SER performance optimisation of AF cooperative communication system based on directional antenna

Ruilian Tan*

Armed Police Engineering University,
Electronic Technology Laboratory of Information Engineering Department,
Xi’an, China
and
College of Equipment Management and Safety Project,
Air Force Engineering University,
Xi’an, China
Email: madamtan@126.com
*Corresponding author

Zhe Li

College of Equipment Management and Safety Project,
Air Force Engineering University,
Xi’an, China
Email: kongyanshi@126.com

Xi Su

Troop 95806,
Beijing, China
Email: suxi60@163.com

Abstract: Aiming at the cooperative communication system with directional antenna, this paper has studied the SER (Symbol Error Rate) performance under AF (Amplify-and-Forward protocol). The model of AF cooperative communication system using directional antenna was firstly established to deduce the closed-form expression of SER in this model as well as the upper limit of SER. Then, the OPA (Optimum Power Allocation) was also analysed on the purpose of minimising SER. Combining specific simulation numerical values, SER performance of established model was thoroughly researched in this paper. Simulation results demonstrate that system SER obviously decreases by adopting cooperative communication system with directional transmitting and directional receiving. Each node’s directional gain, channel quality and power allocation method all have great influence on system’s overall performance. And the OPA is also proved to be superior to EPA (Equal Power Allocation).

Keywords: cooperative communication; directional antenna; amplify and forward; symbol error rate.


Biographical notes: Ruilian Tan is currently a graduate student at the College of Equipment Management and Safety Project of Air Force Engineering University in China. She obtained her master’s degree in 2008. She is interested in the areas of network security, communication network, and cloud computing security.

Zhe Li is now an Associate Professor at the Air Force Engineering University in China. His current research interests include theory of network security, communication network, and cloud computing security.

Xi Su is now an Associate Professor in the Troop 95806 in China. Her current research interests include theory of network security, communication network, and cloud computing security.
1 Introduction

As the research focuses in recent years, cooperative communication has been well studied and acquires numerous achievements in aspects such as cooperative method, capacity, network coding, system synchronisation, relay node selection and resource management (Wang et al., 2016; Zhang et al., 2016; Wu et al., 2015; Puzar and Plagemann, 2011; Morreale et al., 2015; Hammoudi et al., 2015). System performance of dual Rayleigh fading channel has been studied by adopting AF cooperative method (Wang et al., 2016). Based on multi-user cooperative network, we could use a capacity selection method with less complexity to effectively promote system performance (Zhang et al., 2016). And, the optimal coding scheme has been acquired through studying timing coding of OFDM (Orthogonal Frequency Division Multiplexing) asynchronous cooperative network (Wu et al., 2015). A new transmit-receive AF relay network is proposed in Puzar and Plagemann (2011) to realise synchronisation, which demonstrates that system performance is promoted significantly by applying proposed estimating and decoding method. Through researching relay selection and power allocation of two-way relay channel, an efficient energy-saving relay selection scheme is promoted in Morreale et al. (2015) to minimise system’s total transmitting power. With given transmitting rate and SNR (Signal-to-Noise Ratio), the bidirectional cooperative Optimum Power Allocation (OPA) strategy put forward is proved to minimise failure probability and enhance system performance (Hammoudi et al., 2015).

By concentrating and transmitting energy to the same direction, directional antenna can not only decrease interference from other directions but also reinforce spatial multiplexing. Accordingly, directional antenna achieves wider transmitting range with the same power and is widely applied in military field (Li et al., 2012; Ding et al., 2012; Abdullah et al., 2012). For example, six sets of directional antennas have been added to F-22 firefighter to cover the whole airspace to achieve RF (Radio Frequency) stealth. Although there are lots of papers studying cooperative communication and directional antenna, respectively, the reference combining the two technologies is hard to find (Wang et al., 2013). Consequently, great theoretical as well as application value lies in the research combining cooperative communication and directional antenna under the specific application background of military communication domain.

