (pbest_{ij} - x_{ij}^t) + c_2 r_4 (lbest_{ij} - x_{ij}^t) \right] \quad (11)$$

where  $pbest$ ,  $gbest$ , and  $lbest$  denotes the personal best position, global best position, and best position in the neighbourhood of individual swarm, respectively. The value of constriction factor  $\chi$  is taken as 0.72984. The terms  $c_1$  and  $c_2$  denote the cognitive coefficient and social scaling coefficient, respectively. The  $r$  terms denote independent random numbers within  $[0, 1]$ . Combining equations (10) and (11), the aggregate velocity of the search directions is defined as:

$$V_{ij}^{t+1} = u.G_{ij}^{t+1} + (1-u).L_{ij}^{t+1}, \quad u \in [0, 1] \quad (12)$$

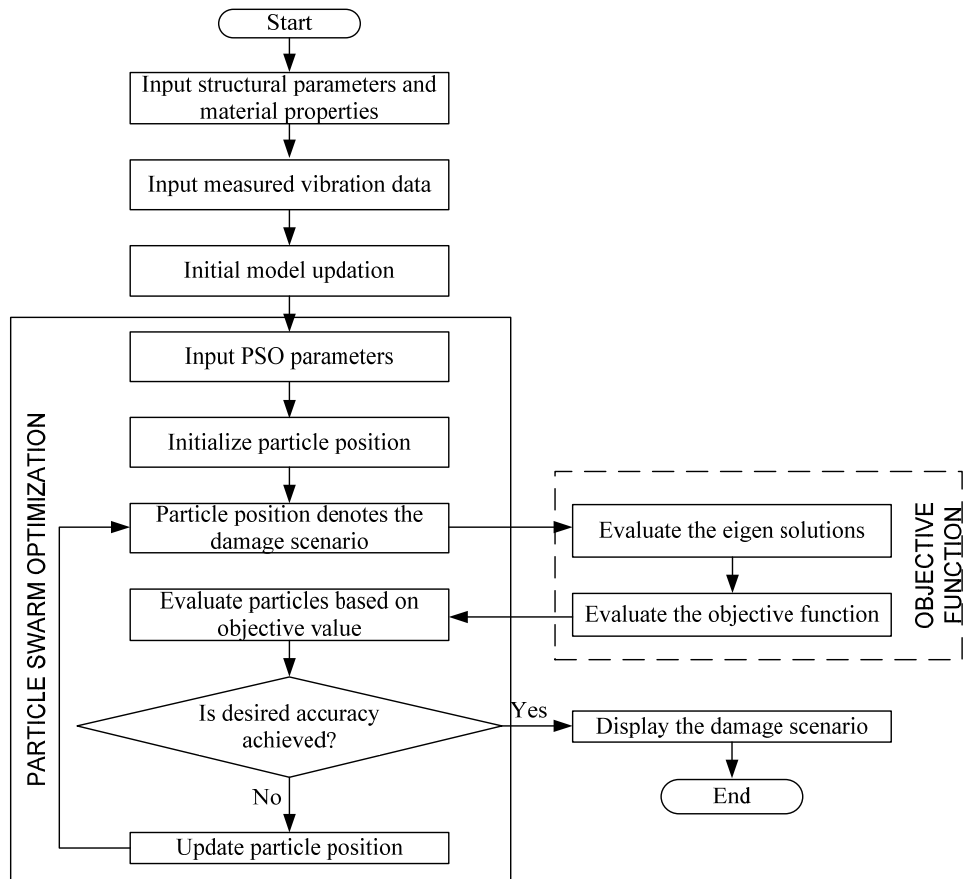
The new position of the particles for  $(t + 1)^{th}$  iteration is:

$$x_{ij}^{t+1} = x_{ij}^t + V_{ij}^{t+1}, \quad \forall i \in P \text{ and } \forall j \in S \quad (13)$$