Through introducing directional antenna into cooperative communication system, the Symbol Error Rate (SER) performance of single node cooperative communication system using AF is studied in this paper. The closed-form expression and upper limit of SER under the directional transmitting and receiving mode of system with directional antenna are acquired by mathematical modelling and theoretical deduction. The relation between SNR and SER under various conditions is ascertained through thoroughly analysing SER performance of system with directional antenna. Furthermore, after analysing power allocation’s influence on system SER, it is proved that each node’s directional gain and channel quality both have evident influence on system SER by optimising power allocation. Therefore, adopting reasonable power allocation scheme according to specific channel environment can significantly promote system’s SER performance.

2 System model

2.1 Directional antenna model

The signal reception model of directional antenna is expressed as (Wang et al., 2013):

\[
P_f = \frac{PG_G}{Kr^2}
\]

in which \(P_f\) denotes transmitting power, \(G\) denotes directive transmitting antenna gain, \(G_r\) denotes directive receiving antenna gain, \(K\) denotes the constant of atmospheric absorption and Ohmic loss and \(V\) denotes path declining coefficient with frequent value range of \([2, 4]\) (Rappaport, 2009).

2.2 Cooperative communication system model based on directional antenna

If both transmitting and receiving terminals use directional antenna, the cooperative communication system with directional antenna is under directional transmitting-directional receiving mode. Compared with isotropic transmitting–isotropic receiving mode, which uses isotropic antennas in both transmitting and receiving terminals, the transmitting and receiving terminals of each single node have two directional antenna gains denoted as \(G_t\) and \(G_r\) respectively. To analyse the performance of system with directional antenna conveniently, the typical three-node cooperative communication model in Rayleigh Flat Fading Channel is studied. Denoting each node’s channel coefficient with \(h_{ij}\), it obeys the index distribution with parameter of \(1/\sigma^2\). Owing to the adoption of directional antenna, after beam forming signal energy can only cover a given spatial angle and accordingly one more communication time slot is needed (Wang et al., 2013) in comparison with isotropic antenna model. Then, the whole communication process illustrated in Figure 1 can be divided into two stages with three time slots.

**Figure 1** Directional and cooperative communication system model
Stage 1 (time slot 1 and time slot 2): source node transmits broadcasting information to destination node and relay node, respectively.

Information received by destination node and relay node are denoted as \( y_{s,d} \) and \( y_{s,r} \), respectively. \( G_s, G_r \) and \( G_d \) represent directional antenna gains of source node, relay node and destination node, respectively. Then \( y_{s,d} \) and \( y_{s,r} \) can be expressed as below:

\[
y_{s,d} = \sqrt{P_s G_s} G_d h_{s,d} x + \eta_{s,d} \tag{2}
\]
\[
y_{s,r} = \sqrt{P_s G_s} G_r h_{s,r} x + \eta_{s,r} \tag{3}
\]
in which \( x \) denotes the transmitted signal, \( P_s \) denotes transmitting power, \( \eta_{s,d} \) and \( \eta_{s,r} \) denote AWGN (Additive White Gaussian Noise) with the mean of zero and variance of \( N_0 \). The channel coefficient from source node to destination node is denoted as \( h_{s,d} \) and channel coefficient from source node to relay node is represented as \( h_{s,r} \). Both of \( h_{s,d} \) and \( h_{s,r} \) are CGRV (Complex Gaussian Random Variable) with the mean of zero and variances of \( \sigma_{s,d}^2 \) and \( \sigma_{s,r}^2 \) which are assumed not equal to zero in this paper.

Stage 2 (time slot 3): under AF protocol, the relay node amplifies and forwards received signal to the destination node.

This paper focuses on cooperative communication system’s performance with directional antenna under AF protocol. Therefore, the ideal fixed AF protocol is adopted to acquire theoretical closed-expression. Assuming relay node’s transmitting power is \( P_d \), with an amplifying factor of \( \beta = \sqrt{\frac{P_s G_s G_d G_r}{\sqrt{P_s G_s} G_r h_{s,r}^2 N_0 + N_0}} \), the relayed signal received by destination node can be expressed as:

\[
y_{r,d} = \beta h_{r,d} y_{s,d} + \eta_{r,d} \tag{4}
\]
Plugging specific values of expression (3) and \( \beta \) into expression (4), it can be expressed as:

\[
y_{r,d} = \frac{\sqrt{P_s P_d G_s G_r G_d h_{s,r}^2}}{\sqrt{P_s G_s} G_r h_{s,r}^2 N_0 + N_0} h_{r,d} h_{s,d} x + \tilde{\eta}_{r,d} \tag{5}
\]
where \( \tilde{\eta}_{r,d} = h_{r,d} \eta_{s,d} \frac{\sqrt{P_s P_d G_s G_r G_d h_{s,r}^2}}{\sqrt{P_s G_s} G_r h_{s,r}^2 N_0 + N_0} + \eta_{r,d} \). Supposing \( \eta_{s,d} \) and \( \eta_{s,r} \) are independent, equivalent noise denoted as \( \tilde{\eta}_{r,d} \) is CGRV with the mean of zero and variance expressed as follow:

\[
\tilde{N}_0 = \left( \frac{P_s G_s G_d h_{s,d}^2}{P_s G_s G_r h_{s,r}^2 N_0 + N_0} + 1 \right) N_0 \tag{6}
\]

With given channel coefficient, MRC (Maximum Ratio Combining) method is frequently adopted at destination node. Terminal node can receive signals from two links: the first one denoted as \( y_{r,d} \) with SNR of \( \gamma_2 \) and the second one denoted as \( y_{s,d} \) with SNR of \( \gamma_1 \). The receiving signal with MRC can be expressed as below:

\[
y = Ay_{r,d} + By_{s,d} \tag{7}
\]
where
\[
A = \sqrt{PPG G h_{s,d}^2} / N_0 \quad \text{and} \quad B = \sqrt{PPG G h_{s,d}^2} / N_0.
\]
Assuming the average energy of transmitted symbol \( x \) is 1, then:

\[
\gamma_1 = \frac{A}{|A|^2 N_0} = \frac{P_s G_s h_{s,d}^2}{P_s G_s h_{s,d}^2 N_0 + N_0} \tag{8}
\]
\[
\gamma_2 = \frac{B}{|B|^2 N_0} = \frac{P_s G_s h_{s,d}^2 + N_0}{P_s G_s h_{s,d}^2 N_0 + N_0} \tag{9}
\]

With given directional gain, \( P_s G_s h_{s,d}^2 / N_0 \) and \( P_s G_s h_{s,d}^2 / N_0 \) both obey index distribution. Referring to expression (9), \( \gamma_2 \) can be approximated as the weighted harmonic mean of \( |h_{s,r}|^2 \) and \( |h_{r,d}|^2 \) with an obedience parameter expressed as:

\[
\xi_2 = \frac{N_0}{P_s G_s G_r h_{s,r}^2} + \frac{N_0}{P_s G_s G_d h_{r,d}^2} \tag{10}
\]

Assuming \( f_{\gamma_1}(x) \) and \( f_{\gamma_2}(y) \) as probability density functions of \( \gamma_1 \) and \( \gamma_2 \), respectively, then \( \xi_1 = \frac{N_0}{P_s G_s G_d h_{r,d}^2} \). Accordingly, expression (11) can be achieved.

\[
f_{\gamma_1}(x) = \begin{cases} \xi_1 \exp(-\xi_1 x), & x > 0 \\ 0, & \text{other} \end{cases} \tag{11}
\]
\[
f_{\gamma_2}(y) = \begin{cases} \xi_2 \exp(-\xi_2 y), & y > 0 \\ 0, & \text{other} \end{cases} \tag{11}
\]

3 SER analysis

3.1 Closed SER analysis

SER plays a key role in analysing communication system’s performance. The conditional SER of nodes using M-PSK can be expressed as below:

\[
P_{\text{psk}}(\gamma) = \frac{1}{\pi} \int_0^{(M-1)\pi} \exp\left( -\frac{by}{\sin\theta} \right) d\theta \tag{12}
\]
in which \( b = \sin^2 \left( \frac{\pi}{M} \right) \) and \( \gamma \) denotes SNR. The theoretical system average SNR can be acquired by averaging expression (12). Therefore, the closed SER in non-cooperative mode can be easily achieved as:

\[
P_{\text{SER}} \left( \gamma \right) = \frac{1}{\pi} \int_{\gamma}^{(M-1)\gamma} E \left( \frac{-b\gamma}{\sin^2 \theta} \right) d\theta
\]

\[
= \frac{1}{\pi} \int_{\gamma}^{(M-1)\gamma} \exp \left( \frac{-b\gamma}{\sin^2 \theta} \right) f_\gamma (x) dx d\theta
\]

\[
= \frac{1}{\pi} \int_{0}^{(M-1)\gamma} \frac{1}{1 + bP_G G \sigma_{r,d}^2} d\theta
\]

The closed SER in AF cooperative mode is expressed as:

\[
P_{\text{SER}} \left( \gamma \right) = \frac{1}{\pi} \int_{\gamma}^{(M-1)\gamma} E \left( \frac{-b\gamma}{\sin^2 \theta} \right) d\theta
\]

\[
= \frac{1}{\pi} \int_{\gamma}^{(M-1)\gamma} \exp \left( \frac{-b\gamma}{\sin^2 \theta} \right) f_\gamma (x) dx
\]

\[
= \frac{1}{\pi} \int_{0}^{(M-1)\gamma} \frac{1}{1 + bP_G G \sigma_{r,d}^2} \sin^2 \theta N_0 d\theta
\]

Expression (14) shows that it is complicated to compute SER’s theoretical value. Even SER is ascertained by numerical computation, it is still difficult to analyse system performance. Consequently, an approximation of SER has been deduced in this paper to improve performance analysis of system based on directional antenna under AF cooperative mode. In AF cooperative mode, it is required that \( 0 < \sin^2 \theta \leq 1 \). Therefore:

\[
P_{\text{SER}} \left( \gamma \right) = \frac{1}{\pi} \int_{\gamma}^{(M-1)\gamma} \frac{N_0 \sin^2 \theta}{1 + bP_G G \sigma_{r,d}^2} \sin^2 \theta \left( P_G G \sigma_{r,d}^2 + P_G G \sigma_{r,d}^2 \right) d\theta
\]

According to expression (15), a loose upper limit of SER can be acquired.

\[
U_{\text{SER}} = \max \left( P_{\text{SER}} \left( \gamma \right) \right)
\]

\[
= \frac{P_G G \sigma_{r,d}^2 + P_G G \sigma_{r,d}^2}{bP_G G \sigma_{r,d}^2} \left( \frac{1}{P_G G \sigma_{r,d}^2} + \frac{1}{P_G G \sigma_{r,d}^2} \right)
\]

\[
\times \frac{1}{\pi} \int_{\gamma}^{(M-1)\gamma} \frac{N_0 \sin^4 \theta d\theta}{bP_G G \sigma_{r,d}^2}
\]

\[
= \frac{C}{(P_G G \sigma_{r,d}^2 + P_G G \sigma_{r,d}^2) + (P_G G \sigma_{r,d}^2 + P_G G \sigma_{r,d}^2)}
\]

where \( C = \frac{1}{\pi b} N_0 \int_{0}^{(M-1)\gamma} \sin^4 \theta d\theta \). With a given value of \( M \), \( C \) is a constant.

### 3.2 Optimum power allocation (OPA)

In the case of limited power, Equal Power Allocation (EPA) is prone to reducing system resource utilisation efficiency. How to reasonably allocate power becomes the study focus (Lee et al., 2012). Present research has demonstrated that rational allocation of transmitting power at both source node and relay node can evidently promote system performance (Solares et al., 2012).

From the numerical optimisation perspective, Lagrange multiplier method is usually applied when the constraint is an equation. With a given total power \( P \), the constraint here is \( P_1 + P_2 = P \). Lagrange multiplier method has been adopted accordingly to optimise power allocation.