The parameter,  $u$  in equation (12) is called unification factor and its value is modified throughout the iterations according to the equation as given follow:

$$u(t) = \exp\left(\frac{t \cdot \log(2.0)}{t_{\max}}\right) - 1.0 \quad (14)$$

**Figure 2** Procedure for damage assessment in structures with semi-rigid connections



### 2.4 Objective function

In the present study, the objective function is defined using changes in natural frequency and mode shape vectors as given below:

$$F = \sqrt{\frac{1}{n} \sum_{i=1}^n \left( \left( \frac{f_i^m}{f_i^c} \right) - 1 \right)^2} + \sum_{i=1}^n (1 - MAC_{ii}) \quad (15)$$



In the above equation,  $f_i^m$  denotes the experimental natural frequencies for the damaged structure,  $f_i^c$  denotes the natural frequencies obtained from numerical simulation and  $n$  denotes the numbers of modes considered in damage assessment. The term  $MAC_{ii}$  denoted modal assurance criterion (MAC) for the  $i^{\text{th}}$  mode, which provides the correlation between two mode shapes  $\varphi_{m,i}$  and  $\varphi_{c,i}$  where the subscript  $m$  and  $c$  represent the measured and the computed mode shapes, respectively. MAC value is a scalar quantity that varies between  $[0, 1]$ , where 0 indicates no correlation, while 1 indicates two completely correlated modes. Mathematically, the MAC value for the  $i^{\text{th}}$  mode shape is given by:

$$MAC_{ii} = \frac{(\varphi_{m,i}^T \varphi_{c,i})^2}{(\varphi_{m,i}^T \varphi_{m,i})(\varphi_{c,i}^T \varphi_{c,i})} \quad (16)$$

The flow chart for element damage assessment in a semi-rigid connected structure using the PSO algorithm is shown in Figure 2.

### 3 Results and discussion

Computer code is developed in MATLAB environment for assessment of element damages in semi-rigid connected structures using the procedures outlined in the previous sections. Its efficacy is demonstrated in the current section in two steps. First, a numerical example problem is solved using a two-bay, ten-story plane frame model. Second, the vibration data extracted for an experimental semi-rigid connected plane frame model used for the detection of elemental damages.

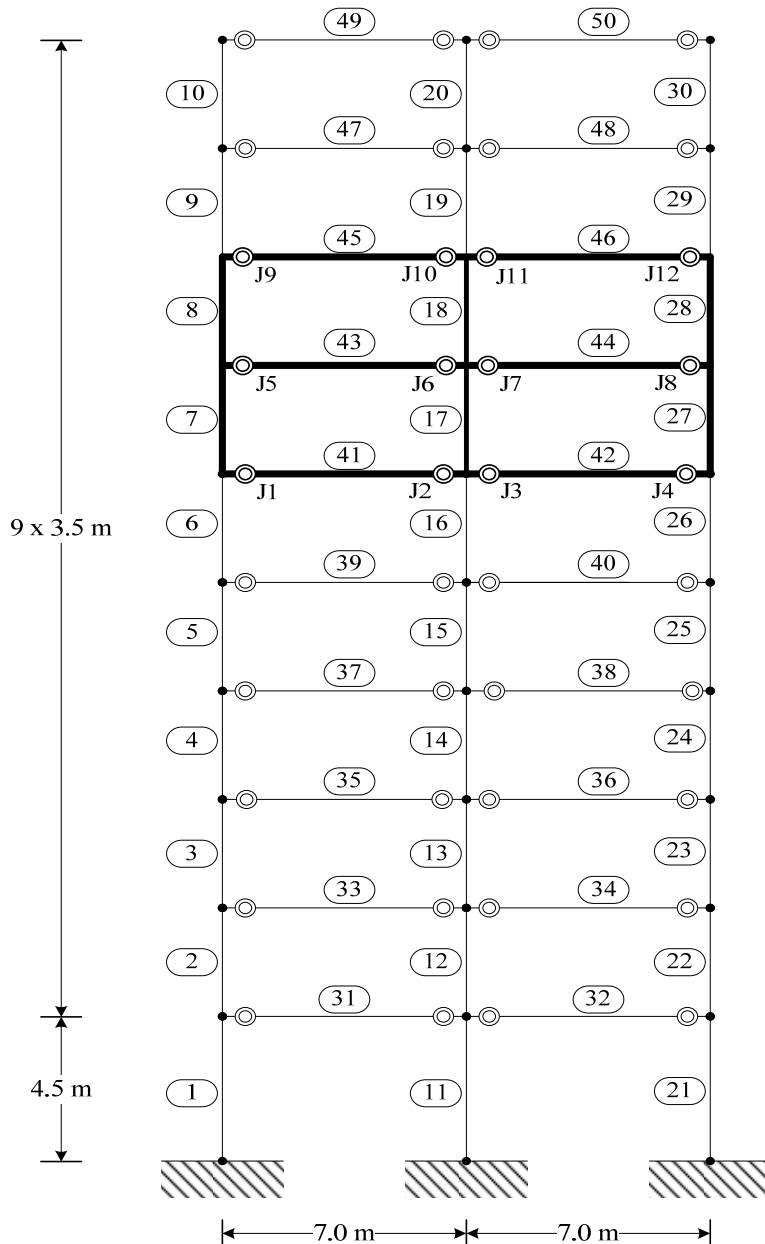
#### 3.1 Damage assessment of semi-rigid connected structural elements