Deeming SER’s upper limit as the optimisation objective, OPA is equivalent to:

\[
\left\{ \begin{array}{l}
\arg \min \left( U_{\text{SER}} \right) \\
\text{s.t.} \quad P_1 + P_2 = P
\end{array} \right.
\]

From expression (16), it is known that \( C \) is a constant with a given value. Then, expression (17) is equivalent to:

\[
\left\{ \begin{array}{l}
\arg \min \left( P_{\text{SER}} \right) \\
\text{s.t.} \quad P_1 + P_2 = P
\end{array} \right.
\]

In the formula, \( P_{\text{SER}} = \frac{P_G G \sigma_{r,d}^2 + P_G G \sigma_{r,d}^2}{P_1 P_G G \sigma_{r,d}^2 + P_2 P_G G \sigma_{r,d}^2} \). Lagrange function is defined as:

\[
L = P_{\text{SER}} + \phi (P_1 + P_2 - P)
\]

In which \( \phi \) denotes Lagrange operator, partial derivatives of \( P_1 \) and \( P_2 \) and \( \phi \) can be obtained according to expression (19). Under the condition of \( \partial L / \partial P_1 = 0 \), \( \partial L / \partial P_2 = 0 \), \( \partial L / \partial \phi = 0 \), it can be expressed as:

\[
\left\{ \begin{array}{l}
1 + \frac{1}{P_1 P_G G \sigma_{r,d}^2} - \phi = 0 \\
1 + \frac{1}{P_2 P_G G \sigma_{r,d}^2} - \phi = 0 \\
P_1 + P_2 = 0
\end{array} \right.
\]
By solving equations above, results are acquired as:

\[
\begin{align*}
\lambda &= \frac{\sqrt{G_s \sigma_{s,r} \sigma_{s,r}} + \sqrt{G_d \sigma_{s,r}^2 + 8G_s \sigma_{r,d}^2}}{3 \sqrt{G_s \sigma_{s,r}^2 + G_d \sigma_{s,r}^2 + 8G_s \sigma_{r,d}^2}} P \\
\lambda_0 &= 2 - \frac{2}{3 + 3/\sqrt{G_s \sigma_{s,r}^2 / G_d \sigma_{s,r}^2}} P
\end{align*}
\]

Expression (21) shows that the OPA of cooperative communication system with directional transmitting and receiving is greatly influenced by channel quality as well as directional gain of each node. With a fixed directional gain, the OPA mainly depends on the two channels which link relay node to source node and destination node, respectively. However, in this case, the OPA is independent to the channel linking source node and destination node. As the channel is ascertained, the OPA is decided by directional gains of source and destination node. Under the above condition, the OPA is independent of the relay node’s gain.

Assuming power allocation factor is \( \lambda \), then \( P_1 = \lambda P \), \( P_2 = (1-\lambda) P \). In this case, \( \lambda \) is the optimum target and the power allocation between \( P_1 \) and \( P_2 \) should be optimised to minimise SER. By transforming \( \lambda \), the following expressions are acquired.

\[
\lambda = \frac{\sqrt{G_s \sigma_{s,r} \sigma_{s,r}} + \sqrt{G_d \sigma_{s,r}^2 + 8G_s \sigma_{r,d}^2}}{3 \sqrt{G_s \sigma_{s,r}^2 + G_d \sigma_{s,r}^2 + 8G_s \sigma_{r,d}^2}}
\]

\[
1 - \lambda = \frac{2}{3 + 3/\sqrt{G_s \sigma_{s,r}^2 / G_d \sigma_{s,r}^2}}
\]

According to expression (22), with the given directional gain of each node, \( \lambda \) gets larger along with the increase of \( \sigma_{s,d}^2 / \sigma_{s,r}^2 \). It leads to a larger \( P_1 \) and the source node will get more transmitting power in this case. Expression (23) shows that \( 0 < 1 - \lambda < 1/2 \) and therefore \( 1/2 < \lambda < 1 \). Then, as \( \sigma_{s,r}^2 > \sigma_{s,d}^2 \), \( 1 - \lambda \) approaches to 0 and \( \lambda \) approaches to 1 which means the overall power should be distributed to source node. As \( \sigma_{s,r}^2 < \sigma_{s,d}^2 \), \( 1 - \lambda \) approaches to 1/2, which means the same power should be allocated to both source and relay nodes.

4 Simulation analysis

The simulation is under the condition of QPSK modulation and transmitting power of each node is normalised to 1. Then it is known that \( N_{s,d} = N_{s,r} = N_{r,d} = 1 \), \( M = 4 \), \( b = 1 \). Considering cooperative communication’s requirement for SNR, the directional gain has been selected as 15 dB.