The proposed procedure is evaluated for assessment of element damages in a two-bay, ten-story plane frame structure comprising 50 elements, as shown in Figure 3. The beam and columns for the plane frame are assumed to be made of  $W24 \times 55$  and  $W14 \times 145$ , respectively. The corresponding sectional area and the moment of inertia for the beams are given by  $1.04 \times 10^{-2} \text{ m}^2$  and  $5.62 \times 10^{-4} \text{ m}^4$  respectively and for the columns given by  $2.78 \times 10^{-2} \text{ m}^2$  and  $7.12 \times 10^{-4} \text{ m}^4$  respectively. The density and young's modulus of steel are taken as  $7,850 \text{ kg/m}^3$  and  $210 \text{ GPa}$ , respectively. The joint fixity factor for all beam-column joints are taken as 0.85. Hence, according to Eurocode 3 (2005), all beam-column joints fall under the semi-rigid connection. Damage is assumed to be located within the substructure containing six beams and six columns of 7th and 8th story as shown in bold lines. Three random cases that correspond to one element damage case, two-element damage case, and three-element damage cases, respectively, are considered for the demonstration purpose. The first nine natural frequencies and corresponding mode shapes are used for the assessment of joint damages. These damage cases and corresponding natural frequencies are presented in Table 1.

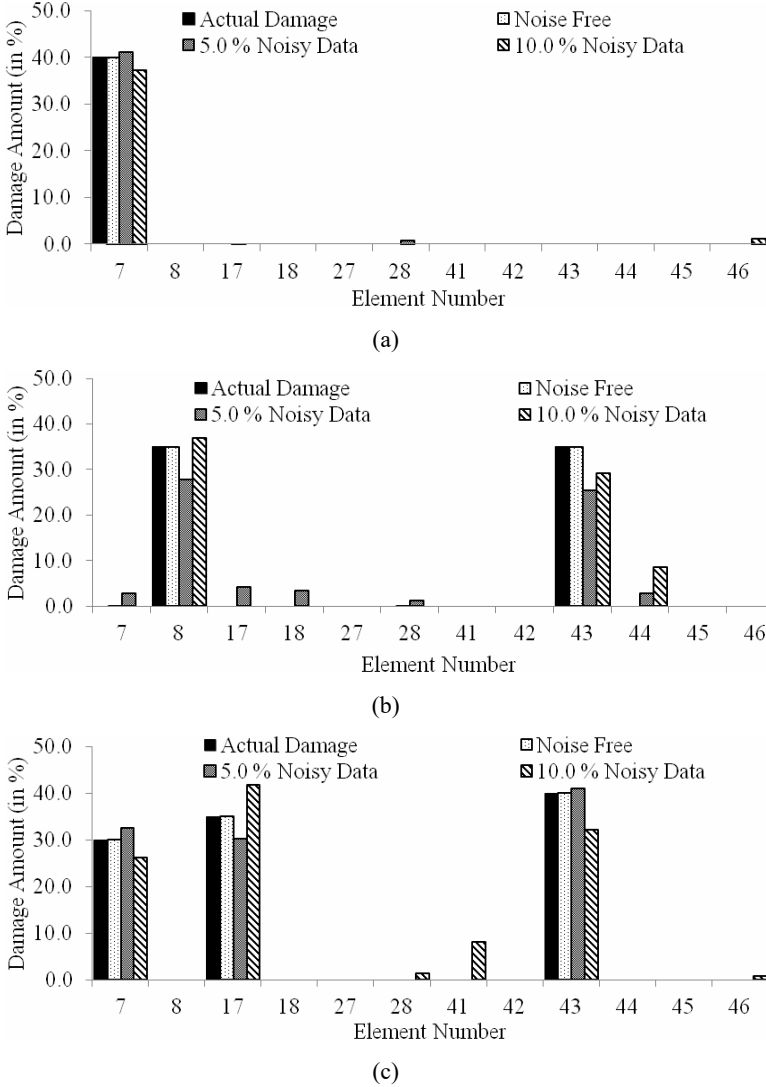
The inverse problem of damage identification is solved using the UPSO algorithm. The number of particles considered is 35. Ring topology with a neighbourhood radius of one particle is considered for local information sharing. Two different noise levels are considered in the analysis. The first noise level contains a 0.5% error in natural frequency

and 5.0% error in mode shape while the second noise level contains 1.0% noise in natural frequency and 10% noise in mode shape. Five numerical experiments are conducted with different initial seeds for each damage case, and the maximum iteration for each experiment is fixed at 1,000. The experiment providing minimum objective function value is considered as the actual damage scenario. The damage assessment results are presented in Figures 4(a)–4(c).

**Figure 3** 50-element plane frame structure with internal substructure (presented in bold line)



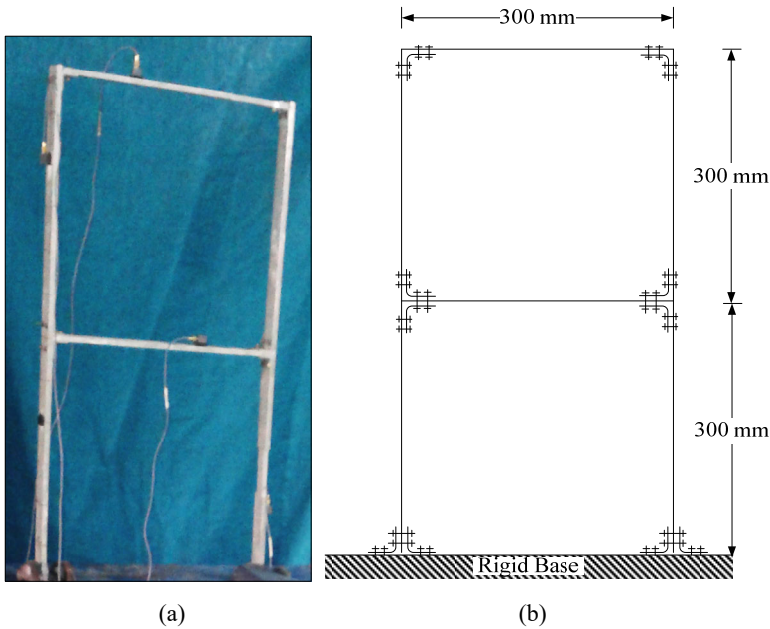
**Figure 4** Element damage assessment in a two-bay ten-story plane frame structure, (a) assessment for one element damage case (b) assessment for two elements damage case (c) assessment for three elements damage case



It is interesting to observe that the methodology can predict accurate results if response data considered for the damage identification are noise-free. However, in multiple element damage cases, some false damages are detected when noisy dataset is used for damage prediction. It is found that maximum false damage predicted is less than 5.0% when the noise level considered in natural frequencies and mode shapes are 0.5% and 5.0%, respectively. Similarly, maximum false damage prediction is less than 9.0% if noise level considered in natural frequencies and mode shapes are 1.0% and 10.0%, respectively. However, it may be regarded as reasonable predictions as it has detected and reasonably quantified all true damage locations.