Firstly, on the big SNR premise, study the upper boundary performance of closed SER and SER under cooperative mode and non-cooperative mode. The parameters are set as below:

\[
\lambda = 0.5 \, (\text{EPA}), \, h = [\sigma_{s,d}^2, \sigma_{s,r}^2, \sigma_{r,d}^2] = [1,1,1], \, \text{system SNR} \, \text{SNR} = 30\, \text{dB}.
\]

1 Under the premise of large SNR, performance of closed SER as well as upper limits of SER deduced under both cooperative and non-cooperative modes are thoroughly studied. Parameters are set as: \( \lambda = 0.5 \) (EPA), \( h = [\sigma_{s,d}^2, \sigma_{s,r}^2, \sigma_{r,d}^2] = [1,1,1] \) and \( \text{SNR} = 30\, \text{dB} \).

Figure 2 SNR and SER relationship comparison
\( G_s = G_r = G_j = 15\, \text{dB} \)

Figure 3 SNR and SER relationship comparison
\( G_s = G_r = G_j = 0\, \text{dB} \)

Giving \( G_s = G_r = G_j = 15\, \text{dB} \), Figure 2 is acquired. And Figure 2 demonstrates following points:

Under isotropic transmitting and isotropic receiving mode, the gap between the SER of cooperative mode and that of non-cooperative mode is small with a comparably low SNR. The SER of cooperative mode decreases rapidly along with SNR’s increase, and therefore better system performance is achieved. As \( \text{SNR} = 30\, \text{dB} \), SER of cooperative mode decreases three orders of magnitude than that of non-cooperative mode.
Since directional gain is introduced by adopting directional antennas at both transmitting terminal and receiving terminal, SER of cooperative mode decreases much faster than that of non-cooperative mode. As SNR = 30 dB, SER of cooperative mode decreases almost six orders of magnitude than that of non-cooperative mode; with the same SNR, the closed SER of directional antenna is much smaller than that of isotropic antenna by theoretical deduction. In other words, directional antenna’s application can promote system performance.

Giving \( G_s = G_r = G_d = 0 \text{dB} \), Figure 3 is achieved. And Figure 3 shows following points:

As directional gain is 0 dB, \( G_s \), \( G_r \) and \( G_d \) in expression (14) are all equal to 1 after unit conversion. In this case, the deductive closed SER of directional mode is the same as that of isotropic mode. Simulation results in Figure 3 demonstrate that the SER curve of isotropic non-cooperative mode overlaps with that of directional non-cooperative mode. On the other side, the SER curve of isotropic cooperative AF mode overlaps with that of directional non-cooperative mode. Accordingly, the correctness of theoretical deduction developed in this paper is validated.

The curve of upper limit of SER parallels upon that of isotropic antenna. Then, giving \( G_s = G_r = G_d = 0 \text{dB} \), the upper limit of SER deduced based on directional cooperative mode is still applicable to isotropic mode.

2 Through studying the closed SER of directional cooperative mode, the influence of power allocation factor and directional gain on system’s SER performance is analysed. Parameters are set as:

\[
h = [\sigma_s^2, \sigma_r^2, \sigma_d^2] = [1, 1, 1] \quad \text{SNR} = 15 \text{dB}
\]

(a) \( G_{s, \text{change}} = [G_s, G_r, G_d] = [i, 15, 15] \text{ (dB)} \)

(b) \( G_{r, \text{change}} = [G_s, G_r, G_d] = [15, i, 15] \text{ (dB)} \)

(c) \( G_{d, \text{change}} = [G_s, G_r, G_d] = [15, 15, i] \text{ (dB)} \)

where \( 0 \text{dB} \leq i \leq 15 \text{dB} \). Simulation results are shown in Figure 4.

**Figure 4** Directive gain and SER relationship comparison of each node

According to Figure 4, several points are known.

The SER of directive AF cooperative communication system decreases along with the increase of directional gain of each node. Compared with the impact of relay node’s directional gain, the promotion of directional gains at both source node and destination node bring down the SER much more rapidly.