**Table 1** Natural frequencies for element damage assessment

Natural frequencies (Hz)	Damage condition			
	Undamaged	I1	I2	I3
		7@40%	8@35% and 43@35%	7@30%, 17@35% and 43@40%
1st	1.771	1.686	1.682	1.676
2nd	5.487	5.183	5.134	5.089
3rd	9.861	9.258	9.234	9.230
4th	14.657	13.624	13.613	13.488
5th	15.519	15.402	15.425	15.287
6th	20.144	18.573	18.434	18.292
7th	22.065	21.560	21.618	21.570
8th	26.498	24.174	24.094	24.097
9th	28.607	28.363	28.290	28.170

**Figure 5** Connection details for the test frame, (a) original test structure (b) details of connections (see online version for colours)

### 3.2 Experimental study

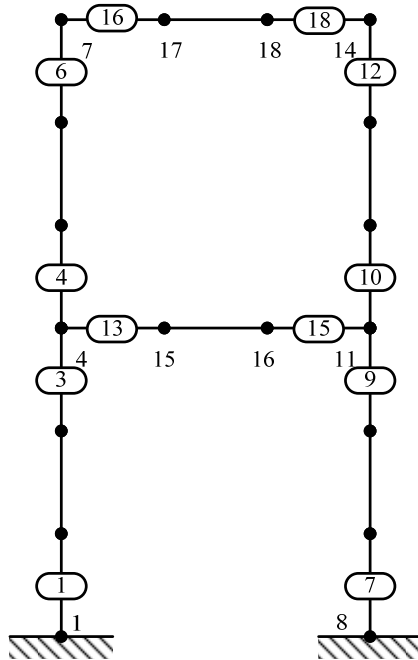
The efficacy of the present damage identification algorithm is validated for an experimental frame structure with semi-rigid connection. A two-story aluminium plane frame structure is fabricated for the experimental study. The joints are made semi-rigid by connecting them through connectors, as shown in Figure 5. The natural frequencies are determined through modal testing, which is carried out in the structural engineering

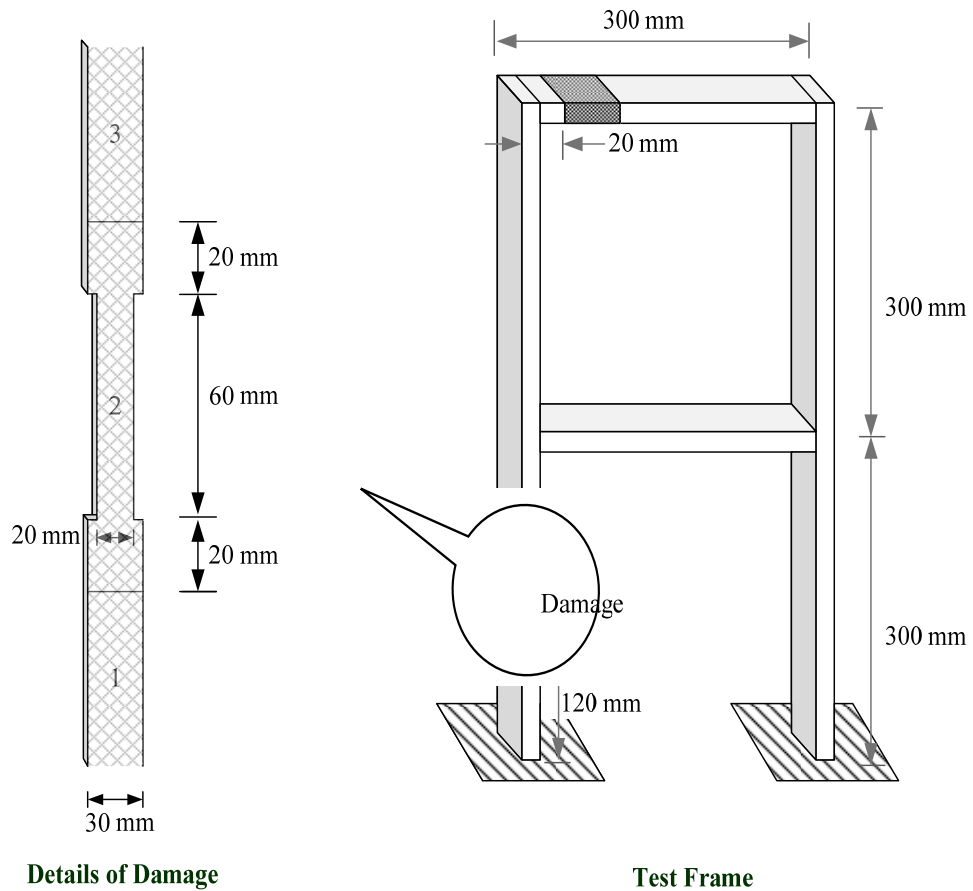
and material testing laboratory of the Indian Institute of Technology, Kharagpur, are used for the determination of damage in the structure. The detailed procedure for carrying out this experiment is elaborated in Nanda et al. (2014b). For numerical simulation, the frame is modelled with 18 Euler-Bernoulli beam elements, as shown in Figure 6.

It is obvious that the modal parameters identified from the experiment does not match with the parameters obtained from numerical experimentation due to the association of several errors like inaccurate modelling, environmental noise, or measurement errors. Thus, model updating is carried out to minimise these discrepancies between measured data and numerical simulation data. The end fixity factors at the beam-column joints and the stiffness modulus of frame materials are considered as the updating variables. It is observed that the updated numerical model is able to exhibit the modal behaviour of the test frame accurately within a 1.0% error limit (Nanda et al., 2014b). From the present model updating, the end fixity factors for four beam-columns joints are estimated as 0.879, 0.678, 0.749, and 0.564, for joint nos. 4, 7, 11, and 14, respectively. As per the euro code, the beam-column joints with end fixity factor less than 0.89 are falling under semi-rigid connection. Hence, these four joints are categorised as semi-rigid connections.

Two damage cases are considered for demonstrating the efficacy of the present algorithm. The first one is a single element damage condition where damage is introduced by cutting element 2. The second one is a two-element damage condition where damage is introduced to element 2 and element 16. Figure 7 presents the schematic diagram showing the introduction of element damage in the experimental frame model. The natural frequencies for the undamaged and damaged structures are measured experimentally and presented in Table 2.

**Figure 6** Finite element discretisation of the experimental frame



**Figure 7** Element damage in the experimental frame model

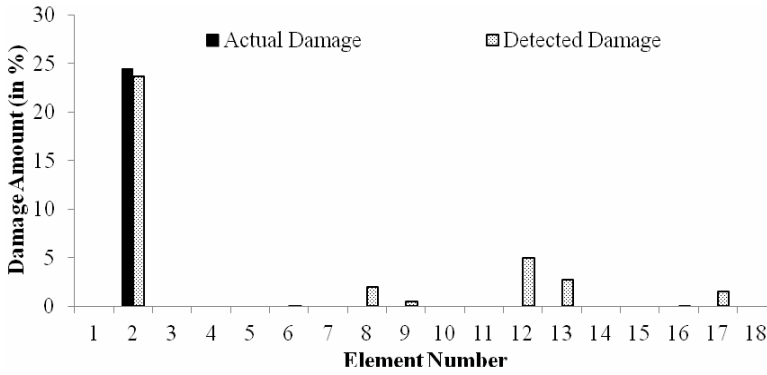
For numerical simulation, the damage is considered as the reduction in the stiffness matrix, which is incorporated using a SRF. To estimate the S.R.F. value for present damage configurations, an inverse procedure is adopted. Two numerical frame models are considered for this purpose. In the first numerical model, the damage is represented by reducing the cross-sectional area and moment of inertia of the element at the damaged location. The frame is discretised in such a way that the entire portion of the frame that contains damage confines to one element. Then, a static load of 100 N is applied at the top-left node of the numerical frame (i.e., at node 7 of Figure 6) in the horizontal direction. With this configuration and loading condition, the displacement along the loading degree of freedom and direction is measured (i.e., at node seven and in the horizontal direction for the present case) and noted down. Then, another numerical frame model, which is discretised in a similar manner to that of Figure 6, is considered for estimation of SRF value. The stiffness value for the elements at damage locations (i.e., elements two and 16 in the present study) is adjusted by multiplying S.R.F values to those elements. With loading conditions similar to the first model, the displacement is measured at a similar loading degree of freedom and direction. In the second frame model by trial and error method, the S.R.F. value is adjusted in such a way that the