Figure 4 (a) shows that SER curves of \( G_s \) and \( G_d \) overlap under EPA. From expression (14), \( G_s \) and \( G_d \) are symmetrical. Therefore, simulation results testify theoretical analysis.

Figure 4 (b) is acquired under OPA in which \( G_s \) and \( G_d \) are asymmetric and present as two different curves. Then, rational allocation of system overall power plays a key role in minimising SER under the condition of a giving directional gain.

The influence of both EPA and OPA on SER performance is also studied. While other conditions remain the same, giving \( G_s = G_r = G_d = 15 \text{dB} \), the optimal power allocation coefficient \( \lambda = 0.6667 \) illustrated in Figure 5 is acquired through plugging giving values into expression (22).

**Figure 5** SER comparison of EPA and OPA

Then, Figure 6 is achieved.

Figure 6 (a) demonstrates that system’s SER performance varies with channel quality. With the same SNR, case (e)’s overall SER is the smallest due to comparably good channel quality from source to destination, while SERs of case (g) and case (f) are similar. As the channel quality from source to
destination is preferable, more power should be allocated to source nodes with low channel quality which is in line with expression (22). Figure 6 shows no matter how channel quality varies, the condition \( 1/2 < \lambda < 1 \) must be ensured to optimise SER performance which is consistent with expression (23).

**Figure 6** Power allocation factor and SER relationship comparison

![Figure 6](image)

Further, Figure 6 (b) (c) and (d) shows that with given channel and directional gain, SER performance is enhanced along with the increase of overall SNR while the OPA factor \( \lambda \) stays the same. As \( \sigma_{d,r}^2/\sigma_{s,r}^2 \) grows from \( \sigma_{d,r}^2/\sigma_{s,r}^2 = 1/10 = 0.1 \), \( \sigma_{r,d}^2/\sigma_{r,s}^2 = 1/1 = 1 \) to \( \sigma_{r,d}^2/\sigma_{r,s}^2 = 10/1 = 10 \), the value of \( \lambda \) grows from 0.5393, 0.6667 to 0.8333 correspondently which is in line with expression (22).

Finally, we continue studying the impact of different channel variances on SER performance under directional antenna mode, other parameters are invariant, make \( h = [\sigma_{r,d}^2, \sigma_{s,r}^2, \sigma_{r,s}^2] = [1, 1, 1] \), \( \text{SNR} = 30 \text{dB} \) and get Figure 7.

Finally, channel variance’s influence on SER is also analysed under directional antenna mode. Keeping other parameters as the same and giving \( h = [\sigma_{r,d}^2, \sigma_{s,r}^2, \sigma_{r,s}^2] = [1, 1, 1] \), \( \text{SNR} = 30 \text{dB} \), Figure 7 is acquired.

**Figure 7** The impact of different channel variance on SER performance

![Figure 7](image)

Figure 7 demonstrates that with the same SNR, case (c)’s SER performance is the best under EPA condition while the SER of both case (f) and case (g) remains the same with a fixed SNR. It can be derived from expression (14) which shows \( \sigma_{r,d}^2 \) and \( \sigma_{r,s}^2 \) are symmetric under EPA. While adopting EPA, \( \sigma_{r,d}^2 \) and \( \sigma_{r,s}^2 \) are asymmetric in expression (14) which conduces to Figure 7 (b). In Figure 7 (b), the overall performance of case (f) is slightly lower than that of case (g) which is consistent with the analysis of Figure 6.

Through the above analysis, we can know channel quality affects SER performance of system directly and rational power allocation according to the actual channel quality may gain superior system performance.

According to analysis above, it concludes that channel quality directly decides system’s SER performance. Consequently, system performance can be obviously enhanced with rational power allocation according to actual channel quality.

**5 Conclusions**

This paper studied cooperative communication system’s SER performance under AF protocol in the presence of directional antenna. The OPA is further researched through theoretically deducing SER expression and its approximate upper limit in the case of directional antenna. It is proved that directive gain can obviously improve a system’s SER performance under cooperative communication mode with directional antenna. On the other hand, due to the great influence of channel quality on SER, efficient power allocation can evidently reduce resource consumption to acquire better service quality. Consequently, adopting OPA in line with practical channel can achieve better SER performance than EPA.
References