resulting deflection will match to the deflection of the first numerical model. This SRF value corresponds to the amount of damage present in corresponding elements of the model. From this simple simulation, it is found that damages at elements two and 16 correspond to an amount of 24.50% (S.R.F = 0.755) in the corresponding element.

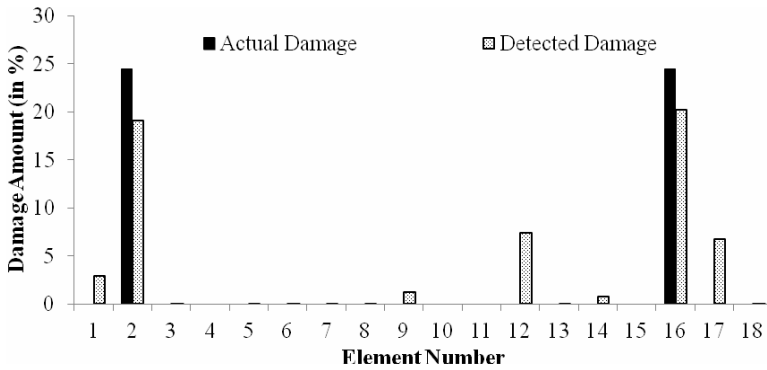
**Table 2** Experimental natural frequencies of the frame with element damage

Mode	Natural frequencies (in Hz)				
	1st	2nd	5th	6th	8th
Undamaged plane frame	19.98	65.60	227.75	287.65	346.10
One element damage	19.85	65.65	225.60	284.05	341.40
Two element damages	19.80	65.00	224.00	283.40	340.20

**Figure 8** Element damage assessment in the experimental frame model, (a) single element damage (b) two element damages



(a)



(b)

Damage identification is carried out with the tabulated frequencies. As discussed earlier, the UPSO algorithm is used for solving the inverse problem. The swarm size selected is 43. Other parameters required for the algorithm are kept equal to previous example. The damage identification results are shown in Figure 8, wherein it may be observed that the present procedure is able to identify damage location and its extent very close to actual

ones in both damage cases. A few false damages are also observed in both cases, which is also obvious considering the presence of noise in measured vibration data. However, their magnitude is quite less than the actual one and may not be considered as harmful as the algorithm is able to detect the actual damaged location with its fair quantification in both cases. Therefore, in conclusion, it may be stated that the proposed methodology is successful in the detection and assessment of damages in a semi-rigidly connected structure.

#### 4 Conclusions

In the present study, the procedure for element damage assessment in semi-rigid connected structures is presented. First, the methodology for the detection of element damages in semi-rigid connected structures is outlined. The beam-column joints are made semi-rigid numerically by reducing their end fixity factor. The efficacy of the present algorithm is demonstrated through a numerical and experimental problems. Model updating is carried out to minimise the discrepancies between the numerical and experimental results. It may be concluded from these studies that the algorithm is able to predict damaged elements in semi-rigid connected structures along with their amount with significant level of accuracy.

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